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Optical single-channel color image asymmetric cryptosystem based on hyperchaotic system and random modulus decomposition in Gyrator domains

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An enhanced optical single-channel **asymmetric** cryptosystem for color image is proposed by using hyperchaotic system and random modulus decomposition (RMD) in gyrator domains. For increasing the security of the encryption scheme, a novel 4D four-winged hyperchaotic system is utilized to generate several hyperchaotic phases in the encryption **approach**. Subsequently, the RMD **is implemented** to obtain the effective trapdoor one-way function in the asymmetric process. Finally, a scrambling encoding algorithm is designed to combine and randomize the intermediate and hyperchaotic data, and the result can be regarded as the ciphertext and private key of the cryptosystem. In the decryption approach, the ciphertext and private key are imported into the input plane of gyrator transform. In fact, the proposed cryptosystem is also applicable in information authentication since the hyperchaotic data is prerequisite both in encryption and decryption approaches. Various numerical simulations are made to demonstrate the validity and capability of the proposed cryptosystem.

Keywords: Asymmetric cryptography, color image encryption, optical transform.

1. Introduction

With the rapid development of the modern communication techniques, the optical information security technology has drawn wide attention in recent decades due to its parallel processing and high speed processing capabilities. Various optical cryptosystems have been reported since Refrégier and Javidi first proposed double random phase encoding (DRPE) system in 1995, which can encrypt the image into **stationary white noise** in $4f$ optical system [1]. However, DRPE scheme has been verified as vulnerable to different kinds of attack, such as chosen plaintext attack and known plaintext attack, due to its inherent linearity property [2-4]. Subsequently DRPE has been deeply researched and various **improved optical cryptosystems** have been proposed in other optical transform domains, such as fractional Fourier transform (FrFT), gyrator transform and Fresnel transform to enhance the security and overcome the inherent linearity property [5-18]. Moreover, some other techniques, such as polarized light

interference [19], randomized lens-phase functions [20] and compressive sensing [21] have also been introduced and utilized to break the linear relationship between the plaintext and ciphertext in optical encryption system. Besides, some random encoding algorithm has also been utilized in the image cryptosystem [22].

Most of the schemes [1-22] described above are symmetric encryption algorithms, in which the decryption key is identical to the encryption key. To overcome the linearity characteristics of DRPE, various asymmetric cryptosystems have been proposed based on phase-truncated Fourier transform (PTFT) [23-25]. However, the PTFT-based asymmetric cryptosystem was proved to be vulnerable to some specific attack using iterative amplitude-phase retrieval algorithm in the further research [26]. Moreover, the iterative phase retrieval algorithm, such as Gerchberg-Saxton algorithm and Yang-Gu algorithm [27-29], have also been deeply researched and introduced in optical encryption system, which show high robustness against various potential attacks. However, the iterative calculation makes the retrieval algorithm based cryptosystem involved large amount of calculation. Besides the retrieval algorithm, several asymmetric encryption schemes based on equal modulus decomposition (EMD) have been presented in recent years [30-32]. Recently, Wang and his colleagues reported that the attacker is able to obtain precise result from EMD-based encryption system by performing a novel designed attack using phase iterative algorithm [33]. To improve the security of EMD, RMD is investigated, in which a random decomposition idea is utilized in creating the effective trapdoor one-way function. RMD can largely reduce the constraints compared with the EMD and it achieves high robustness while resisting the attack of iterative algorithm [33-34]. However, the RMD is evolved from the original EMD, therefore the parameters of RMD may be valuable to exhaustive attack. This attack process may waste much time, but it works in theory. Thus, one can introduce the hyperchaotic system into the cryptosystem, since the high randomness and extreme sensitivity to initial condition of hyperchaotic system can enhance the security of the cryptosystem dramatically. Moreover, some hyperchaotic systems have been employed in some cryptosystem to improve the security since the high sensitive characteristic of the initial value in chaotic system can break the linearity relationship [35-42].

In this paper, a single-channel optical color asymmetric cryptosystem using four-winged hyperchaotic system and random modulus decomposition in gyrator transform domain is proposed. Firstly, a 4D four-winged hyperchaotic system is introduced and utilized to generate the chaotic random phase mask (CRPM), which will be regarded as the public key of asymmetric encryption system. The original color image is encoded into two-dimensional data and then combined with the random phase by propagating CRPM placed in the light path. Subsequently the image data is transformed as the light field in output plane of the gyrator transform. At the output plane of gyrator transform, the scrambled data is decomposed into two masks by performing RMD. Finally, a newly designed scrambling pixels position operation according to another chaotic random phase is employed to obtain the ciphertext and private key synchronously. In the encryption approach, the chaotic random phase serves as the public key in this cryptosystem. The parameter in gyrator transform and the initial conditions of the hyperchaotic system can be regarded as the extra keys to guarantee the security. Some corresponding numerical simulations are made to validate the performance of the proposed asymmetric cryptosystem.

The rest of the paper is organized as follows: In Section 2, the proposed cryptosystem is introduced in detail. In Section 3, numerical simulation results are made to demonstrate the validity of the algorithm. Concluding remarks are summarized in the final section.

2. Optical color image asymmetric cryptosystem

Firstly, the 4D four-winged hyperchaotic system, random modulus decomposition and scrambling pixels position operation are introduced briefly, respectively. Then, the gyrator transform and its optical implementation are presented. Finally, the complete asymmetric encryption algorithm is addressed in detail.

2.1 Hyperchaotic system and scrambling operation

The highly unpredictable and random-like behavior of the hyperchaotic signals are the most attractive features in cryptography. For the hyperchaotic system, the extreme sensitivity to initial conditions and parameters can improve the security of the data against the brute force attack. It is believed that the performance of higher dimensional chaotic systems (with higher dimensional attractors) are much better in image encryption, therefore a 4D four-winged hyperchaotic system is considered and utilized in this paper and the system can be modeled as follows [43-44]

$$\begin{cases} \dot{x} = ax - yz, \\ \dot{y} = byz - cu + xz, \\ \dot{z} = xy - dz, \\ \dot{u} = y + ku. \end{cases} \quad (1)$$

where a, b, c, d and k are constant parameters of the hyperchaotic system. The symbol ‘ $\dot{\cdot}$ ’ represents the operation of derivation. When $a=3, b=-8, c=d=5$ and $k=0.5$, the Lyapunov exponents of the hyperchaotic system are $\lambda_1=0.32, \lambda_2=1.23, \lambda_3=-0.81$ and $\lambda_4=-11.04$. Apparently, the first and second Lyapunov are larger than 0, which means the system is in hyperchaotic state. In calculation, the initial conditions are set as $x_0 = y_0 = z_0 = u_0 = -10$, and the four-winged $(x-y)$ hyperchaotic Lorenz attractors is illustrated in Fig.1.

Fig.1 The hyperchaotic Lorenz attractor in x-y plane.

To obtain the chaotic data for the following encryption process, the hyperchaotic sequences can be generate by the following operation

$$s_i^* = s_i \times 10^4 - \text{round}(s_i \times 10^4), \quad (i=1,2,3,4). \quad (2)$$

where s_i denotes the original chaotic sequence from the hyperchaotic system described in Eq.1, which can be also expressed as x_i , y_i , z_i or u_i . The symbol s_i^* represents the improved sequences corresponding to s_i . Therefore, four improved sequences x_i^* , y_i^* , z_i^* and u_i^* can be obtained by implementing Eq.2. Finally, the obtained improved sequences are converted into two-dimensional format.

To enhance the security of the final ciphertext and private key, the highly unpredictable and random chaotic sequence is considered for designing a scrambling pixels position operation. Here, we suppose the image $I(x, y)$ to be scrambled and the chaotic mask $M(u, v)$ having the same size of 256×256 . Firstly, the chaotic mask $M(u, v)$ is rearranged according to the pixel value, which can be expressed as follows

$$\begin{cases} M'(u_t, v_k) = \text{sort}[|M(u_i, v_j)|] & i, j, t, k = 1, 2, 3, \dots, 256 \\ I'(x_t, y_k) = \text{array}[M'(u_t, v_k)] & t, k = 1, 2, 3, \dots, 256 \end{cases} \quad (3)$$

where the parameters ' i ', ' j ', ' t ' and ' k ' represent the pixel position before and after the rearrangement. The symbol $||$ indicates the pixel values. In addition, ' sort ' and ' array ' denote the operation of rearrangement by size and by the specified position. Therefore, the original image $I(x, y)$ can be scrambled by the position of the random chaotic mask.

In the following step, two threshold pixel value V_1 and V_2 in the chaotic mask $M'(u_t, v_k)$ are selected randomly. According to the position corresponding to the threshold V_1 and V_2 , the scrambled image $I'(x, y)$ is divided into three sections. Finally, the pixels in these sections are synchronously arranged in parallel and vertical, as depicted in Fig.2. The mathematical definition of the scrambling pixels position operation can be written as follows

$$\begin{cases} I_1(x, y) = I'(x_t, y_k), & |M(u_t, v_k)| \leq V_1 \\ I_2(x, y) = I'(x_t, y_k), & V_1 < |M(u_t, v_k)| \leq V_2 \\ I_3(x, y) = I'(x_t, y_k), & |M(u_t, v_k)| > V_2 \end{cases} \quad (4)$$

where $I_1(x, y)$, $I_2(x, y)$ and $I_3(x, y)$ are the three sections of the scrambling image according to the threshold V_1 and V_2 , respectively.

Fig.2 Schematic diagram of the scrambling operation.

2.2 Random modulus decomposition

Different from the equal modulus decomposition, random modulus decomposition can highly reduce the **constraints** since the modulus are not identical according the decomposition theory. The RMD is a kind of unequal modulus decomposition, in which the spectrum is randomly divided into two complex value masks [33]. Therefore, one **two-dimensional** image can be divided into two statistically independent **masks** randomly. To simplify the illustration, one complex number in **two-dimensional** Cartesian coordinate is considered as a **vector** I and it is divided into two vectors randomly as shown in Fig.3.

Fig.3 Schematic (vector) representation of the decomposition process in RMD.

Suppose that $I(u,v)$ in Fig.3 is an image, the function $\alpha(u,v)$, $\beta(u,v)$ and $\varphi(u,v)$ can be expressed as equation

$$\begin{cases} \alpha(u, v) = 2\pi \times \text{rand}(u, v) \\ \beta(u, v) = 2\pi \times \text{rand}(u, v) \\ \varphi(u, v) = \arg\{I(u, v)\} \end{cases} \quad (5)$$

where the function $\text{rand}(u, v)$ denotes a random matrix satisfying normal distribution in the interval $[0, 1]$ and the symbol ‘arg’ represents the argument function. Besides, the amplitude of $I(u, v)$ is given by $A(u, v) = |I(u, v)|$. According to Fig.3, the result of the decomposition masks $P1(u, v)$ and $P2(u, v)$ can be obtained by geometrical deduction as follows

$$P1(u, v) = \frac{A(u, v) \sin[\beta(u, v)]}{\sin[\alpha(u, v) + \beta(u, v)]} \exp\{i[\varphi(u, v) - \alpha(u, v)]\}, \quad (6)$$

and

$$P2(u, v) = \frac{A(u, v) \sin[\alpha(u, v)]}{\sin[\alpha(u, v) + \beta(u, v)]} \exp\{i[\varphi(u, v) + \beta(u, v)]\}. \quad (7)$$

where the $A(u, v)$ and $\varphi(u, v)$ are the amplitude and phase of $I(u, v)$, respectively. In addition, $\theta(u, v)$ is a random function distributed uniformly in interval $[0, 2\pi]$. Besides, the phase function $\alpha(u, v)$ and $\beta(u, v)$ are given by $\alpha(u, v) = 2\pi \times \text{rand}(u, v)$ and $\beta(u, v) = 2\pi \times \text{rand}(u, v)$ as illustrated in Eq.5, respectively.

2.3 Optical gyrator transform

In signal processing filed, the optical gyrator transform can be regarded as a kind of linear canonical integral transform which produces the twisted rotation in position-spatial frequency planes of phase space [45]. In optical implementation, a coherent optical system consisting of six thin **cylinder lenses** can be considered to realize the optical gyrator propagation [46-47]. In fact, the gyrator transform has only **two-dimensional** format and the mathematical definition of gyrator transform for the function $I(x, y)$ can be expressed as

$$\begin{aligned} F(u, v) &= G^\alpha[I(x, y)](u, v) \\ &= \frac{1}{|\sin \alpha|} \iint I(x, y) \exp[i2\pi \frac{(xy + uv) \cos \alpha - xv - yu}{\sin \alpha}] dx dy, \end{aligned} \quad (8)$$

where $I(x, y)$ and $F(u, v)$ represent the input and output of gyrator transform, respectively. x and y are the input coordinates at spatial domain, while u and v denote the output coordinates in gyrator domain. Besides, the parameter ‘ α ’ is the rotation angle of gyrator transform, which usually serves as an additional key in encryption system. **When $\alpha = \pi/2$, the gyrator transform reduces Fourier transform with the rotation of coordinates at $\pi/2$.** Finally, the inverse transform of gyrator transform with rotation angle α is transform with rotation angle $-\alpha$ or $2\pi - \alpha$. Also,

the gyrator transform has the property of index additive and periodic with the parameter α . The optical gyrator transform is utilized to complete the proposed asymmetric cryptosystem.

2.4 Asymmetric cryptosystem

The flowchart of the **intact** proposed cryptosystem is depicted in Fig.4. To implement the proposed cryptosystem, the 4D four-winged hyperchaotic system, optical gyrator transform, RMD and pixels scramble operation are utilized. As shown in Fig.4, the hyperchaotic system **described in Section 2.1** is performed to generate the chaotic data. Part of the chaotic data is encoded into the RGB format which having the same size of the original color image. Note that the chaotic data used here serves as the public key of the proposed encryption system. The secret image is combined with the public key and then encoded into **two-dimensional** format. Subsequently, the intermediate variables are imported into the input plane of gyrator transform and propagate in the gyrator domain.

In the following step, the result of the gyrator transform is **analytically decomposed into two masks P_1 and P_2 by performing RMD**. Finally, with the help of chaotic data and two threshold, the pixels position scrambling operation described in Section 2.1 is performed with P_1 and P_2 synchronously to obtain the ciphertext and private key. The final ciphertext and private key **are both** color random mask as displayed in Fig.4. What is more, the initial condition of the hyperchaotic system, parameter α in gyrator and two **thresholds V_1 and V_2** can be regarded as the extra keys of the encryption scheme to **enhance** the security. Accordingly, the private key and all the additional keys are **required to retrieve** the secret image. In the next section, various experiments are made to test the validity of the private and extra keys in retrieve approach.

Fig.4 The flowchart of the asymmetric cryptosystem

Since the proposed cryptosystem is **asymmetric** scheme, the ciphertext cannot be recovered without the private key and public key. **At the first step of the decryption process, the ciphertext and private key are calculated by the inverse pixels scrambling operation and RMD with two correct thresholds V_1 and V_2 , respectively.** Subsequently, the retrieval $P1$ and $P2$ are transmitted into two spatial light modulator (SLM) and then two beams interfere in a beam splitter (BS). After the beams propagated from the inverse gyrator lenses, the amplitudes and phases are recorded in the output plane and finally decoded back to RGB format image.

The encryption/decryption processes of the intact cryptosystem can be implemented by the electro-optical setups as depicted in Fig.5(a) and Fig.5(b), respectively. The optical gyrator transform and its inverse version are achieved in gyrator lenses system as illustrated in Fig.5(a) [47]. Furthermore, the calculation of hyperchaotic system and scrambling operations is accomplished in the computer. Furthermore, the data communication between the computer and optical system is **achieved** by SLM and CCD as shown in Fig.5. In the output plane of the gyrator transform, the CCD and in-line holography technique is introduced to record the amplitude and phase, respectively. Note that, different from the symmetric encryption algorithm, the decryption process of asymmetric scheme cannot **fulfill** by the inverse version of the encryption process. At the aspect of decryption approach of our proposed cryptosystem, the input plane of the electro-optical setups should be changed to two SLM for modulating the private and ciphertext **synchronously** and then two beams interfere in the **beam splitter (BS)** as displayed in Fig.5(b). The beams, then, transformed by inverse gyrator transform and received by CCD at the output plane. It is worth note that the proposed algorithm can serve in the hardware-based cryptographic.

Fig.5 The electro-optical setup used for the proposed cryptosystem: (a) encryption setup and (b) decryption setup.

3. Numerical simulation

Various numerical simulations are made to testify the security and robustness of the proposed cryptosystem in this section. A test color image ‘Lena’ having 256×256 pixels is considered as the original secret image to be encrypted, which is displayed in Fig.6(a). In addition, the **two-dimensional** encoded image combined with the original color image and the chaotic data is illustrated in Fig.6(b), which has the size of 384×512 . Accordingly, the image can be propagated and transformed in the single channel optical gyrator domain. **At the aspect of chaotic data generated by the hyperchaotic system**, the data format is encoded according to the original image. Therefore, after the newly designed scrambling pixels position operation, the function will be converted back to RGB format. Simultaneously, in calculation the initial condition of the hyperchaotic system is set as $x_0 = y_0 = z_0 = u_0 = -10$ as mentioned before. Besides, the rotation angle in gyrator transform and the two thresholds in scrambling algorithm, which are serve as the extra keys are set as 0.8, -8.1331 and -10.4342 in simulation. The responding ciphertext by using the condition described above is **displayed** in Fig.7(a), which is random pattern. **Concerning to the decrypted process**, the secret image can be retrieval only if the decryption setup and all the keys are correct. By using the proper parameters and keys, the decrypted image is depicted in Fig.7(b), which is identical to the original image in human vision. **It is worth note that** the decrypted image contains speckle when we used the virtual optical system because of the system noise [19]. The result illustrated in Fig.7(b) **is executed with a digital system**. Moreover, the quantitative analysis will be presented in the following subsection.

Since the result of gyrator transform is complex function, the simulation results shown in this paper is amplitude part. In calculation, the experiments results are executed by a computer with 2 Gigabytes memory under Windows 7 system.

Fig.6 (a)The original color image test in the experiments and (b) the encoded image combined with the original image and chaotic data.

Fig.7 The encryption/decryption results: (a) the ciphertext and (b) decrypted result.

Apparently, the secret image cannot be recovered by the inverse encryption approach since the RMD leads to the effective trapdoor one-way function, which highly improves the security of the cryptosystem in resisting the attack of iterative algorithm. One can decrypt the original image by employing the setup illustrated in Fig.5(b) with the **ciphertext, private key and all the extra keys**. Some decryption attempts with the incorrect keys or missing part of the keys are considered to test the proposed scheme in the following subsection. First of all, to distinguish the difference degree between the original and decrypted image, **the peak-signal-to-ratio (PSNR)** function is introduced. The mathematical definition of PSNR can be written as follows as

$$PSNR(I_o, I_d) = 10 \times \log_{10} \frac{255^2 M \times N}{\sum_{\forall x, y} [I_d(x, y) - I_o(x, y)]^2} \text{ (dB)}. \quad (9)$$

where the input function $I_o(x, y)$ and $I_d(x, y)$ represent the original image and its decrypted pattern, respectively. Furthermore, the parameters M and N stand for the size in input

functions. In simulation, the large PSNR value indicates higher similarity between the two input images. Note that the PSNR function will be used to verify the sensitivity of the additional key.

The capability of the private key on protecting the original image is test first. In fact, it is impossible to retrieve the secret image without the private key or using an incorrect key due to its strong randomness. Firstly, two decryption experiments are executed to show the recovered results by using incorrect private key and without private key, respectively. Here, we suppose that the intact encryption/decryption processes are known by the attacker and the cryptosystem is attack using a wrong private key. In calculation, we use a random matrix satisfying uniform distribution in the interval $[0, 1]$ instead of the private key. The corresponding retrieval results are displayed in Fig.8(a) and Fig.8(b), respectively. According to the noise-like images depicted in Fig.8, the original color image is completely under protection. Some other random data are also utilized to decrypt the image and obtain the similar results.

Fig.8 The decrypt results: (a) using incorrect private key and (b) missing private key.

Concerning to the additional key, the security of the thresholds designed in the pixels scrambling algorithm are also examined. As mentioned above, two pixel position 60000 and 110000 are chosen and the corresponding pixels value (thresholds) are $V_1 = -8.1331$ and $V_2 = -10.4342$. Subsequently, the encrypted data is attempted to decrypt by using wrong thresholds and correct keys (including the private key and extra keys). The corresponding decrypted image is illustrated in Fig.9(a), which is blurred image and the secret information is under protected. Moreover, a worse situation is considered. We suppose that the illegal user filch all the private key and extra key, and obtain the two false threshold positions 59999 and 109999 which are very close to the 60000 and 110000. Under these circumstances, the corresponding recovered image is depicted in Fig.9(b), which is a random-like image even the positions are very close to the real one. Apparently, the noise pattern indicates the high security and sensitivity of the thresholds in protecting the information.

Fig.9 The decrypt results: (a) using incorrect thresholds and (b) using false threshold positions which are very close to the real one.

Furthermore, as one of the additional key of the cryptosystem, the contribution of the rotation angle α in optical gyrator transform is also test. When all the other keys are known by the attacker, the extra key α is test in the **decryption process**. In this experiment, the decryption approach is implemented 141 times by using variable α and the corresponding PSNR curve between the secret image and decrypted image is displayed in Fig.10. In calculation, the rotation angle α is changed from 0.5 to 1.2 with the **sampling step** length 0.005. Apparently, the parameter α is **sensitive enough** as an extra key to protect the original image. Two decrypted image by using incorrect keys $\alpha=0.795$ and $\alpha=0.805$ are illustrated in Fig.10, which involved severe noise even the keys are very close to the correct one. Besides, suppose the **illegal user** try to decrypt the image by using a random key, $\alpha=0.595$ and $\alpha=0.995$ for example, the **attacked results** are blurred images and the secret information cannot be recognized completely. The original image is perfectly decrypted when the rotation angle α equal to 0.8.

At the aspect of the contribution of hyperchaotic data, an experiment is design to test the sensitivity of the initial condition. As mentioned above, a minor change of the initial condition in the hyperchaotic system will cause huge and unpredictable change of the attractors. To quantify this unpredictable change caused by the attractors, the **Normalized cross correlation (NCC)** is considered and utilized. Firstly, the definition of NCC can be written as

$$NCC(I_o, I_d) = \frac{\sum I_o(x, y) \square_d(x, y)}{\sqrt{\sum I_o(x, y) \square_o(x, y) \times \sum I_d(x, y) \square_d(x, y)}} \quad (10)$$

where the input functions I_o and I_d are the original image and decrypted image. The value of NCC between two images indicates the image similarity and the higher NCC manifests higher similarity at the range of [0, 1].

In the following experiment, the performance of the initial conditions of the hyperchaotic system is checked. Firstly, the secret image is encrypted by using specify initial conditions described in Section 2.1. Subsequently, the encrypted data is attacked by the tampered chaotic

data generated by different initial conditions, which are very close to real values. Finally, the fake **initial conditions and the NCC values** between the attacked result and the original image are list in the Table.1, respectively. The tiny change of the conditions makes great difference of the decrypted image which implies the information is well protect. The experiment results displayed in Table 1 demonstrates the high sensitive of the parameters in hyperchaotic system. In fact, each parameters of the hyperchaotic system can serve as an independent key because any change of any condition cause huge difference as shown in Table.1. Furthermore, to illustrate the similarity between the decrypted image and the original image, a normalized cross correlation mesh between the correct decrypted image and original image is drawn in Fig.11. **In fact, the normalized cross correlation mesh can be used for image matching, that is, searching for an image with the highest NCC area of a known area as a corresponding match, and then aligning the entire image. The value of NCC equal to 1 indicates the high similarity between these two images. To drawn the mesh, one image is traversing to other image, therefore, the size of the normalized cross correlation mesh is 512×512. The narrow peak of the cross-correlation matrix occurs indicates the high quality of decryption.**

Fig.10 The PSNR curve calculated by variable rotation angle α in gyrator transform.

Table.1 The decryption experiments by using tampered initial conditions.

x_0	y_0	z_0	u_0	$PSNR$	NCC
-10	-10	-10	-10	271.3	1
-9.9999	-10	-10	-10	7.6382	0.1010
-10	-9.9999	-10	-10	9.7318	0.2111
-10	-10	-9.9999	-10	7.2174	0.1003
-10	-10	-10	-9.9999	8.0941	0.0912

Fig.11 The normalized cross-correlation mesh between the decrypted image and original image.

The occlusion attacks are considered to analyze the robustness of the proposed asymmetric cryptosystem. An original image is encrypted firstly and then the corresponding ciphertext is **occluded partly**. Subsequently, the occluded ciphertext is **decrypted** by using all the correct keys (including the private key and additional key) and decryption approach. In calculation, the ciphertext is attacked by occlusion with $1/16$ and $1/8$, respectively. Note that the occluded part of the ciphertext will be replaced by value 0 **in simulation**. Under the circumstances mentioned above, the corresponding attack results of the occluded ciphertext are obtained and illustrated in Fig.12(a) and Fig.12(b). According to the decrypted images, the main information and **silhouette** of the secret image can be recognized in vision, which indicates the well performance of the proposed scheme on resisting the occluded attack. **Note that the decrypted image with larger occlusion part has blurrier attack result as depicted in Fig.12.**

Fig.12 The test of occlusion attack: (a) attack result with $1/16$ occlusion and (b) attack result with $1/8$ occlusion.

4. Conclusion

We have presented a single-channel optical color asymmetric cryptosystem by using 4D hyperchaotic system and RMD in gyrator transform domain. The effective trapdoor one-way function generated by RMD extirpates the constraints used for potential attack algorithm. Various highly unpredictable chaotic **sequences generated** by 4D four-winged hyperchaotic system are utilized in the encoding **operation** to enhance the security. At the aspect of decryption approach, the high sensitive initial conditions in hyperchaotic system and rotation angle in gyrator transform can be regarded as the additional keys to guarantee the secret information. **Various potential attack schemes** are considered in the numerical attack experiments and the results have verified the effectiveness, security and robustness of the proposed **asymmetric** cryptosystem.

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