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Abstract

The Variance Risk Premium (VRP) is estimated directly from synthetic variance swap payoffs. Since variance swap payoffs are highly volatile, the VRP is extracted by using signal extraction techniques based on a state-space representation of the model in combination with a simple economic constraint. The proposed approach, only requiring option implied volatilities and daily returns for the underlying asset, provides measurement error free estimates of the part of the VRP related to normal market conditions, and allows constructing variables indicating agents' expectations under extreme market conditions. The latter variables and the VRP generate different return predictability on the major US indices. A factor model is proposed to extract a market VRP which turns out to be priced when considering Fama and French portfolios.

Keywords: Variance risk premium; Variance swaps; Return predictability; Factor Model; Kalman filter; CAPM

1. Introduction

Financial markets trade several products with exposure only to the volatility of a given underlying asset. In particular, variance swaps, i.e. contracts in which one party pays a fixed amount at a given maturity in exchange for a payment

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equal to the sum of squared daily returns of the underlying asset occurring until that maturity, have become increasingly popular to trade variance. Other popular instruments when the underlying is the S&P500 index are futures and options on the VIX, iPath S&P500 VIX Short-Term Futures ETNs being an important example. See [1] for the use of this type of data for estimating an affine asset pricing model. The prices and payoffs of variance swaps contain useful information on the variance risk premium (VRP), which is defined as the difference between the risk neutral and physical expectations of an asset's total return variation. The empirical features of the VRP are used for validation and development of new asset pricing models, and seem to generate market return predictability. [2] estimate no-arbitrage term structure models for the VRP using proprietary variance swap data and show that the expectation hypothesis does not hold. [3] show that the empirical features of proprietary and synthetic variance swap data are difficult to reconcile with existing structural models for the VRP. See also [4] for a link with higher order moments and the variance term structure. Though clear conceptually, the estimation of the VRP requires multiple sources of data as well as assumptions on the latent volatility processes, rendering its dynamic properties difficult to pinpoint.

This paper proposes a state space model which allows decomposing synthetic variance swap payoff time series into the latent VRP and measures of investors' ability to predict large shocks to the market. Variance swap payoffs are highly noisy series, with time varying variance levels and extreme absolute payoffs occurring during market turmoil and generated by fears or unexpected extreme volatility shocks. To incorporate these features, we allow for regime switching dynamics accounting for normal and extreme market conditions. In fact, expected and realized volatility extremes appear as short lived clusters of volatility bursts in the variance swap series. This feature, if not isolated, distorts the estimation of the continuous component. A particular additive regime structure together with opportunely designed identification constraints allows us to devise a model framework suited to separate the systematic component of the VRP from the discontinuous part. In its simplest specification, e.g. [5], this is achieved by assuming that the measurement error comes from either one of two Normal distributions, and that the transition between these distributions is governed by a Markov Chain. By inflating the measurement error variance, such filter shrinks observations towards their one-step-ahead prediction when a volatility burst occurs neutralising the impact of outlying measurements on the filtered state. A similar result, suggested by [6], is achieved by bounding the effect of the information carried by a new observation according to an influence function. See also [7] for other methods of the state estimation which are robust against unknown outlier measurements. The advantage of our approach is that the model can be written in a linear state space form and estimated by maximum likelihood using the Kalman-Hamilton filter. Since financial theory predicts positivity of the VRP, i.e. risk adverse agents dislike the fact that variance is stochastic, we impose this economic constraint when estimating the model by bounding the signal to noise ratio. Our framework requires data on option implied volatility, e.g. the VIX index for the S&P500, and returns for the underlying, the sources of which are free and readily available for many assets.

The literature has proposed two alternative ways to approximate the VRP directly. First, the variance swap payoff itself has been used as a proxy for the VRP, as e.g. in [8], [9] and [10]. Although this is a model free VRP estimate, it does not constitute an ex-ante expectation as a risk premium should be. Second, the VRP has been estimated as the difference between a option implied variance and a model based forecast of the realized variance, e.g. see [11] and [12]. However, different variance model assumptions can profoundly impact the VRP times series as shown by [13]. While both ways to compute VRP estimates are simple to implement, their drawback stands in an extremely noisy VRP time series, often violating the positivity constraint. Our framework models directly the VRP and is therefore flexible compared to an approach which parametrises the underlying prices and variance dynamics, see for example [14] and [15].

Our setup relates to the expectation hypothesis regressions in [8]. While we allow for idiosyncratic dynamics and stochastic behaviour for the VRP, their dynamics of the variance risk premium reduces to an affine transformation of the risk neutral variance expectations. Combined with an economic constraint on the premium, see [16] and [17] for analogous positivity constraints on the equity premium forecasts, our approach allows to precisely estimate the VRP associated with normal market activity and generates positive and smooth VRPs. The difference between the variance swap payoff and the estimated VRP gives, together with the identification of unusual and extreme episodes of market conditions, rise to two variables related to fear and surprise.

Separating the smooth part of the VRP from the part related to extreme market conditions is first done by [18]. They decompose the VRP in a component that reflects compensation for continuous price moves and a second component that is related to compensation for disaster risk. [19] provide evidence that essentially the second component contributes to explain future return variation. The reported results are for the S&P500 index only and require a large panel of liquidly traded options to estimate nonparametrically jump tails, and intraday high frequency future prices to obtain realized variation measures and the VRP. Our method, on the other hand, only requires option implied volatilities and daily returns for the underlying.

In our empirical application, we consider four major US indices, the S&P500, DJIA, NASDAQ and RUSSELL. We build our dataset of variance swap payoffs by rolling on a weekly basis a synthetic contract with a one month maturity traded at the fair variance swap rate. For all indices, our proposed model provides a good fit to the data, clearly identifies regimes with low and high volatility, and identifies clusters of short-lived extreme market events. The filtered smooth part of the VRP from our model has a high degree of persistence for all indices. Our proposed methodology has implications for market return predictability and for asset pricing. Although the VRP significantly predicts future market returns at short horizons, sizeable increases in predictability are found when the fear and surprise indicators are included in the predictive regressions. Though predictability is improved at all horizons, the largest improvements are found at longer horizons of up to one year.

As expected, for all indices the estimated VRPs move closely together and the episodes of fear and surprise largely overlap in terms of occurrences and duration, and they correspond to and clearly align with well known historical events. We use the four markets in a joint model to filter out a common factor which we interpret as the market variance risk premium (MVRP). When compared to other well-known asset pricing factors, the MVRP is significantly correlated only with the market factor and is priced when considering the returns on most of the five Fama and French (2015) portfolio sorts.

The rest of the paper is organized as follows. Section 2 details the model. Section 3 provides estimation results for four US indices. Section 4 documents predictive return regressions. Section 5 estimates a joint model for retrieving the market VRP. Section 6 concludes.

2. Model

2.1. Variance swaps

We denote by S_t the spot price at time t of an asset defined on a probability space $(\Omega, \mathcal{F}, m)$, $m = P, Q$, with continuous time dynamics described by the stochastic differential equation

$$dS_t = S_{t-}(r_t^f + \lambda_t 1_{\{m=P\}})dt + S_{t-}\sigma_t dW_t^m + \int_{\mathbb{R}} S_{t-}(e^x - 1)\left(\mu(dt, dx) - \nu_t^m(x)dtdx\right), \quad (1)$$

where P and Q represent, respectively, the physical and risk neutral measures, r_t^f is the instantaneous risk-free rate, λ_t is the equity risk premium, σ_t is the instantaneous volatility, W_t^m is a standard Brownian motion, $(\mu(dt, dx) - \nu_t^m(x)dtdx)$ is a compensated counting process with $\mu(dt, dx)$ a measure which takes nonzero values when jumps occur, and $\nu_t^m(x)$ a jump compensator which gives the arrival rate of jumps of size x.

The quadratic variation of the return, i.e. $d\log S_t$, in the interval $[t, t+\tau]$ is

$$QV_{t,t+\tau} = \int_t^{t+\tau} \sigma_s^2 ds + \int_t^{t+\tau} \int_{\mathbb{R}} x^2 \mu(ds, dx), \quad (2)$$

where the first integral represents the portion of quadratic variation due to diffusive price increments (denoted by $CV_{t,t+\tau}$) and the second integral is associated with the jump component (denoted by $JV_{t,t+\tau}$).

The total VRP at time t for a given maturity τ is defined as

$$\widetilde{\Pi}_{t,t+\tau} = E_t^Q[QV_{t,t+\tau}] - E_t^P[QV_{t,t+\tau}]. \tag{3}$$

Given that QV in (2) can be decomposed in diffusive and jump parts, it can be written as

$$\begin{aligned}\widetilde{\Pi}_{t,t+\tau} &= \left(E_t^Q[CV_{t,t+\tau}] - E_t^P[CV_{t,t+\tau}]\right) + \left(E_t^Q[JV_{t,t+\tau}] - E_t^P[JV_{t,t+\tau}]\right) \\ &= \Pi_{t,t+\tau} + \left(E_t^Q[JV_{t,t+\tau}] - E_t^P[JV_{t,t+\tau}]\right), \qquad (4)\end{aligned}$$

where $\Pi_{t,t+\tau}$ is the premium associated with the risk of variance fluctuations under normal market activity.

A consistent and unbiased estimator of $QV_{t,t+\tau}$ is given by the realized variance of [20] and [21], i.e. the sum of squared log returns over $[t, t+\tau]$ and denoted as $RV_{t,t+\tau}$. The riskneutral expectation $E_t^Q[QV_{t,t+\tau}]$, represents the fair strike of a variance swap, and can be represented by a continuum of European call and put options, see [22], [23] and [24]. An estimator is the squared VIX, see [25], and is dentoted $\mathrm{VIX}^2_{t,t+\tau}$.

The payoff generated by a variance swap contract entered at t and held to maturity $t+\tau$ is computed as

$$\mathrm{P}_{t,t+\tau} = \mathrm{VIX}^2_{t,t+\tau} - RV_{t,t+\tau}, \tag{5}$$

where the fixed leg $\mathrm{VIX}^2_{t,t+\tau}$ is calculated at inception and the floating leg $RV_{t,t+\tau}$ is calculated at maturity. In practice, the variance swap payoff is converted in dollar units using a variance notational. Without loss of generality, the variance notional is normalised to one.

2.2. Disentangling the variance swap payoff

The variance swap is designed to hedge against sudden variance fluctuations. As shown in the empirical application, its payoff $\mathrm{P}_{t,t+\tau}$, while stable and slow moving in periods of calm markets, exhibits large and short lasting positive/negative peaks when extreme variance events occur over the life span of the contract. We argue that short lasting expected or unexpected sudden extreme variance events can heavily distort the estimation of the VRP if not adequately identified and measured, giving raise to some contradicting results found in the literature, see e.g. [13] for a discussion.

The variance swap payoff contains relevant information about the latent ex-ante variance risk premium $\widetilde{\Pi}_{t,t+\tau}$ with differences identifiable as the prediction error in computing variance expectations under the physical measure and measurement errors. More explicitly, we can rewrite the variance swap payoff as

$$\begin{aligned}
\mathrm{P}_{t,t+\tau} &= VIX^2_{t,t+\tau} - RV_{t,t+\tau} \\
&= \mathrm{E}^Q_t[CV_{t,t+\tau}] + \mathrm{E}^Q_t[JV_{t,t+\tau}] - (CV_{t,t+\tau} + JV_{t,t+\tau}) + a_t \\
&= \mathrm{E}^Q_t[CV_{t,t+\tau}] - \mathrm{E}^P_t[CV_{t,t+\tau}] + (\mathrm{E}^P_t[CV_{t,t+\tau}] - CV_{t,t+\tau}) + \mathrm{E}^Q_t[JV_{t,t+\tau}] - JV_{t,t+\tau} + a_t \\
&= \Pi_{t,t+\tau} + (\mathrm{E}^Q_t[JV_{t,t+\tau}] - JV_{t,t+\tau}) + e_{t+\tau} \\
&= \Pi_{t,t+\tau} + \mathrm{FS}_{t,t+\tau} + e_{t+\tau}, \qquad (6)
\end{aligned}$$

where the zero mean innovation term $e_{t+\tau}$ reflects measurement error a_t introduced by using VIX and RV plus the prediction error $(E^P_t[CV_{t,t+\tau}] - CV_{t,t+\tau})$.

In equation (6), $\Pi_{t,t+\tau}$ represents the part of the VRP related to normal market conditions, also referred to as the systematic or continuous component of the VRP, while $\mathrm{FS}_{t,t+\tau} = \mathrm{E}^Q_t[JV_{t,t+\tau}] - JV_{t,t+\tau}$ represents the discontinuous component, and more precisely, the extent of fear (> 0) or surprise (< 0) that a realized extreme variance event generates. In particular, fear refers to the situation where agents were expecting an extreme variance event which does not occur or which realises to a limited extent. This can be caused by sudden rumors, sharply increasing market instability or as a reaction to a large unanticipated shock. Surprise refers to an unexpected or underestimated extreme shock which hits the market in the period between t and $t+\tau$. In this sense, $\mathrm{FS}_{t,t+\tau}$ represents the compensation or the extra cost with respect to the "normal" price of hedging against variance fluctuations for the long side of the variance swap contract. The term $\mathrm{FS}_{t,t+\tau}$ can be easily understood by considering the decomposition

$$\mathrm{FS}_{t,t+\tau} = (\mathrm{E}^Q_t[JV_{t,t+\tau}] - \mathrm{E}^P_t[JV_{t,t+\tau}]) + \eta_{t+\tau},$$

where $\mathrm{E}_t^Q[JV_{t,t+\tau}] - \mathrm{E}_t^P[JV_{t,t+\tau}]$ represents the jump risk premium and $\eta_{t+\tau} = (\mathrm{E}_t^P[JV_{t,t+\tau}] - JV_{t,t+\tau})$ represents the jump prediction error under the physical dynamics, which we assume to be Gaussian.

It is well known that variance swap payoffs are subject to different volatility states. To account for this form of heteroskedasticity, we allow different volatility states driven by the realization of a N-state Markov chain $s_{t+\tau}$ with transition probability matrix with elements $p_{ij}, i, j = 1, \ldots, N$. Assuming first order autoregressive dynamics for $\Pi_{t,t+\tau}$, the model can be written as

$$\mathrm{P}_{t,t+\tau} = \Pi_{t,t+\tau} + \mathrm{FS}_{t,t+\tau} + e_{t+\tau}, \tag{7}$$

$$\Pi_{t,t+\tau} = \overline{\Pi} + \phi(\Pi_{t-1,t+\tau-1} - \overline{\Pi}) + \varepsilon_{t+\tau}, \tag{8}$$

The innovations $e_{t+\tau}$ (measurement equation) and $\varepsilon_{t+\tau}$ (state-propagation equation) are assumed to be mean zero Gaussian with state dependent variance σ^2_{e,s_t} and $\sigma^2_{\varepsilon,s_t}$ respectively. Both innovations are defined under the identifying restrictions $\sigma^2_{e,1} < ... < \sigma^2_{e,N-1} = \sigma^2_{e,N}$ and $\sigma^2_{\varepsilon,1} < ... < \sigma^2_{\varepsilon,N-1} = \sigma^2_{\varepsilon,N}$, respectively. The equality restriction imposed on the N-th regime allows to device a modeling strategy suited to identify and capture features of the discontinuous component of the VRP, i.e. the episodes of fear and surprise generated by variance bursts. An alternative way of incorporating time dependence in σ^2_{e,s_t} and $\sigma^2_{\varepsilon,s_t}$ is the stochastic volatility framework, e.g. [26] and [27].

In practice, to exploit the level of decomposition in equation (6), we face limitations due to the heterogeneity, rarity and sparsity of the extreme variance events making it difficult to identify the dynamic properties of the jump risk premium. For these reasons, we opt for an agnostic approach which assumes $\mathrm{FS}_{t,t+\tau}$ to be a realization of a non-zero mean Gaussian process, centered around the unconditional jump risk premium, with occurrence driven by the realization of the Markov chain $s_{t+\tau}$. The measurement equation becomes $\mathrm{P}_{t,t+\tau} = \Pi_{t,t+\tau} + \mathrm{I}_{s_t=N}\mu + \tilde{e}_{t+\tau}$ where $\mathrm{I}_{s_t=N}$ is an indicator function which takes value one if $s_t = N$ and zero otherwise, μ represents the unconditional jump risk premium, $\tilde{e}_{t+\tau} = e_{t+\tau} + \mathrm{I}_{s_t=N}\eta_{t+\tau}$ is a Gaussian noise with state dependent variance $\sigma^2_{\tilde{e}} = \sigma^2_{e,s_t} + \mathrm{I}_{s_t=N}\sigma^2_\eta$. Thus, given the restrictions imposed above, the variance of $\mathrm{FS}_{t,t+\tau}$ is identified as a marginal increase from the high volatility regime. This modeling strategy allows us to model variance bursts as the realisations of a random occurrence and size process the persistence of which is generated by the state variable s_t.

Despite the fact that the ex-post payoff, $\mathrm{P}_{t,t+\tau}$, must be negative for some t, as postulated by financial theory we require $\Pi_{t,t+\tau}$ to be positive for all t. This positivity constraint is analogous to [16] and [17] who impose positivity on their equity premium forecasts. The main idea is that in periods where data are very noisy it becomes hard to extract the underlying quantity of interest. Imposing economic constraints alleviates this identification problem. The positivity constraint is implemented by bounding the signal to noise ratio such that $\Pi_{t,t+\tau} > 0$ point-wise. The linear state space form in (7) and (8) is estimated by maximum likelihood using the Kalman-Hamilton filter, see [28] and [29] for more details. See also [30] for a similar estimation technique to fit a VRP term structure model.

3. Empirical application to four US indices

3.1. Data

We consider four US stock market indices: S&P500, Dow Jones Industrial Average (DJIA), NASDAQ and RUSSELL 2000 (RUSSELL). For each market, we compute variance swap payoffs using realized variance computed from daily squared returns, and we obtain the risk neutral variance expectations which we denote respectively by $\mathrm{VIX}^2_{t,t+\tau}$, $\mathrm{VXD}^2_{t,t+\tau}$, $\mathrm{VXN}^2_{t,t+\tau}$ and $\mathrm{RVX}^2_{t,t+\tau}$ from CBOE. In this paper, we consider τ to be equal to the one month horizon. Following [31] and [32], the data is sampled weekly every Wednesday and starts according to availability on February 1, 1990, for S&P500, November 26, 1997, for DJIA, August 26, 2003, for NASDAQ, and February 3, 2004, for RUSSELL. The sample ends on July 29, 2016, for all indices. A weekly frequency allows for new information to update the measures, especially the RV, and avoids local over-smoothing due to the rolling of the variance swap contract over overlapping 30-day windows.

Table 1 reports descriptive statistics. We find that on average the risk neutral variance expectation is higher than the realized variance implying a positive average variance swap payoff, the latter ranging between 8.33 for the NASDAQ and 10.73 for the S&P500, respectively. All time series are highly volatile, in particular the payoff series have standard deviations which are at least three times larger than their respective averages. In fact, variance swap payoffs have a range of at least 600, an example being the RUSSELL with a minimum of -432.66 and a maximum 216.58. All time series are highly persistent, with the highest autocorrelation in the realized variance series, equal to about 0.96 for all indices, which is expected given the overlapping nature of the data.

Table 1: Properties of the risk-neutral, physical variances and unconditional VRP

Index	Variable	Mean	Stddev	Minimum	Maximum	AR(1)	N
S&P500	$\mathrm{VIX}^2_{t,t+\tau}$	37.184	35.781	7.926	376.04	0.919	1354
	$RV_{t-\tau,t}$	26.452	47.928	1.850	579.09	0.959	1354
	$\mathrm{P}_{t,t+\tau}$	10.733	34.465	-450.81	212.2	0.852	1354
DJIA	$\mathrm{VXD}^2_{t,t+\tau}$	37.823	35.04	7.418	324.77	0.929	956
	$RV_{t-\tau,t}$	28.264	47.309	0.986	526.15	0.955	956
	$\mathrm{P}_{t,t+\tau}$	9.559	34.722	-413.31	194.77	0.863	956
NASDAQ	$\mathrm{VXN}^2_{t,t+\tau}$	45.536	44.127	11.796	383.53	0.932	664
	$RV_{t-\tau,t}$	37.207	61.084	3.759	582.02	0.960	664
	$\mathrm{P}_{t,t+\tau}$	8.3286	43.851	-466.71	190.16	0.859	664
RUSSELL	$\mathrm{RVX}^2_{t,t+\tau}$	59.703	58.773	17.52	478.62	0.935	641
	$RV_{t-\tau,t}$	51.226	81.697	5.9827	661.34	0.960	641
	$\mathrm{P}_{t,t+\tau}$	8.4777	52.065	-432.66	216.58	0.822	641

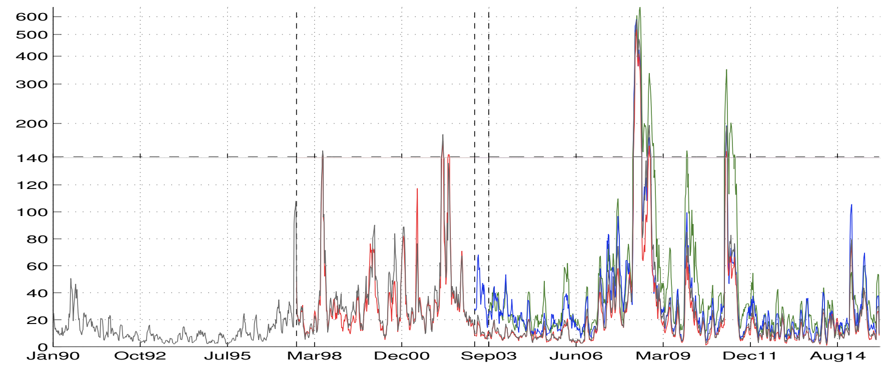
Notes: This table reports descriptive statistics for implied variance, e.g. $\mathrm{VIX}^2_{t,t+\tau}$, realized variance, $RV_{t-\tau,t}$, and variance swap payoff $\mathrm{P}_{t,t+\tau}$ with τ equal to one month. Stddev means standard deviation, AR(1) is the sample autocorrelation of order one and N denotes the number of observations. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSS. The sample ends on July 29, 2016 for all indices.

Figure 1 displays realized variances, risk neutral variance expectations, and variance swap payoffs respectively over the available time span for each market. From February 2004, when data for all indices is available, we see that the time series share the same patterns. The realized variances have upward peaks around the same dates for all indices. The risk neutral variance expectation series also have large positive jumps around the same periods of the realized variance but at different dates as can be seen from the payoff series. Interestingly, the payoff series have peaks up and down with similar amplitude noting that the left tail events of an absolute size larger than the observed maximum occur with frequency lower than one percent. These tail episodes coincide mainly with events related to the peak of the global financial crisis. To stress the weight of these extreme negative points on the overall sample implying a downward bias in the unconditional variance risk premium estimated by sample means, we re-compute the mean and standard deviation of the variance swap-payoffs excluding the months of October and November 2008. We obtain respectively (standard deviation between brackets), S&P500 12.52 (23.01), DJIA 11.08 (25.57), NASDAQ 11.56 (26.18), RUSSELL 13.37 (21.31), i.e. a sensible increase in the average coupled with a striking reduction of the standard deviation.

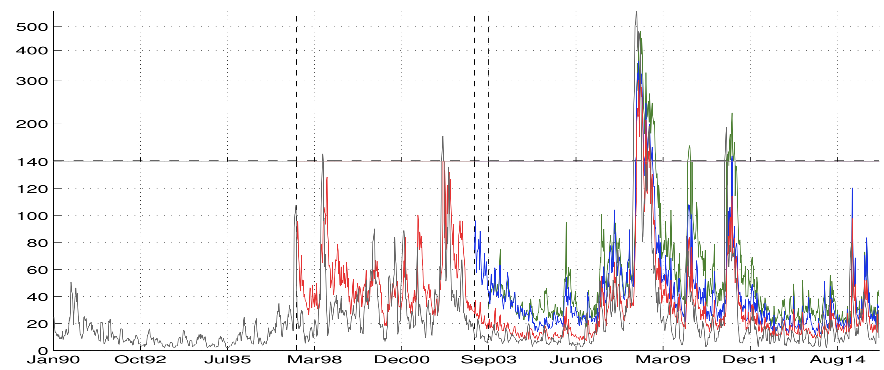
3.2. Estimation results

In this section, we provide results for the model described by (7) - (8) in a three regime specification, i.e. $s_{t+\tau} \in [l \text{ (low)}, h \text{ (high)}, j \text{ (jump)}]$. In fact, the first regime refers to low (l) noise and signal variance, the second to high (h) noise and signal variance while the third regime (j) identifies the occurrence and, up to measurement error, size of fear/surprise episodes. Using standard likelihood ratio tests, we reject specifications with additional regimes.

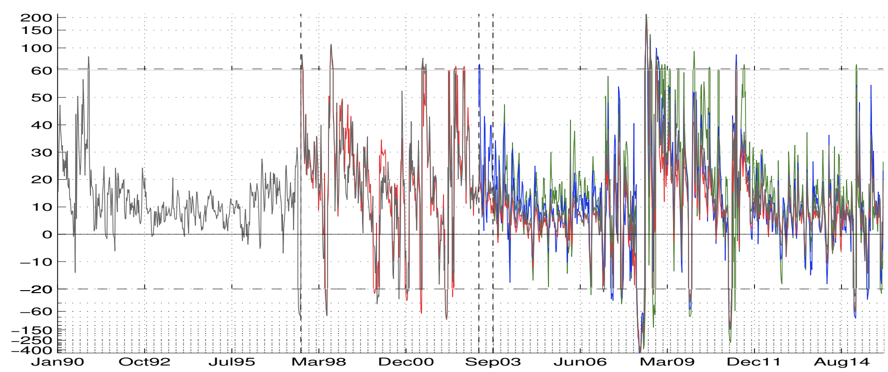
Table 2 provides parameter estimates for the four indices. The heterogeneity in the prediction error deriving from heteroskedasticity is clear for all the indices. In the high volatility regime, the estimated standard errors are homogenous among indices and about four times higher than the base regime for the S&P500, DJIA and NASDAQ, and two times higher for the RUSSELL. The



(a) $RV_{t,t+\tau}$



(b) $VIX^2_{t,t+\tau}$, $VXD^2_{t,t+\tau}$, $VXN^2_{t,t+\tau}$, $RVX^2_{t,t+\tau}$



(c) $\mathrm{P}_{t,t+\tau}$

Figure 1: $RV_{t,t+\tau}$, implied variance, e.g. $VIX^2_{t,t+\tau}$, and $\mathrm{P}_{t,t+\tau}$. S&P500 (grey), DJIA (red), NASDAQ (blue) and RUSSELL (green). The vertical dashed lines mark the beginning of the sample of each market.

Table 2: Quasi-maximum likelihood estimates of (7) - (8)

Parameter	S&P500	DJIA	NASDAQ	RUSSELL
$\sigma_{e,l}$	2.71	3.56	3.28	7.34
$\sigma_{e,h}$	11.27	12.00	12.14	14.06
σ_η	43.27	69.27	52.64	51.41
$\sigma_{\epsilon,l}$	1.66	0.25	1.38	2.31
$\sigma_{\epsilon,h}$	1.87	1.41	1.54	3.03
$\overline{\Pi}$	13.72	8.09	14.03	16.79
ϕ	0.94	0.97	0.97	0.94
p_{ll}	0.82	0.85	0.75	0.84
p_{lh}	0.18	0.15	0.25	[0.01]
p_{lj}	[0.00]	[0.00]	[0.00]	0.15
p_{hl}	0.17	0.16	0.11	0.06
p_{hh}	0.58	0.71	0.76	0.84
p_{hj}	0.25	0.13	0.13	0.10
p_{jl}	0.35	0.01	0.34	[0.00]
p_{jh}	0.29	0.34	0.35	0.73
p_{jj}	0.36	0.65	0.31	0.27
LLF	-2.8261	-2.9326	-3.1656	-3.2929
N	1354	956	664	641
Steady-state probability				
l	0.55	0.45	0.37	0.24
h	0.32	0.40	0.53	0.62
j	0.13	0.15	0.10	0.14
Expected duration				
l	5.68	6.74	3.95	6.43
h	2.39	3.42	4.20	6.06
j	1.56	2.86	1.44	1.36

Notes: LLF denotes the average loglikelihood and N the number of observations. Steady state probabilities are computed according to (4.49) in [29]. Expected duration is computed as $1/(1-p_{ii})$, $i \in [l$ (low), h (high), j (jump)]. The [] brackets indicate insignificant parameters at the five percent significance level. The parameter μ is set to zero as it turns out to be insignificant for all indices. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

large marginal increase in the measurement error variance between the second and third regime, measured by σ_η, together with transition probabilities which imply relatively short expected durations for the third regime, proves that the latter identifies rare and short-lived extreme market events. The parameter μ is not reported in Table 2 as it turns out to be insignificant for all indices. This indicates that fears and surprises tend to compensate on average.

The implied expected durations for extreme payoff events range between 1.36 (RUSSELL) and 2.86 (DJIA) weeks. The transition probability matrix, however, reveals a persistent low volatility regime, with expected durations of 5.68, 6.74, 3.95 and 6.43 weeks, for the four indices respectively. The high volatility regime is more heterogenous, with expected durations ranging between 2.39 weeks for the S&P500 and 6.06 weeks for the RUSSELL, and acts in most cases as the layer of transition between the low and extreme volatility regimes. In fact, except for the RUSSELL, the probability p_{lj} is virtually zero. This evidence is less striking in the opposite direction with p_{jl} smaller than one percent only for DJIA and RUSSELL.

We estimate the occurrence of the extreme variance events as the observations for which the jump state posterior probability is the highest. Using the notation in [29], the latter probability is computed as $\max\Big(P(s_{t+\tau}|\psi_{t+\tau})\Big) = P(j|\psi_{t+\tau})$, where $P(s_{t+\tau}|\psi_{t+\tau})$ is the posterior probability of the state $s_{t+\tau} \in [l, h, j]$ and $\psi_{t+\tau}$ is the information set up to $t+\tau$. Table 3 provides details on the major events that generated extreme variance swap payoffs. The longest lasting event is the global financial crisis with an estimated length of about seven months, with average variance swap payoff favouring the long side of the contract for all indices, particularly so for the S&P500 with average gains of 158 times the notational. The global financial crisis thus constitutes an example of a period where the effect of surprises dominate over fears. The collapse in oil prices in August 2015 is an example of relatively short lasting episode of 1.5 months. Regarding this event, three out of the four indices exhibit gains for the long position up to 33.4 times the notational, with the exception of the RUSSELL for which the average payoff favours the short side of the swap contract with average gains of 8.45 times the notational.

The latent state $\Pi_{t,t+\tau}$ has an unconditional level in line with Section 3.1 and a high degree of persistence for all indices. The estimated autoregressive coefficient ϕ is substantially higher than the one estimated on the raw payoff data, stressing the downward bias due to the presence of extreme payoff realisations, see Table 2.

Figure 2 displays variance swap payoffs (grey), the smooth part of the VRP (red), and the detected extreme variance events (vertical grey lines) for the four indices. As indicated by the parameter estimates in Table 2, $\Pi_{t,t+\tau}$ is slowly moving and above its mean in volatile periods and below its mean in periods of calm financial markets. Over the period for which we have data for all indices, we see from Figure 3 that the respective $\Pi_{t,t+\tau}$ estimates are moving closely together. The pairwise correlations between the variance premia is the highest between S&P500 and DJIA (90 percent) and the lowest between the NASDAQ and the RUSSELL (76 percent). Figure 3 also shows that episodes of extreme variance events, indicated by the shaded grey areas, largely overlap both in terms of occurrences and duration.

3.3. Fear and surprise

Once endowed with the filtered VRP and the posterior probabilities associated to the states s_t, we infer the occurrence of abnormal variance swap payoffs and determine whether and to what extent they are caused by fears or surprises. Figure 4 displays the difference between the payoff and the smooth part of the VRP, i.e. $\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}$, for all indices. This quantity represents the measurement error for regimes l and h, while it represents the sum of measurement error and the extent of the fear/surprise for regime j. The difference in magnitude between normal and extreme regimes (indicated by the shaded areas) is striking as the magnitude of $\mathrm{FS}_{t,t+\tau}$ dominates the measurement error. We see that positive deviations, associated to fears (red), mirror negative deviations, associated to surprises (green) both in absolute size and magnitude, with the exception of the unique events coinciding with the peak of the financial crisis in

Table 3: Extreme variance clusters

Event	Market	Start	End	N	$\bar{P}_{t,t+\tau}$	$\bar{\Pi}_{t,t+\tau}$
Asian Crisis	S&P500	1997-10-29	1997-12-24	8	0.58	16.29
Russian Crisis	S&P500	1998-08-12	1998-12-02	14	23.35	14.99
	DJIA	1998-09-02	1998-12-09	13	27.86	13.23
09/11	S&P500	2001-09-26	2001-12-05	10	45.50	17.45
	DJIA	2001-09-19	2001-12-12	11	23.08	15.00
Dot-com bubble burst	S&P500	2002-07-24	2002-11-27	13	-8.83	9.57
	DJIA	2002-07-24	2002-12-11	16	5.22	8.49
Global financial crisis	S&P500	2008-09-17	2009-05-27	28	-158.54	10.35
	DJIA	2008-09-17	2009-05-27	30	-40.34	3.59
	NASDAQ	2008-09-24	2009-05-27	27	-49.05	15.08
	RUSSELL	2008-09-17	2009-05-27	30	-80.04	7.26
Flash crash	S&P500	2010-05-12	2010-07-28	8	6.72	15.31
	DJIA	2010-05-12	2010-06-02	4	-28.54	14.83
	NASDAQ	2010-05-12	2010-07-07	6	-16.30	18.92
	RUSSELL	2010-05-05	2010-07-07	8	-12.15	17.73
US debt downgrade	S&P500	2011-08-10	2011-11-02	8	-46.38	9.76
	DJIA	2011-08-10	2011-11-02	7	-43.79	9.10
	NASDAQ	2011-08-10	2011-11-02	11	-19.38	6.68
	RUSSELL	2011-08-10	2011-11-02	8	-118.68	9.70
Collapse in oil prices	S&P500	2015-08-26	2015-09-30	6	-20.46	7.89
	DJIA	2015-08-26	2015-09-30	6	-18.63	7.39
	NASDAQ	2015-08-26	2015-09-30	6	-33.44	6.53
	RUSSELL	2015-08-26	2015-09-23	5	8.45	6.65

Notes: This table reports extreme variance swap payoff clusters over the period January, 1990, to September, 2015. N denotes the number of extreme variance weeks in the period between Start and End. The variables $\bar{P}_{t,t+\tau}$ and $\bar{\Pi}_{t,t+\tau}$ are respectively the average payoff and smooth VRP over the extreme variance weeks. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

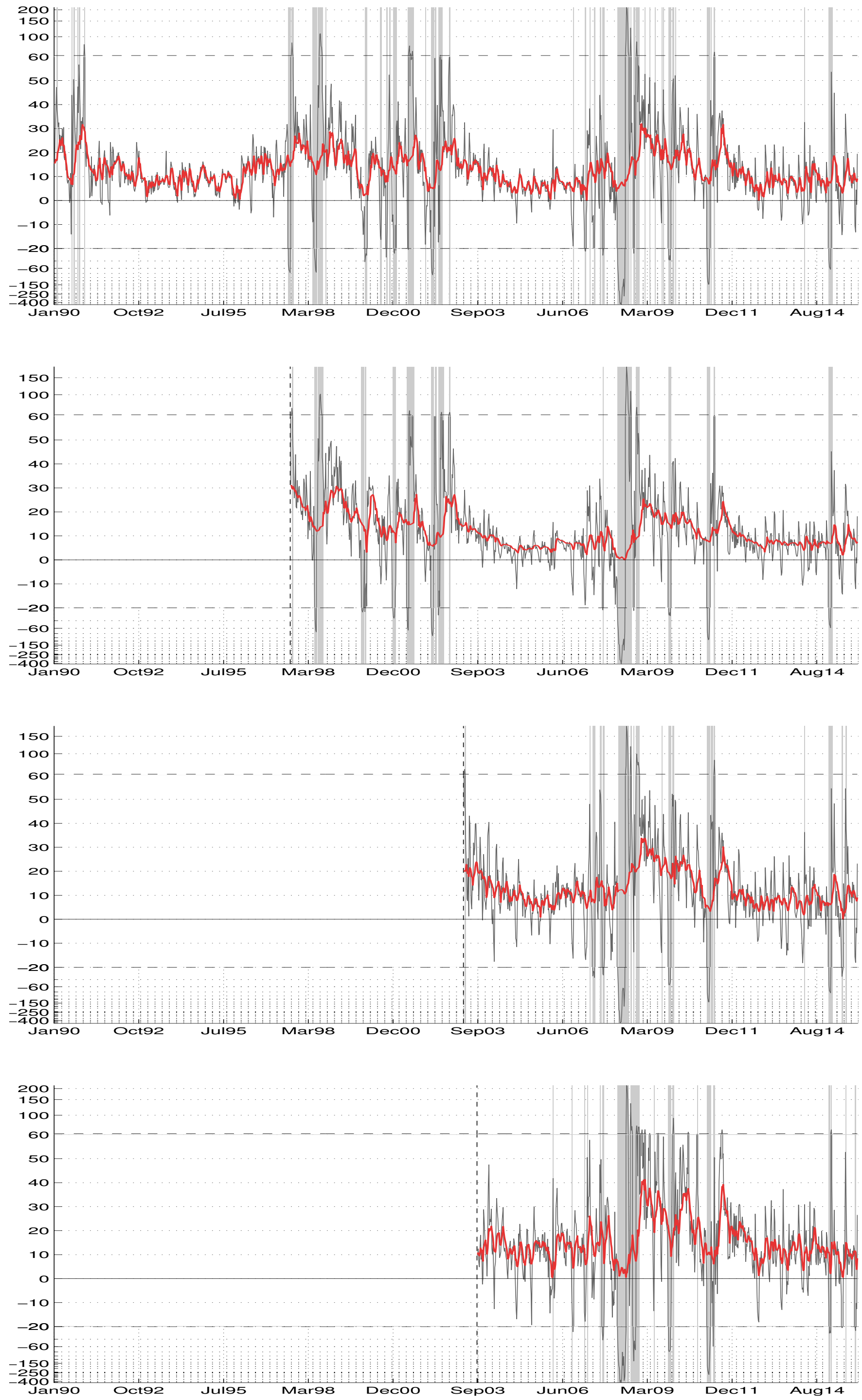


Figure 2: Variance swap payoffs (grey), $P_{t,t+\tau}$, the smooth part of the VRP (red), $\Pi_{t,t+\tau}$, and detected extreme variance events (vertical grey areas for the four indices. From the top we plot S&P500, DJIA, NASDAQ and RUSSELL, respectively.

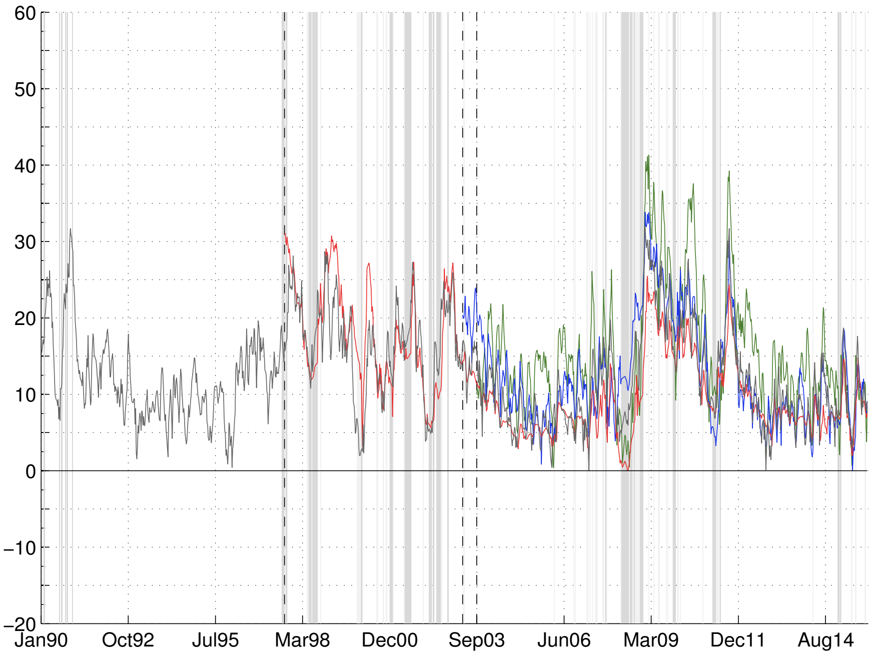


Figure 3: The smooth part of the VRP, $\Pi_{t,t+\tau}$, for all for indices together. S&P500 (grey), DJIA (red), NASDAQ (blue) and RUSSELL (green). The vertical dashed lines mark the beginning of the sample of each market. The dark grey areas indicate extreme variance events detected in the four markets. The light grey areas indicate extreme variance events detected in 3 markets or less.

September and October of 2008.

To isolate the extent of fears and surprises, we offset the normal regime by intersecting $\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}$ with the indicator $\mathrm{I}_{s_t=j}$. The resulting variable represents the extent of the fear or surprise generated by a realized extreme shock on the market occurred in the period between $t-\tau$ and t. More precisely, we define fear as $\mathrm{F}_{t,t+\tau} = (\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau})\mathrm{I}_{\{s_t=j\,\cap\,(\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})>0\}}$ and surprise as $\mathrm{S}_{t,t+\tau} = |\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}|\mathrm{I}_{\{s_t=j\,\cap\,(\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})<0\}}$. We switch the sign of the surprise effect in the following so that its coefficient in the predictive regression analysis below represents the direction of the pricing of the risk factor.

Table 4 gives descriptive statistics for the $\mathrm{F}_{t,t+\tau}$ and $\mathrm{S}_{t,t+\tau}$ variables. Fears and surprises show a similar number of occurrences, as seen in Figure 4, with the

Figure 4: Positive (negative) part of $\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}$ in red (green) and detected extreme variance events (vertical grey lines) for the four indices. From the top we plot S&P500, DJIA, NASDAQ and RUSSELL, respectively.

Table 4: Fear and surprise variables associated with extreme market conditions

		Mean	Std. dev.	Minimum	Maximum	N
Fear ($F_{t,t+\tau}$)	S&P500	45.716	33.351	18.813	203.88	73
	DJIA	50.040	33.587	19.655	192.695	51
	NASDAQ	54.137	34.529	24.562	177.491	33
	RUSSELL	73.889	53.625	33.313	214.128	21
Surprise ($S_{t,t+\tau}$)	S&P500	87.161	98.015	21.613	458.156	69
	DJIA	87.923	86.096	23.574	414.373	53
	NASDAQ	119.10	121.79	32.085	478.893	36
	RUSSELL	138.05	121.42	22.830	434.604	41

Notes: Fears and surprises are computed as $\mathrm{F}_{t,t+\tau} = (\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau})\mathrm{I}_{\{s_t=j\,\cap\,(\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})>0\}}$ and $\mathrm{S}_{t,t+\tau} = |\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau}|\mathrm{I}_{\{s_t=j\,\cap\,(\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})<0\}}$ respectively. N is the number of nonzero values over which the descriptive statistics are computed.

exception of RUSSELL where surprises are twice as frequent as fears. The fear variable is on average between four and five times larger than the estimated unconditional level of the VRP reported in Table 2, and at the minimum at least twice as large. The surprise variable exhibits in general a more extreme behaviour. As discussed in Section 3.1, these statistics are largely affected by a handful of extremely negative variance swap payoffs observed during the peak of the global financial crisis.

4. Predictive return regressions

4.1. Smooth component of the VRP

The importance of the VRP as a predictor for future aggregate market returns has been pointed out by many authors, see [11], [12], [33], and [13] among others. We use the variance swap payoffs $\mathrm{P}_{t,t+\tau}$ as a proxy for the VRP as in [9] and [10] to assess its contribution to future market returns.

Return predictability is quantified by estimating the following predictive regression model

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} = a_0(h) + a_1(h)X_{t,t+1} + u_t(h), \qquad (9)$$

where h denotes the horizon, r_t denotes the excess return for week t and X a predictor. To correct for the highly overlapping dependent variable, Hansen-Hodrick corrected standard errors are reported. Using Datastream, we construct the weekly aggregated market returns in excess of the three-month T-bill rate over horizons from one week (h=1) up to one year (h=52). Predictability is measured by the adjusted R^2 of the regression.

Table 5 provides parameter estimates and adjusted R^2s for the regression model in (9) using $\mathrm{P}_{t,t+\tau}$ as a predictor. For the S&P500 and NASDAQ, the slope coefficients are significant from short to medium-long horizons (one to six months). The significant coefficients are associated with R^2s reaching 3.19 and 4.40 for the S&P500 and DJIA respectively. Similarly, the DJIA shows significant slope coefficients at medium-long horizons (three to nine months), while for the RUSSELL coefficients are significant at the medium horizon only (three to five months). In the four cases, we observe the usual inverse U-shape found in the literature.

Comparing the return predictability of $\Pi_{t,t+\tau}$ with that of the variance swap payoffs $\mathrm{P}_{t,t+\tau}$, a different pattern appears. In general, we find that signs of return predictability emerge at rather short horizons and peak at the two-three months mark. For the S&P500, we find positive significant slope coefficients $a_1(h)$ up to two months with the highest adjusted R^2s showing the aforementioned inverse U-shape peaking at 2.12. For the DJIA, R^2s reach 3.50 percent at the three month horizon, although it is significant at five percent only up to the two month horizon. The NASDAQ is the only index that does not show any significant slope coefficients. The RUSSELL has only significant R^2s at the three to five month horizons, with the largest R^2 of 3.06 percent at the four month horizon.

Table 5: Smooth VRP predictive regressions

Horizon	1	4	8	12	16	20	24	36	52
					S&P500				
$\Pi_{t,t+\tau}$	**1.951**	**2.148**	**1.785**	1.384	0.933	0.665	0.536	0.277	0.224
	(0.973)	(0.943)	(0.878)	(0.840)	(0.724)	(0.645)	(0.634)	(0.532)	(0.486)
R^2	0.21	1.51	2.12	1.92	1.14	0.67	0.48	0.14	0.11
$P_{t,t+\tau}$	0.306	**0.361**	**0.202**	**0.265**	**0.278**	**0.260**	**0.176**	0.069	0.016
	(0.359)	(0.182)	(0.102)	(0.041)	(0.050)	(0.040)	(0.029)	(0.039)	(0.027)
R^2	0.14	1.28	0.78	2.16	3.19	3.41	1.77	0.33	-0.05
					DJIA				
$\Pi_{t,t+\tau}$	**1.982**	**2.086**	**2.024**	1.719	1.361	1.133	0.919	0.363	0.268
	(0.984)	(1.019)	(1.003)	(1.030)	(0.890)	(0.773)	(0.746)	(0.655)	(0.616)
R^2	0.23	1.50	3.03	3.50	3.07	2.60	1.94	0.36	0.26
$P_{t,t+\tau}$	0.119	0.172	0.092	**0.216**	**0.288**	**0.284**	**0.215**	**0.100**	0.026
	(0.390)	(0.235)	(0.156)	(0.069)	(0.070)	(0.055)	(0.024)	(0.042)	(0.025)
R^2	-0.07	0.18	0.07	1.41	3.65	4.40	2.88	0.84	-0.03
					NASDAQ				
$\Pi_{t,t+\tau}$	0.863	0.982	1.535	1.707	1.549	1.567	1.428	0.887	0.686
	(1.067)	(1.188)	(1.204)	(1.243)	(1.181)	(1.114)	(1.106)	(1.009)	(0.919)
R^2	-0.10	0.12	1.10	2.13	2.37	3.06	2.99	1.76	1.55
$P_{t,t+\tau}$	0.512	**0.480**	**0.198**	**0.247**	**0.239**	**0.225**	**0.137**	0.022	-0.028
	(0.347)	(0.138)	(0.067)	(0.071)	(0.073)	(0.049)	(0.041)	(0.040)	(0.016)
R^2	0.62	2.59	0.74	1.88	2.41	2.67	1.09	-0.12	-0.05
					RUSSELL				
$\Pi_{t,t+\tau}$	1.875	2.203	2.307	**2.104**	1.235	0.878	0.713	0.317	0.236
	(1.288)	(1.244)	(1.152)	(1.210)	(1.049)	(0.747)	(0.503)	(0.201)	(0.216)
R^2	0.09	1.35	3.15	4.05	1.87	1.09	0.78	0.14	0.08
$P_{t,t+\tau}$	0.143	0.396	0.152	**0.268**	**0.261**	**0.196**	0.088	0.022	-0.031
	(0.516)	(0.226)	(0.126)	(0.056)	(0.090)	(0.077)	(0.065)	(0.029)	(0.026)
R^2	-0.10	1.87	0.44	2.72	3.67	2.47	0.45	-0.11	0.01

Notes: Estimation results for predictive return regressions $\frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} = a_0(h) + a_1(h)X_{t,t+\tau} + u_{t+\tau}(h)$, with $X_{t+\tau}$ equal to $\Pi_{t,t+\tau}$ or $P_{t,t+\tau}$ respectively and where h denotes the horizon. Hansen-Hodrick standard errors are in brackets. Coefficients significant at five percent are in boldface. Adjusted R^2s in percentages. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

4.2. Fears and surprises

Besides the smooth VRP ($\Pi_{t,t+1}$), our model infers the agents' reaction to realized extreme variance events, namely fears ($\mathrm{F}_{t,t+1}$) and surprises ($\mathrm{S}_{t,t+1}$). The contribution of these variables is expected to generate significantly larger return predictability in periods when such events occur. To test this hypothesis, we augment the predictive regression model in (9) as follows

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} = a_0(h) + a_1(h)\Pi_{t,t+\tau} + a_2(h)\mathrm{F}_{t,t+\tau} + a_3(h)\mathrm{S}_{t,t+1} + u_{t+\tau}(h). \quad (10)$$

Table 6 reports estimates for the parameters $a_1(h)$, $a_2(h)$ and $a_3(h)$ of the model in (10) as well as their associated adjusted R^2s. The smooth VRP slope estimates, $a_1(h)$, hardly change with respect to Table 5 for all indices, confirming the orthogonal nature of the regressors. Fear is positive and significant at medium and long horizons for all indices. RUSSELL also shows significant negative coefficients at very short horizons. This suggests that fear for large volatility shifts although possibly driving down returns in the very short term, constitutes a risk factor, to which agents do not react immediately, but that they distinctly price for long periods. Surprise is typically significant and negative at the medium horizons but they switch sign at the long horizons.

In sum, our results show evidence of systematic longer lasting response of the expected average returns triggered by unexpected (at least in their magnitude) realized large shocks and fear of future extreme shocks. Comparing the decomposition proposed in our model with the predictability generated by $\mathrm{P}_{t,t+\tau}$, we find substantial increases in the R^2s at all horizons for all indices. While the S&P500 preserves the inverse U-shape, the remaining indices show longer term predictability, which does not die out before the one year horizon.

We have shown that fears and surprises, i.e., direction and size of agents' reaction to extreme shocks to the market, have a relevant effect on future market performances. We argue that such an effect is likely to be asymmetric and systematically related to the current market conditions at the moment the shock occurs. To test this hypothesis, we further extend the predictive re-

Table 6: Premium, fear & surprise predictive regression

Horizon	1	4	8	12	16	20	24	36	52
S&P500									
$\Pi_{t,t+\tau}$	**1.828**	**2.010**	**1.746**	1.278	0.796	0.527	0.446	0.248	0.223
	(0.893)	(0.876)	(0.838)	(0.801)	(0.683)	(0.621)	(0.633)	(0.530)	(0.496)
$\mathrm{F}_{t,t+\tau}$	-0.207	-0.232	-0.388	0.129	**0.283**	**0.304**	**0.306**	0.226	0.202
	(0.608)	(0.402)	(0.466)	(0.235)	(0.098)	(0.082)	(0.060)	(0.144)	(0.113)
$\mathrm{S}_{t,t+\tau}$	-0.317	**-0.356**	**-0.181**	**-0.193**	**-0.219**	**-0.217**	**-0.114**	0.006	0.051
	(0.360)	(0.099)	(0.058)	(0.062)	(0.074)	(0.068)	(0.051)	(0.035)	(0.027)
R^2	0.24	2.36	2.87	2.70	2.96	2.97	1.69	0.60	0.81
DJIA									
$\Pi_{t,t+\tau}$	**1.866**	**1.883**	**1.917**	1.643	1.251	1.035	0.905	0.447	0.398
	(0.907)	(0.909)	(0.952)	(0.962)	(0.804)	(0.711)	(0.753)	(0.729)	(0.720)
$\mathrm{F}_{t,t+\tau}$	-0.336	-0.508	-0.539	-0.007	**0.229**	**0.304**	**0.385**	**0.424**	**0.380**
	(0.853)	(0.531)	(0.501)	(0.236)	(0.074)	(0.080)	(0.104)	(0.106)	(0.110)
$\mathrm{S}_{t,t+\tau}$	-0.095	-0.181	-0.040	-0.105	**-0.199**	**-0.198**	-0.099	0.023	**0.010**
	(0.377)	(0.127)	(0.101)	(0.098)	(0.103)	(0.095)	(0.063)	(0.053)	(0.046)
R^2	0.07	1.86	3.74	3.53	4.46	4.72	3.69	2.79	3.81
NASDAQ									
$\Pi_{t,t+\tau}$	0.752	0.829	1.487	1.625	1.459	1.476	1.354	0.855	0.659
	(1.014)	(1.131)	(1.188)	(1.204)	(1.132)	(1.071)	(1.074)	(0.999)	(0.919)
$\mathrm{F}_{t,t+\tau}$	-0.462	0.347	-0.049	**0.477**	**0.668**	**0.673**	**0.714**	**0.529**	**0.414**
	(0.553)	(0.285)	(0.335)	(0.208)	(0.051)	(0.051)	(0.051)	(0.145)	(0.118)
$\mathrm{S}_{t,t+\tau}$	-0.662	**-0.533**	**-0.223**	**-0.180**	**-0.144**	**-0.122**	0.004	**0.090**	**0.120**
	(0.353)	(0.100)	(0.081)	(0.051)	(0.036)	(0.029)	(0.042)	(0.029)	(0.019)
R^2	0.65	2.67	1.68	3.51	4.94	6.07	6.08	5.04	5.64
RUSSELL									
$\Pi_{t,t+\tau}$	1.112	1.664	**1.962**	1.932	1.123	0.816	0.798	0.520	0.478
	(1.040)	(1.034)	(1.027)	(1.113)	(0.959)	(0.722)	(0.586)	(0.404)	(0.328)
$\mathrm{F}_{t,t+\tau}$	**-1.607**	-0.227	**-0.694**	0.114	**0.398**	**0.365**	**0.394**	**0.464**	**0.403**
	(0.716)	(0.263)	(0.361)	(0.212)	(0.113)	(0.103)	(0.060)	(0.046)	(0.067)
$\mathrm{S}_{t,t+\tau}$	-0.356	**-0.444**	**-0.164**	**-0.180**	**-0.187**	**-0.009**	0.072	**0.117**	**0.001**
	(0.449)	(0.150)	(0.082)	(0.072)	(0.077)	(0.068)	(0.052)	(0.034)	(0.035)
R^2	0.73	2.94	4.48	4.76	3.97	2.68	1.64	2.85	4.54

Notes: Estimation results for predictive return regressions $\frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} = a_0(h) + a_1(h)\Pi_{t,t+\tau} + a_2(h)\mathrm{F}_{t,t+\tau} + a_3(h)\mathrm{S}_{t,t+\tau} + u_{t+\tau}(h)$, where h denotes the horizon. Hansen-Hodrick standard errors are in brackets. Coefficients significant at five percent are in boldface. Adjusted R^2s in percentages. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

gression model by discriminating fears and surprises according to the signed jump variation defined as the difference between positive and negative realized semivariances, i.e., the differential between partial sums of squared negative and positive jumps, denoted by $\Delta_J = \sum_{i=1}^{I} r_{t+i}^2 (\mathrm{I}_{\{r_{t+i}>0\}} - \mathrm{I}_{\{r_{t+i}<0\}})$, see [34]. This hypothesis, which relates to the concept of good versus bad volatility developed in [35] and [36], allows us to capture the asymmetry with respect to the type of the fear/surprise triggering shock. Defining $\mathrm{F}_{t,t+\tau}^{+} = (\mathrm{P}_{t,t+\tau} - \Pi_{t,t+\tau})\mathrm{I}_{\{s_t=j\,\cap\,(\mathrm{P}_{t,t+\tau}-\Pi_{t,t+\tau})>0\,\cap\,\Delta_J>0\}}$ and other variables accordingly, the extended predictive regression model becomes

$$\begin{aligned} \frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} \quad &= \quad a_0(h) + a_1(h)\Pi_{t,t+\tau} + a_2^{+}(h)\mathrm{F}_{t,t+\tau}^{+} + a_2^{-}(h)\mathrm{F}_{t,t+\tau}^{-} \\ &\quad + a_3^{+}(h)\mathrm{S}_{t,t+\tau}^{+} + a_3^{-}(h)\mathrm{S}_{t,t+\tau}^{-} + u_{t+\tau}(h). \end{aligned} \tag{11}$$

Table 7 reports the results for the model defined in (11). It turns out that when conditioning on the sign of the jump variation more fear and surprise variables coefficients become significant. This is particularly the case for NASDAQ and RUSSELL, where most of the return predictability, if not all of it, stems from fear and surprise. In case of the S&P500, the fear variable associated with negative jump variation, i.e. $\mathrm{F}_{t,t+\tau}^{-}$, is significant and positive from the four month horizon onwards while in contrast $\mathrm{F}_{t,t+\tau}^{+}$ is significant only at the five and six month horizons. This result is in line with the notion of leverage effect and its implication in terms of intertemporal risk-return tradeoff. The surprise variable related to the positive jump variation, $\mathrm{S}_{t,t+\tau}^{+}$, is significant and negative between the medium and long horizon, while $\mathrm{S}_{t,t+\tau}^{-}$ is significant and negative between the one and five month horizon, becoming positive at the one year mark. Except for the S&P500, the inverse U-shape pattern for the R^2s becomes milder as the long horizon R^2s stay as high as, or sometimes even higher than, the medium horizon R^2s.

Table 7: Premium, signed fear & surprise (signed jump variation) predictive regression

Horizon	1	4	8	12	16	20	24	36	52
					S&P500				
$\Pi_{t,t+\tau}$	**1.870**	**2.013**	**1.745**	1.281	0.798	0.527	0.448	0.247	0.222
	(0.901)	(0.880)	(0.834)	(0.799)	(0.683)	(0.621)	(0.633)	(0.528)	(0.496)
$F^+_{t,t+\tau}$	-0.601	-0.233	-0.531	-0.008	0.168	**0.226**	**0.201**	0.146	0.139
	(0.582)	(0.360)	(0.496)	(0.236)	(0.124)	(0.103)	(0.094)	(0.162)	(0.122)
$F^-_{t,t+\tau}$	1.731	-0.219	0.288	0.779	**0.830**	**0.669**	**0.807**	**0.600**	**0.498**
	(1.255)	(0.784)	(0.494)	(0.434)	(0.188)	(0.143)	(0.078)	(0.169)	(0.149)
$S^+_{t,t+\tau}$	0.040	-0.102	-0.977	**-0.759**	**-0.703**	-0.596	-0.566	-0.519	**-0.309**
	(0.956)	(0.343)	(0.525)	(0.195)	(0.193)	(0.376)	(0.359)	(0.304)	(0.149)
$S^-_{t,t+\tau}$	-0.327	**-0.365**	**-0.154**	**-0.173**	**-0.203**	**-0.204**	-0.098	0.012	**0.064**
	(0.359)	(0.094)	(0.070)	(0.072)	(0.087)	(0.081)	(0.057)	(0.039)	(0.025)
R^2	0.35	2.23	3.36	3.22	3.45	3.22	2.30	1.40	1.35
					DJIA				
$\Pi_{t,t+\tau}$	**1.939**	**1.858**	**1.962**	1.664	1.278	1.045	0.923	0.477	0.424
	(0.914)	(0.910)	(0.967)	(0.967)	(0.816)	(0.716)	(0.765)	(0.740)	(0.734)
$F^+_{t,t+\tau}$	-0.520	-0.348	-0.688	-0.080	0.135	**0.258**	0.310	**0.302**	**0.266**
	(0.607)	(0.408)	(0.544)	(0.235)	(0.131)	(0.126)	(0.164)	(0.124)	(0.097)
$F^-_{t,t+\tau}$	0.272	-1.005	-0.060	0.230	**0.530**	**0.450**	**0.621**	**0.812**	**0.741**
	(1.992)	(0.969)	(0.450)	(0.295)	(0.080)	(0.061)	(0.063)	(0.108)	(0.128)
$S^+_{t,t+\tau}$	**-0.609**	**-0.261**	-0.258	**-0.204**	**-0.315**	**-0.216**	**-0.154**	-0.057	0.048
	(0.312)	(0.059)	(0.157)	(0.060)	(0.052)	(0.043)	(0.060)	(0.046)	(0.047)
$S^-_{t,t+\tau}$	0.262	-0.129	0.114	-0.035	-0.117	-0.185	-0.059	0.081	**0.139**
	(0.452)	(0.201)	(0.186)	(0.155)	(0.152)	(0.154)	(0.101)	(0.073)	(0.053)
R^2	0.18	1.80	4.24	3.57	4.76	4.57	3.75	3.60	4.67
					NASDAQ				
$\Pi_{t,t+\tau}$	0.773	0.820	1.474	1.614	1.454	1.470	1.350	0.862	0.662
	(1.038)	(1.129)	(1.188)	(1.201)	(1.129)	(1.067)	(1.065)	(0.998)	(0.921)
$F^+_{t,t+\tau}$	**-0.730**	0.139	-0.283	0.313	**0.543**	**0.612**	**0.593**	**0.558**	**0.429**
	(0.379)	(0.208)	(0.313)	(0.174)	(0.017)	(0.058)	(0.038)	(0.081)	(0.102)
$F^-_{t,t+\tau}$	0.697	1.238	**0.958**	**1.180**	**1.206**	**0.931**	**1.215**	0.410	**0.352**
	(1.860)	(0.840)	(0.273)	(0.321)	(0.098)	(0.130)	(0.129)	(0.624)	(0.100)
$S^+_{t,t+\tau}$	**-1.168**	**-0.600**	**-0.260**	**-0.181**	**-0.187**	**-0.088**	-0.008	0.016	**0.080**
	(0.105)	(0.085)	(0.067)	(0.032)	(0.018)	(0.016)	(0.024)	(0.016)	(0.013)
$S^-_{t,t+\tau}$	-0.457	**-0.504**	**-0.207**	**-0.178**	**-0.126**	**-0.135**	0.010	**0.120**	**0.136**
	(0.494)	(0.119)	(0.096)	(0.065)	(0.043)	(0.040)	(0.049)	(0.037)	(0.020)
R^2	0.69	2.62	1.96	3.61	4.99	5.87	6.19	4.92	5.35
					RUSSELL				
$\Pi_{t,t+\tau}$	1.124	1.671	**1.964**	**1.936**	1.127	0.824	0.806	0.524	0.484
	(1.047)	(1.054)	(1.057)	(1.128)	(0.969)	(0.732)	(0.597)	(0.423)	(0.345)
$F^+_{t,t+\tau}$	**-1.194**	-0.43	**-1.086**	-0.108	**0.210**	**0.237**	**0.222**	**0.350**	**0.314**
	(0.620)	(0.281)	(0.387)	(0.245)	(0.070)	(0.094)	(0.056)	(0.060)	(0.079)
$F^-_{t,t+\tau}$	**-3.531**	**0.820**	**1.196**	**1.193**	**1.310**	**1.014**	**1.232**	**1.006**	**0.822**
	(1.102)	(0.294)	(0.440)	(0.235)	(0.167)	(0.171)	(0.221)	(0.130)	(0.132)
$S^+_{t,t+\tau}$	-0.052	0.580	0.746	0.503	0.383	**0.715**	**0.718**	0.331	**0.364**
	(1.375)	(1.134)	(0.632)	(0.280)	(0.282)	(0.218)	(0.220)	(0.366)	(0.140)
$S^-_{t,t+\tau}$	-0.375	**-0.494**	**-0.212**	**-0.216**	**-0.217**	**-0.182**	**-0.047**	0.058	**0.105**
	(0.473)	(0.133)	(0.063)	(0.080)	(0.075)	(0.080)	(0.072)	(0.033)	(0.040)
R^2	0.66	3.56	7.14	6.29	5.48	5.25	4.41	3.71	5.45

Notes: Estimation results for predictive return regressions $\frac{1}{h}\sum_{j=1}^{h} r_{t+\tau+j} = a_0(h) + a_1(h)\Pi_{t,t+\tau} + a_2^+(h)\mathrm{F}^+_{t,t+\tau} + a_2^-(h)\mathrm{F}^-_{t,t+\tau} + a_3^+(h)\mathrm{S}^+_{t,t+\tau} + a_3^-(h)\mathrm{S}^-_{t,t+\tau} + u_{t+\tau}(h)$ where h denotes the horizon. Hansen-Hodrick standard errors are in brackets. Coefficients significant at five percent are in boldface. Adjusted R^2s in percentages. The sample frequency is weekly and starts on February 1, 1990 for S&P500, November 26, 1997 for DJIA, August 26, 2003 for NASDAQ, and February 3, 2004 for RUSSELL. The sample ends on July 29, 2016 for all indices.

5. Market VRP and CAPM regressions

Our choice of the four US stock market indices is not coincidental. Although heterogenous in terms of size, composition and degree of diversification, they all ultimately convey information about the aggregate US stock market. From the previous analysis, we find a high degree of similarity among the variance swap payoffs of the four indices. In fact, besides the high degree of correlation, a principal component analysis reveals that the first component explains 95 percent of the total payoff variation. To avoid the impact of the extreme payoff realisations, we perform the principal component analysis on the filtered VRP's and we find that the relative weight of the first component amounts to 88 percent.

The previous evidence suggests existence of a common and dominant source of variance risk driving the four indices, i.e. a market variance risk premium (MVRP). We estimate this common variance risk factor by building a joint model that exploits the intra-market cross-sectional dimension. The state space form and the Kalman filter provide advantage because they allow to exploit information coming from multiple measurements to improve the estimation accuracy of a common latent factor.

Defining $\mathrm{P}^i_{t,t+\tau}$ the variance swap payoff for the market index $i =$S&P500, DJIA, NASDAQ and RUSSELL, the model in (7) of Section 2 with $N = 3$, can be written as

$$\begin{aligned} \mathrm{P}^i_{t,t+\tau} &= (\mathrm{VIX}^i_{t,t+\tau})^2 - RV^i_{t,t+\tau} \\ &= \Pi^i_{t,t+\tau} + \mathrm{I}_{s_t=j}\mu_i + e^i_{t+\tau} + \mathrm{I}_{s_t=j}\eta^i_{t+\tau}. \end{aligned} \tag{12}$$

The variance premium $\Pi^i_{t,t+\tau}$ can be written as the sum of an affine transformation (centered and rescaled) of a common factor $G_{t,t+\tau}$ and an idiosyncratic i.i.d. factor $\varepsilon^i_{t+\tau}$, i.e. $\Pi^i_{t,t+\tau} = \overline{\Pi}^i + \beta_i G_{t,t+\tau} + \varepsilon^i_{t+\tau}$. Then the measurement equation above becomes

$$\mathrm{P}^i_{t,t+\tau} = \overline{\Pi}^i + \beta_i G_{t,t+\tau} + \mathrm{I}_{s_t=j}\overline{\mathrm{FS}}_i + \tilde{e}^i_{t+\tau}, \tag{13}$$

where $\tilde{e}^i_{t+\tau} = \varepsilon^i_{t+\tau} + e^i_{t+\tau} + \mathrm{I}_{s_t=j}\eta^i_{t+\tau}$ is a Gaussian noise term with state dependent variance $\sigma^2_{\tilde{e}^i} = (\sigma^2_{\varepsilon^i} + \sigma^2_{e^i} + \mathrm{I}_{s_t=j}\sigma^2_{\eta^i})$. Note that the variances of $\varepsilon^i_{t+\tau}$ (idiosyncratic component) and $e^i_{t+\tau}$ (prediction error) are identified in the sum, which is sufficient for the purpose of our model, but not individually. Also, identification restrictions on the variance of the common factor require $\beta_{S\&P500} = 1$. The common factor $G_{t,t+\tau}$, denoting the MVRP, is assumed to evolve as a first order autoregressive process, i.e. $G_{t,t+\tau} = \phi G_{t-1,t+\tau-1} + \xi_{t+\tau}$ with variance of $\xi_{t+\tau}$ equal to σ^2_ξ. The MVRP is not a proper VRP since it is centered and normalized. However, up to the affine transformation $\overline{\Pi}^i + \beta_i G_{t,t+\tau}$ it becomes the linear predictor for the VRP of index i. Heteroskedasticity is accounted for in the same fashion as in Section 2.

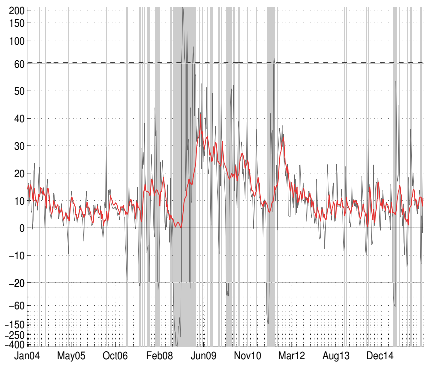
Table 8 reports quasi-maximum likelihood parameter estimates for the common factor model. Compared to the S&P500, the estimated common factor β_i loads less than proportionally on the other indices. The estimated persistence, measured by ϕ, amounts to 0.98 confirming slowly evolving VRP's for all indices, see also Figure 5 which shows the VRP's implied by the common factor model. The expected durations for the normal, high and extreme regimes are respectively 17.11 , 2.68 and 2.07 weeks. Figure 5 shows that the third regime representing episodes of fear and surprises, with a steady state probability equal to 0.14, exhibits three major clusters associated with, respectively, the global financial crisis, the flash crash and the US debt downgrade.

In an intertemporal CAPM framework of [37], we test whether the estimated MVRP contains relevant information about the perceived level of variance risk that is actually priced in financial assets, see e.g. [38]. The three factor model of [39] and the five factor model of [40] extend the CAPM of [41] and [42] to describe patterns in the return variation left unexplained by the market risk factor. More elaborated models and factor selection tools exist, see e.g. [43], but are beyond the scope of this paper. The model is designed to capture the relation between the average return and factors like size (market capitalisation), price ratios like book-to-market, profitability (the difference between the returns on diversified portfolios of stocks with robust and weak profitability) and investment (the

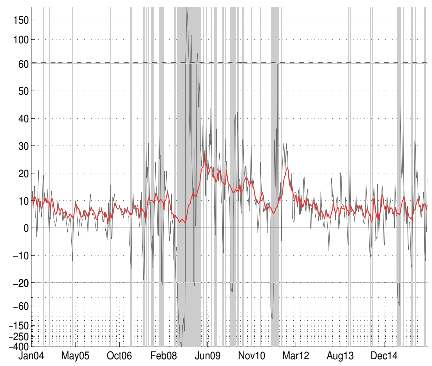
Table 8: Quasi-maximum likelihood estimates of the common factor model

Parameter	Common	S&P500	DJIA	NASDAQ	RUSSELL
$\sigma_{e^i,l}$		1.84	2.06	5.27	5.13
$\sigma_{e^i,h}$		11.46	6.38	9.04	12.51
σ_{η^i}		39.00	35.15	31.03	44.75
$\overline{\Pi}^i$		11.53	5.79	11.86	14.50
β_i		1.00	0.62	0.93	0.88
$\sigma_{\epsilon,l}$	0.74				
$\sigma_{\epsilon,h}$	1.01				
ϕ	0.98				
p_{ll}	0.94				
p_{lh}	[0.04]				
p_{hh}	0.63				
p_{hj}	0.23				
p_{jj}	0.52				
p_{jh}	0.48				
LLF	-11.0547				
T	641				
Steady-state prob.					
l	0.62				
h	0.24				
j	0.14				
Expected duration					
l	17.11				
h	2.68				
j	2.07				

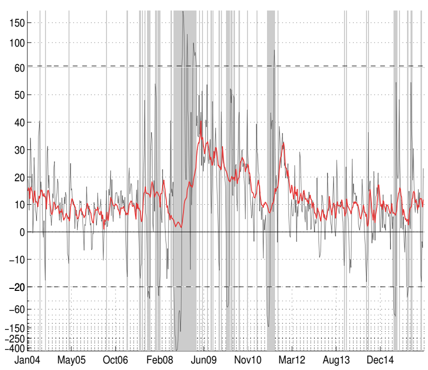
Notes: LLF denotes the average loglikelihood and T the number of observations. Steady state probabilities are computed according to (4.49) in [29]. Expected duration is computed as $1/(1-p_{ii})$, $i \in [l$ (low), h (high), j (jump)]. Identification restrictions on the variance of the common factor require $\beta_{S\&P500} = 1$. The [ ] brackets indicate insignificant parameters. The parameter $\overline{\mathrm{FS}}$ is set to zero as it turns out to be insignificant. The sample frequency is weekly between February 3, 2004 and July 29, 2016.



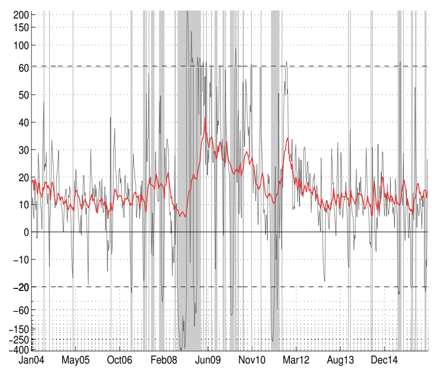
(a) S&P500



(b) DJIA



(c) NASDAQ



(d) RUSSELL

Figure 5: VRP's implied by the common factor model.

difference between the returns on diversified portfolios of the stocks of low and high investment firms).

Although not being a risk factor immediately comparable in essence to the five Fama-French factors, as it is not the return on a tradable portfolio, the MVRP represents the level of agents' volatility risk aversion and it is directly proportional to the cost of hedging against volatility risk. Correlations between the five Fama-French factors and the MVRP, show that while the Fama-French factors are substantially correlated among themselves, the MVRP is significantly correlated only with the market factor at 28%. Hence, the MVRP potentially contains novel information to capture the variation in the expected return of financial securities and portfolios.

Following [40], we consider returns on five portfolio sorts based on the following four characteristics: beta, variance, residual variance and net share issues. Details on the construction of the portfolios and data are available from Kenneth R. French's online data library. Being standardised by construction, adding the MVRP in the Fama-French regression preserves the comparability with [40] and does not introduce distortion in the estimation of the intercepts. Following practice in the CAPM literature, e.g. [44], we resample the data at a monthly frequency.

Tables 9 and 10 report parameter estimates of the Fama-French five-factor model and the MVRP. The latter contributes significantly to explain the portfolios return variations for the sorts based on beta, variance and residual variance, and only marginally for the ones based on net share issues. More specifically, Table 9 Panel A reports results for the portfolio sorted with respect to the individual companies' beta. Portfolios of stocks with large betas, thus more sensitive to market fluctuations, show positive exposure to variance risk with respective loadings significant at the 1% level. For a given level of perceived variance risk, portfolios more exposed to the market incorporate a higher remuneration. Conversely, portfolios with market exposure smaller than one, react in the opposite direction showing the existence of a negative premium for stability. Panel B of Table 9 shows that, as expected, for the five portfolios sorted according to the assets' variance, the exposure to variance risk increases as we move from low to high. Positive exposure to volatility risk reflects the compensation required to face the higher cost of hedging against such risk. Table 10 Panel A shows that similar patterns hold for portfolios sorts based on residual variance. The results in Table 10 Panel B show a weaker link between the expected return of portfolios based on growth, measured by net share issues, and the MVRP.

Table 9: CAPM regressions

***Panel A*: Portfolio sorts based on βs**

	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{MVRP}	R^2
Low	0.043	0.805	-0.266	-0.047	0.174	0.254	-0.034	0.88
	(0.427)	(25.653)	(-5.393)	(-0.700)	(2.668)	(2.856)	(-2.574)	
2	0.096	0.982	-0.044	0.120	0.153	-0.192	-0.015	0.97
	(1.537)	(41.924)	(-1.340)	(3.330)	(3.220)	(-3.406)	(-1.751)	
3	-0.006	1.059	0.060	0.097	0.053	-0.164	0.034	0.97
	(-0.085)	(35.296)	(1.704)	(1.872)	(0.818)	(-2.148)	(3.328)	
4	-0.049	1.203	0.231	0.124	-0.128	-0.320	0.050	0.95
	(-0.377)	(26.644)	(4.034)	(1.388)	(-1.562)	(-3.908)	(3.436)	
High	-0.107	1.292	0.406	-0.093	-0.551	0.070	0.089	0.90
	(-0.462)	(17.366)	(4.246)	(-0.506)	(-3.554)	(0.435)	(3.328)	

***Panel B*: Portfolio sorts based on Variance**

	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{MVRP}	R^2
Low	0.130	0.786	-0.204	-0.067	0.136	0.248	-0.011	0.91
	(2.006)	(33.74)	(-4.700)	(-1.261)	(2.575)	(3.100)	(-1.143)	
2	0.030	1.043	0.037	-0.008	0.097	-0.051	-0.002	0.97
	(0.421)	(54.41)	(0.949)	(-0.195)	(2.282)	(-0.704)	(-0.181)	
3	-0.106	1.207	0.089	0.044	-0.014	-0.172	0.004	0.94
	(-1.060)	(33.85)	(1.683)	(0.448)	(-0.148)	(-1.112)	(0.282)	
4	-0.065	1.309	0.326	0.148	-0.152	-0.281	0.040	0.91
	(-0.420)	(23.94)	(3.131)	(1.435)	(-1.116)	(-2.173)	(2.039)	
High	-0.246	1.308	0.610	0.384	-0.765	-0.748	0.049	0.88
	(-1.180)	(12.97)	(5.430)	(2.335)	(-4.516)	(-2.828)	(2.470)	

Notes: Estimation results CAPM regressions. MKT is the market return in excess of the risk-free interest rate, SMB is small minus big, HML is high minus low, RMW is robust minus weak and CMA is conservative minus aggressive. The β sorts are based on univariate market beta. The variance sorts are based on individuals assets' variance. The sample frequency is monthly between February, 2004 and July, 2016. The t-statistics (in parentheses) are based on Newey-West standard errors.

Table 10: CAPM regressions

***Panel A*: Portfolio sorts based on Residual variance**

	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{MVRP}	R^2
Low	0.061	0.853	-0.200	-0.044	0.130	0.259	-0.020	0.94
	(1.070)	(46.15)	(-5.769)	(-1.130)	(2.583)	(3.959)	(-2.071)	
2	0.035	1.034	0.046	0.087	0.034	-0.041	0.007	0.96
	(0.492)	(46.99)	(1.064)	(2.101)	(0.861)	(-0.701)	(0.660)	
3	-0.082	1.134	0.108	0.059	-0.067	-0.305	0.012	0.95
	(-0.996)	(34.90)	(1.940)	(1.072)	(-0.842)	(-2.955)	(0.853)	
4	0.030	1.269	0.238	0.061	-0.169	-0.450	0.031	0.91
	(0.195)	(25.85)	(2.981)	(0.464)	(-1.658)	(-3.216)	(1.609)	
High	-0.299	1.304	0.630	0.144	-0.539	-0.535	0.078	0.87
	(-1.317)	(16.29)	(5.493)	(0.687)	(-2.730)	(-2.221)	(2.405)	

***Panel B*: Portfolio sorts based on Net Share Issues**

	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{MVRP}	R^2
Low	0.031	0.952	-0.090	-0.001	-0.081	0.044	0.023	0.91
	(0.285)	(29.447)	(-1.713)	(-0.010)	(-0.889)	(0.455)	(1.580)	
2	0.047	0.998	0.111	-0.062	-0.042	0.011	-0.008	0.92
	(0.370)	(23.686)	(2.058)	(-0.800)	(-0.429)	(0.112)	(-0.529)	
3	0.185	1.089	0.126	-0.095	-0.011	-0.302	0.004	0.92
	(1.480)	(29.497)	(1.544)	(-1.295)	(-0.098)	(-2.295)	(0.263)	
4	0.000	1.079	0.213	-0.232	-0.133	-0.349	0.068	0.91
	(0.003)	(21.593)	(3.553)	(-2.717)	(-1.299)	(-3.225)	(3.522)	
High	0.063	1.015	0.118	0.098	-0.670	-0.377	0.010	0.93
	(0.400)	(24.123)	(1.768)	(1.096)	(-5.684)	(-3.864)	(0.526)	

Notes: Estimation results CAPM regressions. MKT is the market return in excess of the risk-free interest rate, SMB is small minus big, HML is high minus low, RMW is robust minus weak and CMA is conservative minus aggressive. The residual variance sorts are based on variance of the residuals from the Fama-French three-factor model. The Net Share Issues sorts are shares' growth rate. The sample frequency is monthly between February, 2004 and July, 2016. The t-statistics (in parentheses) are based on Newey-West standard errors.

6. Conclusion

This paper estimates the Variance Risk Premium (VRP) directly from synthetic variance swap payoffs. Our approach provides measurement error free estimates of the part of the VRP related to normal market conditions, and allows constructing variables indicating agents' expectations under extreme market conditions. In particular, though the VRP significantly predicts future market returns at shorter horizons, across S&P500, DJIA, NASDAQ and RUSSELL indices, sizeable increases in predictability are found when the agents' reactions to extreme events are included in the predictive regressions. This return predictability substantially dominates the one obtained from using the variance swap payoff as a benchmark proxy. Finally, we filter out a common factor interpretable as a market variance risk premium (MVRP). The MVRP shares the properties that the individual VRPs have and allows identifying common extreme events. When compared to other well-known asset pricing factors, the MVRP is significantly correlated only with the market factor and it is priced when considering the returns on several of the five Fama and French (2015) portfolio sorts.

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