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Lily Battershill, Colin N. Whittaker, Emily M. Lane, Stéphane Popinet, James D. L. White, et al.. Numerical simulations of a fluidized granular flow entry into water: insights into modeling tsunami generation by pyroclastic density currents. *Journal of Geophysical Research: Solid Earth*, 2021, 10.1029/2021jb022855 . hal-03419617

HAL Id: hal-03419617

<https://hal.science/hal-03419617>

Submitted on 8 Nov 2021

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1 Numerical simulations of a fluidized granular flow entry
2 into water: insights into modeling tsunami generation
3 by pyroclastic density currents

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13 **Key Points:**

- 14
- 15 • A Newtonian fluid model is able to reproduce the wave generation mechanism at
16 the interaction between a fluidized granular flow and water.
 - 17 • The basal shear stress has a first-order control on the interaction dynamics and
flow/wave energetics.

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This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process, which may lead to differences between this version and the [Version of Record](#). Please cite this article as [doi: 10.1029/2021JB022855](https://doi.org/10.1029/2021JB022855).

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Abstract

The tsunami generation potential of pyroclastic density currents (PDCs) entering the sea is poorly understood, due to limited data and observations. Thus far, tsunami generation by PDCs has been modeled in a similar manner to tsunami generation associated with landslides or debris flows, using two-layer depth-averaged approaches. Using the adaptive partial differential equation solver Basilisk and benchmarking with published laboratory experiments, this work explores some of the important parameters not yet accounted for in numerical models of PDC-generated tsunamis. We use assumptions derived from experimental literature to approximate the granular, basal flow component of a PDC as a dense Newtonian fluid flowing down an inclined plane. This modeling provides insight into how the boundary condition of the slope and the viscosity of the dense granular-fluid influence the characteristics of the waves generated. It is shown that the boundary condition of the slope has a first-order impact on the interaction dynamics between the fluidized granular flow and water, as well as the energy transfer from the flow to the generated wave. The experimental physics is captured well in the numerical model, which confirms the underlying assumption of Newtonian fluid-like behaviour in the context of wave generation. The results from this study suggest the importance of considering vertical density and velocity stratification in wave generation models. Furthermore, we demonstrate that granular-fluids more dense than water are capable of shearing the water surface and generating significant amplitude waves, despite vigorous overturning.

Plain Language Summary

When a volcano erupts, it ejects large quantities of volcanic rock, gas, ash and debris. These ejected materials can flow very rapidly down the side slopes of the volcano—these flows are called pyroclastic density currents (PDCs). When PDCs enter the sea, they displace water and can generate tsunami waves with enormous destructive potential. One method of understanding this potential is by mathematically modelling the flow and its interactions with water, and confirming these model results against laboratory data. The present study compares numerical model results with published laboratory experiments of PDC generated tsunamis, to understand how our assumptions about the flow and its motion along the boundary can affect the amount of energy transferred to the generated waves. We approximate a PDC generated tsunami as a dense, viscous fluid moving down a slope into water. The amount of friction on the slope and the properties of the dense fluid lead to different interaction dynamics between the PDC and the water. The interaction dynamics lead to a wide range of wave breaking behaviours. Our results show the importance of the boundary conditions and fluid properties in correctly capturing experimental observations and in predicting how PDCs generate tsunamis.

1 Introduction

1.1 Volcanic tsunamis

Around 80% of tsunamis are triggered by underwater earthquakes which cause a sudden and rapid displacement of the water surface. Due to the wavelengths associated with the large horizontal scale of the fault rupture (tens to hundreds of *kms*), this displacement results in long period waves capable of propagating across ocean basins (Center, 2006). Tsunamis can also be generated through sub-aerial and submarine landslides, meteorite impacts and volcanic eruptions. Volcanic eruptions themselves can generate waves through a number of mechanisms, including volcano-tectonic earthquakes, slope instabilities, pyroclastic density currents (PDCs), underwater explosions, shock waves and caldera collapse (Paris, 2015). There have been a number of geologically recent examples of such events. In 1996, the subaquatic explosive eruption near the northern shore of Karymskoye Lake in Kamchatka, Russia, generated multiple tsunamis (Belousov & Belousova, 2000). Locally to the source ($r < 1.3 \text{ km}$), wave heights reached up to 30 *m* but were

68 rapidly attenuated, leading to average runup heights of 2-3 *m* at locations 3 *km* from
69 the source. Tsunamis generated by PDCs entering the sea were observed during the Montser-
70 rat 1997 and 2003 eruptions, with maximum run-up heights of 4 *m* in Montserrat (Narcisse
71 et al., 2004), as well as the Rabaul 1994 eruption, where run-up heights reached 8 *m* in
72 Rabaul Bay (Nishimura et al., 2000). The eruption of Krakatau volcano in 1883 triggered
73 a tsunami that generated localized runup as high as 45 *m* and killed 36,000 people, un-
74 derstood to be as a result of voluminous PDCs entering the sea (Carey et al., 1996; Egorov,
75 2007; Maeno & Imamura, 2011).

76 Globally, around 20% of deaths associated with erupting volcanoes are a result of
77 tsunamis generated directly by the eruption (Center, 2006). Despite the fact that over
78 half of these deaths are thought to be a result of pyroclastic density currents (PDCs) en-
79 tering the sea, the tsunami generation potential of PDCs is still poorly understood. Not
80 only are there limited observations, but experimental as well as theoretical studies are
81 rare, due to the complexities involved in the modeling and observations of such phenom-
82 ena (Paris, 2015).

83 Both the potential impact and the probability of occurrence of these mechanisms
84 are often not included in tsunami hazard assessments, which are most often primarily
85 focused on earthquake generated tsunamis. Coastal communities living close to active
86 volcanoes may be unprepared for the possibility of tsunamis generated by volcanic erup-
87 tions (Paris, 2015). In 2002 a tsunami generated from lava and flank sliding at Strom-
88 boli volcano produced up to 10 *m* local runup and damage to buildings, but tourist fa-
89 cilities were seasonally unoccupied (e.g. Bonaccorso et al., 2003; Fornaciai et al., 2019).
90 In July and August 2019, a sequence of pyroclastic density currents at Stromboli entered
91 the sea, generating wave heights of up to 1 *m* near the entrance point (Giordano & De Astis,
92 2021; Giudicepietro et al., 2020). Stromboli is only one of several coastal volcanoes that
93 threaten the southern Tyrrhenian Sea. Another recent volcanic event which affected coastal
94 communities was the December 2018 flank collapse of Anak Krakatau, Indonesia, which
95 generated tsunami waves and killed over 400 people.

96 1.2 Pyroclastic density currents

97 PDCs are density currents made up of volcanic gas and particles. They are capa-
98 ble of transporting micrometer size ash particles to clasts larger than 1 *m* and can vary
99 in temperatures from a few tens of °C up to 800°C (Sulpizio et al., 2014). Currents of
100 interest in this study are ground-hugging, move at speeds of up to 100 *m/s* down-slope
101 away from their source (Legros & Druitt, 2000; Freundt, 2003) and generally exhibit runout
102 lengths of 10¹ *km* or less. PDCs are among the most hazardous volcanic phenomena on
103 Earth (Dufek, 2016; Lube et al., 2020). They form when hot mixtures of fragmented vol-
104 canic ash, rock and gas remain or become negatively buoyant with respect to the sur-
105 rounding air, forming a particle-driven gravity current. PDCs originate by collapse of
106 eruption columns (e.g. Sparks et al., 1978), by breakup and collapse of effusing domes
107 above volcanic slopes (e.g. Ui et al., 1999), from inclined or laterally directed eruptive
108 jets (e.g. Belousov et al., 2007) or from sustained pyroclastic fountaining (e.g. Báez et
109 al., 2020). How PDCs form affects concentration, rheology and steadiness. For the present
110 analysis of PDC-generated tsunamis, we focus on the currents typically generated on cone
111 volcanoes, which commonly flow downslope and cross the shoreline (Edmonds & Herd,
112 2005; Mattioli et al., 2007). PDCs are often subdivided according to their flow density
113 (particle concentration). Lithofacies of PDC deposits reflect which type of flow emplaced
114 them, and record transformations from one flow type to another (Fisher, 1979; Dufek
115 et al., 2015; Lube et al., 2020):

- 116 • Dilute PDCs (pyroclastic surges), in which most of the mass and momentum are
117 transported by a dilute, fully turbulent current, whose interaction with the sub-
118 strate feeds a relatively thin bedload in which particles are transported by trac-
119 tion (e.g. Andrews & Manga, 2012; Dellino et al., 2019; Fauria et al., 2016)

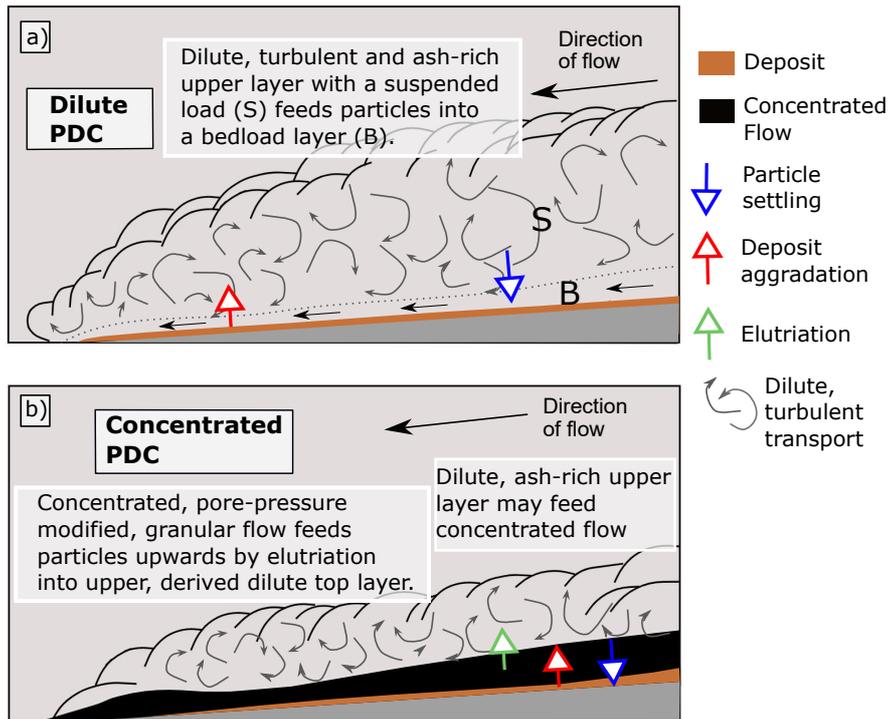


Figure 1. Diagram depicting the two key end-members of PDC: (a) a dilute PDC and (b) a concentrated PDC. The present study focuses on the concentrated end-member and ignores any momentum contribution from the dilute component.

- Concentrated PDCs (pyroclastic flows), comprising a gas-pore-pressure-modified granular flow (Wilson, 1980) overlain by a dilute cloud.

Figure 1 illustrates these two end members. Currents vary in velocity and temperature, and there are transitional regimes between these two end members, as well as down-flow transformations from one to another. The boundary between dilute-upper and concentrated-lower parts of a PDC may be diffuse, or may have a relatively sharp interface, i.e. a steep density gradient (Branney & Kokelaar, 2005).

There are no direct observations of PDC interiors from the field, due to the hostile nature of the currents, the unpredictability of eruptive events and the dynamics of the events themselves (Baxter et al., 2005; Cas & Wright, 1991; Legros & Druitt, 2000). Experimental study of PDCs using particles and gas encompasses laboratory-scale studies of concentrated granular currents (Roche et al., 2002; Delannay et al., 2017; G. Smith et al., 2020) and dilute ones (Andrews & Manga, 2012; Andrews, 2019), large-scale studies of hopper-fed confined currents (Lube et al., 2015, 2019, 2020) and of unconfined currents generated from experimental eruptions (Dellino et al., 2007, 2020). Of particular interest to our study, Freundt (2003) experimentally produce small, hot, pyroclastic flows that enter and interact with water.

The high mobility and near-frictionless behavior of PDCs observed both in the field and experimentally is an important characteristic of these currents, but understanding the source of this phenomenology is an ongoing area of research. While Lube et al. (2019) demonstrate the presence of an air-lubrication layer generated by an internal shear mechanism, other studies show that differential motion between particles and gas in the impact zone of a collapsing pyroclastic fountain can increase the pore fluid pressure within

143 the PDC (Wilson, 1980; Roche et al., 2010; Valentine, 2020; Fries et al., 2021), which
144 is then advected downstream by the flow, thus lowering the basal friction. Roche et al.
145 (2008) show experimentally that pore-fluid pressure leads to Newtonian fluid-like behav-
146 ior of fluidized granular flows. They conclude that efficient gas-particle interactions in
147 dense, ash-rich pyroclastic flows may promote a water-like behavior during most of their
148 propagation. The presence of pore-fluid pressure and the low-apparent basal friction co-
149 efficient of these currents are important considerations in the present study, when ap-
150 proximating the concentrated, basal component of a PDC.

151 The following section focuses on the ability of PDCs to generate tsunamis and out-
152 lines previous work on the subject including field studies and theoretical modeling, as
153 well as numerical and experimental works.

154 **1.3 Pyroclastic density current generated tsunamis: current understand-** 155 **ing and previous works**

156 Stratigraphic reconstruction, mapping of PDC deposits and observations of past
157 events all suggest that in the past these currents have initiated tsunamis (e.g. Maeno
158 & Imamura, 2011; Nishimura et al., 2000; Nomikou et al., 2016; Sulpizio et al., 2014; Waythomas
159 & Watts, 2003). Geological investigation of sub-aqueous PDC deposits has concluded
160 that when PDCs enter water they are generally disrupted explosively and/or ingest wa-
161 ter and transform into water-supported mass-flows (e.g. Cas & Wright, 1991; Jutzeler
162 et al., 2017; Carey et al., 1996).

163 Theoretical studies also assume that PDCs are capable of passing into, over or un-
164 der the water. Watts (2003) argues theoretically that the most energetic and coherent
165 water waves are produced by the impact of the dense, basal, granular flow component
166 of the PDC with water, assuming that the violent dynamics of the splash zone or ver-
167 tical ejection of debris at interaction have negligible effects on wave generation. Other
168 phenomena such as steam explosions, flow pressure, shear, and pressure impulse were
169 considered, but the authors conclude that these mechanisms would generate smaller waves.
170 Previous numerical works (e.g. Maeno & Imamura, 2011; Nomikou et al., 2016), includ-
171 ing the present study, accept this hypothesis and only consider the dense, basal compo-
172 nent.

173 Laboratory experiments allow physical processes to be investigated in a controlled
174 and (relatively) repeatable environment. This is particularly useful in the case of PDC
175 modeling, where access to field data is limited. Furthermore, key parameters for numer-
176 ical modeling must first be obtained from laboratory experiments.

177 Earlier experimental studies of tsunami generation by granular flows focus on ini-
178 tial parameters such as geometry and mass of an analog landslide (e.g. Fritz et al., 2003;
179 Heller, 2009; Mohammed & Fritz, 2012). The work of Fritz et al. (2003) explores land-
180 slide generated impulse waves and the associated generation of hydrodynamic impact
181 craters. It identifies three different regimes associated with the interaction zone and shows
182 that the amount (and rate) of water displacement is governed by the slide Froude num-
183 ber prior to impact (see Equation 2), the relative slide volume and the relative slide thick-
184 ness (both with respect to the water depth). In the separated slide regime identified, a
185 hydrodynamic impact crater forms, which is either outwards or backwards collapsing in
186 nature.

187 Freundt (2003) addresses the interaction of a PDC with water, but primarily fo-
188 cuses on thermodynamic behaviour in the flow-water interaction zone. A series of ex-
189 periments was conducted, where granular flows of heated ignimbrite ash (20 - 400°C)
190 and of bulk density near that of water, ran down a smooth chute and enter a water-filled
191 tank at an angle of 26°. For lower temperatures, the majority of material penetrated the
192 surface and mixed with water, creating a forward-directed ash fountain, a turbulent mix-
193 ing zone and a water-supported mass flow. As the temperature was increased, most of
194 the flow was redirected across the surface of the water, mixed with the water and gen-
195 erated steam explosions. No water-supported mass flow was generated in this latter case,

196 but waves were generated as a result of steam explosions. Although waves were recorded
 197 during these experiments, their characteristics were not explored in detail.

198 More recent experiments on tsunami generation have pioneered research in the im-
 199 pact of cool, fluidized granular flows (representing the dense basal component of a PDC)
 200 into water and their effect on wave generation (Bougouin et al., 2020). Fluidized, micro-
 201 meter ($d = 65 \pm 10 \mu m$) spherical glass beads are released from a lock and propagate
 202 down a ramp (with a continuous supply of air flow to maintain pore fluid pressure), be-
 203 fore interacting with water. This fluidization is used to replicate the high mobility/low
 204 friction behavior and the interstitial gas pore pressure of dense PDCs observed exper-
 205 imentally and in the field (e.g. Wilson, 1980; Roche et al., 2010; Lube et al., 2019; Valen-
 206 tine, 2020; Fries et al., 2021), while also helping to overcome scaling issues (Rowley et
 207 al., 2014; G. M. Smith et al., 2018). Notable features of the mixing zone include the gen-
 208 eration of a vertical granular jet, a leading wave and a turbulent mixing zone, similar
 209 to that observed by Freundt (2003). The vertical granular jet redirects a small amount
 210 of material across the surface of the water, while the remaining flow forms a gravity cur-
 211 rent on the slope underwater. Spilling behaviour in the breaking wave is also observed.
 212 The equivalent experiments were conducted using dense salt water flows and yielded sim-
 213 ilar results to cases when fluidized grains were used. The Newtonian fluid-like behav-
 214 ior of fluidized granular flows proposed by Roche et al. (2008) is one possible explana-
 215 tion for this. Another reason for the similarity in results is the small particle size, which
 216 makes the granular flow nearly impermeable on impact.

217 According to previous works (e.g. Fritz et al., 2004; Heller, 2009; Zweifel et al., 2006),
 218 the wave amplitude associated with wave generation by granular-flows or rock-avalanches
 219 is primarily considered a function of a number of dimensionless parameters; the Froude
 220 number Fr , the relative slide thickness S and the relative slide mass M . The recent study
 221 of Bougouin et al. (2020) suggests a product impulse parameter ζ for waves generated
 222 by highly fluidized granular flows:

$$\zeta = \left(\frac{u_f}{\sqrt{gH_i}} \right) \left(\frac{h_f}{H_i} \right) \left(\frac{\rho_f}{\rho_0} \right) \left(\frac{v}{H_i^2} \right) \sin \theta = FrSM \sin \theta \quad (1)$$

223 with u_f the front velocity at impact, H_i the initial water depth, h_f the height of the flow
 224 front 10 cm from the head at impact, v the volume per width $v = (H_0 - h_r)L_i$ (where
 225 H_0 is the initial column height, h_r is the remaining granular flow in the reservoir and
 226 L_i is the width of the reservoir), ρ_f the effective density of the grains and ρ_0 the water
 227 density. The dimensionless amplitude scales as $A/H_i = f(\zeta)$, where A is the wave am-
 228 plitude in this setup 2.4 m from the shoreline. Bougouin et al. (2020) demonstrate how
 229 data for fluidized granular flows and dense salt water flows collapse onto the same curve,
 230 whereas dry granular flows show a notably lower amplitude for the same value of ζ .

231 Bougouin et al. (2020) analyze features of the leading wave in the near-field region
 232 and its primary conclusion is that in the case of fine-grained fluidized flows, the mass
 233 flux and volume of granular material are the primary parameters affecting the amplitude
 234 of the resulting wave. This is analogous to the findings from sub-aerial and submarine
 235 landslide literature, including Fritz et al. (2003) and the recent study by Robbe-Saule
 236 et al. (2020), which shows that the density has a second order effect on the wave ampli-
 237 tude.

238 Numerically modeling the interaction of a PDC with water and the resulting wave
 239 generation relies upon many simplifications. This includes approximating the density strat-
 240 ification and flow dynamics, as well as the sub-aqueous transport of the flow following
 241 its initial entry to the water. Previous numerical studies (e.g., Maeno & Imamura, 2011;
 242 Nomikou et al., 2016) of PDC generated tsunamis assume the dilute component of a PDC
 243 to be negligible in terms of its effect on wave generation and focus on the dense, basal
 244 layer. Generally, these studies use depth-averaged approaches when considering both the
 245 PDC and the water (where vertical velocity stratification is ignored). Three dimensional
 246 numerical simulation can be used to capture the more complex physical processes oc-
 247 ccurring, but has been avoided in simulations of tsunami generation by PDC, primarily

for computational efficiency when considering large scales. Capturing these physical processes is, however, a desirable next step towards improving our understanding of this phenomenon and improving the capabilities of present hazard assessment models.

In the context of volcanic landslide-generated tsunamis, the numerical benchmark study carried out by Esposti Ongaro et al. (2021) on the 2002 eruption of Stromboli, shows that the resulting waveform is highly sensitive to the source description. Furthermore, it shows that the use of non-hydrostatic models, coupled with a multilayer approach (where vertical velocity stratification is considered), allow for the most accurate hazard estimation. This highlights the importance of exploring similar levels of complexity in the context of PDC-generated tsunamis.

1.4 Context of present study

Modeling and predicting the behaviour of granular flows remains a challenging goal, since granular flows are characterized by a large diversity of behaviours depending on their environment and conditions (MiDi, 2004; Delannay et al., 2017). Creating a generic continuum granular rheology is still very much an active area of research, challenges including the identification of a relevant variable to describe the transition from arrest to flow and the understanding of non-local effects. A PDC adds further complexity, with basal friction effects and transient pore pressure complicating the modeling further (Breard et al., 2020). Lube et al. (2020) also note that the vertical velocity profile remains somewhat parabolic as well as transient. Furthermore, the velocity at the slope boundary is not necessarily zero and there is a broad range of velocity configurations within these currents.

The present study numerically models the interaction of a laboratory-scale dense PDC with water in a flume (the experiments of Bougouin et al. (2020)) and the associated waves generated using a two-dimensional numerical model, in order to investigate the potential of our model to capture some of the more complex physical processes occurring. This enables us to determine some of the key parameters involved in capturing the important physics. The definition of a boundary condition for the slope, in particular, is non-trivial. Our numerical study replicates the laboratory experiments of Bougouin et al. (2020), comparing with their experimental results and confirming that the adopted model is adequate to simulate laboratory experiments. Bougouin et al. (2020) propose that the granular flow can be approximated as a dense, single-phase Newtonian fluid from a wave generation perspective, which is a useful assumption to make numerically when simplifying the granular continuum rheology. Our numerical model is a useful means of testing this assumption. The modeling is achieved by numerically solving the Navier-Stokes equations on an adaptive grid, using the Basilisk flow solver (Popinet & collaborators, 2013–2020). The granular-fluid is modelled as a Newtonian fluid, denser than water.

The numerical simulation outputs show a strong agreement with the experimental results. Figure 2 shows a direct comparison for different times, for two initial column heights and resulting granular-fluid Froude numbers. The Froude number for the granular-fluid is defined as:

$$Fr = \frac{u_f}{\sqrt{gH_i}} \quad (2)$$

where u_f is the depth-averaged u_x velocity over the height of the granular-fluid front at the moment of impact (or in the case of the laboratory experiments, the calculated front velocity), H_i is the initial water depth and g is the gravitational acceleration. In the numerical snapshots, we present two-dimensional vertical slices at the scale of the laboratory domain. The red represents the granular-fluid, the yellow the water and the blue the air. The granular-fluid is miscible in the water, but a sharp Volume of Fluid interface separates the granular-fluid and water from the air (see Section 2 for more details). Features of interaction including the generation of a granular jet, a plunging breaker and the retardation of the granular-fluid upon interaction with water are all captured in the numerical model. A characterization of interaction dynamics is discussed in Section 3.2

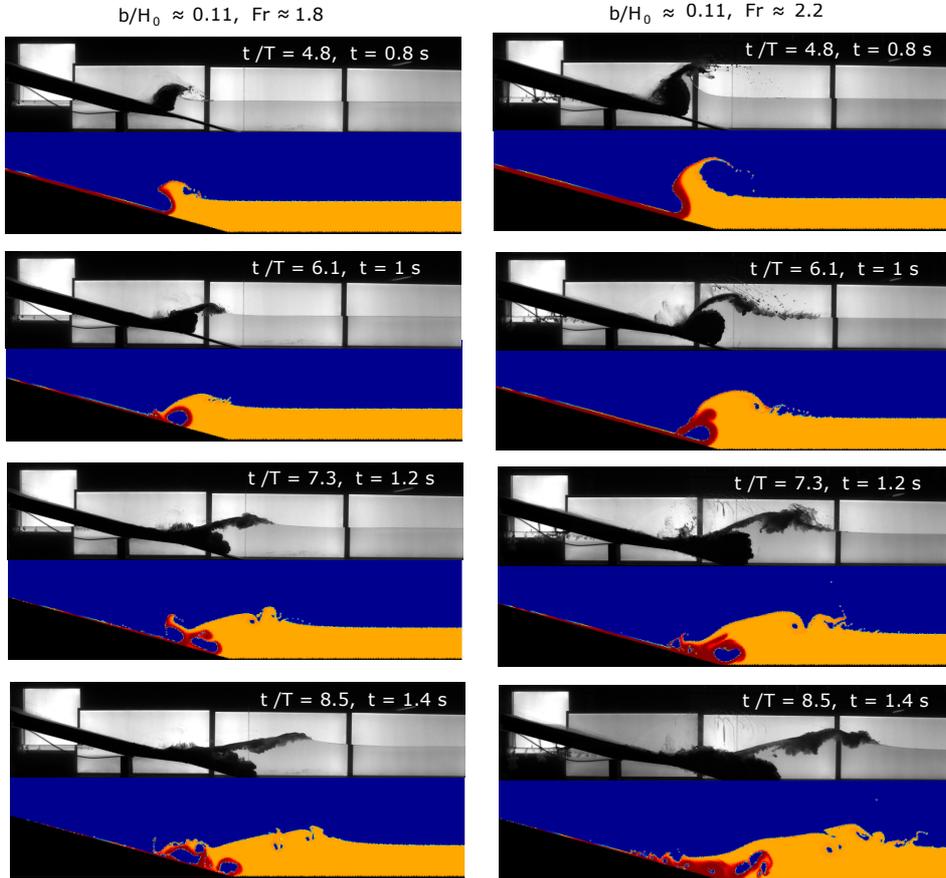


Figure 2. A comparison between numerical and experimental results (Bougouin et al., 2020), at four different times t/T (where $T = H_i/\sqrt{(gH_i)}$). The initial column heights in the experiments are 22.5 cm and 42.5 cm in the left and right columns, respectively. The numerical heights are initialized at 3 cm lower, to account for the residual grains left in the reservoir. The resulting Froude numbers are 1.8 and 2.2 in the left and right columns, respectively. More details on the setup information and outputs are discussed in methodology, Section 2.

and a detailed discussion of the experimental/numerical comparison is presented in Section 3.3. The strength of agreement between the numerical results and the experiments presented is remarkable, but this is highly sensitive to the boundary condition. Hence, the present study also investigates the effect of variability in the boundary condition of the slope (i.e. the boundary friction) on the vertical (perpendicular to the slope) x velocity profile of the granular-fluid, u_x , the wave generation process and the resulting far-field wave characteristics. A range of slope boundary conditions is explored and a detailed characterization of the associated granular-fluid/water interaction dynamics is presented. Furthermore, we investigate how different boundary conditions and associated interaction dynamics show different efficiencies of energy transfer from the granular-fluid to the water and the far-field wave. We first outline the methodology used, followed by an extensive discussion and presentation of our results in the following section.

2 Methodology

The following sections outline our numerical methodology. Section 2.1 gives the assumptions made and the governing equations solved, Section 2.2 provides details of the Basilisk flow solver and the numerical setup and Section 2.3 discusses the outputs analyzed.

2.1 Assumptions made and governing equations solved

We assume the fluidized grains from the experiments of Bougouin et al. (2020) to behave as a continuum. This takes the form of a dense, viscous and incompressible Newtonian fluid. The dense fluid and the water are assumed to be miscible with one another, but immiscible with air: they are separated from the air by a sharp interface. Surface tension is assumed to have negligible effect on interaction dynamics and wave propagation, due to the contrast of scales. The limitations of these assumptions are discussed in the discussion of results.

These assumptions lead to the applicability of the variable-density, multi-phase, incompressible Navier-Stokes equations:

$$\partial_t \mathbf{u} + \nabla \cdot (\mathbf{u}\mathbf{u}) = \frac{1}{\rho} [-\nabla p + \nabla \cdot (\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T))] + \mathbf{g} \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (4)$$

$$\partial_t f + \mathbf{u} \cdot \nabla f = 0 \quad (5)$$

$$\partial_t c + \mathbf{u} \cdot \nabla c = D \nabla^2 c \quad (6)$$

with p , \mathbf{u} , μ , ρ , D and \mathbf{g} representing the pressure field, velocity field, dynamic viscosity, density, diffusion coefficient and acceleration due to gravity respectively. f is the volume fraction tracer in our VoF approach that delineates between air ($f = 0$) and the variable density fluid ($f = 1$). The density of the water/granular-fluid mixture can vary continuously between ρ_{water} (where $c = 0$) and ρ_{grains} (where $c = 1$). The same applies to the viscosity. The overall density ρ and the viscosity μ are therefore functions of c and f , i.e. $\rho(f, c)$, $\mu(f, c)$. The diffusion term in Equation 6 expresses the diffusion of the granular fluid in water. In practice we set the diffusion coefficient D to zero but use a standard, diffusive numerical scheme (in contrast with the non-diffusive, geometric VOF scheme used to approximate Equation 5). The effective diffusion of the granular fluid in water is thus controlled by the properties of the numerical scheme and the spatial resolution. The method described is an alternative to an immiscible three-phase approach, where three fluids are separated by an interface (e.g., Joubert et al., 2020).

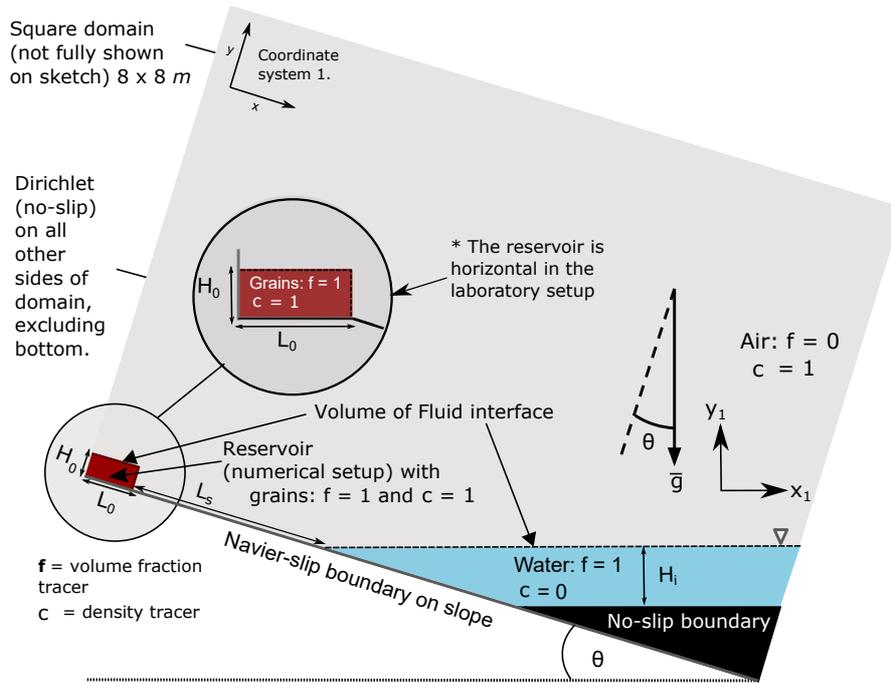


Figure 3. Setup of the initialized numerical domain, labelling the boundary implementations, tracer initialization and reference heights. The circular inset highlights the difference between the laboratory reservoir and the Basilisk initialization: the reservoir is horizontal in the laboratory setup, and tilted in the numerical setup. In our numerical simulations (and the physical experiments studied for comparison), $L_0 = 0.338 \text{ m}$, $\theta = 15^\circ$, $H_i = 0.265 \text{ m}$ and L_s (length of exposed slope) = 1 m .

2.2 Numerical implementation

Equations 3–6 are solved using the adaptive partial differential equation solver Basilisk (Popinet & collaborators, 2013–2020), developed as the successor to Gerris by the same authors (Popinet, 2003, 2009, 2015). In Basilisk, an adaptive tree-grid structure is implemented which facilitates local refinement and coarsening, for computational efficiency. The Navier-Stokes solver has been successfully used in a number of two-phase problems to model splashing (Thoraval et al., 2012) and wave breaking in both two and three dimensions (Deike et al., 2015). A two-phase Volume of Fluid (VoF) approach is used to capture the interface between the air and the variable density fluid. Basilisk uses a conservative, non-diffusive, geometric VoF scheme (Scardovelli & Zaleski, 1999). The consideration of a dense granular fluid (rather than a dilute phase) leads us to assume this single velocity model. The momentum equation is solved using the Bell Colella Glaz projection method (Bell et al., 1989), and we adapt the momentum-conserving scheme for VoF advection to account for variable density on the water/granular fluid side of the VoF interface.

In the present study we consider a two-dimensional vertical slice. This two dimensional approximation of a three dimensional process will lead to the generation of more coherent vortical structures, which has implications when considering wave breaking and overturning that must be considered. Since Basilisk works primarily with square or cubic domains, the length of the domain L is set to be 8 m and the domain is rotated by angle $\theta = 15^\circ$, to represent the slope (see Figure 3). This is in order to capture the dimensions of the experimental setup, see Bougouin et al. (2020) for details. This rotation

of the domain leads to the definition of two coordinate systems; x, y before rotation (where x is in the downwards direction of the slope, with y perpendicular) and x_1, y_1 after rotation. The bottom of the tank is implemented by masking the equivalent part of the numerical domain (i.e. setting the normal and tangential velocity components of each grid cell to zero).

A maximum grid resolution of 4096^2 is used, leading to a minimum cell size of $L/4096 \approx 1.4$ mm. For the initial column heights considered, this allows the vertical velocity profile of the granular-fluid and the interface boundary to be adequately resolved (see Figures S1 and S2 in supplementary material).

In order to compare our results with the experiments of Bougouin et al. (2020), it is necessary to determine the parameters most representative of the experimental setup. The maximum density of the granular fluid is set to 1400 kgm^{-3} and the density of the water is 997 kgm^{-3} . We have no information on the equivalent dynamic viscosity of the dense granular-fluid and the boundary condition on the slope in the experimental setup, since these conditions are non-trivial to define. Section 3 therefore presents an exploration of this parameter range (and the associated granular-fluid velocity profiles) in order to determine the most representative conditions and to explore how these parameters control granular-fluid/water interaction dynamics. We choose a range of viscosities which lead to similar impact velocities to those observed in the laboratory experiments. This comparison gives us a benchmark against which to check depth-averaged or multi-layer approaches (e.g., Audusse, 2005; Popinet, 2020).

We choose to explore a range of friction (boundary) conditions within our numerical model for the granular-fluid. We therefore use a Navier-slip boundary for the slope boundary condition, viz.,

$$u_t + b \frac{\partial u_t}{\partial z} = 0 \quad (7)$$

where b represents the Navier-slip length of the granular-fluid. The choice of this Navier-slip length allows us to vary this boundary condition between no-slip, partial-slip and free-slip. Varying the boundary condition between the free-slip and no-slip end members allows us to explore the full range of slip lengths to better constrain the model.

The Navier-slip boundary condition is set along the bottom x boundary (i.e. the slope). For the implementation of the tank bottom, the velocity field is set to zero at all time-steps, leading to a no-slip (Dirichlet) boundary condition, as depicted by the shaded black area in Figure 3. This implementation is limited by the current capabilities of embedded boundaries in Basilisk, however the primary focus of our analysis is associated with initial wave generation and propagation before the current interacts with the bottom boundary. The vertical u_x profile of the granular-fluid is dependent on the granular-fluid viscosity and boundary condition. The boundary-layer thickness (denoted in Figure 4 by δ_x) represents the distance normal to the wall to a point where the velocity of the granular-fluid has reached a certain percentage of the outer velocity u_{max} , e.g. 99%. (Schlichting & Gersten, 2016). There is no unique boundarylayer thickness, since the effect of the viscosity in the boundary layer decreases asymptotically as we move outwards from the wall.

2.3 Outputs

Following Bougouin et al. (2020), we evaluate the front height h_f and output the front velocity profile $u_{x,front}$ at 10 cm from the head of the granular-fluid at the time of impact (i.e. 10 cm from the slope-water intersection). The constant front velocity u_f is defined in our numerical experiments as the depth-averaged velocity at this location. We also consider the energy of the system. We calculate the total energy:

$$E = E_k + E_g \quad (8)$$

412 as the sum of gravitational potential energy;

$$E_g = \int \rho g y dx dy - E_{rest} \quad (9)$$

413 and the kinetic energy:

$$E_k = \frac{1}{2} \int \rho u^2 dx dy \quad (10)$$

414 for the granular-fluid, water and air. The components are calculated at each location
 415 using the respective volume fraction f and granular-fluid tracer g values. At initial-
 416 ization, the kinetic energy is 0 and the total energy of the domain is stored in the po-
 417 tential energy of the granular-fluid, i.e. $E_{init} = E_{g,init}$. The constant E_{rest} is the min-
 418 imal gravitational potential energy achievable by the system, i.e. the potential energy
 419 corresponding to a horizontal layer of granular fluid overlaid by a horizontal layer of wa-
 420 ter.

421 3 Results

422 3.1 Vertical profiles of the horizontal velocity component of the granular- 423 fluid, at impact

424 Figure 4 shows the vertical profiles of the horizontal velocity component of the granular-
 425 fluid at impact (the time-step at which the granular-fluid first interacts with the water)
 426 as we vary the boundary between the no-slip and free-slip end members. Velocity pro-
 427 files are shown for two different values of dynamic viscosity: $\mu = 0.01 Pa s$ and $\mu = 0.1$
 428 $Pa s$, for initial column heights $H_0 = 18.5 cm$ and $H_0 = 39.5 cm$. The dimensionless
 429 slip length is defined as: b/H_0 .

430 For a given dynamic viscosity μ and given column height H_0 , the thickness of the
 431 boundary layer δ_x at impact remains approximately the same for all values of slip length
 432 b , whereas the depth-averaged velocity across the flow front u_f is highly dependent on
 433 b . As the dynamic viscosity of the granular flow increases, the boundary layer thickness
 434 δ_x increases. As the initial column height H_0 increases, the flow front height h_f at im-
 435 pact also increases, but only a small increase of δ_x is observed. For higher dynamic vis-
 436 cosities and lower initial column heights (i.e. $\mu = 0.1 Pa s$, $H_0 = 18.5 cm$), we observe
 437 a well-resolved boundary layer, as shown in Figure 4. As dynamic viscosity is reduced
 438 (i.e. $\mu = 0.01 Pa s$), a higher resolution is required to resolve a similar number of grid
 439 cells over the boundary layer.

440 3.2 Snapshots of interaction dynamics

441 Longer slip lengths (higher basal slip velocities) exhibit notably different interac-
 442 tion dynamics between the granular-fluid and the water, compared with shorter slip lengths.
 443 This leads to different wave generation mechanisms associated with different slip lengths,
 444 as illustrated in Figure 5. The interaction dynamics depicted are present across the range
 445 of column heights and dynamic viscosities considered, however Figure 5 presents snap-
 446 shots from the case where $\mu = 0.1 Pa s$ and $H_0 = 18.5 cm$.

447 The interaction dynamics are differentiated qualitatively by the amount of granular-
 448 fluid directed across the water surface versus down-slope, and the resulting breaking wave
 449 characteristics. The breaking characteristics are described by the amount of overturn-
 450 ing of the free surface.

451 At the no-slip end-member (where the basal slip velocity is equal to zero), the granular-
 452 fluid is generally directed across the water surface at impact and the breaking wave most
 453 often exhibits violent overturning (Figure 5 subfigures a-c). No water surface closure be-
 454 hind the wave front is observed in this case and water rushes back towards the ramp un-
 455 der the influence of gravity. This type of collapse behind the leading wave is referred to

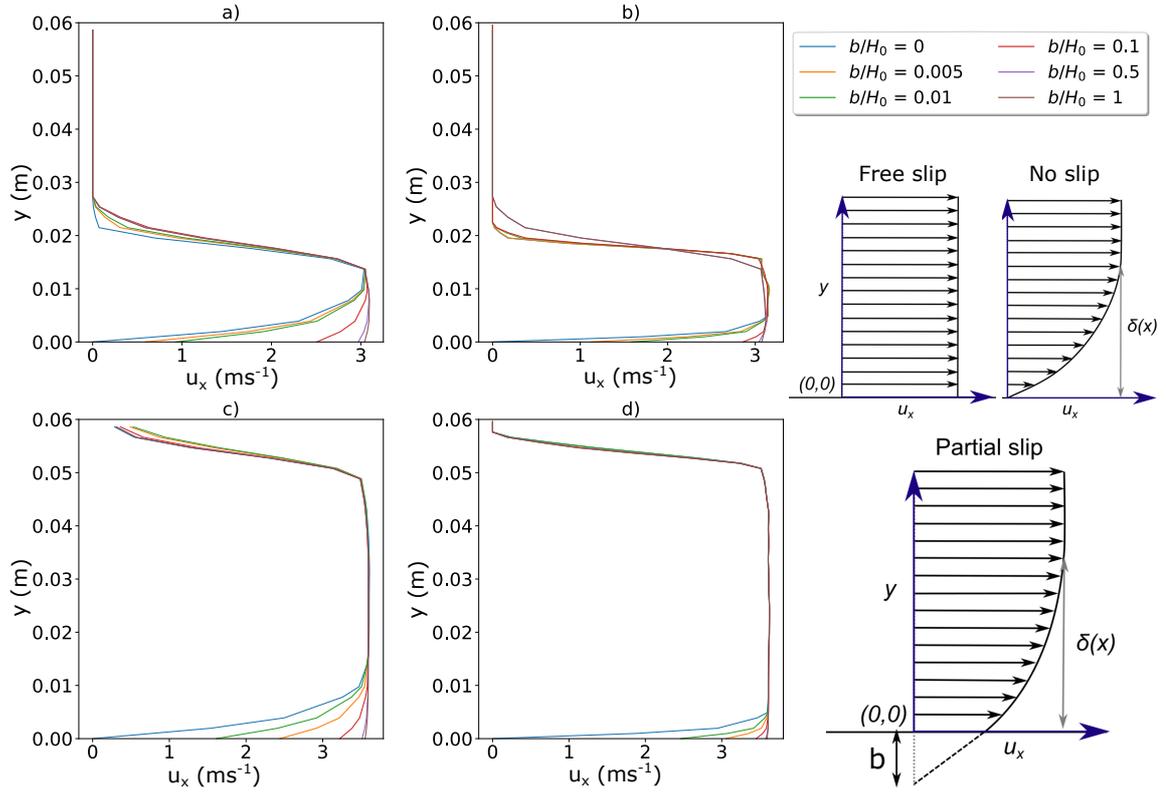


Figure 4. Velocity profiles of the granular-fluid (10 cm from the head) at the time of impact, for a range of dimensionless slip lengths b/H_0 . a) $H_0 = 18.5$ cm, $\mu = 0.1$ Pa s, b) $H_0 = 18.5$ cm, $\mu = 0.01$ Pa s, c) $H_i = 39.5$ cm, $\mu = 0.1$ Pa s, d) $H_0 = 39.5$ cm, $\mu = 0.01$ Pa s. Light blue inset shows how boundary conditions on the slope affect the boundary layer thickness δ_x at a time t . Graphical depiction of slip length b . $b = 0$ for no-slip and $b = \infty$ for free-slip.

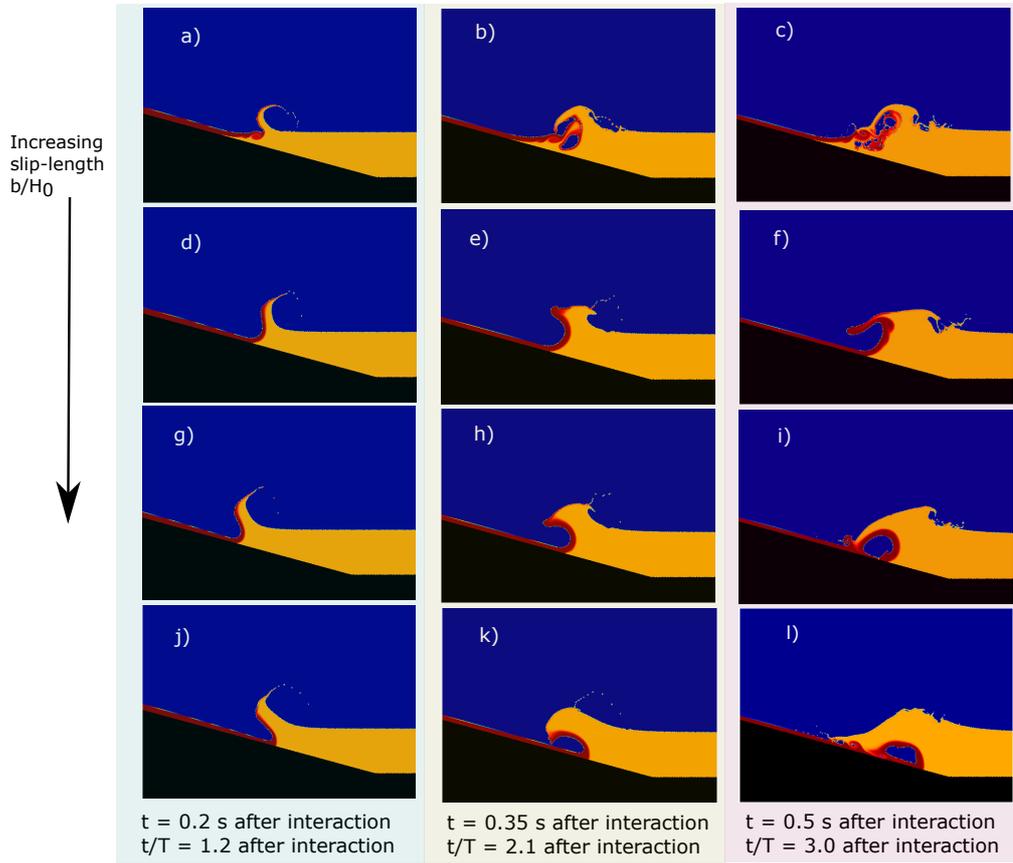


Figure 5. These snapshots are for the initial conditions $\mu = 0.1 \text{ Pa s}$ and $H_0 = 18.5 \text{ cm}$. The b/H_0 values shown are specific to these initial conditions, leading to $\text{Fr} = 1.8$ (Equation 2). a) - c) No-slip/low values of slip: the granular-fluid shears the water surface leading to violent overturning. d) - f) Low to medium values of slip: the granular-fluid partly shears the water surface, and partly propagates downslope, leading to a steep, plunging breaker. g) - i) Medium to high values of slip: most of the granular-fluid is directed downslope, with some shearing of the water surface. This leads to the generation of a plunging breaker. j) - l) Free slip: granular-fluid fully propagates downslope, with little/no overturning in resulting wave.

456 as an outwards collapsing impact crater, considering the terminology used in the land-
 457 slide tsunami generation experiments of Fritz et al. (2003). This type of interaction does
 458 not generally support the immediate generation of a gravity current and most material
 459 therefore remains near the interaction zone. In some cases, however, at the no-slip end-
 460 member, the granular-fluid lifts fully off the bottom boundary and then re-attaches to
 461 the ramp. This reattachment reduces the amount of overturning observed in the break-
 462 ing wave. Figure S3 in the supplementary material demonstrates an example when no-
 463 slip leads to reattachment and the generation of a gently spilling wave (Figure S3 sub-
 464 figures a-c).

465 For higher basal-slip velocities, more granular-fluid becomes directed down-slope
 466 rather than across the water surface. This leads to the generation of a plunging breaker
 467 (Figure 5 subfigures d-f). As the basal-slip velocity is increased further, the wave steep-
 468 ness and amount of overturning/breaking decreases. Under these conditions, a backwards
 469 collapsing impact crater is observed (Fritz et al., 2003). This type of impact crater is gov-
 470 erned by a surface closure resulting in the inclusion of air in the form of a cavity.

471 At the free-slip end-member (where the basal slip velocity is equivalent to the free
 472 stream velocity), the granular-fluid appears to initially expel the water upwards, then
 473 punches through the water, also forming a backwards collapsing impact crater (Figure
 474 5 subfigures g-i). A spilling breaker is generated from the initial uplift, showing no sig-
 475 nificant breaking overturning.

476 For each viscosity and column height, the change in interaction dynamics gener-
 477 ally follows the same pattern as the dimensionless slip length b/H_0 is increased. Figure
 478 S3 in supplementary material shows the equivalent of Figure 5, for $H_0 = 39.5 \text{ cm}$ $\mu =$
 479 0.01 Pa s . The values of b/H_0 corresponding to the transition between different inter-
 480 action dynamics vary depending on the Froude number of the granular-fluid (see Equa-
 481 tion 2), but qualitatively similar dynamics are observed.

482 These results and observations imply that the boundary condition plays an impor-
 483 tant role in determining the interaction dynamics and the characteristics of the gener-
 484 ated wave.

485 3.3 Experimental comparison

486 Figure 2 in Section 1.4 demonstrates excellent qualitative agreement between the
 487 observed behaviour in the numerical simulations and physical experiments. Sections 3.1
 488 and 3.2 show how the boundary condition influences this agreement, demonstrating the
 489 sensitivity of the velocity profiles and granular-fluid/water interaction dynamics to the
 490 choice of boundary condition. In Section 3.3.1, we explore the qualitative comparisons
 491 in more detail and discuss the limitations of the numerical model in capturing the ex-
 492 perimental physics. Section 3.3.2 expresses the numerical results in terms of the prod-
 493 uct impulse parameter (Equation 1), and shows the applicability of this parameter as
 494 a predictor of the dimensionless wave amplitude in our simulations. This also provides
 495 a quantitative measure of the agreement between the numerical results and laboratory
 496 measurements over a range of boundary conditions, viscosities and column heights.

497 3.3.1 Granular-fluid/water interaction dynamics

498 The experimental results cover a range of Froude numbers comparable to our nu-
 499 merical simulations, information related to the friction condition is limited. In the lab-
 500 oratory experiment snapshots (Bougouin et al., 2020), the details of the interaction be-
 501 hind the granular-fluid front (e.g. evidence of a hydrodynamic impact crater) cannot be
 502 observed due to vigorous mixing of the granular material. However, a number of key fea-
 503 tures associated with the wave generation, the granular-fluid separation and the propa-
 504 gation of the gravity current may still be identified. As the granular-fluid impacts wa-
 505 ter, some momentum is directed across the water surface, causing the generation of an
 506 outwards projecting granular jet. This behaviour is also reported in the cool volcanic ash

507 experiments of Freundt (2003). The majority of the granular-fluid undergoes mixing af-
 508 ter impact and forms a water-supported mass flow which travels down-slope, at a slower
 509 velocity to that of the leading wave. For experimental Froude numbers > 2.0 , the wave
 510 generated in the initial 0.4 seconds after interaction displays features of a steeply plung-
 511 ing breaker. As the Froude number decreases, the plunging breaker becomes increasingly
 512 gentle and for lower Froude numbers (e.g. $Fr \approx 1.6$), a spilling breaker is generated. The
 513 Froude number considered uses the granular-fluid front velocity at the moment of im-
 514 pact (i.e. prior to mixing and deceleration) with respect to the shallow water wave speed
 515 (see Equations 1 and 2).

516 Figure 5 shows that high values of partial-slip best reproduce the interaction dy-
 517 namics observed in the laboratory experiments. Work on granular flows with pore fluid
 518 pressure and/or on inclines shows that there is significant slip on a smooth substrate (e.g.
 519 Roche et al., 2010; Brodu et al., 2015; Lube et al., 2019), so this inference is in line with
 520 these experimental observations.

521 Figure 2 shows directly how the numerical results (for a high value of partial slip,
 522 $b/H_0 = 0.1$) capture the generation of the granular jet, the plunging breaker behaviour
 523 with associated splashes and overturning, as well as the approximate shape and veloc-
 524 ity of the gravity current in the first 1.2 s.

525 While the overall comparison is favourable, there are some discrepancies. At the
 526 interaction zone, where the numerical results show the formation of an impact crater,
 527 the experimental snapshots show vigorous mixing. Furthermore, the numerical results
 528 show increased breaking and overturning of the breaking wave at later time-steps in com-
 529 parison with the experimental snapshots. We attribute these discrepancies to the lim-
 530 itations surrounding the immiscibility of air and granular-fluid in our simulations, and
 531 the two-dimensional representation of an intrinsically three-dimensional process. Given
 532 that our model assumes the grains to behave as a continuum (in the form of a Newto-
 533 nian fluid) confined to one side of the interface, we do not capture the dynamics of in-
 534 dividual grains and we therefore do not capture the same level of mixing observed in the
 535 physical experiments. This means the numerical model is likely to overemphasize the for-
 536 mation of an air pocket/cavity. Furthermore, the two-dimensionality of this model leads
 537 to enhanced vortical structures, in both the interaction zone and the breaking wave.

538 In addition, after the gravity current reaches the base of the tank, some lift is ob-
 539 served from the boundary in our numerical simulations. This is likely a result of the bot-
 540 tom (horizontal) boundary implementation, which is limited to no-slip. As shown in the
 541 interaction of the granular-fluid with water for no-slip conditions, granular-fluid lift is
 542 often observed. These limitations do not impact our overall conclusions, however, and
 543 the numerical/experimental agreement is of good quality. The wave generation appears
 544 qualitatively similar and the propagation and shape of the leading wave are well captured.

545 In the high temperature experiments of Freundt (2003), all of the granular-fluid
 546 is redirected across the surface of the water, leading to violent overturning, similar to
 547 what is observed in Figure 5 a-c. Localized waves of smaller amplitude are observed, which
 548 are associated with steam explosions occurring near the surface of the water. Although
 549 temperature is not considered in our numerical simulations, we observe a similar inter-
 550 action behaviour for the no-slip end member, whereby the granular-fluid is redirected
 551 across the surface of the water, leading to violent breaking behaviour. This granular-fluid
 552 redirection can be attributed to a number of potential factors, including changes in den-
 553 sity or buoyancy, boundary behaviour and shear. The present study does not explore the
 554 effects of temperature, but this is an interesting area for future research.

555 *3.3.2 Product impulse parameter and dimensionless wave amplitude*

556 We compare our numerical results with the experiment results of Bougouin et al.
 557 (2020) using the product impulse parameter ζ , which they showed to be a good predic-
 558 tor of the dimensionless wave amplitude (Figure 6). Our numerical data match the lab-
 559 oratory results well over a range of column heights, which provides confidence in the nu-

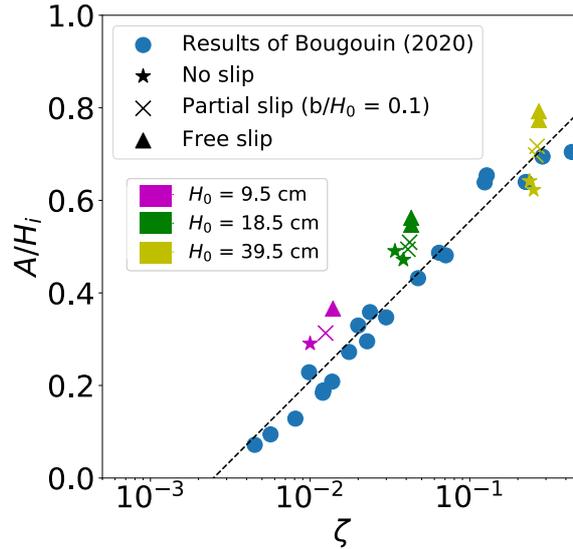


Figure 6. Normalized amplitude A/H_i at $x_1 = 2.4 m$ from the shoreline as a function of the impulse parameter ζ (Equation 1), for the laboratory experiments (of fluidized glass beads) and the numerical experiments considered in the present study. Results from the numerical simulations are shown for multiple slip conditions and initial column heights (represented by the marker style and color, respectively). For $H_0 = 39.5 cm$ and $H_0 = 18.5 cm$, markers are shown for both $\mu = 0.1 Pa s$ and $\mu = 0.01 Pa s$. Dotted line (---) $A/H_i = 0.15 \ln(\zeta) + 0.88$.

merical model (and therefore the Newtonian-fluid approximation of the granular-fluid). The numerical wave amplitudes are slightly higher than the experimental results; this is likely due to a number of factors, including the two-dimensional approximation of three-dimensional turbulence, the differences in flow rheology, the immiscibility between the granular-fluid and air, and the difference in reservoir geometry (see Figure 3). Furthermore, subtle differences between the numerical and laboratory methodologies for calculating u_f (we consider depth-averaged velocity over the height of the flow h_f , where the laboratory experiments use the constant flow front velocity) may lead to small differences in ζ . Finally, the calculation of wave amplitude in both experimental and numerical methodologies has a degree of uncertainty.

Across the range of initial conditions considered in our numerical results, a change in boundary condition leads to a considerable difference in the resulting wave amplitude. Partial-slip boundary conditions lead to wave amplitudes closer to the laboratory data points, compared with free-slip conditions. This agrees with our qualitative inferences, which show partial slip conditions to better capture the interaction dynamics observed in the laboratory snapshots. Free-slip conditions lead to little to no breaking (Figure 5 a-c), which is one reason for these larger amplitudes.

While the inferences drawn from this Figure are useful, further analysis on far-field ($x_1 > 5m$ from the shoreline) amplitudes and overall energetics is required to constrain further the effect of boundary condition (see Section 3.4).

3.4 Energy transfer and wave characteristics

3.4.1 Energy evolution and transfer

It has been demonstrated across multiple experimental and numerical studies (e.g., Deike et al., 2015) that wave breaking has a significant effect on energy dissipation and

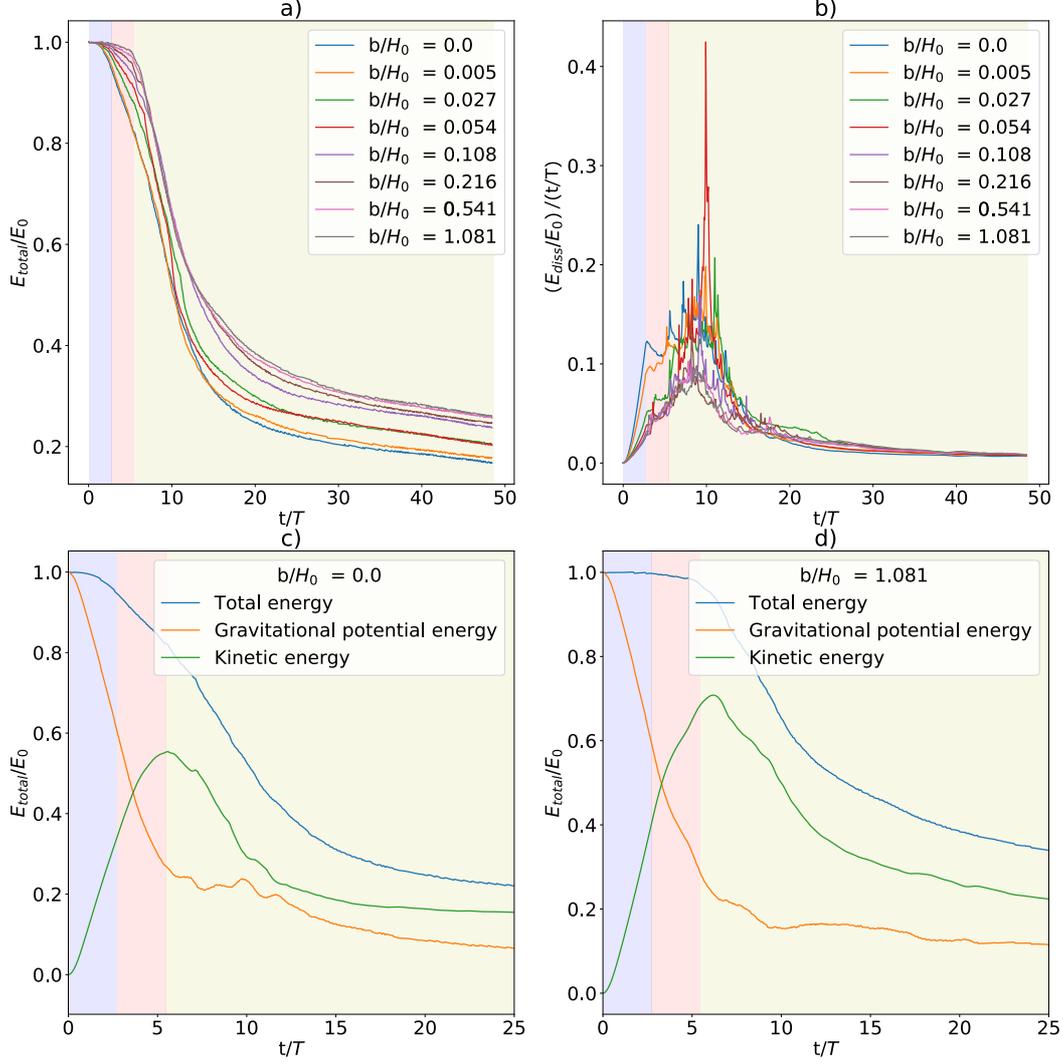


Figure 7. a) Normalized total energy of domain as a function of time t/T (where $T = H_i/\sqrt{(gH_i)}$), for a range of slip lengths b/H_0 . b) Energy dissipation rate (relative energy lost per T) of entire domain as a function of time t/T . c) and d) Normalized total energy of domain, including the normalized gravitational and kinetic components as a function of time t/T for $b/H_0 = 0.0$ and $b/H_0 = 1.081$. The background colors broadly represent the different stages of the simulation: purple = granular-fluid propagation on slope prior to interaction, pink = interaction and initial wave generation and yellow = wave breaking and impact crater collapse.

584 momentum transfer. In our numerical simulations there are a number of significant dis-
 585 sipation processes occurring, including wave breaking, collapse of the hydrodynamic im-
 586 pact crater, and air entrainment at the mixing zone. In the case where a spilling wave
 587 is observed and less dissipation associated with wave breaking would be expected (for
 588 some of the high slip conditions, no breaking is observed), there may be increased dis-
 589 sipation elsewhere in the domain; i.e. in the impact crater collapse or the propagation
 590 of the gravity current. Similarly, in the case of violent overturning there is no significant
 591 impact crater collapse or propagation of a gravity current. Thus, beyond the initial granular-
 592 fluid propagation and water impact, differentiating between energy dissipation mecha-
 593 nisms is non-trivial. Exploring the energy evolution of the domain (and its components)
 594 does, however, allow us to determine the amount and timing of energy dissipation as-
 595 sociated with the different boundary conditions and the relative granular-fluid/water in-
 596 teraction dynamics.

597 Figure 7a shows the evolution of the normalized total energy of the domain and
 598 Figure 7b shows the dissipation rate for a range of slip lengths b/H_0 , for the initial con-
 599 dition where $H_0 = 18.5 \text{ cm}$ and $\mu = 0.1 \text{ Pa s}$. Figures 7c and 7d present the evolution
 600 of the normalized total energy of the domain, along with the gravitational and poten-
 601 tial components, for $b/H_0 = 0.0$ and $b/H_0 = 1.081$. Figure 8 presents the kinetic, po-
 602 tential and total energy evolutions for the granular-fluid and the water components, as
 603 well as their dissipation rates. Similar evolution patterns can be seen across a range of
 604 initial conditions; an example for $H_0 = 39.5 \text{ cm}$ and $\mu = 0.01 \text{ Pa s}$ is shown in the sup-
 605plementary material (Figure S4).

606 As the granular-fluid propagates down the slope, the total energy of the domain
 607 begins to decrease. This decrease is greater for high friction cases (i.e. smaller values of
 608 b/H_0) and is most clearly depicted in the total energy dissipation plot (Figure 7b), which
 609 shows the increased dissipation rate for high friction conditions during the initial prop-
 610 agation. Figures 8b and 8d show the transfer of potential energy to kinetic energy within
 611 the granular-fluid as it propagates down-slope, with Figure 8h showing that the dissi-
 612 pation occurring at this stage is driven by the granular-fluid.

613 As the granular-fluid impacts the water and the water surface is uplifted (as de-
 614 picted in Figure 5, $t/T = 1.2$ after interaction), the total energy of the domain contin-
 615 ues to decrease for all values of b/H_0 . At the transition between uplift and the onset of
 616 wave breaking (and/or impact crater collapse), the kinetic energy of the domain and the
 617 kinetic energy of the granular-fluid reach a maximum for all values of b/H_0 and the rate
 618 of change in potential energy decreases, as the granular-fluid slows down within the col-
 619 lapsing region (i.e. Figure 5, $t/T = 2.1$ after interaction). When the granular-fluid shears
 620 the water surface, the potential energy of the granular-fluid remains higher than for other
 621 interaction styles. The maximum value of total kinetic energy observed is $\approx 20 \%$ higher
 622 for larger slip conditions (i.e. $b/H_0 = 1.081$), since in these cases less energy has been
 623 dissipated in the initial propagation, interaction and wave generation stages. Once wave
 624 breaking starts and/or the impact crater collapses, the granular-fluid is slowed and the
 625 kinetic energy of the granular-fluid decreases abruptly, which corresponds with a decrease
 626 of the total energy in the domain. However, during this stage, the total energy of the
 627 water and its components continue to increase. Generally, the dissipation rate at this
 628 stage increases as the amount of overturning increases in the wave breaking and entrain-
 629 ment of air at the shoreline during the impact crater collapse.

630 The total energy of the water reaches a maximum at $t/T \approx 10$ and is greatest for
 631 lower friction conditions (see Figure 8e). The time of maximum energy in the water cor-
 632 responds to the time at which the dissipation rate of the granular-fluid begins to slow,
 633 and the potential energy of the granular-fluid flattens, suggesting that the grains stop
 634 imparting significant energy to the water at this stage. It can be observed that after t/T
 635 ≈ 20 , the energy dissipation rates for all slip conditions begin to tend towards a steady
 636 rate, as breaking ceases and the granular-fluid has undergone significant mixing.

637 For later times (i.e. $t/T > 20$), the total energy of the water tends towards a more
 638 constant value. When considering tsunami generation potential, this observation sug-

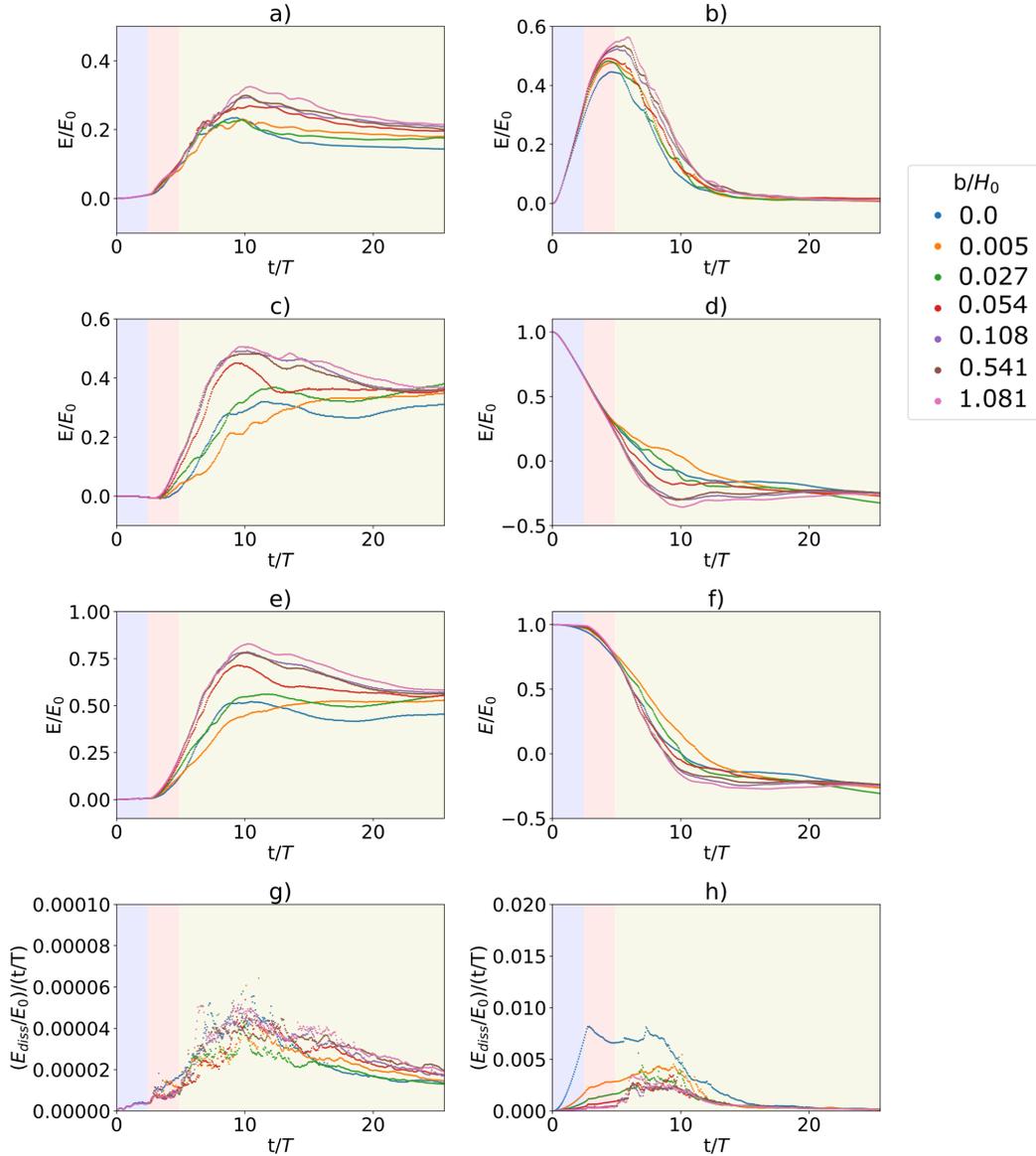


Figure 8. a) Kinetic energy evolution of water. b) Kinetic energy evolution of granular-fluid. c) Potential energy evolution of water. d) Potential energy evolution of granular-fluid. e) Total energy evolution of water. f) Total energy evolution of granular-fluid. g) Dissipation rate of water. h) Dissipation rate of granular-fluid. Results show a range of dimensionless slip lengths, for the initial condition where $\mu = 0.1 \text{ Pa s}$ and $H_0 = 18.5 \text{ cm}$.

gests that the wave is carrying sufficient energy to propagate significantly further, without considerable dissipation, if it were to continue in an infinite domain. Generally, the total energy of the water is greatest for lower friction conditions, with a few exceptions where an increase in total energy is observed for values of b/H_0 at $t/T \approx 20$. We hypothesize this to be a result of granular-fluid propagation: in these cases, snapshots and energy evolution plots demonstrate that the granular-fluid remains attached (or reattaches) to the bottom boundary, thus imparting more energy to the water in the near-field than simulations where the granular-fluid lifts off the tank bottom. This suggests that energy plots for the separate components, when considering the entire domain, cannot tell us all the information about the far-field wave if the gravity current does not behave in a consistent manner between simulations. For this reason, far-field wave gauges are used to reflect the relationship between the slip length and the total energy of the generated wave in the far-field, without considering the energy at the interaction or granular-fluid propagation zone.

In summary, the boundary condition influences the energy dissipated in the granular-fluid on the slope prior to interaction, and hence the energy available for wave generation. It also determines the detailed mechanism by which this available energy is transferred to the generated wave. As viscosity is decreased, or initial column height is increased, the boundary condition has a less significant impact on the energy dissipation within the granular-fluid prior to impact. The boundary condition does, however, continue to affect the interaction dynamics (see Figure S3). Therefore under these conditions, despite similar overall energy in the granular-fluid at impact, significant discrepancies still exist between energy evolutions associated with different boundary conditions (see Figure S4).

Understanding the energy transfer mechanisms also helps to interpret qualitative observations from Section 3.2 and quantitative results from Section 3.3.2, which showed that free-slip conditions (associated with efficient energy transfer and little to no wave breaking) overestimate near-field laboratory wave amplitudes, compared with partial-slip conditions.

3.4.2 Far-field dimensionless wave amplitude

Figures 9 a-d show the influence of the slip condition and the boundary velocity, on the far-field wave amplitude (considered at $x_1 = 6$ m from the shoreline) and maximum total energy of the water. These figures are for the initial conditions $H_0 = 18.5$ cm and $\mu = 0.1$ Pa s. The far-field wave amplitude and the maximum total energy of the water follow an almost identical pattern of dependence on slip length. Between $b/H_0 = 0$ and $b/H_0 = 0.1$, there is a sharp increase in both the maximum total energy of the water and the far-field amplitude. At $b/H_0 > 0.2$, this increase becomes more gentle. The maximum total energy of the water occurs at $t/T \approx 10$, suggesting that most of the initial energy transfer from the granular-fluid to the wave occurs in this time. Figure 9b and Figure 9d show how maximum total energy of the water and maximum amplitude, respectively, vary with the dimensionless slope boundary velocity. As the boundary velocity increases, it is only once $u_{x,boundary}/u_{x,max} > 0.50$ that we observe a significant increase in energy transferred to the wave and likewise, a significant increase in the resulting wave amplitude. This result is surprising, since between $u_{x,boundary}/u_{x,max} > 0$ and $u_{x,boundary}/u_{x,max} = 0.50$, we expect the largest change in the shape of the boundary layer, and thus energy dissipation. We therefore attribute this behavior to the change in interaction dynamics; as basal slip is increased, energy is more efficiently transferred into the total energy of the wave.

These observations enable us to quantitatively determine the relationship between slip length and energy transfer. For the initial conditions presented, we can infer for a given slip length, what the resulting energy transfer or far-field amplitude would be. Upon changing the initial conditions, if the influence of the slope boundary condition on the granular-fluid velocity profile decreases (i.e. viscosity decreases or column height increases),

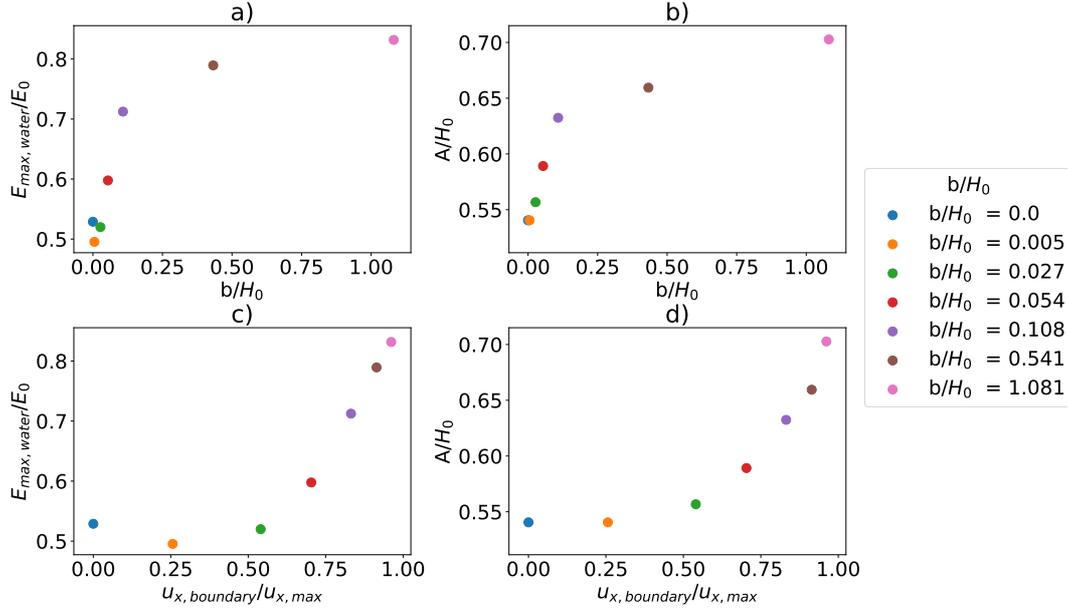


Figure 9. Effect of boundary condition on the far-field amplitude ($x_1 = 6$ m from the shoreline) and maximum total energy of the water. a) Total energy of water variation with slip condition. b) Dimensionless far-field (6 m from source) amplitude relative to the slip condition b/H_0 . c) Total energy of water variation with dimensionless boundary velocity at impact $u_{boundary}/u_{max}$. d) Dimensionless far-field amplitude variation with $u_{boundary}/u_{freestream}$. These relationships are for $H_0 = 18.5$ cm and $\mu = 0.1$ Pa s.

692 the same relationships are generally observed, with less difference between maximum and
693 minimum total energy or amplitude.

694 The overall energy dissipated in the interaction and wave generation process can
695 be a result of viscous friction, air entrainment, mixing processes and the directionality
696 of the granular-fluid front, which influences the amount of overturning associated with
697 the wave breaking. These observations highlight the importance of exploring the pro-
698 cesses occurring at (and before) PDC-water interaction in more detail, in order to cap-
699 ture more accurate initial conditions when performing a numerical hazard assessment
700 and exploring a wider range of possible scenarios. Accounting for processes such as mix-
701 ing at the shoreline might also be a vital step in understanding the characteristics of as-
702 sociated tsunamis.

703 4 Conclusions

704 Numerical experiments on the entrance of fluidized granular flows into water have
705 been carried out and compared against laboratory results, in order to explore how the
706 slope boundary condition plays a role in determining the vertical u_x velocity profile of
707 the flow, the associated wave generation mechanism and the far-field wave characteris-
708 tics.

709 It is shown that the boundary condition of the slope heavily determines to what
710 extent the granular-fluid shears the water surface or propagates down-slope, which has
711 important implications for the wave generation and breaking process. For low values of
712 slip, the granular-fluid shears the water surface at impact, leading to considerable over-
713 turning and wave breaking at the interaction zone. For higher slip conditions, the granular-
714 fluid momentum is directed increasingly down-slope (beneath the water surface) and the

715 overturning associated with the wave breaking decreases. It is concluded that high val-
 716 ues of partial-slip lead to the closest qualitative representation of the experimental re-
 717 sults.

718 We consider the relationship between the dimensionless product impulse param-
 719 eter proposed by Bougouin et al. (2020) and the dimensionless wave amplitude A/H_i (at
 720 $x = 2.4 m$ from the interaction zone) and show that the numerical results display a good
 721 quantitative agreement with the laboratory experiments, helping to confirm our New-
 722 tonian fluid approximation. Partial-slip conditions appear to be a closer predictor of di-
 723 mensionless wave amplitude than free-slip conditions, which is in agreement with the qual-
 724 itative comparison.

725 Energy transfer is considered, in order to understand the far-field impact of the dif-
 726 ferent slope boundary conditions and associated interaction dynamics in greater detail.
 727 The timing of the different phases of the simulation (e.g. granular-fluid propagation, im-
 728 pact, initial uplift, impact crater collapse and breaking) is inferred from our outputs. The
 729 importance of energy dissipation and transfer processes in influencing the wave charac-
 730 teristics is demonstrated, confirming why significant differences in near and far-field wave
 731 amplitudes are observed between slip conditions. We demonstrate a non-linear relation-
 732 ship between the dimensionless slip length (or basal slip velocity) and the maximum to-
 733 tal energy of the water, as well as the far-field amplitude: increasing the boundary ve-
 734 locity at the time of impact makes no significant difference to the energy transfer in the
 735 resulting wave until $u_{x,boundary}/u_{x,max} > 0.5$. It is concluded that as basal slip is in-
 736 creased, energy is more efficiently transferred into the total energy of the wave.

737 When considering large scales, these observations may have significant implications
 738 for numerical simulations of PDC tsunami hazard. Firstly, these experiments confirm
 739 the assumption that a fluidized granular-flow can be modeled as a viscous Newtonian
 740 fluid, particularly in the context of wave generation. Using this assumption, our exper-
 741 iments demonstrate the importance of using an adequate boundary condition for the slope
 742 in order to capture the physics of wave generation and the associated far-field wave char-
 743 acteristics. Our results also highlight the sensitivity of the wave generation process to
 744 vertical variations in the horizontal velocity components within the granular-fluid, which
 745 is dependent on the relative importance of viscosity. This suggests that exploring the
 746 impact of vertical velocity stratification within highly-mobile PDCs may be an impor-
 747 tant next step when considering large-scale impacts of these flows with seawater. This
 748 would require the use of a multi-layer model. Furthermore, our results confirm that denser-
 749 than-water, fluidized granular-flows are capable of shearing the water surface and still
 750 generating waves of significant amplitude.

751 Acknowledgments

752 We would like to acknowledge the University of Auckland Doctoral Scholarship for its
 753 generous contribution of PhD fees and stipend of Lily Battershill. The project is also
 754 supported by the Marsden Fund Council from Government funding, managed by Royal
 755 Society Te Aprangi.

756 We thank Alexis Bougouin, Raphal Paris and Olivier Roche for the sharing of their
 757 experimental results, including the snapshots presented in Figure 2.

758 The authors also wish to acknowledge the use of New Zealand eScience Infrastruc-
 759 ture (NeSI) high performance computing facilities, consulting support and/or training
 760 services as part of this research. New Zealand's national facilities are provided by NeSI
 761 and funded jointly by NeSI's collaborator institutions and through the Ministry of Busi-
 762 ness, Innovation and Employment's Research Infrastructure programme. [https://www](https://www.nesi.org.nz)
 763 [.nesi.org.nz](https://www.nesi.org.nz).

764 Thanks also to the two anonymous reviewers whose insightful comments were in-
 765 valuable in increasing the clarity and impact of this paper.

766 Basilisk is a freely available, multi-purpose tool to solve partial differential equa-
 767 tions, developed by Popinet and collaborators (2013–2020). It has its own website, which
 768 provides general information including installation instructions and a tutorial: <http://>

769 www.basilisk.fr. The code contains solvers for Saint-Venant problems, the Navier-Stokes
 770 equations and more, see <http://basilisk.fr/src/README>. The Basilisk setup code used
 771 for the numerical simulations is freely available at [https://doi.org/10.5281/zenodo](https://doi.org/10.5281/zenodo.5601492)
 772 [.5601492](https://doi.org/10.5281/zenodo.5601492) and the data used in the manuscript can be easily generated by running these
 773 scripts with the Basilisk software.

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