OATAO
Open Archive Toulouse Archive Ouverte

Open Archive Toulouse Archive Ouverte (OATAO)

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in: http://oatao.univ-toulouse.fr/20025

Official URL: https://doi.org/10.1016/S1474-6670(17)38992-9

To cite this version:

Rotella, Frédéric and Carrillo, Francisco Javier and Ayadi, Mounir *Polynomial controller design based on flatness.* (2001) IFAC Proceedings Volumes, 34 (13). 213-218. ISSN 2405-8963

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

POLYNOMIAL CONTROLLER DESIGN BASED ON FLATNESS

F. Rotella, F.J. Carrillo, M. Ayadi

LGP-ENIT, 47 av. d'Azereix, BP48,
65016 Tarbes CEDEX, France.
Email: {rotella, carrillo, ayadi}@enit.fr

Abstract: By the use of flatness the problem of pole placement, which consists in imposing closed loop system dynamics can be related to tracking. Polynomial controllers for finite-dimensional linear systems can then be designed with very natural choices for high level parameters design. This design leads to a Bezout equation which is independent of the closed loop dynamics but depends only on the system model.
Copyright © 2001 IFAC

Keywords: Flatness, two-degrees-of-freedom controllers, linear systems, polynomial controllers.

1. INTRODUCTION

For finite-dimensional linear systems, a well-known control design technique is constituted by polynomial two-degrees-of-freedom controllers (Aström, and Wittenmark, 1990; Franklin, *et al.*, 1998; Kučera, 1991), which have been introduced forty years ago by Horowitz (1963). Whatever the chosen design method, this powerful method is based on pole placement and presents one deficiency: it needs to know *a priori* where to place all the poles of the closed loop system. Following (Aström *et al.*, 1991): "the key issue is to choose the closed loop poles. This choice requires considerable insight...". This can be done, for instance, through LQR design, but the problem is then replaced by the correct choice of the weighting matrices in the cost functions.

In order to overcome the drawback of this design technique, it will be seen, in the following, that the use of a new method for system control, namely with a flatness point of view, enlightens the choice of the high level parameters and brings physical meanings to obtain a clear guideline for polynomial pole placement design. Following (Fliess, *et al.*, 1995; Fliess, *et al.*, 1999), flatness is a very interesting property of processes to design a control, specially for trajectory planning and tracking for nonlinear systems.

The paper is organized as follows. Section 2 and 3 are devoted to survey very quickly, the design of polynomial controllers for the first one and, for the second one, the flatness property and the control design implied for a flat system. At the end of the section 3, a methodology for the control of flat systems is proposed and illustrated on finite-dimensional linear systems in the section 4. This point of view leads to propose a flatness-based two-degrees-of-freedom controller which is realized in the section 5.

In the following, the paper will be developed in a continuous time formulation, but all the developments can be extended to discrete time linear systems. For $n \in \mathbb{N}$, the following notations will be used , $u^{(n)}(t) = \frac{d^n u(t)}{dt^n} = p^n u(t)$, where p denotes the differential operator.

The paper will also be developed, for the sake of shortness, for SISO linear systems, but all the results can be adapted to MIMO linear systems.

2. POLYNOMIAL CONTROLLERS

This section offers a short description of the design principles of the polynomial two-degrees-of-freedom controllers for linear systems. More details are given in (Aström, and Wittenmark, 1990; Franklin, *et al.*, 1998; Kučera, 1991) and the references therein, and in the following these controllers will be denoted as RST controllers (Landau, 1993).

Consider the finite-dimensional SISO linear system described by the input-output model:

$$Ay = Bu, \tag{1}$$

where y and u are the output and control signals, A is monic and A and B are coprime polynomials.

For (1), the RST (two-degrees-of-freedom) controller (Aström and Wittenmark, 1990) is given by:

$$Ru = -Sy + Tr, \tag{2}$$

where r is the reference to track, and R, S and T are polynomials in the considered operator. These polynomials are given by the following rules: R and S are solutions of the Diophantine (or Bezout) equation:

$$P = AR + BS, \tag{3}$$

where the roots of the polynomial P are constituted by the desired closed loop and observer poles, and P and R are monic; T is given by the desired closed loop transfer such that:

$$PB_m = TBA_m. \tag{4}$$

When all these conditions are fulfilled, the closed loop behavior is obtained:

$$A_m y = B_m r. \tag{5}$$

Some remarks for the design:

(i) Denoting by $\deg P$ the degree of a polynomial P. For realizability conditions, the RST controller must be such that (Aström and Wittenmark, 1990):

$$\deg T \leq \deg R, \tag{6}$$

$$\deg S \leq \deg R. \tag{7}$$

(ii) Another remark is that it has been used, for the choice of T, the point of view developed in (Aström and Wittenmark, 1990), where (B_m, A_m) was a model-to-follow, but it can be also chosen the proposed one in (Landau, 1993), where r is given by:

$$A_m r = R_m \tag{8}$$

where (B_m, A_m) defines a trajectory-to-follow or a trajectory generator of $r(t)$. In this last point of view, T is designed such that:

$$TB = P. \tag{9}$$

(iii) For the implementation, the RST controller (2) must be written in the proper operator (p^{-1}) which leads to write the RST control as:

$$R^*(p^{-1})u(t) = -S^*(p^{-1})y(t) + T^*(p^{-1})r(t), \tag{10}$$

with $R^*(0) = 1$. As a remark it can be noted that all the design of a RST controller can be performed with all the polynomials written in realizable operators, as in (Landau, 1993), and in this case, if $A(0) = R(0) = P(0) = 1$, the realizability conditions (6) and (7) disappear.

3. SHORT SURVEY ON FLATNESS

The flat property, which has been introduced recently (Fliess, *et al.*, 1992; Fliess, *et al.*, 1993; Fliess, *et al.*, 1995) for continuous time nonlinear systems, leads to interesting points of view for control design. In the following, a short review about flatness of systems and the application of this property to design a controller will be given. The interested reader may find more details in the quoted literature and the references therein.

A system described by:

$$x^{(1)} = f(x, u), \tag{11}$$

where x is the state vector of dimension n, and u is the control vector, possesses the flatness property (or is flat) if there exists a vector z:

$$z = h(x, u, u^{(1)}, \ldots, u^{(\alpha)}), \tag{12}$$

where $\alpha \in \mathbb{N}$, such that the components of z are differentially independent, and such that there exist two functions $\mathbf{A}(.)$ and $\mathbf{B}(.)$, and an integer β such that:

$$x = \mathbf{A}(z, \ldots, z^{(\beta)}), \tag{13}$$

$$u = \mathbf{B}(z, \ldots, z^{(\beta+1)}). \tag{14}$$

The selected output z is called a flat output and, obviously, there is no uniqueness. But, as it has been observed on numerous examples, the flat output has a simple and physical meaning.

Roughly speaking, the implications of flatness are of very importance in several ways for control. For motion planning, by imposing a desired trajectory on the flat output, the necessary control to generate the trajectory, can be obtained explicitly (without any integration of the differential equations). The desired trajectory, $z_d(t)$, must be $(\beta+1)$-times continuously differentiable. For feedback

control which only ensures a good stabilization around the desired motion $z_d(t)$.

All these points, which have been formalized through the Lie-Bäcklund equivalence of systems in (Fliess, *et al.*, 1993; Fliess, *et al.*, 1999), lead to propose a nonlinear feedback which ensures a stabilized tracking of a desired motion for the flat output. This methodology has been applied on many industrial processes as it has been shown previously, for instance, on magnetics bearings (Lévine *et al.*, 1996), chemical reactors (Rothfuß *et al.*, 1996), cranes or flight control (Lévine, 1999) or turning process (Rotella, and Carrillo, 1998; Rotella, and Carrillo, 1999), among many other examples.

To be more precise the use of flatness leads to the following methodology to design a control. The main objective of this control is to insure an asymptotic tracking of a desired trajectory and can be ensured through the following steps :

(i) Explicit the flatness: namely, the analytic expressions are obtained here:

$$z = h(x, u, u^{(1)}, \dots, u^{(\alpha)}), \tag{15}$$

$$x = \mathbf{A}(z, \dots, z^{(\beta)}), \tag{16}$$

$$u = \mathbf{B}(z, \dots, z^{(\beta+1)}), \tag{17}$$

where u is the chosen control variables, and x, the whole set of internal variables. It must be noted here that the relationship which gives x will not be used in the following, but it is necessary to confirm z as a flat output. Indeed, if one of the internal variable is not defined by z, then z is not a flat output.

(ii) Linearization: by the control:

$$u = \mathbf{B}(z, \dots, z^{(\beta)}, v), \tag{18}$$

where v is a new control, the linear system $z^{(\beta+1)} = v$ is obtained. It must be noted here that this step is an intermediate one and must be followed by the next ones.

(iii) Motion planning: it consists in the design of a trajectory defined by $z_d(t)$, which must be differentiable at the order $(\beta + 1)$.

(iv) Motion tracking: by the control:

$$v = z_d^{(\beta+1)}(t) + \sum_{i=0}^{\beta} k_i (z_d^{(i)}(t) - z^{(i)}(t)), \tag{19}$$

where the k_i ensure that the polynomial $K(p) = p^{\beta+1} + \sum_{i=0}^{\beta} k_i p^i$ is Hurwitz, the complete control is then as follows:

$$u = \mathbf{B}(z, \dots, z^{(\beta)}, z_d^{(\beta+1)}(t) + \sum_{i=0}^{\beta} k_i (z_d^{(i)}(t) - z^{(i)}(t))) = \Phi(z, \dots, z^{(\beta)}, K(p) z_d(t)), \tag{20}$$

which leads to an asymptotic tracking of the desired trajectory.

It must be noted here that the information needed by this control can be obtained through observers, and a major advantage of this controller with respect to other nonlinear strategies is that it overcomes the problems generated by non stable zeros dynamics (Isidori, 1989; Nijmeijer, and Van der Schaft, 1990).

4. IMPLICATION FOR LINEAR SYSTEMS: TOWARDS RST CONTROLLERS

Despite the fact that flatness has been firstly developed for nonlinear systems, it has been applied to finite-dimensional linear systems (Bitaud, *et al.*, 1997; Fliess, *et al.*, 1998) and extended for infinite-dimensional ones (Fliess and Mounier, 1998). It will be seen, in this section, that applying the guideline induced by a flatness based control to a linear system leads to express it in a natural RST form.

The previous methodology will be applied now to a linear lumped parameter SISO system defined by the transfer:

$$A(p)y(t) = B(p)u(t), \tag{21}$$

where the notations have been previously defined but with:

$$A(p) = p^n + \sum_{i=0}^{n-1} a_i p^i = p^n + A^*(p), \tag{22}$$

$$B(p) = \sum_{i=0}^{n-1} b_i p^i. \tag{23}$$

From coprimeness, it has been shown in (Bitaud, *et al.*, 1997; Fliess, *et al.*, 1999), that this system is flat with a flat output defined by:

$$z(t) = N(p)y(t) + D(p)u(t), \tag{24}$$

where $N(p)$ and $D(p)$ are the polynomial solutions of the following Bezout equation:

$$N(p)B(p) + D(p)A(p) = 1. \tag{25}$$

Due to coprimeness, existence of $N(p)$ and $D(p)$ are guaranteed and the minimum degree solution is, for $n > 1$, such that $\deg N = n - 1$ and $\deg D = n - 2$.

The explicit expressions of the output $y(t)$ and the control $u(t)$ are given by:

$$u(t) = A(p)z(t), \tag{26}$$

$$y(t) = B(p)z(t). \tag{27}$$

which allows to relate the flat output of a linear system to the partial state defined by several authors (Kailath, 1980).

Following the step (iv) of the methodology, the control is given by:

$$u(t) = v(t) + A^*(p)z(t), \tag{28}$$

where:

$$v(t) = z_d^{(n)}(t) + \sum_{i=0}^{n-1} k_i(z_d^{(i)}(t) - z^{(i)}(t)). \tag{29}$$

and by introducing the polynomials:

$$K(p) = p^n + \sum_{i=0}^{n-1} k_i p^i = p^n + K^*(p), \tag{30}$$

the control $u(t)$ is given by:

$$u(t) = K(p)z_d(t) + [A^*(p) - K^*(p)]\, z(t). \tag{31}$$

Taking into account that $z(t) = N(p)y(t) + D(p)u(t)$, then it can be written:

$$u(t) = K(p)z_d(t) + [A^*(p) - K^*(p)] \\ [N(p)y(t) + D(p)u(t)], \tag{32}$$

which leads to:

$$[1 - [A^*(p) - K^*(p)]\, D(p)]\, u(t) \\ = K(p)z_d(t) + [A^*(p) - K^*(p)]\, N(p)y(t). \tag{33}$$

This appears as a RST controller form with:

$$R(p) = 1 - [A^*(p) - K^*(p)]\, D(p), \tag{34}$$

$$S(p) = -[A^*(p) - K^*(p)]\, N(p), \tag{35}$$

with the difference that here the trajectory to follow is directly integrated to the controller with the term $K(p)z_d(t)$. An important property of this controller can be also deduced, due to the fact that $P = AR + BS$. From the previous definitions of $R(p)$ and $S(p)$, and with the help of $N(p)B(p) + D(p)A(p) = 1$, and $A^*(p) - K^*(p) = A(p) - K(p)$, it follows:

$$A(p)R(p) + B(p)S(p) = K(p). \tag{36}$$

From (36), it is then obtained that the closed loop dynamics of the RST controllers are those designed for the tracking of the desired flat output trajectory. The choice of these poles is then lighted. But as:

$$\deg(1 - [A^* - K^*]\, D) = \deg([A^* - K^*]\, N) - 1, \tag{37}$$

it is not realizable. The realization of this controller will be the subject of the next part.

5. REALIZATION

To implement the control (31), it can be used an observer of the vector $Z = \left[\, z(t) \;\; \dots \;\; z^{(n-1)}(t) \,\right]^T$ which is the state vector of the controllable Luenberger realization of $u(t) = A(p)z(t)$, $y(t) = B(p)z(t)$, namely:

$$Z^{(1)} = AZ + bu, \tag{38}$$

$$y = cZ, \tag{39}$$

where:

$$A = \begin{bmatrix} & 1 & & \\ & & \ddots & \\ & & & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \\ c = \left[\, b_0 \;\; b_1 \;\; \cdots \;\; b_{n-1} \,\right]. \tag{40}$$

A (full order) observer of Z is given by:

$$\hat{Z}^{(1)} = (A - \Gamma c)\hat{Z} + bu + \Gamma y, \tag{41}$$

where Γ is chosen such that the eigenvalues of $F = A - \Gamma c$ are with negative real part. This leads to:

$$\hat{Z} = (pI - F)^{-1}(bu + \Gamma y) \tag{42}$$

By introducing $a = \left[\, a_0 \;\; a_1 \;\; \cdots \;\; a_{n-1} \,\right]$ and $k = \left[\, k_0 \;\; k_1 \;\; \cdots \;\; k_{n-1} \,\right]$, the control (31) is implemented by:

$$u(t) = K(p)z_d(t) + (a - k)\hat{Z}(t), \tag{43}$$

as in (Fliess *et al.*, 1998). But, in this solution the difficulty is the choice of the observer poles. To overcome this point the enlightening ideas suggested in (Fliess, 2000) and applied in (Marquez *et al.*, 2000) can be used. In the one hand, from (Kailath, 1980):

$$Y = O_{(A,c)}Z + M_{(A,b,c)}U, \tag{44}$$

where $Y = \left[\, y \;\; y^{(1)} \;\; \cdots \;\; y^{(n-1)} \,\right]^T$,

$U = \left[\, u \;\; u^{(1)} \;\; \cdots \;\; u^{(n-2)} \,\right]^T$, $O_{(A,c)}$ is the observability matrix:

$$O_{(A,c)} = \begin{bmatrix} c \\ cA \\ \vdots \\ cA^{n-1} \end{bmatrix} \tag{45}$$

and $M_{(A,b,c)}$ is given by:

$$M_{(A,b,c)} = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ cb & \ddots & & \vdots \\ cAb & cb & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ cA^{n-2}b & \cdots & cAb & cb \end{bmatrix}. \tag{46}$$

From this equation, and due to the fact that $A(p)$ and $B(p)$ are coprime, thus rank $O_{(A,c)} = n$, it becomes:

$$Z = O_{(A,c)}^{-1} \left\{ Y - M_{(A,b,c)} U \right\}. \quad (47)$$

As the first component of Z is $z(t)$, it can be seen that the first line gives the flat output expressed in terms of the derivatives of $y(t)$ and $u(t)$. Namely, a solution of the Bezout identity (25) is obtained with:

$$N(p) = \left[1\ 0\ \cdots\ 0 \right] \times O_{(A,c)}^{-1} \mathrm{diag} \left\{ 1, p, \ldots, p^{n-1} \right\}, \quad (48)$$

$$D(p) = -\left[1\ 0\ \cdots\ 0 \right] O_{(A,c)}^{-1} \times M_{(A,b,c)} \mathrm{diag} \left\{ 1, p, \ldots, p^{n-2} \right\}. \quad (49)$$

In the other hand, from (Fliess, 2000):

$$\forall \mu \in N,\ Z = A^{\mu} p^{-\mu} Z + \sum_{i=1}^{\mu} A^{i-1} b p^{-i} u, \quad (50)$$

where p^{-1} stands for the integration operator:

$$p^{-1} x(t) = \int_{-\infty}^{t} x(\tau) d\tau, \quad (51)$$

with $x(-\infty) = 0$. This last hypothesis ensures commutativity between p and p^{-1}. As a particular case, it comes that for $\mu = n - 1$:

$$Z = A^{n-1} p^{-(n-1)} Z + \sum_{i=1}^{n-1} A^{i-1} b p^{-i} u, \quad (52)$$

then, by combining (47) and (52), it follows:

$$Z = A^{n-1} O_{(A,c)}^{-1} p^{-(n-1)} Y - A^{n-1} O_{(A,c)}^{-1} M_{(A,b,c)} p^{-(n-1)} U + \sum_{i=1}^{n-1} A^{i-1} b p^{-i} u. \quad (53)$$

By replacing this expression in the control (31), it follows the control:

$$u(t) = K(p) z_d(t) - S^*(p^{-1}) y(t) - Q^*(p^{-1}) u(t), \quad (54)$$

where:

$$S^*(p^{-1}) = [k - a]\, A^{n-1} O_{(A,c)}^{-1} \Pi, \quad (55)$$

$$Q^*(p^{-1}) = [a - k] \times \left\{ \begin{array}{c} A^{n-1} O_{(A,c)}^{-1} M_{(A,b,c)} \\ -\left[A^{n-2} b \ \cdots\ b \right] \end{array} \right\} \times \Pi^*, \quad (56)$$

with $\Pi = \left[p^{-(n-1)}\ p^{-(n-2)}\ \cdots\ p^{-1}\ 1 \right]^T$ and $\Pi^* = \left[p^{-(n-1)}\ p^{-(n-2)}\ \cdots\ p^{-1} \right]^T$. By denoting $R^*(p^{-1}) = 1 + Q^*(p^{-1})$, this control can be written in the RST form:

$$R^*(p^{-1}) u(t) = K(p) z_d(t) - S^*(p^{-1}) y(t). \quad (57)$$

As a remark, it follows also, with $h = \left[1\ 0\ \cdots\ 0 \right]$:

$$z(t) = h \left\{ A^{n-1} p^{-(n-1)} Z + \sum_{i=1}^{n-1} A^{i-1} b p^{-i} u \right\}, \quad (58)$$

thus:

$$z(t) = h \left\{ A^{n-1} O_{(A,c)}^{-1} \begin{bmatrix} p^{-(n-1)} y \\ p^{-(n-2)} y \\ \vdots \\ y \end{bmatrix} \right\} + h \left\{ \sum_{i=1}^{n-1} A^{i-1} b p^{-i} u \right\} - h \left\{ A^{n-1} O_{(A,c)}^{-1} M_{(A,b,c)} \begin{bmatrix} p^{-(n-1)} u \\ p^{-(n-2)} u \\ \vdots \\ p^{-1} u \end{bmatrix} \right\}, = N^*(p^{-1}) y(t) + D^*(p^{-1}) u(t), \quad (59)$$

which defines the flat output in terms of the proper operator p^{-1}.

6. CONCLUSION

This paper showed that the use of a flatness point of view allows a simplification in the design of high level parameters of RST controllers. The main feature of the flatness approach for RST controller design is to avoid the problem of the closed loop poles choice which are constituted of the observer poles and those obtained with a state feedback (Aström and Wittenmark, 1990). Now the design is focused in the choice of the trajectory z_d to follow and the tracking dynamics with $K(p)$. These developments were done in a continuous time framework, but are transposable for discrete time systems.

In the case where a constant output perturbation, for instance, is to be rejected, an integral action must be added in R. This can be achieved by multipliing the original A by p which gives a new denominator from which the method can be applied again. The robustness, by introducing fixed polynomials H_R and H_S as proposed in (Landau *et al.*, 1998), could be treated in the same way.

A direction of future works will be to attack the problem of the numerical implementation of such controllers as it has been underlined in (Rotella

and Carrillo, 1999) and to tackle with adapted implementations of the proposed control to overcome uncertainty in model parameters.

7. REFERENCES

Aström, K.J., B. Bernardson and A. Ringdhal (1991). Solution using robust adaptive pole placement. *ECC91*, pp. 2341–2346, Grenoble.

Aström, K.J. and B. Wittenmark (1990). *Computer Controlled Systems, Theory and design*, Prentice Hall.

Bitaud, L., M. Fliess and J. Lévine (1997). A flatness based control synthesis of linear systems and application to windshield wipers, *ECC'97*, Bruxelles.

Dubois. F., N. Petit and P. Rouchon (1999). Motion planning and nonlinear simulations for a tank containing a fluid, *ECC'99*, Karlsruhe.

Fliess, M. (2000). Sur des pensers nouveaux faisons des vers anciens, *CIFA2000*, pp. 26–36, Lille.

Fliess, M., J. Lévine, Ph. Martin and P. Rouchon (1992). Sur les systèmes non linéaires différentiellement plats, *C. R. Acad. Sci. Paris*, I-315, pp. 619-24.

Fliess, M., J. Lévine, Ph. Martin and P. Rouchon (1993). Linéarisation par bouclage dynamique et transformées de Lie-Backlund, *C. R. Acad. Sci. Paris*, I-317, pp. 981-6.

Fliess, M., J. Lévine, Ph. Martin and P. Rouchon (1995). Flatness and defect of nonlinear systems: introductory theory and applications, *Int. J. Control*, **61**, n. 6, pp. 1327-61.

Fliess, M., J. Lévine, Ph. Martin and P. Rouchon (1999). A Lie-Bäcklund approach to equivalence and flatness of nonlinear system, *IEEE Trans. Aut. Control*, **44**, pp. 922-937.

Fliess, M. and H. Mounier (1998). Controllability and observability of linear delay systems: an algebraic approach, *ESAIM: Control, Optimzation and Calculus of Variations*, **3**, pp. 301–314.

Fliess, M., H. Sira-Ramirez and R. Marquez (1998). Regulation of non minimum phase outputs: a flatness approach, *Perspectives in Control - Theory & Applications*, Coll. in honor of I.D. Landau, Paris, Springer-Verlag.

Franklin, G.F., J.D. Powell and M. Workman (1998). *Digital Control of Dynamic Systems*, Addison-Wesley.

Isidori, A. (1989). *Nonlinear Control Systems*, Springer-Verlag.

Horowitz, I.M. (1963). *Synthesis of Feedback Systems*.

Kailath, T. (1980). *Linear Systems*, Prentice-Hall.

Kučera. V. (1991). *Analysis and Design of Discrete Linear Control Systems*, Prentice Hall.

Landau, I.D. (1993). *Identification et Commande des Processus*, Hermès.

Landau, I.D., R. Lozano and M. M'Saad (1998). *Adaptive Control*, Springer-Verlag.

Laroche, B., P. Martin and P. Rouchon (1998). Motion planning for a class of partial differential equations with boundary control. *37th IEEE Conf. on Dec. and Control*, pp. 3494–3497, Tampa.

Lévine, J., J. Lottin and J.C. Ponsart (1996). A nonlinear approach to the control of magnetic bearings, *IEEE Trans. on Control Systems Techn.*, **4**, n. 5, pp. 524-44.

Lévine, J. (1999). Are there new industrial perspectives in the control of mechanical systems? In: *Advances in Control* (Paul M. Frank, Ed.), pp. 197–226, Springer.

Marquez, R., E. Delaleau and M. Fliess (2000). Commande par PID généralisé d'un moteur électrique sans capteur mécanique, *CIFA2000*, pp. 453–458, Lille.

Mikusiński, J. (1959). *Operational Calculus*, Pergamon Press.

Mounier, H., J. Rudolph, M. Fliess and P. Rouchon (1998). Tracking control of a vibrating string with an interior mass viewed as a delay system, *ESAIM: Control, Optimzation and Calculus of Variations*, **3**, pp. 315-321.

Nijmeijer H. and Van Der Schaft (1990). *Nonlinear Dynamical Control Systems*, Springer-Verlag.

Rotella, F. and F.J. Carrillo (1998). Flatness based control of a turning process, *CESA'98*, **1**, pp. 397–402, Hammamet, Tunisie.

Rotella, F. and F.J. Carrillo (1999). Flatness approach for the numerical control of a turning process, *ECC'99*, Karlsruhe.

Rotella, F. and I. Zambettakis (1996). Mikusiński operational calculus for distributed parameter systems, *IEEE-SMC Conf. CESA'96*, pp. 975–979.

Rothfuß, R., J. Rudolph and M. Zeitz (1996). Flatness based control of a nonlinear chemical reactor, *Automatica*, **32**, n. 10, pp. 1433-39.