Title A minimal and alternative model to dark matter. An illustrative application to the galactic rotation problem

G. Pascoli

emails : pascoli@u-picardie.fr; sciences.univers@gmail.com

Faculté de sciences

Département de physique

Université de Picardie Jules Verne (UPJV)

33 rue saint Leu, Amiens, France

Abstract The measurement of the couple velocity/acceleration and of the gravitational field intensity at a given location in a galaxy could potentially be induced in an unexpected manner by the environment of the observer, for instance the local mean mass density in the galaxy. This contingency, mathematically supported by the asymmetric distance concept, is here illustrated by a study bearing on the rotation of spiral galaxies. This suggestion is new in the astrophysics field (it is named in the following κ-model) and could help to mimic the main effects seen in the MOND theory, in models of modified gravity (MOG), or in other related ones built with the aim to eliminate the dark matter, and which are already well established theories. Thus starting from two selected examples of galaxies, we show in the paragraph 5 that there is an equivalence between MOND and the κ-model.

However, the question about whether the paradigm considered in the present paper is eventually just a simple mathematical inquisitiveness and nothing more, or is a real physical insight, may legitimately be put; but its great interest is that it is not artificially parametrized and is sufficient in itself, with the only presence of the baryonic matter and it is a major issue. In this sense it is more easily refutable than any other theory postulating the existence of a large variety of exotic particles with unknown, and which can be changed at will !, characteristics. Especially, at the opposite side, there is the speculative nature of the dominant paradigm, the elusive dark matter, a matter whose the properties remain always undefined, in spite of intense theoretical and observational efforts for the past half century and more !

Keywords : galaxy, asymmetric distance, galactic rotation, spiral pattern, dark matter

1 Introduction

The alternative theories to dark matter attribute the flatness of the galactic rotation curve either to a departure from the inertia law (MOND) in the very low acceleration regime [1, 2] or to a failure of the Newtonian gravity at a large scale, emulated by a variable gravitational constant G [3, 4], both without invoking an additional and very huge quantity of dark matter. An interesting variety of alternative theories without dark matter have also been proposed [5, 6, 7]; even though some of them, for instance those based upon the conformal gravity, seem ambiguous as for the conclusions [8]. Negative masses have also been invoked, after admitting a drastic modification to Einstein's theory of general Relativity to allow negative masses to exist [9, 10] !, but what is the point to replace an unobserved exotic entity (the dark matter) by another unobserved, even more exotic, entity ?

So far, the assumption that both the gravitational field intensity and the measurement of the couple velocity/acceleration of a particle could depend on the mean mass density estimated in a given location has not been considered. Yet this idea, even though very simple, offers new perspectives by its original nature and its various consequences [11]. The key strengths of the concept are

1. The basic elements are very natural, and reside within the framework of the Newtonian mechanics. Let us recall that the Newtonian mechanics is only relevant when the velocities are low and the gravitational field is weak, but this is the case here, even though the model can still be derived from a relativistic context[1]. The concept, hereinafter named kappa model, postulates an enhancement, resp. a diminution, of the gravity (and of the self-gravity) in the regions where the mean mass density is weak (resp. high) in a galaxy. An associated effect is that the measured distances become now apparent and asymmetric. This statement is not explicit in the context of both the MOND paradigm [1, 2] and modified gravity (MOG) theories [3, 4]. In addition, the kappa model remains in the strict framework of a preserved Newtonian law of gravity, at least from a formal point of view. More generally all the physic laws, locally expressed, remain unmodified in the kappa model.

2. The persistence of a spiral substructure in the "grand design" galaxies and the flatness of the rotation curves are shown to be interrelated in an unexpected manner.

3. Some problems like the Bullet cluster, seemingly difficult to solve in the framework of the various alternatives to the Dark Matter paradigm (for instance MOND), could maybe find here a natural explanation. The reason is that the gravity, where it is acting, is now modulated by the local mean density. Another item is relative to the accelerating expansion of the Universe [12, 13] which is today explained by the existence of a mysterious fluid : the dark energy. Yet this problem could also be solved in the framework of the kappa model, eventually leading from two distinct paradigms (dark matter and dark energy) to a single one. All these important issues, where a possible density-dependent aspect of the gravitational force is put in evidence, will be considered later.

Let us eventually specify that the kappa model is not another "revolutionary" theory proposing once again a new physics with exotic particles appearing out of nowhere or still starting from physical laws, skillfully manoeuvred with addition of supplementary terms. On the contrary in its framework the classical laws of physics are left formally unchanged, and this is where its originality lies.

2 The κ-model

The equations which we are dealt with are the usual Newtonian (non-relativistic) equations weighted by a coefficient κ [2]. For a system composed of N identical particles of mass m (index $i = 1, ..., N$), we write

$$\frac{d}{dt}\left(\kappa_i[\bar{\rho}]\frac{d\boldsymbol{\sigma}_i}{dt}\right) = -Gm \sum_{j=1,\quad j\neq i}^{N} \frac{\kappa_i[\bar{\rho}](\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)}{\left[\kappa_i[\bar{\rho}]\ \ \|\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j\|\ \right]^3} \tag{1}$$

2.1 A formal deduction of eq. 1 from the gravitational Newtonian law

The machinery is supplied in the appendix A. Here we start with the Newton gravitational law as it is testified to be accurate at the local level (the solar system). Let two very distant particles M and M' of respective masses m and m', marked by the vectors $\mathbf{R}$ and $\mathbf{R}'$ (arbitrary origin). We write formally for the particle M, assuming that the gravitational constant G is universal

$$\frac{d\mathbf{P}}{dt} = -Gmm'\frac{(\mathbf{R} - \mathbf{R}')}{\|\mathbf{R} - \mathbf{R}'\|^3}$$

[1] See [11] for an attempt to develop the kappa model in a relativistic context.

[2] The relativistic expression is not useful here, even though the relativistic transcript would be easy to make in the kappa model framework.

where $\mathbf{P}$ is the momentum of the particle M and we can write a similar equation for M'. At this stage there is no observer (no reference frame) to measure the couple velocity/acceleration (and as it is well known in Mechanics without a reference frame both the velocity and acceleration are undetermined. Likewise $\frac{d\mathbf{P}}{dt}$ cannot be added to $\frac{d\mathbf{P}'}{dt}$ apart from a purely formal manner). In addition in the kappa model the norms of $\mathbf{R}$ and $\mathbf{R}'$ are now themselves left undetermined. Thus in order to solve this couple of equations, we must choose a representation : a fictitious inertial observer A is assumed to be located near M (resp. B near M'), each of these observers is equipped with a scale factor κ (resp. κ'). Then A locally expresses $\mathbf{R}, \mathbf{R}'$ and $\mathbf{P}$: $\mathbf{R} \longrightarrow \kappa\boldsymbol{\sigma}$, $\mathbf{R}' \longrightarrow \kappa\boldsymbol{\sigma}'$ and $\mathbf{P} \longrightarrow \kappa(m\frac{d\boldsymbol{\sigma}}{dt})$ and B does the same thing but with a scale factor κ'. We can note that there exists some – even though obviously very remote – mathematical analogy with the quantum mechanics, when we pass from the operational notation to the Schrödinger representation for the position $\mathbf{R}$ and momentum $\mathbf{P}$ of a point particle ($\hat{\mathbf{R}} \longrightarrow \mathbf{R}$, $\hat{\mathbf{P}} \longrightarrow -\mathbf{i}\hbar\boldsymbol{\nabla}$).

The scale factor (the mathematics) is then linked to the mean density by the equation 2 introduced below (the physics). Eventually we assume that the fictitious (inertial) observers A and B are motionless to each other, a procedure which is easily checked by spectroscopic measurements.

2.2 The coefficient κ

In the equation 1 the coefficient κ is no longer a constant (equal to κ_E) as in the basic Newtonian equations. On the contrary κ is now defined as a functional of the mean local density $\bar{\rho}$[3]. Obviously locally for us, i.e. at the scale of the Earth or the solar system we can take $\kappa_i = \kappa_E = Const$, $\forall i$, and analysing a group of particles contained in the solar system (planets, asteroids, etc), these equations exactly express the usual Newtonian law.

The system 1 of N coupled equations seems to be able to be "easily" treated by any standard and well-known method of smoothed-particles hydrodynamics and by using a software available on line. Unfortunately, the process of solving is much more complex given that the factor κ is now a functional of the mean density. This is a self-consistent problem and we were now forced to make a special iterative program for that. Unfortunately the CPU time considerably increases in the latter situation. Independently of the physics, the study of this equation is very interesting in itself and arguably only a part of its complexity is revealed in this paper. First, by taking into account the fact that κ is assumed to be a smooth function of linear-type, and otherwise that $\bar{\rho}$ is rather an exponential-type function in a typical galaxy [14], it appears intuitive to impose a natural and simplest form, i.e. a logarithmic relationship between κ and $\bar{\rho}$, let

$$\frac{\kappa[\rho_0]}{\kappa\,[\bar{\rho}]} = 1 + Ln(\frac{\rho_0}{\bar{\rho}}) \tag{2}$$

where the index 0 labels the maximum value of the density distribution (Ln is the symbol for the Napierian logarithm). This law is assumed to be universal and available for any galaxy.

At this level the system of equations 1 and 2 can be admitted from the outset as a postulate of the model and we could simply stick to that, leaving the deduction from first principles aside. However, the reader can still refer to the appendix A for a synthetic demonstration of the dynamics equation 1 starting from a simple mathematical concept within the framework of a half-metric space.

Let us examine the equation 1. The inertia term is modulated by the factor κ (as in MOND). On the other hand from the right-hand side, we can see that wherever the mean density is high (resp. low) the gravitational "feeling" between two masses is low (resp. high). Two masses are more strongly gravitationally linked with each other in the outer regions (low density, low κ)

[3]It should be noted that the densities in equation 2 are the densities estimated by the same observer or, more precisely, that ρ_0 and $\bar{\rho}$ are simultaneously measured by this observer.

of a galaxy than in the inner regions (high density, high κ). This bears some resemblance with the electromagnetism in the situation where two charged particles are placed in a medium of relative dielectric permittivity ϵ_r different from the unit ($\epsilon_r > 1$). The electric field between these charges is decreased relative to vacuum. However the analogy must not go too far. There exists an important difference : no background medium is existing in the κ-model and the coefficient κ induces no refractive effect. The light still propagates in straight line with the same celerity c for all frequencies; and κ is frequency-independent contrarily to ϵ_r which is depending on frequency. Likewise the trajectory of any free particle is rectilinear in the κ-model (when the gravitational force is eliminated).

3 Computational details

In order to solve the system of equations 1 and 2 a 6th order Runge Kutta ODE algorithm under MATLAB was used throughout. This software is currently implemented on a SGI Altix UV 100 (MatriCS Platform) at UPJV.

A damping term has been added to each equation of the system of equations 1 in order to simulate a radial pseudoviscosity effect when the density is larger than a fixed threshold (the inner region). This term has the simple form

$$-\alpha_i[\kappa_i \frac{d\boldsymbol{\sigma}_i}{dt} . \frac{\kappa_i \boldsymbol{\sigma}_i}{\|\kappa_i \boldsymbol{\sigma}_i\|}] \frac{\kappa_i \boldsymbol{\sigma}_i}{\|\kappa_i \boldsymbol{\sigma}_i\|} \tag{3}$$

where the damping coefficient $\alpha_i = \alpha \sqrt{\frac{4GM}{\|\kappa_i \boldsymbol{\sigma}_i\|^3}}$ (M is defined in the following and α is a numerical coefficient, possibly adjusted, which we have taken equal to 1)[4].

The characteristic time is taken to be equal to the free fall of an outer particle, $\sim 10^8$ years (reference unit taken for the time in the figures 1, 3, 6). We start with an initial discoidal configuration of radius $\sim 10\ r_G$ where $r_G \sim 10\ kpc$ (reference unit taken for the distances in the figures 1, 3, 6), this radius is estimated by a terrestrial observer provided with a coefficient $\kappa = \kappa_E$. The total mass of gas M is taken equal to $10^{44}\ g$ (this mass is only baryonic in the κ-model). The initial disk configuration, of thickness $\sim 0.6\ kpc$, is assumed to be cold and homogeneous. The mean density $\bar{\rho}$ is automatically recalculated after 10 time steps. This mean density is obtained by counting the number of particles of mass m in a volume of $(0.6\ kpc)^3$. Eventually an initial low shear velocity field of the form $v_x = 10\ \frac{\kappa_E \sigma_x}{10}$ ($\kappa_E \sigma_x$ in kpc and v_x in km/s) is assumed to pervade the disk. The coefficient κ is a constant $= \kappa_{ini}$ in the initial, assumed to be homogeneous, distribution of baryonic matter, and is such as $\frac{\kappa_{ini}}{k_E} \sim 0.15$, which appears as a reasonable value after trial. This is for the content of physics, then the equations have been properly normalized.

The cartesian coordinates (x, y) are used. An initial uniform distribution of $10\ 000 = 100 \times 100$ identical particles of mass m has been selected[5]. In order to distinctly exhibit the asymmetric substructures, the quantity reported in the figures 3.a, 3.b and 6.a, 6.b is not directly $\bar{\rho}\,(x, y)$ but rather the difference

$$\delta\bar{\rho}(x, y) = \bar{\rho}(x, y) - \bar{\rho}\left(\sqrt{x^2 + y^2}\right) \tag{4}$$

where $\bar{\rho}\left(\sqrt{x^2 + y^2}\right)$ is the density averaged on the polar angle. The latter one gives a circular distribution in the galactic plane. Eventually only a particle out of 10 is reported in the figures in order to clearly distinguish the substructures.

[4]From a physical point of view, we can still imagine that when the system of particles shrinks, the energy of the free fall is rapidly transformed in infrared radiations which are on the spot evacuated from the system.

[5]The study should be resumed with a larger number of particles, but the excessive CPU time as due to the self-consistent process precludes it for the moment.

4 Results

When the simulation is running, the disk rapidly shrinks by self-gravity and a weak spiral substructure naturally quickly appears, just after $\sim 6\ 10^8$ years[6]. This weak substructure is shown in figure 1, the axisymmetric background being excluded. In figure 2 the trajectories of some individual particles are also displayed and we can observe a rapid and chaotic fall toward the centre[7] for these particles and the consecutive circularization of the orbits.

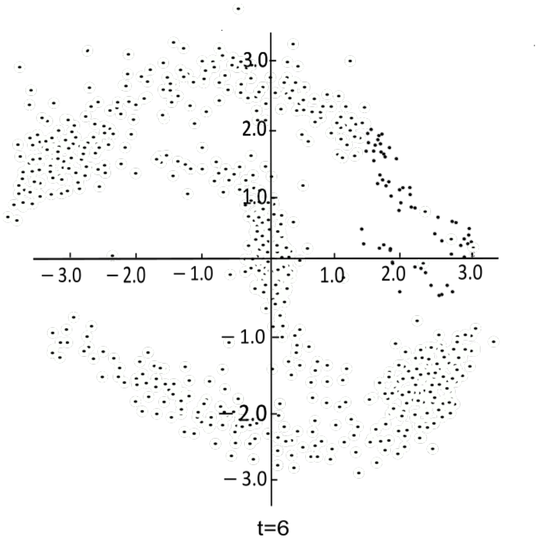


Figure 1 Formation of the spiral substructure

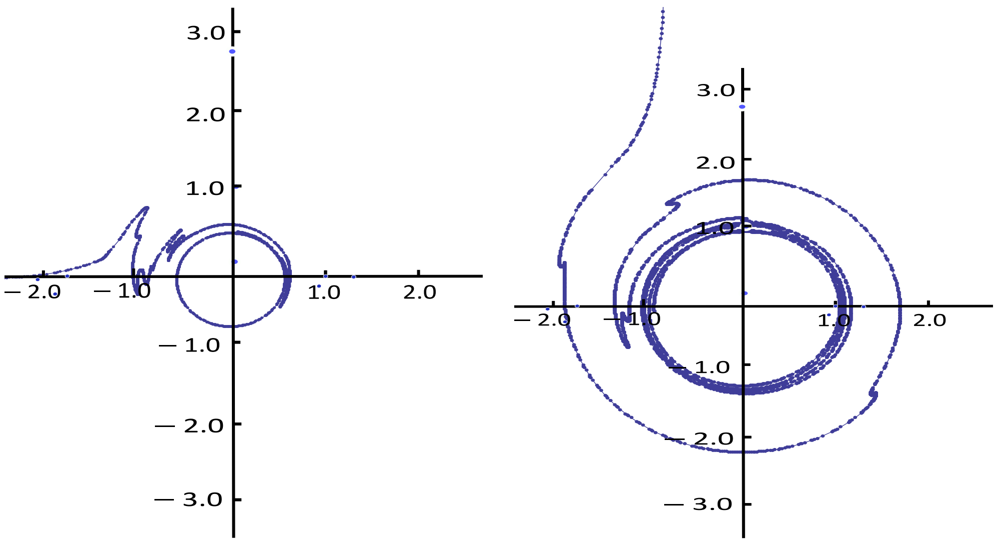
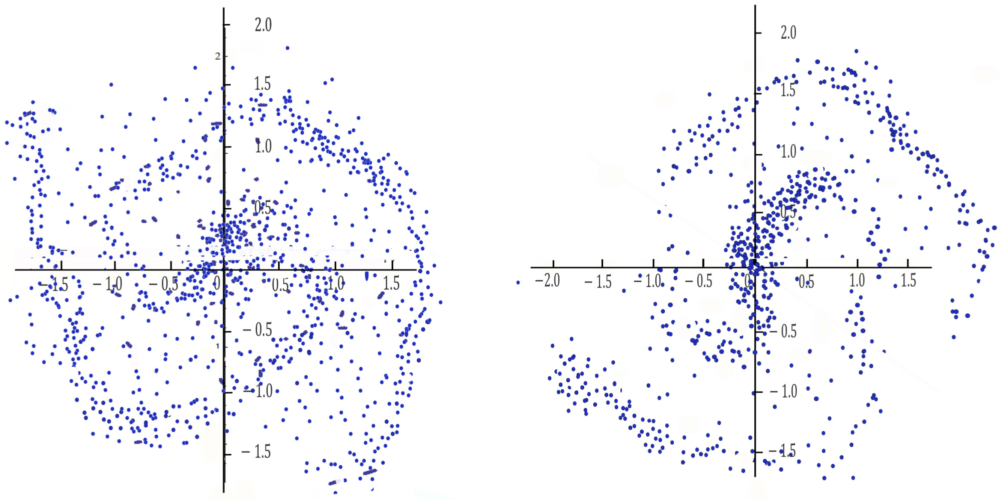


Figure 2

The axisymmetric disk on which a spiral pattern is superimposed then stabilizes and the system becomes quasi-steady after about $2\ 10^9$ years ($t = 20$). An impressive large-scale coherent "grand design" galaxy appears. This is evidenced in figures 3.b and 3.c.

[6]This characteristic time is almost twice as large in the dark matter model where the shrinkage is lower. The galaxies form much more rapidly when considering the κ-model than when considering the dark matter paradigm.

[7]The irregularities seen along on the trajectories are due to close encounters between particles over the free fall process. During this process the particles which are ejected from the galaxy are no longer taken into account.



a t=20 b t=30

Figure 3 The quasi-permanent spiral in the κ-model

That the spiral pattern and the central bulge are self-maintained with time is also clearly seen in these figures[8]. This result is quite remarkable and it is not the case within the framework of the dark matter paradigm where the spiral substructures are just transient phenomena over a few rotational periods of the galaxy. In parallel a quasi flat rotation curve for the velocities[9] is also obtained (figure 4; $r = \kappa_E \sigma$)[10]. We have also reported on the same figure the Keplerian velocity curve (2) making $\kappa = \kappa_E$ everywhere. However for $r \gtrsim 5$, κ comes again constant and the rotation curve (1) falls off in a Keplerian manner (not shown on the figure).

[8]However some local and relatively strong deformations, as due to the self-gravity are still persisting even after two milliards of years, and the galaxy after formation does not rotate as a rigid body, contrarily to what one might think in the framework of the κ-model. There remain a very large number of stars with elliptic motions, not fully circularized, a phenomenon which also contributes to the deformations. In the framework of the κ-model, a galaxy is a "living" object, and it should not be seen as a perfect and immutable wheel ! Accordingly the substructure spiral always evolves, even though much more slowly than in other contexts, as for instance in the density wave theory where the lifetime of the spiral substructure appears very short unless the density wave is constantly feeded.

[9]For practical (observational) reasons we do not report in figure 4 the velocities, but rather the measured shift of the frequencies ν. Let us specify that the radial velocity measurements supply the true velocities (in fact the radial part), i.e. those measured by an inertial observer on site.

[10]The curve is not properly covered for $r \leq 0.3$. A three dimensional model for the bulge is needed. The curve (2) is obtained from the mass density derived by the resolution of the equations 1 and 2 and then by recalculating the velocities.

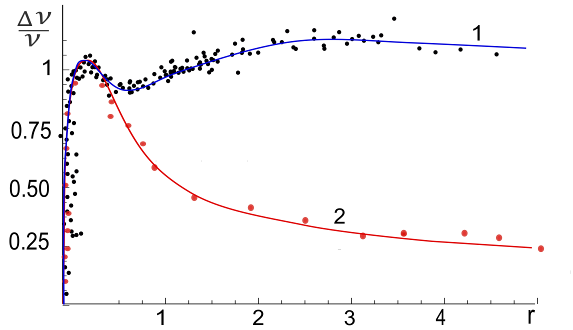


Figure 4 Galactic rotation curve :
(1) κ-model (2) pure Newtonian model

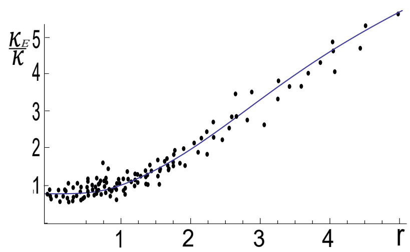


Figure 5
Variation of κ_E/κ along a galactic radius

4.1 The resolution of the winding dilemma

The winding problem has always been recurrent in the simulation of "grand design" galaxies [15], and remains partially unsolved whatever may be said. Yet, against all odds the two interlinked effects, i.e. the flatness of the rotation curve and the almost-steady nature of the spiral design, can easily be understood in the framework of the κ-model. Let two inertial observers A and B situated along a galactic radius, and each of them measuring the (true) velocity of one particle, resp. M for A and M' for B, located near them. We have

$$v_A(M) = v_B(M')$$

expressing the flatness of the rotation curve as measured by spectroscopy[11]).

However $v_A(M) = \kappa_A \sigma(M)\dot{\theta}(M)$ and $v_B(M') = \kappa_B \sigma(M')\dot{\theta}(M')$ where θ is the polar angle defined from a radial baseline with origin at the galaxy center (in the κ-model a given direction is well identified and is the same for all observers). Following the figure 3 the coefficient κ (or more rigorously the measurable ratio $\frac{\kappa}{\kappa_E}$) is approximately proportional to $\frac{1}{\kappa_E \sigma}$ in the outer regions of a galaxy[12], we obtain thus

$$\dot{\theta}(M) = \dot{\theta}(M') \tag{5}$$

for any couple of points located along a galactic radius. The points remain steadily aligned with time along the galactic radius (a straight line plotted from the galactic centre) ! This noteworthy outcome enables us to now understand why we can simultaneously observe a quasi-steady "grand design" substructure and a flat rotation curve, two effects which would seem to contradict each other before any thought. Let us note however that the persistence of the spiral substructure is not absolute and ultimately the latter one may slowly distort with time. The main reason is that the orbits of stars are not perfect circles, but rather ellipses with slight excentricities. On

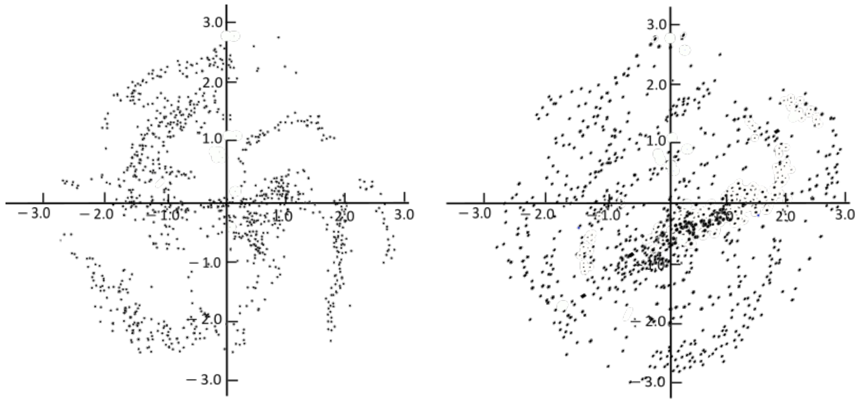
[11] In the Universe the radial velocities are estimated from spectroscopic measurements. These velocities are the same as those measured by a local inertial observer. On the other hand the tangential velocities are deduced from proper motions and parallaxes measurements. These apparent quantities are linked to the terrestrial observer who measures them by trigonometry postulating an unique, and most likely imaginary background. These two methods are very different and this difference is expressed in the framework of the κ-model. The first method (spectroscopy) supplies true (radial) velocities, whereas the second method (observations of proper motions) supplies apparent (tangential) velocities.

[12] More rigorously, it is a mean κ obtained by curve fitting (the curve in figure 5).

the other hand the factor κ fluctuates by accompanying the variation of the density within the spiral substructure itself.

In this respect the comparison with the results obtained in the framework of the dark matter paradigm is quite interesting. In the latter simulations we have obviously taken the same initial conditions for the baryonic matter. We can conclude from this comparison that the κ-model gives something similar to the dark matter paradigm in many aspects, but without dark matter; however the spiral substructure is self-maintained and much more impressive.

In order to perform this comparison we have added a halo of dark matter with a density distribution proportional to r^{-2} in the outer regions (halo mass) and we have taken $M_{DM} \sim 10\ M_B$ and $\kappa = \kappa_E$ everywhere, i.e. the orthodox point of view rehabiliting the common background, where we can put $r = \kappa_E \sigma$. Then with the dark matter superimposed to the baryonic component taken in account, we can see that a spiral pattern with a central bar effectively appears as due essentially to the self-gravity, but the outer arms are now multiple, transient and discontinuous, and their lifetime is also very short (figures 6.a, 6.b).



a t=20 b t=30

Figure 6 The transient spiral in the dark matter model

In any case a flocculent galaxy pattern seems to emerge from the simulations, but definitely not a permanent "grand design". Certainly at $t = 20$, the figure 6a clearly mimics a spiral-type structure, but with four arms instead of two; however at $t = 30$ the situation becomes confusing while the galaxy core is getting denser and the arms are more winded and unrecognizable. This effect is well known, and the same conclusion results from the other published N-body simulations within the spherical dark matter models, even using very sophisticated smoothed particle hydrodynamics (SPH), grid-based procedures, including complicated physics like cooling, star formation, energy injection from supernovae, etc [16]. We can compare the figure 6 of the present paper and the figures 2 of [16] obtained after a series of state-of-the-art simulations; on the one hand we see that they are fairly similar, but on the other hand that we are far removed from the set objective for a realistic "grand design" galaxy. Obviously additional ingredients can be artificially introduced in the calculations such as a permanent density wave within the gas which is driven by a rotating central bar [15], a triaxial distribution of dark matter forming a rotating halo with angular momentum [17] or still a Yukawian gravitational potential [18]; however even though all these proposals are interesting a solid experimental/observational basis is still lacking. The interaction with a galaxy companion to sustain a "grand design" substructure is maybe much more realistic [19], even though there exist attested cases of "grand design" galaxies without the presence of a companion.

By contrast, starting from an initial homogeneous cold gas pervaded with a shear velocity field, it is quite remarkable that, in a very simple way, the κ-model naturally directly leads to a quasi-steady "grand design" spiral galaxy with very clearly well-formed and distinct arms. A bar also appears in the central region and the arms are formed by the self-gravity which is much higher in the outer regions (weak mean density) than in the inner ones (high mean density) of the galaxy. Another noticeable point is that the spiral substructure is becoming stronger with time instead of a loose phenomenon as generated in other models with dark matter, or even in MOND or in MOG. These results provide a strong support for the self-consistent κ-model, the important conclusion being that the knowledge of the sole distribution of visible baryonic matter is sufficient to understand the dynamics of a galaxy and that the introduction of exotic particles is not needed.

5 Two selected examples

The preceding simulation can however appear as a toy model. Most interesting is the reality check. For that we have chosen two examples, a high luminosity galaxy (NGC 6946) and a low luminosity galaxy (NGC 1560). These two examples are taken from the review [20] (see the references therein for the original data sources).

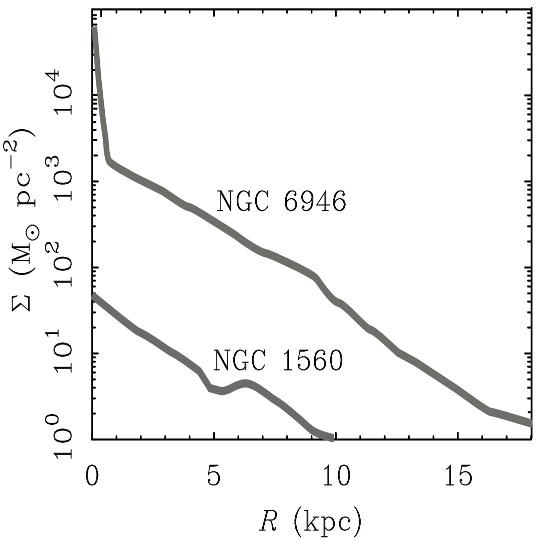


Figure 7: Surface density profiles (stars + gas) of two galaxies: the HSB spiral NGC 6946 and the LSB galaxy NGC 1560 (from [20], figure 13)

We assume here that the galaxies to be studied are stabilized and that the orbits of the stars are circular. The Newtonian velocities v_{Newt} could then be obtained from equation 1 making $\kappa_i \equiv \kappa_E$, in it[13]. The true velocities v (measured by spectroscopy) are then deduced from the Newtonian velocities v_{Newt} by the following "magnification" relation (its deduction is made in the appendix C)

$$v = \left(\frac{\kappa}{\kappa_E}\right)\left(\frac{\kappa_E}{\kappa}\right)^{\frac{3}{2}} v_{Newt} = \left(\frac{\kappa_E}{\kappa_0}\right)^{\frac{1}{2}}\left(\frac{\kappa_0}{\kappa}\right)^{\frac{1}{2}} v_{Newt} \tag{6}$$

where

$$\frac{\kappa_0}{\kappa} = 1 + Ln(\frac{\Sigma_0}{\Sigma}\frac{\delta}{\delta_0}) \tag{7}$$

The latter relationship 7 directly derives from equation 2, Σ represents the surface density and δ the mean thickness of the matter (stars + gas) along the line of sight. The index 0 labels

[13]We can see that in equation 1 the factor $(\frac{\kappa_E}{\kappa_i})$ est factorizable in front of the sum on j. This sum expresses the Newtonian force acting on the particle with index i.

the maximum value of Σ for a given galaxy. Assuming an uniform thickness, we have taken $\delta = \delta_0 = Const$ throughout the calculations. The figure 8 displays the ratio $(\frac{\kappa_0}{\kappa})^{\frac{1}{2}}$ for the two selected galaxies calculated with equation 7.

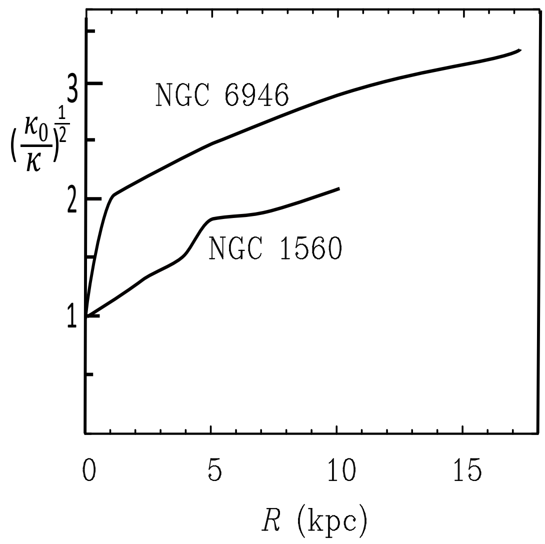


Figure 8 : Variation of $(\frac{\kappa_0}{\kappa})^{0.5}$ as a function of the radial distance R

The global ratios $\frac{\kappa_E}{\kappa_0}$ are unknown, but we can attempt to supply an estimate by again employing equation 2[14]. With $\Sigma_\odot = 60\ M_\odot\ pc^{-2}$ (from [20], figure 19; [21], figure 5), we find respectively $\left(\frac{\kappa_E}{\kappa_0}\right)^{0.5} = 0.36$ for NGC 6946 and 1.24 for NGC 1560. We have however noted that a better fit of the v-curves is realized by taking $\left(\frac{\kappa_E}{\kappa_0}\right)^{0.5} = 0.43$ (instead of 0.36) for NGC 6946 and 1.30 (instead 1.24) for NGC 1560. In any way the calculation of these global magnification factors is certainly vitiated by various biases.

For instance, we know that the inclination angle i of a galaxy can strongly impact the velocity curves [22]. For NGC 6946 a variation of 7° of the inclination angle, starting from $i = 38°$ for this galaxy [22], is equivalent to increase the factor $\left(\frac{\kappa_E}{\kappa_0}\right)^{0.5}$ from the value predicted by the κ-model (0.36) to its fit value (0.43). Eventually, even though to a lesser extent, the distance of this galaxy is highly uncertain [23]. This important parameter intervenes in the estimate of the absolute luminosity, subsequently on that of the surface density and eventually on that of the ratio $\left(\frac{\kappa_E}{\kappa_0}\right)^{0.5}$.

The estimate of $\Sigma_\odot$ can also play a role. Thus for NGC 1560 passing from $\Sigma_\odot = 60$ to $70\ M_\odot\ pc^{-2}$ increases the numerical value for $\left(\frac{\kappa_E}{\kappa_0}\right)^{0.5}$ from 1.24 to the value taken for the fit in figure 10, let 1.30 (with insignificant change for NGC 6946). Note that a value $\sim 90\ M_\odot\ pc^{-2}$ is given in [24, Table 4]. However this value is well above the estimates of other authors [20] and [21] and is very likely overestimated.

Fortunately while some incertainty is attached to the global ratio $\left(\frac{\kappa_E}{\kappa_0}\right)^{0.5}$, the resulting effect is a global shift of the mean height of the velocity curve[15] it does not heavily affect the

[14]The relation which has then been used is

$$\frac{\kappa_1}{\kappa_2} = 1 + Ln(\frac{\Sigma_1}{\Sigma_2}\frac{\delta_2}{\delta_1})$$

respecting the condition $\Sigma_1\delta_2 > \Sigma_2\delta_1$.

[15]In the literature the mean height of the observational velocity curve can also vary substantially; for instance for the Milky Way, we can compare [24, figure 1] and [26, figures 5,6].

details seen on the curve, such as bumps, wiggles, or still the typical flatness in the outer regions. The final results are presented in figure 9 (NGC 6946) and in figure 10 (NGC 1560).

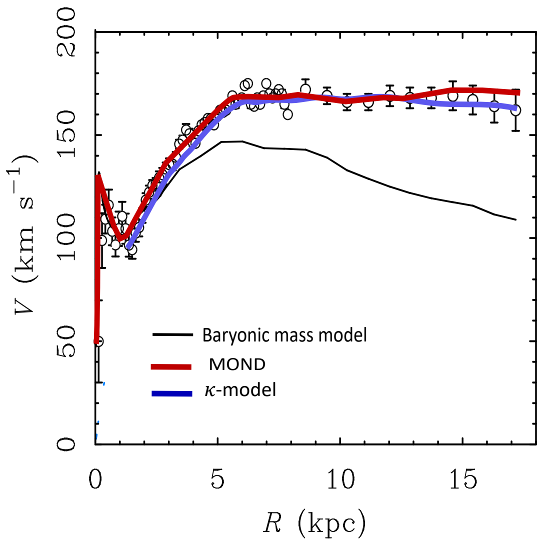


Figure 9 : NGC 6946 : the observed rotation curve (black circles) is displayed together with those respectively predicted by the Baryonic mass model (black line), MOND (red line) and κ-model (blue line)

Examining the curves reproduced in figure 9, we can see that MOND and the κ-model give very similar values for the velocities between $6 > r > 13$ kpc. For $r < 6$ kpc, MOND is slightly better than the κ-model, and conversely for $r > 13$ kpc, where the MOND curve does not initiate a slow decrease whereas this decrease is well noticeable on the curve of the κ-model. However the differences between MOND and the κ-model are small.

Regarding now the galaxies of lower baryonic surface densities, it is well known that the Newtonian prediction is very bad everywhere, even at smaller radius [25]. The galaxy NGC 1560 illustrates this situation whereas both MOND and the κ-model are fairly consistent with the observed curve (figure 10).

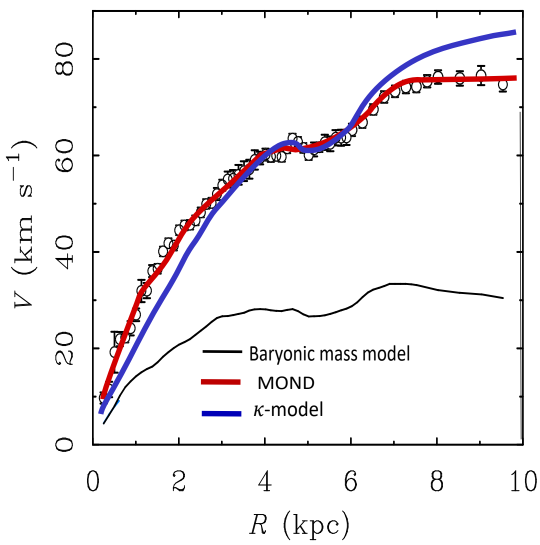


Figure 10 : NGC 1560 : the observed rotation curve (black circles) is displayed together with the theoretical curves predicted by the Baryonic mass model (black line), MOND (red line) and κ-model (blue line)

The results from both MOND and the κ-model are well superimposed between $3 > r > 6$ kpc, even though outside this interval MOND seems slightly better. However an optimal overlap of the κ-model versus the observation could eventually be realized by an ajustment of the thickness δ. Thus the κ-model could predict that the thickness along the line of sight, δ, is larger than δ_0 for $1 > r > 3$ kpc by a factor 1.5, and conversely smaller by the same factor for $6 > r > 10$ kpc.

However let us still notice that a comparison with the observational curve derived from other sources [25] can lead to another interpretation of the results. The x-axis of figure 10 has been suited to figure 11. In this case we can see that within the intervals $0 > r > 3$ kpc and $8 > r > 10$ kpc the observational data are now located in the area bounded by MOND and the κ-model, namely resp. above by MOND and below by the κ-model for $0 > r > 3$ kpc, and conversely for $6 > r > 10$ kpc.

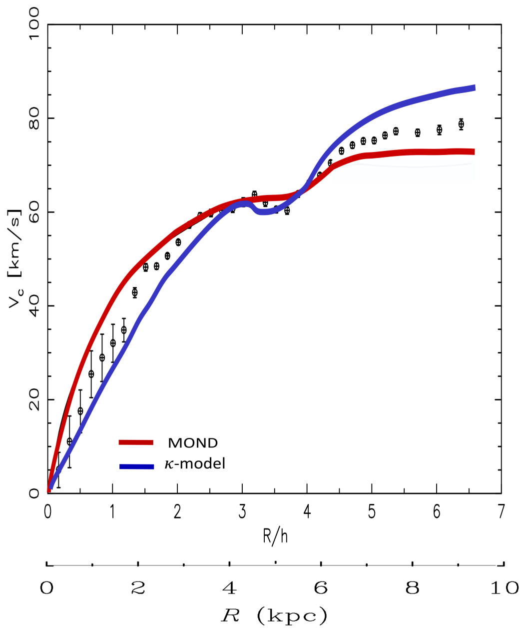


Figure 11 : NGC 1560 : Another observed rotation curve (black circles), derived from [25], is displayed together with the theoretical curves predicted by MOND (red line, reproduced from [25]) and the κ-model (blue line)

Taking into account the various uncertainties regarding the measured velocities, the surface density profiles, the inclination of the galaxies and the estimate of the distances, the analysis of the two individual examples which has been performed clearly shows that from the sole consideration of the observed distribution of baryonic matter, both MOND and the κ-model can predict satisfactorily the observed rotation curves of galaxies. Such a prediction is not possible using the dark matter paradigm, except if we take a fine-tuned distribution of dark matter especially suited to each type of galaxy.

Likewise we can make a very interesting remark. We can notice than between $3 > r > 6$ kpc, the peculiarities seen on the Newtonian curve at a location are nicely transcribed on the observed curve at the same location. It is a rather astonishing fact, already pointed out by other authors [20], [26] and which is named the Sancisi's law [27][16]. Here again this phenomenon seems to be very difficult to mimick using the dark matter paradigm, given the gravitational force is a long range force. A mass removal at a place does not create at the same place a trough in

[16] The MDAR formula [26] has been a first attempt to quantify the phenomenon. Unfortunately, and even tough very noteworthy, this relation is purely empirical and is not based on a specific theoretical background.

the observed velocity curve. An unlikely conspiracy between dark matter and baryonic mass is then needed in order that the trough in the Newtonian curve may be replicated on the observed velocity curve at the same place. This fact is clearly in favour of models such as MOND or the κ-model, the latter one assuming a "simple" magnification of the velocity curve produced on the spot from the baryonic model. Thus following the couple of equations 6 – 7 this magnification is the larger as the mean density (or, in an equivalent manner, the factor κ) in the galaxy is weak. This circumstance is very well illustrated when comparing a high luminosity galaxy (e.g. NGC 6946) where the magnification is relatively weak and a low luminosity galaxy (e.g. NGC 1560) where the magnification is strong.

6 Conclusion

The κ-model is a nice application of the notion of - here apparent - asymmetric distance well known in mathematics and which essentially leads to privilege a local physics rather than a global one. Undoubtedly the κ-model is no longer the whole of the story and even might finally be a pure mathematical construction or still a simple exercise of the mind. However regardless of these considerations, it should not be swept away with a wave of the hand, mainly as due to the strong support of its conclusions, i.e. the unification of some properties of galaxies under a sole umbrella, mainly an unexpected, but however possible, correlation between the appearance of a quasi-steady spiral substructure and the flat rotation curve in galaxies.

An extension of this work is requisite. For the elliptical galaxies and globular clusters a three dimensional model is needed. On the other hand a forthcoming analysis of the data obtained on both the Train Wreck and Bullet clusters should help to understand better the density-dependent character of the measured gravitational force which has here been assumed. Maybe the κ-model could also help to get rid of the dark energy. After the formation of the galaxies the mean value of the factor κ increases[17]. This phenomenon leads to a slow decline of the mean gravitational forces between the galaxies which could eventually be interpreted as an apparent aceleration of the Universe.

Data availability statement : The author confirms that the data supporting the findings of this study are available within the article and the reference list.

Acknowledgements : I thank the reviewer for his fruitful comments which helped to improve the manuscript.

Conflicts of Interest: The author declares no conflict of interest.

7 References :

[1] Milgrom M., 1983, ApJ, 270, 365
[2] Milgrom M., 2019, arXiv: 1910.04368v3
[3] Moffat J.W., 2006, J. Cosmol. Astropart. Phys., 2006, 4
[4] Moffat J.W., 2008, Reinventing Gravity, Eds. HarperCollins
[5] Capozziello S., and De Laurentis M., 2012, Ann. D. Physik, 524, 545
[6] O'Brien J.G., and Moss R.J., 2015 J. Phys.: Conf. Ser. 615, 012002
[7] Ludwig, G.O., 2021, The European Physical Journal C, 81, 186
[8] Hobson M. and Lasenby A., 2021, arXiv: 2103.13451v1
[9] Farnes J.S., 2018, A&A, 620, A92

[17] The harmonic mean of κ for a set of particles (a spiral galaxy, an elliptical galaxy or a cluster of galaxies) can be defined by $\frac{\kappa_E}{<\kappa>} = \sqrt{\frac{\int \kappa_E^3 d^3\sigma \ \rho(\kappa_E \boldsymbol{\sigma}) \ (\frac{\kappa_E}{\kappa})^2}{\int \kappa_E^3 d^3\sigma \ \rho(\kappa_E \boldsymbol{\sigma})}}$ The two integrations under the square root must be performed over the volume containing the set of particles.

[10] Socas-Navarro H., 2019, A&A, 626, A5
[11] Pascoli G., and Pernas L., 2020, hal.archives-ouvertes.fr/hal-02530737
[12] Frieman J.A., Turner M.S. and Dragan H., 2008, Annual Review of Astronomy and Astrophysics, 46 (1), 385
[13] Weinberg S., 2008, Cosmology, Oxford University Press
[14] Binney J., and Merrifield M., 1998, Galactic Astronomy, Princeton University Press
[15] Shu F., 2016, Annual Review of Astronomy and Astrophysics, 54, 667
[16] Guedes J., Callegari S., Madau P., and Mayer L., 2011, ApJ, 742, 76
[17] Martinez-Medina L.A., Bray H.L., and Matosa T., 2015, JCAP, 1512
[18] Brandao and de Araujo J.C.N., 2012, ApJ, 750, 29
[19] Dobbs C.L., Theis C., Pringle, J.E. and Bate M.R., 2010, Mon. Not. Astron. Soc., 403, 625
[20] Famaey B., and McGaugh S.S., 2012, Living Rev. Relativity, 15, 10
[21] Moni Bidin C., Carraro G., Méndez R.A., and R. Smith R., 2012, ApJ, 751, 30
[22] Carignan C., Charbonneau P., Boulanger F., and Viallefond F., 1990, A&A, 234, 43
[23] Elridge J.J., and Xiao L., MNRAS, 2019, 485, L58
[24] Sofue Y., Honma M., and Omodaka T., 2009, PASJ, 61, 227
[25] Sánchez-Salcedo F.J., and Hidalgo-Gámez A.M., 1999, A&A, 345, 36
[26] McGaugh S.S., 2016, ApJ, 816, 42
[27] Sancisi, R., 2004, The visible matter – dark matter coupling, in Ryder, S., Pisano, D., Walker, M. and Freeman, K., eds., Dark Matter in Galaxies, IAU Symposium 220, p. 233, Astronomical Society of the Pacific, San Francisco
[28] https://breakthroughinitiatives.org//initiative/3

Appendix A

We start this appendix by summarizing the paper [11]. A large part of this paper being a mathematical essay, we begin with some elementary questions. Let an "empty" space, or more specifically the vacuum in its ground state. We introduce in this space a point particle A, with no extension or with zero dimension. We put the question : what is the velocity (direction and norm) of this point particle ? The response is : undetermined, given that it is well known that the vacuum cannot serve as a reference system. Now we introduce a second point particle B and we assign a label σ to the segment joining A and B. Here σ represents the bipoint (A, B). We put the question : what is the distance between A and B (by abuse called "length" of the label σ) ? The response is : undetermined[18]. But with two particles we can define a geometric element : the unnormed vector $\boldsymbol{\sigma}$ supplies a fixed direction $A \longrightarrow B$. Now we introduce a third point particle C, assumed to be not aligned with A and B. Then the situation drastically changes. We have two fixed non-collinear directions and we can now measure the angles, taking for instance (AB, AC) as a reference. Likewise the ratio $\frac{AC}{AB}$ (the distance AC) is determined, taking AB as a reference even though this reference remains by itself undetermined. If AB and AC homothetically change, this evolution will pass unnoticed for an hypothetical observer as the ratio $\frac{AC}{AB}$ remains unchanged.

Next the physics must be introduced. The hypothesis of a local mean mass density effect on the measurements of both the couple velocity/acceleration of a remote particle and of the apparent gravity where this particle is located, is simulated through the use of a mathematical ansatz, a half-metric empty vector space. In this "empty" space the vectors have undetermined

[18] Let us note that the word "undetermined" in no way means that we cannot assign a length to the vector $\boldsymbol{\sigma}$, but rather we can assign to it any length (an infinity), i.e. the length depends on the observer who measures it. A length is assigned after measurement by a given observer. The measurements of the very large lengths or distances are relative and no longer absolute in the κ-model.

norms, but the angle between any two vectors is well-defined, measurable and the same for all observers. The figure 12 can help to understand the main idea of the mathematics behind the κ-model. We have represented a Euclidean space with fictitious observers, E (Earth), A, B,..., but each of them is equipped with a different cartesian grid pattern. To be clear it is assumed that these observers assign the same direction to a vector, or to the rectilinear trajectory of a particle; but the measured norm of this vector, or that of any length picked along the rectilinear trajectory, is observer-dependent.

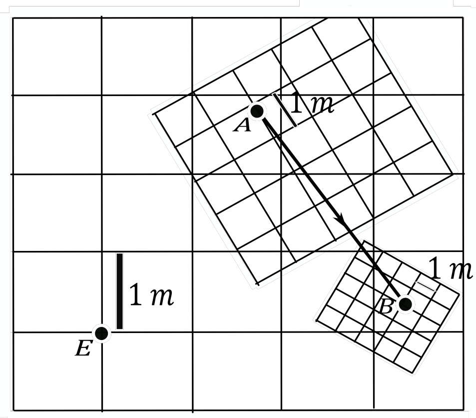


Figure 12 Half metric space. Each square of the grid attached to a given observer has a side equal to 1, it is the reference length for this observer, even though this reference length looks different for any other observer. For instance for E the apparent velocity of the particle changes when this particle goes from A to B whereas in reality this velocity remains unchanged.

Obviously in this framework, no distance is originally attached to the fabric of space. It is somewhat as if the space was without "common background", only the relationships between the objects are maintained and the scenery is removed. The length of an object is only meaningful relative to the length of another object taken as a reference. However the directions are still retained, this condition is important, the parallelism is saved, any displacement of a vector moved parallel to itself retains its usual meaning as in the Euclidean geometry. Once the constraint of absolute distance is relaxed, we can envisioned that the measurement of very large (interstellar) distances (i.e. $\gtrsim 0.1\ kpc$) estimated by any observer[19] may now be, for instance, a functional of the mean density of matter surrounding this observer; a bit like a magnification produced by a perfect lens, even though this image should not be taken at face value because obviously in the κ-model there is no refractive medium.

A cartesian mesh is drawn on the fabric of space. This operation is possible because both the notions of direction and parallelism remain still relevant, especially the notion of right angle subsists. We can even subsidiarily provide a norm to this mesh, but this norm is observer-dependent. The position of any point is specified by a label-vector $\boldsymbol{\sigma}$ drawn from an arbitrary origin. The direction of $\boldsymbol{\sigma}$ is determined, the same for all observers, but this vector is left unnormed before measurement. Nevertheless each observer is able to allocate to it a norm (i.e. to assign it a length by applying to σ a scale factor κ which depends on the local mean density of matter surrounding him. In other words for any observer A at a given place M, we have $d\mathbf{M}|_A = \kappa_A d\boldsymbol{\sigma}$ and this observer, measuring the oriented distance between M (label $\boldsymbol{\sigma}$) and another distant point M' (label σ'), finds $\mathbf{MM'}|_A = \kappa_A (\boldsymbol{\sigma}' - \boldsymbol{\sigma})$.

[19]Let us recall that for us the distances in space are always measured by trigonometry, by photometry or by other more or less related methods using local devices. Unfortunately we have just one point of reference in the Universe ! For two objects very close to each other there is no difficulty, but for two objects very distant to each other the situation is much more problematic, especially when a common background is suppressed as it is assumed here.

This distance is a "true" length for him, even though it is an apparent one[20]. The Universe perceived by this observer is not deformed, apart from a homothety, given the factor κ_A is the same for any (infinitesimal or finite) distance which is measured by him. Once again let us note that neither κ nor the norm of $\boldsymbol{\sigma}$ can be separately determined, only the product $\kappa\boldsymbol{\sigma}$ can be estimated (or still the ratios $\frac{\kappa'}{\kappa}$). In imaginative ways the vectors $\boldsymbol{\sigma}$ represent the bare (and hidden) Universe – a kind of skeleton – and this hidden substructure has yet to be dressed by a given observer (scale factor κ) to eventually supply the apparent Universe watched by him. a common error is to imagine that there exists an absolute reference length attached to an hypothetical universal vacuum. In fact, in a given location, a reference length is always linked to a material system, i.e. a device composed of atoms, especially for instance an interferometer, which is itself immersed, not in an hypothetical universal vacuum, but in a given environment. Any other length is then compared to this reference length in a local manner. Let a meter-stick located very far from us in the Galaxy. How do we know whether it is really identical to our terrestrial meter-stick ? To check this statement two methods are conceivable. We can move that invariable meter-stick and approach it to our meter-stick. Obviously then the two meter-sticks would be trivially found identical, as representing the real situation; but unfortunately such a method is unmanageable at the present time. Otherwise we can use the trigonometry and make a parallax measurement. However in the latter case we could observe that the two (yet identical) meter-sticks are different, even though it would be an illusion and not a real effect ! It is certainly a bold hypothesis (uncheckable at the present time, however maybe testable in the not too distant future[21]), but this is the point of view adopted in this paper. Maybe there exists a Relativity for the measured lengths in the Universe[22], as there exists a Relativity for the velocities in Kinematics.

For two distinct observers A and B the ratio $\frac{\kappa_A}{\kappa_B}$ is determined. In the following κ_E is the terrestrial reference. This trick would allow for a terrestrial observer to compare his measurements with those of a distant observer or for two distant observers in a galaxy to compare their measurements with each other, if it were possible ! In the following we admit these considerations as a natural postulate.

A The laser distance

Let us specify that, even though we cannot define an absolute distance in the κ-model, we can at least define a symmetric distance between two distant observers A and B. The so-called laser distance [11] is obtained by adding small segments locally measured by a series of observers aligned between A and B. Let $\int_A^B \kappa d\sigma = \int_B^A \kappa(-d\sigma)$. Dividing this quantity by c, i.e.the universal celerity of light, locally measured by each observer, and multiplying by two the integral we obtain a round trip time delay, indifferently measured by A or B. More specifically in the apparent background of the observer A, we can write

$$\frac{1}{c}\int_A^B \kappa d\sigma = \int_A^B \frac{1}{c_{ap}|_A}\kappa_A d\sigma$$

where $c_{ap}|_A = \frac{\kappa_A}{\kappa}c$ is the apparent celerity of light measured by A, and a very similar integral for B. This time expressed by a number is obviously the same for A and B. It is an universal number. However we put the question: what is the physical signification of this number ? Before answering to this question, let us imagine that we wish to measure the velocity of the barycenter of a system of particles. To realize this operation we must add the velocities of the individual particles composing the system (weighted by the masses). However in the recipe we implicitly

[20]Another distant observer B would assign to it the vector $\mathbf{MM}'|_B = \kappa_B(\boldsymbol{\sigma}' - \boldsymbol{\sigma})$.

[21]Maybe the Breakthrough Starshot program initiated by Stephen Hawking of sending nano-spacecrafts represents an opportunity to confirm (or to refute) this idea [28].

[22]This type of Relativity applies to the very large distances ($\gtrsim 1\ pc$); by contrast in the solar system there is no problem, the distances ($\lesssim 10^{-3}\ pc$) can be considered as absolute.

admit the fact that all the velocities need to be measured in the same reference frame, otherwise the result has no value. This is the same thing here, we can always write an integral of the type $\int_A^B \kappa d\sigma$, but by proceeding in this manner we add in fact quantities measured by different observers located in different places. Without a "common background" the notion of "absolute" distance, when the latter one is very large (to fix ideas $\gtrsim 1\ pc$), loses its meaning. In that sense the laser distance is no more absolute than the apparent distance measured by A or B. A question can however be put. What would be the difference between the apparent trigonometric distance and the laser distance, if the latter one could be measured ? For Proxima b, the apparent distance measured by any local device, for instance the trigonometric and photometric data derived from the Gaia satellite, is 1.301 pc. In any way the measurement of the laser distance would give an insignificant difference of 0.001 pc compared to the apparent estimate taking into account the equation 2. However unfortunately today the only reachable distance is $\kappa_E \sigma$ (with for instance $A = E$); the laser distance is not measurable with our state-of-the art technology.

However in this case what is the trajectory of a free particle or a photon ? We shall see in the following that the trajectory of a free massive particle or of a photon is always a straight line in the κ-model even though, paradoxically enough, κ varies from place to place. However we have checked that when transcribed in the general relativity context the trajectories of the photons are indeed curved like it should be and that a gravitational lens effect is existing. The subsection C.3 of the appendix A gives some additional details on how to proceed to optimize an integral of the type $\int dM$ and to define a kind of "geodesic" even in the framework of the κ-model.

B Does the meter-stick vary with the location in a galaxy ?

At first glance the idea of asymmetric distances could appear unreasonable. However as an afterthought, this idea is not more odd than saying that the celerity of light is the same, regardless of the motion of the light source or of any observer, which is yet the today admitted and repeatedly testified base of the Relativity !

By the way we could think, but misleadingly, that the κ-model eventually leads to the following preposterous conclusion that the meter-stick varies from place to place in a galaxy ! However this question can legitimately be put because when measuring a length the factor κ (which is variable) intervenes. Obviously the meter-stick does not vary from place to place in a galaxy, for instance the size of a hydrogen atom, taken in its fundamental level, is an immutable and universal invariant[23]

It is its measurement carried out from a distant place which can change. It is an apparent - or still a perspective - effect, just as when we observe an object through a simple lens; we do not magnify the object itself but its image !, even though this quick comparison should not be taken at face value. Let us recall again that the size of any object is always estimated from the size of another object taken as a standard length.

The trouble experienced here results from the removal of a common background (the scenery)[24] assimilated to an imaginary extension made by some observer of his proper local background, i.e. the solar system for us. However the space remains Euclidean with the same standard meter-stick everywhere.

In order to be clear, let two very distant observers A and B. The observer A allocates to a

[23]The particles (protons and HZE ions) produced by the supernovae and active galactic nuclei and which impact the Earth are exactly the same than their terrestrial equivalents which are produced in the particle accelerators. The physical laws and the fundamental constants (speed of light, gravitational constant, etc) are assumed here to be universal.

[24]There exists some analogy with the Aether. This elusive medium, involving contradictory properties, invented in the nineteenth century before the Relativity, was seen as a kind of absolute frame of reference. It has been definitively eliminated from the physics at the very beginning of the twentieth century by A. Einstein.

meter-stick the length l. We can put $l = k_A \Delta\sigma$, but this machinery is hidden from the observer view, given that $\Delta\sigma$ is unreachable for him. After transport to B we have always l but now with $l = k_B \Delta\sigma'$, taking into account that the length l of a meter-stick is an invariant. Obviously $\Delta\sigma$ has changed but the observers are unaware of that since no length is linked to this quantity. Now if the observer A from its place had the possibility to remotely measure the meter-stick located at B, he would assign to it an apparent length $l' = \frac{k_A}{k_B} l$.

This is the same thing with the velocities and this point will be considered at a later stage. Assuming an observer L who locally measures the velocity of an ultrarelativistic particle, let v_L. We have $v_L = \kappa_L \frac{\Delta\sigma}{\Delta t}$ where $\Delta\sigma$ is a label (the same for all observers). In order to simplify the study we assume that this velocity is perpendicular to the line of view. Then the terrestrial observer E measures a pure proper motion which leads by calculation to the apparent velocity $v_E = \kappa_E \frac{\Delta\sigma}{\Delta t}$ (the label $\Delta\sigma$ is the same). We obtain the relation $v_E = \frac{\kappa_E}{\kappa_L} v_L$. Then if $\frac{\kappa_E}{\kappa_L} > 1$ and $v_L \sim c$ (c is the celerity of light, an universal constant), the terrestrial observer can now record on the sky background a superluminal velocity for the particle, even though this effect is only apparent, locally the celerity of a particle is always smaller than c in accordance with the special Relativity.

Let us specify again that neither κ nor σ can separately be measured. We can just reach the ratios $\frac{\kappa_E}{\kappa_L}$ and the products of the type $\kappa\sigma$, given the label σ is a hidden variable in the κ-model (the common background is suppressed).

A figure is presented to better understand the situation (figure 13). On this figure we have reported a reference length ($l = 1$ meter) travelled by a moving particle with a constant velocity ($v = 1\ m/s$).

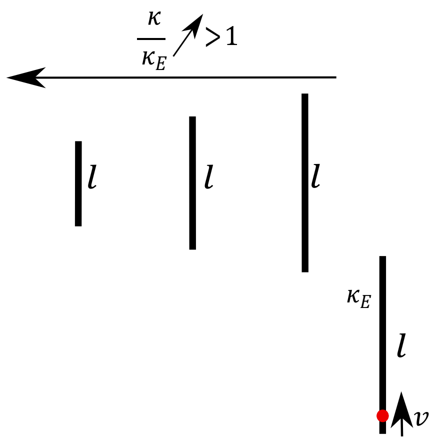


Figure 13 Apparent measurement of a unit of length as a function of $\frac{\kappa}{\kappa_E}$

This reference length is parallel-transported from the Earth (observer E) to different points along a galactic radius without changing (invariant transport). We see that the corresponding lengths projected on the sky background of the terrestrial observer appear now different for him. On the other hand the true velocity v, namely the radial velocity or the component measured by spectroscopy, is left unchanged. However the time being universal in the κ-model at the Newtonian level, the length l is travelled in the same time interval and the apparent velocity, the latter one being supplied by the proper motion measurement or namely the tangential part of the velocity, is perceived differently by the observer E. We assist thus to a forced perspective as due to an environmental mean mass density effect. Let us note however that the effect is not so marked as represented in Figure 8. We can compare for instance the apparent radius r of two stars of solar-type ($\odot$), the first one picked in the direction of the center of the Galaxy (C) and the other in the direction of the anticenter (A) for a same distance of 1 *pc*. With equation 2, we find $\frac{r_A - r_C}{r} \sim 0.5$ percent. A statistical measurement of this ratio could allow to validate (or still to refute) the κ-model, but today it appears tricky to perform it.

C The dynamics equation

We explain here the origin of the form (1) for the dynamics equation.

C.1 First demonstration for the term of inertia

Let a pair of infinitely close observers A and B. Let M a test particle of mass m passing in front of them at the respective instants t and $t+dt$. The velocity measured by A is

$$\mathbf{v}_A(M) = \kappa_A \frac{d\boldsymbol{\sigma}}{dt} \tag{8}$$

It is not necessary to index the label-vector σ because, even though it is hidden, this is an universal item and it is assumed to be the same for all observers. Likewise for the velocity measured by B

$$\mathbf{v}_B(M) = \kappa_B \frac{d\boldsymbol{\sigma}'}{dt} \tag{9}$$

Translating now the vector v_B parallel to itself at the point where A is placed[25], we can give a natural definition of the acceleration of M at this point[26]

$$\begin{aligned}\mathbf{a}(M) &= \frac{\kappa_B \frac{d\boldsymbol{\sigma}'}{dt} - \kappa_A \frac{d\boldsymbol{\sigma}}{dt}}{dt} = \frac{(\kappa_A + d\kappa_A)\frac{d\boldsymbol{\sigma}'}{dt} - k_A \frac{d\boldsymbol{\sigma}}{dt}}{dt} \\ &= \frac{\kappa_A\left(\frac{d\boldsymbol{\sigma}'}{dt} - \frac{d\boldsymbol{\sigma}}{dt}\right) + d\kappa_A \frac{d\boldsymbol{\sigma}}{dt}}{dt} \simeq \kappa_A \frac{d^2\boldsymbol{\sigma}}{dt^2} + \frac{d\kappa_A}{dt}\frac{d\boldsymbol{\sigma}}{dt}\end{aligned} \tag{10}$$

$$\mathbf{a}(M) = \frac{d}{dt}(\kappa \frac{d\boldsymbol{\sigma}}{dt}) \tag{11}$$

By multiplication by the mass m we obtain the term of inertia in equation 1. Without an applied force, the true (locally measured) acceleration is null ($a(M) = 0$) and the true velocity ($\mathbf{v}(M) = \kappa \frac{d\boldsymbol{\sigma}}{dt}$) is a constant of motion (inertia principle). By contrast for a distant observer (a terrestrial observer E for instance) the (apparent) acceleration is

$$\mathbf{a}_E(M) = \kappa_E \frac{d^2\boldsymbol{\sigma}}{dt^2} = \frac{d}{dt}\left(\frac{\kappa_E}{\kappa}\kappa\frac{d\boldsymbol{\sigma}}{dt}\right) = \frac{d}{dt}\left(\frac{\kappa_E}{\kappa}\right)\mathbf{v}(M) + \frac{\kappa_E}{\kappa}\mathbf{a}(M) = \frac{d}{dt}\left(\frac{\kappa_E}{\kappa}\right)\mathbf{v}(M) \neq 0\ ! \tag{12}$$

Now the particle seems to be accelerated for the terrestrial observer. Let us note however that this apparent acceleration is collinear to the velocity and that the apparent trajectory is still a straight line coinciding with the true trajectory.

The faulty reasoning made today is that the terrestrial observer measures $\kappa_E\boldsymbol{\sigma}$ or $\kappa_E d\boldsymbol{\sigma}$, while he should use $\kappa\boldsymbol{\sigma}$ or $\kappa d\boldsymbol{\sigma}$. In that respect two relations can be useful. Starting from the measured (apparent) couple ($\mathbf{v}_E(M)$, $\mathbf{a}_E(M)$) we can deduce the true quantities, i.e. those measured by a local observer, there where the particle is situated at a time t

$$\mathbf{v}(M) = \tfrac{\kappa}{\kappa_E}\mathbf{v}_E(M)$$

$$\mathbf{a}(M) = \tfrac{\kappa}{\kappa_E}\left[\tfrac{dLn\ \kappa}{dt}\mathbf{v}_E(M) + \mathbf{a}_E(M)\right]$$

[25] The parallel displacement is not ambiguous in the κ-model where the orientation is well defined (there is no curvature at the Newtonian level).

[26] The observer B communicates to A its measurement and then A calculates the difference.

the factor κ being supplied by equation 2. Then the equation of dynamics is (without a dark matter term)

$$m\mathbf{a}\left(M\right)=\mathbf{F} \tag{13}$$

and definitely not as today

$$m\mathbf{a}_E\left(M\right)=\mathbf{F}+? \tag{14}$$

the ? representing the dark matter effect.

In equation 13 $\mathbf{F}$ is the gravitational force produced by the other (baryonic) masses and applied to the given particle. Additionally this force must imperatively be calculated where the mass is located (dressed force).

The equation 13 indicates that a particle travels in space with a constant velocity $\mathbf{v}$ when no force is applied, adding now the restrictive condition : this velocity must be measured locally[27]. Likewise a photon travels with a constant wave vector $\mathbf{k}$, this vector being measured locally. For a distant observer the direction of $\mathbf{v}$ remains constant, likewise for the direction of $\mathbf{k}$ for the photon, but not the norm. From the point of view of a distant observer a photon could surprisingly be seen accelerating (or decelerating) even though it is an apparent effect.

C.2 Second demonstration starting from the variational principle

We introduce the formal action

$$S=\int dt[\frac{1}{2}m\left[\left(\frac{d\mathbf{M}}{dt}\right)^2-V(M)\right] \tag{15}$$

where m is the mass of a test particle and $V(M)$ the potential as experienced by this particle located at a given point M. An arbitrary variation $\delta\mathbf{M}$ from M gives

$$\delta S=\delta\int dt\left[\frac{1}{2}m\left(\frac{d\mathbf{M}}{dt}\right)^2-V\left(M\right)\right]=\int dt\;\delta\left[\frac{1}{2}m\left(\frac{d\mathbf{M}}{dt}\right)^2-V\left(M\right)\right] \tag{16}$$

$$=\int dt\;\left[m\frac{d\mathbf{M}}{dt}\delta\left(\frac{d\mathbf{M}}{dt}\right)-\delta V\left(M\right)\right] \tag{17}$$

In order to contiue the calculations, we must now exchange d and δ.

The exchange of d and δ

Let three observers A, B and C located in a plane (figure 14). We can define this plane by imagining a common direction perpendicular to $\mathbf{M}_0\mathbf{M}_1$ and $\mathbf{M}_0\mathbf{M}_2$. This operation is possible because any orientation is well defined in the κ-model. With the help of this figure, we write[28]

$$d\mathbf{M}\triangleq\mathbf{M}_0\mathbf{M}_1\left[d\boldsymbol{\sigma}\right]\Big|_A\equiv\;d\mathbf{M}_{\|}=\mathbf{M}_2\mathbf{M}_{1\|}\Big|_C\longrightarrow\kappa d\boldsymbol{\sigma}\quad(a)$$

The first expression signifies that the observer A measures $\mathbf{M}_0\mathbf{M}_1$ and obtains $\kappa d\boldsymbol{\sigma}$ and that the observer C measures $\mathbf{M}_2\mathbf{M}_{1\|}$ and obtains the same value. Other very similar relations follow

[27]The notion of velocity could appear ambiguous at this level. In order to remove this ambiguity one can place in space a series of fictitious observers, at rest from each other. To check that it is a simple matter to make spectroscopic measurements. Each fictitious observer can then measure the velocity of the particle when this one passes in front of him (obviously this velocity is a relative velocity measured by an observer taken in the chosen series).

[28]Notation : The arrow $\longrightarrow$ indicates that a given observer makes a measurement of the corresponding bipoint. The symbol $\triangleq$ indicates a definition, the symbol $\equiv$ signifies that the two compared vectors have the same orientation and the same norm, but each of them is seen by a distinct observer.

$$\delta\mathbf{M} \triangleq \mathbf{M_0M_2}\left[\delta\boldsymbol{\sigma}\right]|_A \equiv \delta\mathbf{M}_{||} \triangleq \mathbf{M}_1\mathbf{M}_{2||}\left[\delta\boldsymbol{\sigma}\right]\Big|_B \longrightarrow \kappa\delta\boldsymbol{\sigma} \quad (b)$$

$$\mathbf{M}_2\mathbf{M}_4^{"}\;\left[d\boldsymbol{\sigma'}\right]\Big|_C \longrightarrow (\kappa+\delta\kappa)d\boldsymbol{\sigma'} \equiv \mathbf{M}_2\mathbf{M}_4\left[d\boldsymbol{\sigma}+\delta d\boldsymbol{\sigma}\right]|_A \longrightarrow \kappa\left(d\boldsymbol{\sigma}+\delta d\boldsymbol{\sigma}\right) \quad (a')$$

$$\mathbf{M}_1\mathbf{M}_4^{'}\;\left[\delta\boldsymbol{\sigma'}\;\right]\Big|_B \longrightarrow (\kappa+d\kappa)\delta\boldsymbol{\sigma'} \equiv \mathbf{M}_1\mathbf{M}_4\left[\delta\boldsymbol{\sigma}+d\delta\boldsymbol{\sigma}\right]|_A \longrightarrow \kappa\left(\delta\boldsymbol{\sigma}+d\delta\boldsymbol{\sigma}\right) \quad (b')$$

We must remark that $d\mathbf{M}_{||}$ is $d\mathbf{M}$ parallel-displaced to itself respecting the conservation of the length during the transport (likewise for $\delta\mathbf{M}_{||}$ *vs* $\delta\mathbf{M}$). The observer C located at M_2 sees $d\mathbf{M}_{||}$ exactly as the observer A located at M_0 sees $d\mathbf{M}$; thus $d\mathbf{M}$ and $d\mathbf{M}_{||}$ have the same orientation and the same norm, but their origins are distinct. However these two vectors are perceived differently by the observer A located at M_0. The figure 14 is the projection of the full set of vectors on the background of this observer. Let us note that $\mathbf{M}_0\mathbf{M}_1$ is the real path and $\mathbf{M}_2\mathbf{M}_4^{"}$ is the corresponding varied path. Now we put

$$d\delta\mathbf{M} \triangleq \mathbf{M}_{2||}\mathbf{M}_4^{'} \qquad \delta d\mathbf{M} \triangleq \mathbf{M}_{1||}\mathbf{M}_4^{"}$$

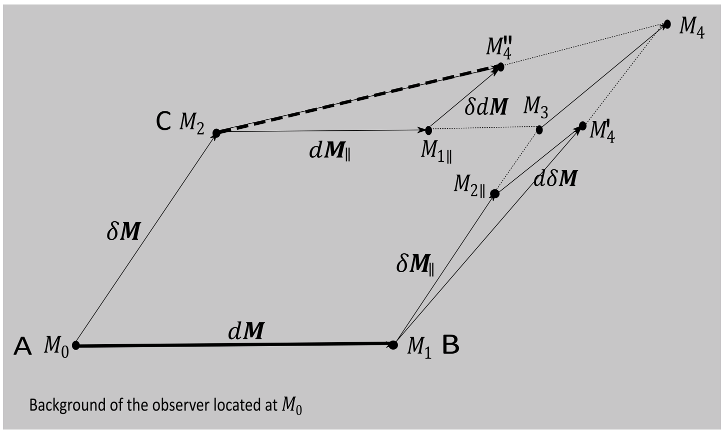


Figure 14 Diagram of real and virtual paths : the thick full segment represents the real path, the virtual path is indicated by a thick-dotted line.

After soustraction $(a')-(a)$ and $(b')-(b)$ it yields respectively

$$\mathbf{M}_{1||}\mathbf{M}_4^{"}\;\left[\delta d\boldsymbol{\sigma'}\right]\Big|_C \longrightarrow \kappa\delta d\boldsymbol{\sigma} \qquad\qquad \mathbf{M}_{2||}\mathbf{M}_4^{'}\left[d\delta\boldsymbol{\sigma'}\right]\Big|_B \longrightarrow \kappa d\delta\boldsymbol{\sigma}$$

We have naturally $\delta d\boldsymbol{\sigma} = d\delta\boldsymbol{\sigma}$, thus (omitting the indexes)

$$d\delta\mathbf{M} = \delta d\mathbf{M}$$

Let us specify however that for the observer A located at M_0 the vectors $d\delta\mathbf{M}$ and $\delta d\mathbf{M}$ (projected on his proper background) are parallel, but their (apparent) lengths are seen different, resp. $\frac{\kappa}{\kappa+d\kappa}d\delta\boldsymbol{\sigma}$ and $\frac{\kappa}{\kappa+\delta\kappa}\delta d\boldsymbol{\sigma}$, but for him $\mathbf{M}_3\mathbf{M}_4[d\delta\boldsymbol{\sigma}] \longrightarrow \kappa d\delta\boldsymbol{\sigma}$, even though these three vectors only differ by a small third order term. Let us remark that $(\kappa+\delta\kappa)\,d\boldsymbol{\sigma'} - \kappa d\boldsymbol{\sigma} = \kappa\left(d\boldsymbol{\sigma'} - d\boldsymbol{\sigma}\right) + \delta\kappa d\boldsymbol{\sigma'} = \kappa\delta d\boldsymbol{\sigma'} + \delta\kappa d\boldsymbol{\sigma'}$. This quantity is equal to $\kappa\delta d\boldsymbol{\sigma}$ (in both orientation and norm), but $\kappa\delta d\boldsymbol{\sigma'} + \delta\kappa d\boldsymbol{\sigma'}$ (origin $M_{1||}$) is evaluated by C and $\kappa\delta d\boldsymbol{\sigma}$ (origin M_3) is evaluated by A.

After exchanging of d and δ in equation 17, we obtain

$$\int dt\ \left[m\frac{d\mathbf{M}}{dt}\frac{d}{dt}\delta\mathbf{M}-\boldsymbol{\nabla}_{M}V\left(M\right)\delta\mathbf{M}\right]$$

$$=m\frac{d\mathbf{M}}{dt}\delta M\Big|_{extremities}+\int dt\ \left[-\frac{d}{dt}\left(m\frac{d\mathbf{M}}{dt}\right)-\nabla_{M}V\left(M\right)\right]\delta M \tag{18}$$

For a stationary value of S we have $\delta S=0$. We take also $\delta M=0$ at both the extremities of the portion of the real trajectory. Eventually we obtain

$$\frac{d}{dt}\left(m\frac{d\mathbf{M}}{dt}\right)+\boldsymbol{\nabla}_{M}V\left(M\right)=\mathbf{0} \tag{19}$$

This is the same expression as the usual dynamics equation in Newtonian mechanics. The physics is left formally unchanged. It is a very interesting item. For practical (computational) reasons however we rewrite this equation

$$\frac{d}{dt}\left(m\kappa\frac{d\boldsymbol{\sigma}}{dt}\right)+\boldsymbol{\nabla}_{(\kappa\boldsymbol{\sigma})}V\left(\kappa\boldsymbol{\sigma}\right)=0 \tag{20}$$

This is the equation 1. The potential $V\left(M\right)$ is the gravitational potential. The dressed potential, experienced by an observer located at a point M and produced by a point source (mass M) located at an arbitrary origin (labeled by $\boldsymbol{\sigma}=\mathbf{0}$), is[29]

$$V\left(\kappa_{M}\boldsymbol{\sigma}\right)=-G\mathsf{M}m\frac{1}{\left(\kappa_{M}\boldsymbol{\sigma}\right)} \tag{21}$$

We have explicitly indexed by κ the point where the potential is measured[30]. For a shifted origin (at O) we have likewise

$$V\left(\kappa_{M}\sigma\right)=-G\mathsf{M}m\frac{1}{\left(\kappa_{M}\ \left\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_{O}\right\|\right)} \tag{22}$$

The understanding of the relationships 21 and 22 needs an explanation. The coefficient κ_{M}, which is linked to the observer located at M, determines the intensity of the potential measured by this observer where he stands. It would seem that this is the measurement process itself which imposes to the observer the distance which separates him from the attractive mass M! This circular reasoning may sound crazy. In reality both the measured distance and the apparent gravitational potential felt by the observer depends on the mean density $\bar{\rho}$ at M. This is the environment of the observer (and obviously not the observer himself) which affects the measurements. There is nothing strange about this. Thus we can equivalently reason by admitting that the attractive mass M is perceived by the observer as $\frac{\mathsf{M}}{\kappa_{M}}$, the smaller κ_{M}, the higher the apparent attractive mass and conversely (however the true mass is always M).

C.3 The variational principle for the light

A similar formulation can be made for the eikonal equation. Let a given observer A, equipped with a common (apparent) background. He thinks right to optimize the following integral, after having developed the inner machinery, let $d\mathbf{M}\longrightarrow\kappa d\boldsymbol{\sigma}$

[29] For two masses m and m' respectively located at M and M' the interaction potential is asymmetric and $V_{M}=-Gmm'\frac{1}{(\kappa_{M}\ \|\sigma-\sigma'\|)}\neq V_{M'}=-Gmm'\frac{1}{(\kappa_{M'}\ \|\sigma-\sigma'\|)}$. The principle of reciprocal action seems to be altered, but it must be kept in mind that the two masses are not isolated and are not located in the same environment. Thus this fundamental principle is not violated, but it is not directly applicable here. In contrast the bare potential, $V_{b}=-Gmm'\frac{1}{\|\sigma-\sigma'\|}$, is symmetric (even though it is not measurable, given that $\|\boldsymbol{\sigma}-\boldsymbol{\sigma}'\|$ is hidden !).

[30] Let us specify that we cannot directly measure a potential (this quantity is defined up to a constant), but its gradient (the force on a test particle of unit mass). But this remark is simply a detail here.

$$T = \frac{1}{c}\int dM \longrightarrow \int \frac{1}{c_{app}}\sqrt{\left(\frac{\kappa_A d\boldsymbol{\sigma}}{\kappa_A d\sigma}\right)^2}\kappa_A d\sigma \tag{23}$$

where $c_{app} = \frac{c}{\kappa}$ (c is the universal celerity of light). However c_{app} depends on $\boldsymbol{\sigma}$ (by κ), also the minimization now gives an eikonal equation which implies a curvature for the trajectory of the photons in the vacuum ! This paradox results from the spurious reasoning by this observer which unconsciously introduces a common and illusive background, namely the space of vectors $\boldsymbol{\sigma}$ multiplied by κ_A. Even though difficult to visualize, a radical solution then consists to imagine a space, filled with various objects (stars, galaxies) of defined size, but deprived of a common background, and mostly to cut ourselves from the notion of absolute distance at a very large scale. It is maybe the most delicate issue of the κ-model. It underlies the intrinsic limitation of the ability for any observer to estimate the very large distances in the Universe.

In fact an observer, by placing himself in different locations in space, would see everywhere the same environment. For him the space would be an euclidean manifold where the trajectory of a photon must be a straight line in the vacuum. Thus in order to take account of this fact and to perform the "optimization" of the integral $\int dM$, the right way is not to develop the inner machinery for $d\mathbf{M}$, but rather to reason in a formal manner. Thus for a photon we must directly "optimize" the integral

$$T = \frac{1}{c}\int dM = \frac{1}{c}\int \sqrt{\left(\frac{d\mathbf{M}}{\kappa_A d\sigma}\right)^2}\kappa_A d\sigma \tag{24}$$

where c is the celerity of light (an universal constant locally measured). With a formal dM and a corresponding T, a number with dimension of time, the Fermat's principle now imposes $\delta T = 0$. This immediateley leads to the eikonal equation

$$\frac{d}{\kappa_A d\sigma}\left(\frac{d\mathbf{M}}{\kappa_A d\sigma}\right) = 0 \tag{25}$$

and eventually to its practical equivalent, established at the very end of the calculation chain:

$$\frac{d}{d}\left(\kappa\frac{d\boldsymbol{\sigma}}{d\sigma}\right) = 0 \tag{26}$$

We find then that the trajectory of a photon seen by the observer A is eventually a straight line in the vacuum as it should be at the Newtonian level. The directions being universal in the κ-model any observer sees a homothetic straight line for this trajectory.

Appendix B

The comparison with other alternative theories (MOND and MOG)

MOND and MOG are noteworthy theories which are based on two very different formalisms. They are fundamentally distinct, but their aim is to solve the same problem, i.e. the flatness of the rotation curve of galaxies. In fact the κ-model provides such an incentive, but within a pure Newtonian context, the appeal to the Relativity being not needed at this level.

MOND theory

MOND affects the inertia term [1, 2]. This effect is incorporated here in the same manner. In the special case where a particle describes a circular orbit around the galaxy centre, assuming the galaxy approximately axisymmetric, the factor κ is independent of time. We have for the inertia term

$$\frac{d}{dt}\left(m\kappa\frac{d\boldsymbol{\sigma}}{dt}\right) = m\kappa\frac{d^2\boldsymbol{\sigma}}{dt^2} \tag{27}$$

The naked acceleration $\frac{d^2\boldsymbol{\sigma}}{dt^2}$ is modified in the outer region of a galaxy by the factor κ. In MOND this factor is arbitrarily parametrized. Rather here it is interpreted as a density effect and it is not parametrized.

Modified Gravity (MOG)

The Modified Gravity (MOG) is a remarkable relativistic theory of gravitation that is derived from a relativistic action principle involving scalar, tensor, and vector field ([3, 4]. Presented in a very basic way, this theory eventually leads to a spatially varying gravitational coupling. The latter effect is easy to reproduce in the context of the κ-model, however without leaving Newton[31] ! We can still write the relationship (20) as

$$V\left(\kappa_M\sigma\right) = -\frac{\kappa_E}{\kappa_M}G\mathsf{M}m\frac{1}{\left(\kappa_E\left\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_O\right\|\right)} \tag{28}$$

The k-effect is now perceived as a modification of G itself which becomes $G_{mod} = \frac{\kappa_E}{\kappa_M}G$ (κ_E is the value taken by κ in our solar system or on the Earth)[32]. The κ-model is automatically transformed in a kind of modified-G theory. We find that there is some equivalence between the MOG theory and the κ-model even if the formalisms are obviously very far from each other (thus we insist on the fact that in the κ-model the modification of G is not real but just fictitious, G is here an universal constant. If two distant observers in a galaxy measure locally the gravitational constant, they obtain exactly the same value).

Eventually the κ-model can also easily mimic a dark matter effect by putting $\mathsf{M}+\mathsf{M}_{\mathsf{DM}} = \frac{\kappa_E}{\kappa_M}\mathsf{M}$ in the potential (M baryonic mass) with $\frac{\kappa_E}{\kappa_M} \sim 6-10$.

Appendix C

The magnification formula

We assimilate a galaxy to a steady and axisymmetric thin disk. The stars travel in pure uniform circular motion and the coefficient κ is independent of time. After removal of the index i for a test particle of unit mass and taking $\kappa_E = 1$[33], the equation 1 can be simplified to

$$\frac{d}{dt}\left(\frac{d\boldsymbol{\sigma}}{dt}\right) = \frac{\mathbf{F}_{New}}{\kappa^3} \tag{29}$$

where the Newtonian force $\mathbf{F}_{New}$ acting on the test particle is

[31] A relativistic version of the kappa model is possible, even though not needed here.

[32] Let us note that when the force is calculated from the potential and given that G_{mod} is a "variable" constant which depends on σ, we must formally use $G_{app} = \left(\frac{\kappa_E}{\kappa_M}\right)^3 G$ in the expression of this force. Thus there exists a little ambiguity in the definition of a "variable" G. This ambiguity is not perceptible because the potential is a mathematical entity which cannot directly be measured. In any way in the κ-model there is no ambiguity because G is firmly determined and is everywhere the same as the G measured on Earth.

[33] The full equations with the coefficient κ_E are

$$\frac{d}{dt}\left(\kappa_E\frac{d\boldsymbol{\sigma}}{dt}\right) = \left(\frac{\kappa_E}{\kappa}\right)^3\mathbf{F}_{\mathbf{New}}$$

and

$$\mathbf{F}_{New} = -Gm\sum_{j=1}^{N-1}\frac{\kappa_E(\boldsymbol{\sigma}-\boldsymbol{\sigma}_j)}{\left[\kappa_E\ \left\|\boldsymbol{\sigma}-\boldsymbol{\sigma}_j\right\|\right]^3}$$

$$\mathbf{F}_{New} = -Gm \sum_{j=1}^{N-1} \frac{(\boldsymbol{\sigma} - \boldsymbol{\sigma}_j)}{\|\boldsymbol{\sigma} - \boldsymbol{\sigma}_j\|^3} \tag{30}$$

The trajectory of the test particle being circular and the force acting on it purely radial, the equation 29 immediately gives

$$\frac{(\sigma\dot{\theta})^2}{\sigma} = \frac{F_{New}}{\kappa^3} \tag{31}$$

where θ designates the polar angle in the galactic plane from a reference direction taken in this plane. This leads to

$$\sigma\dot{\theta} = \frac{1}{\kappa^{\frac{3}{2}}}(F_{New}\ \sigma)^{\frac{1}{2}} = \frac{1}{\kappa^{\frac{3}{2}}}\ v_{New} \tag{32}$$

where v_{New} is the Newtonian velocity. Eventually the true velocity[34] v is obtained by multiplying by κ the equation 32. We obtain the magnification formula 6 after reinserting in it the coefficient κ_E.

[34]The true velocity refers to the local velocity which is reachable by spectroscopy.