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A new empirical model to calculate bubble size in froth flotation columns

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Abstract

In flotation, bubbles are responsible for carrying the hydrophobic particles to the concentrate and bubble size is one of the most effective parameters in this process. The usual method to estimate the bubble size is measuring the diameter of rising bubbles, which requires special equipment and it is time consuming. In some cases, the bubble size can be estimated by mathematical methods. In this regard, parameters such as gas holdup, superficial gas velocity, superficial liquid velocity, and fluid density are required. A new model to estimate the bubble size in flotation was proposed in this paper. It was shown that the estimated values for bubble sizes obtained from this model have higher accuracy than those obtained using previously published models.

Keywords: Flotation; bubble size; mathematical model

1. Introduction

Bubble size is one of the most important parameters of gas distribution in flotation process with a significant effect on the flotation efficiency (Deglon et al., 2000; Reis et al., 2016). In fact, the superficial levels of the gas bubble and gas holdup, which are related to the transfer capacity of minerals are both affected by bubble size (Kracht et al., 2005). Bubble size also influences the efficiency of collision and binding the particles and bubbles (Verrelli et al., 2011).

The bubble diameter has a significant impact on the collision and attachment efficiency so that the collision efficiency decreases with increasing bubble diameter. However, other factors, such as particle diameter, turbulent dissipation rate, also affect the collision efficiency (Tao, 2004; Yoon et al., 2013; Wan et al., 2020). Although small bubbles increase collision and attachment efficiency. It should be noted, it doesn't always increase recovery (Reis et al., 2019). The presence of large bubbles is required to provide coarse particle–bubble aggregate (Tao, 2010). But, it is known that the effect of bubble velocity to remove the fine particles inertial forces in particle-bubble interaction is greater than the bubble diameter (Hassanzadeh et al., 2017).

Various factors can affect the bubble diameter. For example decreasing the superficial gas velocity (Vinnett et al., 2014), increasing the impeller velocity (Gorain et al., 1995), increasing pressure (Han et al., 2002), increasing temperature (Zhang, 2014), increasing frother (Wei et al., 2014; Resi et al., 2016; Zhu et al., 2019) and increasing the solid concentration (Shabalala et al., 2011) decreases the bubble diameter.

Considering the importance of bubble size in froth flotation, it has been the subject of research for a long time (Tucker et al., 1994). Typically, there are direct and indirect methods for determining the bubble size in flotation. In the direct methods, bubbles are measured one by one to determine their size distribution, but to do that, a sample from the whole bubble population in

the cell must be taken (Sovechles & Waters, 2015). The bubble size analyzer of University of Cape Town (Grau & Heiskanen, 2005), and the McGill University analyzer (Hernandez-Aguilar et al., 2004; Leiva et al., 2010) are examples of direct methods to measure the bubble size. However, the most common method for directly determine the bubble size is probably the machine vision technique. In this method, bubbles are collected with a sampling tube and inserted into a sloping glass vessel. Then, their captured images or videos are analyzed with special software (Leiva et al., 2010; Vinnett et al., 2012; Pugh, 2009; Hosseini et al., 2015). Another direct method of measuring bubble size is the acoustic method (Zhang at al., 2012; Pandit & Varley, 1992; Wu & Chahine, 2010). In this method, bubble diameter is determined based on the measurement of the dispersion or excessive attenuation of acoustic waves and changes in the sound velocity (Pandit & Varley, 1992; Wu & Chahine, 2010). It should be noted that, in all direct methods, using an external probe may disrupt the sampling process and increases the measurement error. In addition, there is still a lack of a suitable and sensitive tool for direct measurement of bubble size at industrial conditions, especially in real time (Sovechles & Waters, 2015).

Bubble size can be also indirectly calculated by mathematical methods using a number of operating parameters (Dobby et al., 1988). In this method, data are collected with no direct effect on flotation performance. The required parameters are gas holdup, superficial gas velocity, superficial liquid velocity, and fluid density (Dobby et al.; 1988; Yianatos et al., 1988; Banisi & Finch, 1994).

In froth flotation, if the size of different bubbles are assumed to be uniform, the relative velocity of the gas and liquid slip in the opposite flow system (upstream and downstream liquid flow), U_{sb} , can be obtained from Equation 1 (Dobby et al., 1988; Yianatos et al., 1988; Banisi & Finch,

1994) where J_g and J_l are the superficial gas velocity and liquid velocity, respectively (cm/s), and ε_g is the gas holdup.

$$U_{sb} = J_g / \varepsilon_g + J_l / (1 - \varepsilon_g) \quad (1)$$

The limit velocity (U_t) has a direct relation with the bubble size (Tan et al., 2013). The slip velocity is related to a single bubble's terminal rise velocity in an infinite pool. A frequently used relationship for U_{sb} is Equation 2 (Dobby et al., 1988; Yianatos et al., 1988; Banisi & Finch, 1994):

$$U_{sb} = U_t (1 - \varepsilon_g)^{m-1} \quad (2)$$

where 'm' is the function of the Reynolds number which can be calculated from Equations 3 and 4:

$$m = (4.45 + 18d_b/d_c) Re_b^{-0.1} \quad 1 < Re_b < 200 \quad (3)$$

$$m = 4.45 Re_b^{-0.1} \quad 200 < Re_b < 500 \quad (4)$$

Reynolds number is a dimensionless quantity that shows the ratio of the inertial force to the viscous force in a fluid. The important application of this number is to determine the slow flow or fluctuation. The amount of Reynolds number for a single bubble is calculated from Equation 5 (Dobby et al., 1988; Yianatos et al., 1988; Banisi & Finch, 1994) where μ_f is the fluid viscosity (g/cm.s) and ρ_f is the fluid density (g/cm³):

$$Re_b = (d_b U_t \rho_f) / \mu_f \quad (5)$$

If the bubble density is known, then the Reynolds number for a bubble swarm (Re_s) can be calculated using Equation 6, and the slip velocity can be related to the Reynolds number and the bubble diameter using Equation 7:

$$Re_s = d_b U_{sb} \rho_f (1 - \varepsilon_g) / \mu_f \quad (6)$$

$$U_{sb} = g d_b^2 (1 - \epsilon_g)^{(m-1)} \Delta \rho / 18 \mu_f (1 + 0.15 Re_s^{0.687}) \quad (7)$$

As stated, different models have been developed by the researchers to determine the bubble diameter as outlined below:

Dobby et al. (1988)

In this method, a value is initially estimated for ‘m’. U_{sb} is calculated from Equation 1, and then ‘ U_t ’ is determined using Equation 2. A value for the bubble diameter is assumed, and the Reynolds number is calculated using Equation 6. The bubble diameter is then calculated using Equation 7, and the previous steps (‘m’ assumption, and Reynolds number calculation) are repeated by using the newly obtained bubble diameter. The value of ‘m’ is determined using Equations 3 and 4, and it is compared to the initial assumed value. The previous steps are repeated until the assumed and calculated values for ‘m’ are equal (or close) (Dobby et al., 1988).

Yianatos et al. (1988)

In this method, an initial value for the bubble diameter is considered. Then a value for ‘m’ is considered and ‘ U_t ’ is calculated from Equations 2. By using the value of U_t , the Reynolds number is calculated from Equation 5 and ‘m’ is determined from Equation 2. The initial estimated ‘m’ is replaced by the new calculated ‘m’ and all steps are continuously repeated until both ‘m’ are the same (or adequately close). The bubble diameter is calculated from Equation 7. By substituting the new bubble diameter, the previous steps are repeated until the new and old diameters are both the same (Yianatos et al., 1988).

Xu & Finch (1990)

In this model, the value of 'm' is initially considered as 2, and a value for the bubble diameter is assumed. Then 'U_{sb}' is calculated using Equations 2. The Reynolds number is calculated using Equation 8 and the diameter of the new bubble is calculated using Equation 7. The previous steps are continuously repeated until the initially assumed bubble diameter is equal (or adequately close) to the calculated diameter (Xu & Finch, 1990). Researchers (Banisi & Finch, 1994) have stated that a simplified version of Xu & Finch model is of particular interest for modelling gas holdup versus the gas rate when bubble size changes with the gas rate.

$$Re_s = d_b U_t \rho_f / \mu_f \quad (8)$$

Banisi & Finch (1994)

Banisi & Finch (1994) have stated that the discrepancies among the above mentioned models are resolved in the model they presented in 1994. This method is similar to Xu & Finch (as explained above) but the value of 'm' is considered as 3. Then 'U_{sb}' and 'U_t' are calculated using Equations 1 and 2, respectively. The Reynolds number is also calculated using Equation 6 and the new bubble diameter is obtained from Equation 7. The previous steps are repeated until the initial bubble diameter is equal (or close) to the calculated bubble diameter (Banisi & Finch, 1994).

The structures of the above mentioned models are different, but they have an acceptable error for calculation of the bubble diameter. In these models, the authors have assumed the bubble are uniformly distributed in the flotation cell. This assumption is not always valid in the practice, and therefore, the experimental error could increase. The aim of the current paper is proposing a new model with rather simpler and less complex calculation to obtain less error in the bubble diameter measurements.

2. The proposed model

In the proposed model, an initial value for the bubble diameter and ‘m’ is assumed, and the limit velocity is estimated using Equation 2. By having the limit velocity, the bubble diameter is calculated using Equation 9. Further, ‘n’ which is a dimensionless number and it is related to the bubble diameter and Reynolds number, is calculated from Equation 10.

$$U_t' = g d_b^2 ((1 - \varepsilon_g)^{(n-1)} / 18 \mu_f (1 + 0.15 \text{Re}_s^{0.687})) \quad (9)$$

$$n = (4.45 + 18 d_b / d_c) \text{Re}_s^{-1} \quad (10)$$

One of the parameters in the proposed model is ‘m’ which is the function of the Reynolds number, as explained. The best value for ‘m’ needs to be identified to obtain the optimum results from the proposed model. Therefore, the bubble diameter for 23 different test conditions was calculated using the proposed model, and the data were compared to the experimental diameters as reported by Yianatos et al., (1988) (Table 1) to obtain the coefficient of determination (R^2) for different ‘m’ values. The results show that the highest R^2 and sum error is obtained when the value of ‘m’ is 4 (Figure 1).

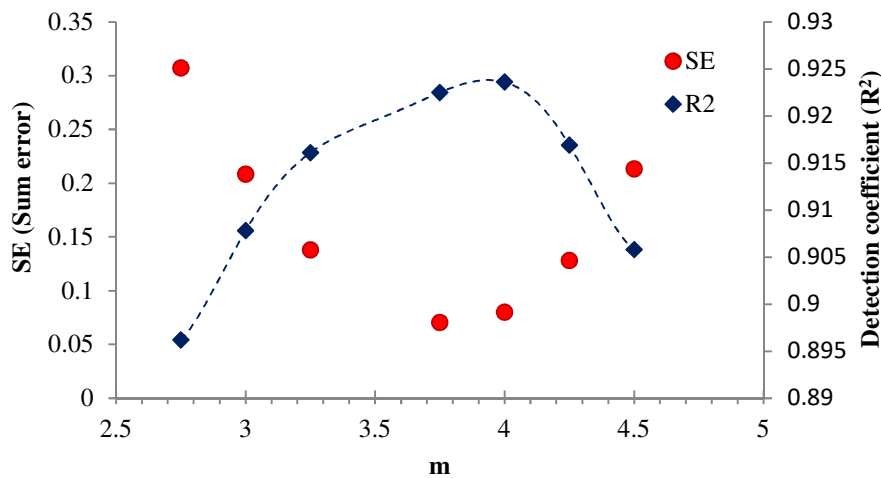


Figure 1: The coefficient of correlation (R^2) and sum error for different ‘m’ values

Considering the above discussion, the structure of the proposed model is as follows:

- (I) assuming an initial bubble diameter,
- (II) calculating U_t using Equation 2 by considering $m=4$,
- (III) calculating U_t' using Equation 9, and
- IV) calculating the bubble diameter using U_t is equal to U_t' .

The above method requires fewer calculations than previous methods and the bubble diameter can be obtained faster.

In Table 1, the estimated bubble diameters obtained from the proposed model are compared with those obtained from the previously published models. The error (E) is from the difference of the bubble between the values measured by visual method (by Yianatos et al.,1988), and those calculated from the current model (in the absolute or unsigned form). The conditions for the experiments are set out in Table 2 (Yianatos, et al., 1989).

Table 1: Comparison of the estimated bubble diameters using different models (J_g : superficial gas velocity, J_l : superficial liquid velocity, ϵ_g : gas holdup, d_c : column diameter. The fluid density was considered as 1 g/cm^3).

No.	dc (cm)	J_g^* (cm/s)	J_l^* (cm/s)	ϵ_g^* %	d_m^* (mm)	Yianatos et al. (1988)		Banisi & Finch (1994)		Current model	
						d_b (mm)	E (%)	d_b (mm)	E (%)	d_b (mm)	E (%)
1	3.8	1.0	0.91	9.5	1.20	1.11	0.09	1.14	0.06	1.14	0.06
2	3.8	1.0	0.85	12.9	0.86	0.87	0.01	0.88	0.02	0.89	0.03
3	3.8	1.0	0.82	15.8	0.77	0.76	0.01	0.77	0	0.78	0.01

4	3.8	1.0	0.85	15.5	0.69	0.77	0.08	0.78	0.09	0.79	0.1
5	3.8	1.0	0.77	16.2	0.73	0.74	0.01	0.75	0.02	0.76	0.03
6	10.3	2.1	0.30	15.7	1.51	1.40	0.11	1.47	0.04	1.48	0.03
7	10.3	1.5	0.30	14	1.13	1.11	0.02	1.14	0.01	1.15	0.02
8	5.7	0.5	1.00	12.3	0.62	0.55	0.07	0.54	0.08	0.55	0.07
9	5.7	0.8	1.00	17.0	0.67	0.64	0.03	0.64	0.03	0.65	0.02
10	5.7	1.0	1.00	20.0	0.70	0.69	0.01	0.70	0	0.71	0.01
11	5.7	1.2	1.00	23.4	0.74	0.74	0	0.75	0.01	0.77	0.03
12	5.7	1.5	1.00	28.0	0.81	0.80	0.01	0.83	0.02	0.85	0.04
13	5.7	1.8	1.00	32.0	0.88	0.87	0.01	0.93	0.05	0.95	0.07
14	3.8	1.0	0.96	11.2	0.97	0.98	0.01	1.00	0.03	1.00	0.03
15	3.8	1.0	0.88	13.2	0.85	0.86	0.01	0.87	0.02	0.88	0.03
16	3.8	1.0	0.91	14.4	0.85	0.81	0.04	0.83	0.02	0.83	0.02
17	3.8	1.0	0.87	17.7	0.82	0.72	0.1	0.72	0.1	0.74	0.08
18	3.8	1.0	0.83	21.5	0.71	0.65	0.06	0.66	0.05	0.67	0.04
19	3.8	1.0	0.90	13.2	0.78	0.86	0.08	0.88	0.1	0.88	0.1
20	3.8	1.0	0.90	13.3	0.75	0.86	0.11	0.87	0.12	0.88	0.13
21	3.8	1.0	0.91	13.6	0.80	0.84	0.04	0.86	0.06	0.86	0.06
22	3.8	1.0	0.91	15.3	0.73	0.78	0.05	0.79	0.06	0.80	0.07
23	3.8	1.0	0.96	18.0	0.67	0.72	0.05	0.73	0.06	0.74	0.07
Average Error					-	-	0.04	-	0.05	-	0.05
Standard deviation (SD)					-	-	0.04	-	0.03	-	0.03
R ² : Coefficient of determination (%)					100	92.2	-	92.2	-	92.4	-

* The values were obtained from Yianatos et al. (1988).

Table 2: Conditions for the experiments (Yianatos et al.,1988).

Test number (Table 1)	Frother		Cell		
	Type	ppm	Diameter (cm)	Sparger	Height (m)
1,2,3,4,5	DOW	5, 10, 15, 20, 25	3.81	Ceramic	200
6,7	DOW	10, 15	2.5*10	Steel	180
8, 9, 10, 11, 12, 13	DOW	15	5.71	Ceramic	450
14, 15, 16, 17, 18	TEB	5, 10, 15, 20, 25	3.81	Ceramic	200
19, 20, 21, 22, 23	MIBC	20, 30, 45, 60, 75	3.81	Ceramic	200

Table 1 shows that the detection coefficient (or the coefficient of determination, R^2) in the proposed method is higher than those obtained by Banisi & Finch (1994), or Yianatos et al. (1988). In addition, it has other advantages such as considering the effect of cell diameter in determining the bubble diameter, reducing the dependence on the initial value of 'm', and its calculation is relative, simpler and easier. All models have acceptable accuracy, but the current model seems to have less complex calculations.

For all the tests, ranging up to a bubble diameter of 1.5 mm, Figure 2 shows a good agreement between the measured and calculated bubble size. The middle line is the predicted bubble diameter (d_b in Table 1) and the other two lines in both sides represent the value within $\pm 15\%$ of the measured data, which is about the limit of the experimental error. This accuracy is adequate for most purposes. A typical upper size for bubbles in flotation is about 2-2.5 mm (Yianatos 1989, Yianatos, et al., 1989; Rulyov et al. 2015), therefore the presented model is well suited to flotation studies with less than 15% measurement error.

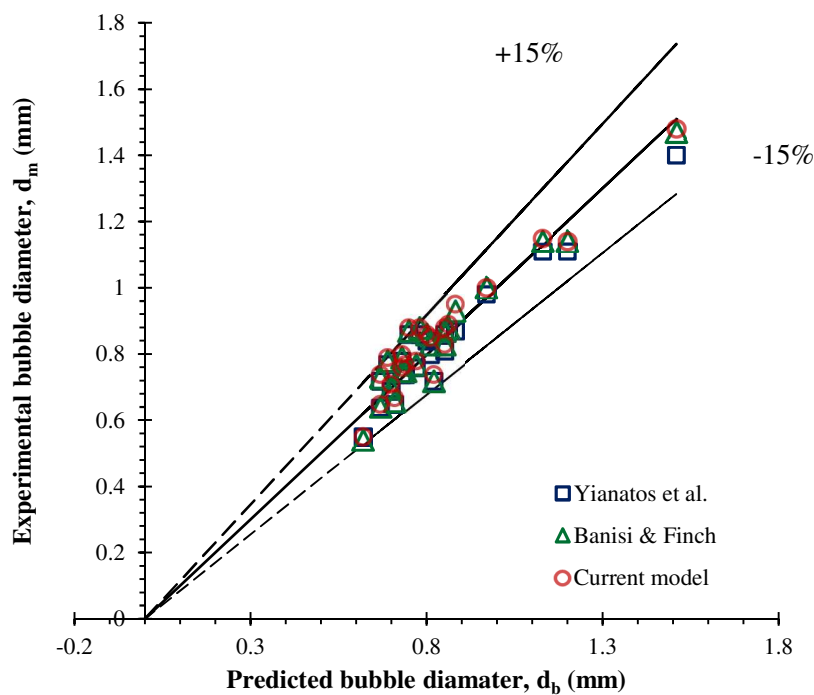


Figure 2: Comparison of the predicted bubble diameters using different models with the experimental data reported by Yianatos et al. (1988) (middle line). The dashed lines in both sides represent the value within $\pm 15\%$ of the measured data.

Table 1 shows that the model developed in the current study has been compared with only two models of Yianatos et al. (1988) and Banisi & Finch (1994) (and not with those provided by Dobby et al. (1988) and Xu & Finch (1990)). In order to justify, it should be highlighted that the models developed by Yianatos et al. (1988) and Banisi & Finch (1994) are the more complete and broader version of those provided earlier by Dobby et al. (1988) and Xu & Finch (1990), respectively. Also, Banisi & Finch (1994) have already compared all 4 models and concluded that their model is more complete and it is more suitable than the other 3 models for applications.

3. Factors affecting the bubble size

Using the proposed model in this paper, the bubble diameter was calculated for various values of fluid density, cell diameter, gas stagnation, gas and liquid superficial velocity. It was found that increasing the superficial gas velocity causes an increase in the bubble diameter. Researchers have shown that bubble size increases upon increasing the aeration rate (Grau & Heiskanen, 2005; Ostadrahimi et al, 2019). The superficial liquid velocity is directly related to the bubble size (Finch & Dobby, 2007). It was also observed that (though with a less intensity) the bubble size increases when the liquid velocity increases. Decreasing the bubble size could often result in a higher gas holdup in the pulp phase as smaller bubbles exhibit slower rising velocity. Interestingly, when the gas holdup is higher than a certain limit (about 32%, see Table 1), the bubble size starts to increase. This can be due to the enhanced probability of bubble collision, and coalescence when the gas holdup increases. For example, it has been reported that in the froth phase in flotation, the bubble size increases as the gas holdup increases (Yianatos et al., 1986). Furthermore, it was observed that bubble size reduces when the fluid density increases

which is in agreement with the previously published data (Luo et al., 1999; Gulden et al., 2018). The density of the fluid (i.e. water, ρ_f) was considered as 1 g/cm³ (Table 1). However, in Figure 3, this parameter is between 0.8 to 1.4 g/cm³. These values were used to show that the bubble diameter can decrease if the fluid density increases, independently from other parameters. It is worth considering the effect of slurry density, which is always higher than water, in future work and include the ‘pulp density’ in the model. Increasing the temperature decreases the fluid viscosity and consequently decreases the bubble diameter (Zhang, 2014). Figure 3 also shows that the bubble size slightly reduces when the diameter of the column (in column flotation) increases.

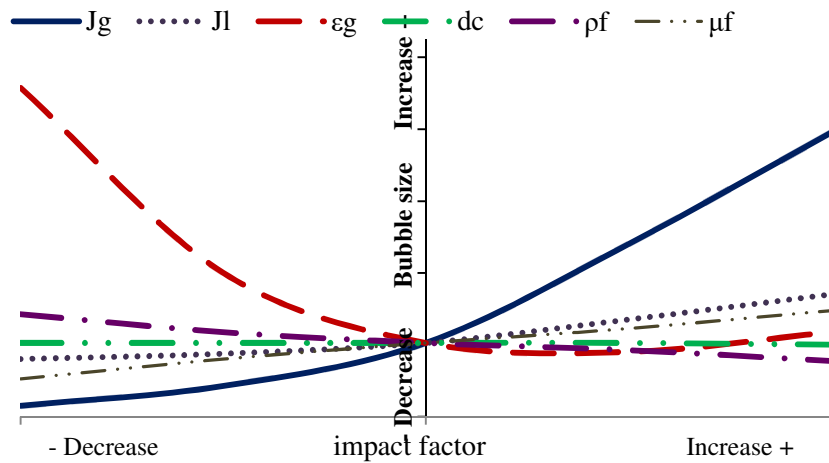



Figure 3: Effect of different factors on the bubble size according to the proposed model (See Table 3 for the data, J_g : superficial gas velocity, J_l : superficial liquid velocity, ϵ_g : gas holdup, d_c : column diameter, ρ_f : fluid density).

Table 3: The impact factors used in Figure 3

impact factor	d_c (cm)	J_g (cm/s)	J_l (cm/s)	ϵ_g (%)	μ_f (g/cm.s)
Decrease  Increase	1.25	0.25	0.25	0.05	0.004
	2.5	0.5	0.5	0.1	0.007
	5	1	1	0.2	0.01
	10	2	2	0.3	0.013
	20	3	3	0.4	0.018

4 Conclusions

In this paper, a new model to calculate bubble size in froth flotation was proposed. The bubble diameter for 23 different test conditions was calculated by the proposed model. While the difference between the obtained coefficient of determination (R^2) and those previously provided in the literature is not significant enough to claim a better method, the current model includes much simpler calculations. In addition, based on the proposed model, the apparent velocity of gas and water are directly related to the increase in the bubble diameter. However, the apparent velocity of water has less effect than the apparent velocity of the gas. Furthermore, increasing the fluid density and gas holdup results in a reduction in the bubble size. However, the apparent velocity of water has less effect compared to the apparent velocity of gas on the bubble size. Furthermore, increasing gas holdup in the flotation cell normally (but not always) result in a reduction in the bubble size. When the gas holdup is higher than a certain limit, the bubble size may raise. In this case, the gas holdup is too high and the pulp behaves differently from what is normally used in flotation.

It should be highlighted that based on the current and previously published data, the current model is valid for column flotation. It needs to be further validated for other types of flotation cells.

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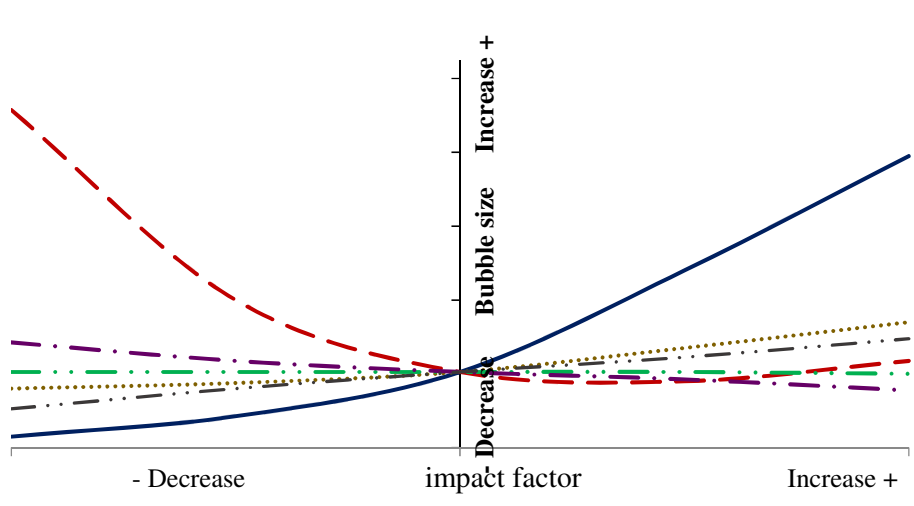
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J_g : superficial gas velocity
 J_l : superficial liquid velocity
 ϵ_g : gas holdup
 d_c : column diameter
 ρ_f : fluid density