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The dispersive velocity of compressional waves in magmatic suspensions

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14 **Abstract:**

15 The geophysical detection of magma bodies and the estimation of the dimensions, physical
16 properties, and the volume fraction of each phase is required to improve the forecasting of volcanic
17 hazards and understanding of transcrustal magmatism. We develop an analytical model to calculate the
18 speed of sound in a magma consisting of crystals and gas bubbles suspended in a viscous melt. We
19 apply our model to calculate the speed of sound as a function of the temperature in three magmas with
20 different chemical compositions, representative of the diversity that may be encountered in arc
21 magmatism. The model employs the coupled phase theory that explicitly accounts for the exchange of
22 momentum and heat between the phases. We show that the speed of sound varies non-linearly with the
23 frequency of an acoustic perturbation between two theoretical bounds. The dispersion of the sound in a
24 magma results from the exchange of heat between the melt and dispersed phases that affects the
25 magnitude of their thermal expansions. The lower bound of the sound speed occurs at low frequencies
26 where all the constituents can be considered in thermal equilibrium, whereas the upper bound occurs at
27 high frequencies for which the exchange of heat between the phases may be neglected. The presence of
28 gas in a magma produces a sharp decrease in the velocity of compressional waves and generates
29 conditions in which the dispersion of the sound is significant at the frequencies usually considered in
30 geophysics. Finally, we compare the estimates of our model with the one from published relationships.
31 Differences are largest at higher frequencies and are <10% for typical magma.

32

33 **Key words:** Acoustic properties; Body waves; Physics of magma and magma bodies; Volcano
34 seismology.

35 **I introduction**

36 The possible reintrusion of crystal-rich magma reservoirs with crystal-poor magmas (e.g.
37 Caricchi et al. 2014, Annen et al. 2015, Wiebe 2016, Carrara et al. 2020), the crystal content of eruptive
38 products (e.g. Pallister et al. 1996, Eichelberger & Izbekov 2000, Takahashi & Nakagawa 2013), and
39 the occurrence of mechanisms leading to the extraction of melt or exsolved volatiles from crystal-rich
40 reservoirs (e.g. Bachmann & Bergantz 2004, Huber et al. 2011, Parmigiani et al. 2016, Holness 2018,
41 Bachmann & Huber 2019, Degruyter et al. 2019) are evidences of the episodic existence in upper
42 crustal magma reservoirs of volumes dominated by the presence of fluids. The crystals and exsolved
43 volatiles of a magma affect its rheology (e.g. Caricchi et al. 2007, Petford 2009, Mader et al. 2013) and
44 can dramatically change eruptive styles (e.g. Karlstrom et al. 2012, Cassidy et al. 2018). When the solid
45 volume fraction reaches a threshold, crystals start to touch each other, forming a semi-rigid skeleton
46 inhibiting magma flow (Bergantz *et al.* 2017). As a result, the volumes dominated by the presence of
47 fluids likely represent the eruptible portions of the reservoir. Therefore, the detection of such volumes
48 and the estimation of the volume fraction of each phase and is of paramount importance to enhance our
49 ability to predict the occurrence and style of eruptions and to best assess volcanic hazards.

50 Among geophysical methods, tomography of seismic wave velocities and attenuations has been
51 widely employed to map magma reservoirs but has not clearly evidenced the presence of fluid-
52 dominated bodies (Waite & Moran 2009, Paulatto *et al.* 2012, De Siena *et al.* 2014, Huang *et al.* 2015,
53 Delph *et al.* 2017, Kiser *et al.* 2018, Hooft *et al.* 2019). Tomography images are computed with the first
54 wave arrival at the stations, which corresponds to the fastest travel from the source. The velocities of
55 compressional waves (also called sound speed) are lower in fluid-rich magmas. As a result, the ray
56 paths of the first arrivals may circumvent and undersample such volumes in the resulting images. The
57 spatial averaging of the seismic properties resulting from tomography may also smooth the effects of

58 the presence of small fluid bodies, which are then interpreted as partially molten rocks. Finally, seismic
59 waves may be attenuated during their propagation across the magma reservoir. Hence, improving our
60 knowledge of the acoustic properties of the materials constituting the magma reservoir can reduce the
61 uncertainties in interpretation.

62 While models exist to compute the speed of sound and/or the associated attenuation coefficient
63 in partially molten rocks (e.g. Mavko 1980, Hammond & Humphreys 2000, Takei 2002, Hier-
64 Majumder 2008, Carcione et al. 2020) or in bubbly melts (e.g. Chouet 1996, Kumagai & Chouet 2000,
65 Morrissey & Chouet 2001, Neuberg & O’Gorman 2002, Collier et al. 2006, Karlstrom & Dunham
66 2016), no model addresses the acoustic properties of magmas composed of crystals and gas bubbles
67 suspended in the melt. Here we aim at calculating the velocity of a compressional wave at the
68 frequencies used in volcano seismology (~ 0.001 -1000 Hz) in a three-phase suspension composed of
69 solids and gas bubbles suspended in a viscous liquid. Previous models for the acoustic properties of a
70 suspension employed methods based on the effective medium theory (e.g. Kuster & Toksöz 1974,
71 Berryman 1980) because they are applicable at any frequency, and account for the presence of an
72 unlimited number of phases. However this approach neglects, the influence of the liquid phase
73 viscosity, the relative motion (or relative velocity) between the constituents, the evolution of the
74 temperature of the phases, and the interaction between neighboring solids grazing each other.
75 Alternatively, methods using the coupled phase theory (e.g. Harker & Temple 1988, Atkinson &
76 Kytömaa 1992, Margulies & Schwarz 1994, Kytömaa 1995, Evans & Attenborough 1997, Valier-
77 Brasier et al. 2015) can capture all these effects for bi-phasic suspensions (e.g. solids in a liquid or
78 bubbles in a liquid) but are restricted to the long-wavelength approximation (wavelength of the
79 perturbation much larger than the size of the discrete phases). The coupled phase theory is, therefore,
80 suitable to compute the acoustic properties of magmas because the long-wavelength approximation is

81 always valid at the frequencies used in geophysics. To be extended to a three-phase magma, the method
82 requires modifications to account for both viscous and thermal effects and the joint presence of crystals
83 and gas bubbles.

84 Here, we adapt the coupled phase theory to the computation of the velocity of compressional
85 waves traveling in magmas where the crystals are not touching each other and where the melt is the
86 carrier phase. We first introduce the conservation equations controlling the propagation of an acoustic
87 perturbation in a suspension and present the calculation of the speed of sound using the coupled phase
88 theory. The resulting model allows us to compute both the velocity and intrinsic attenuation coefficient
89 of compressional waves. In this work, we focus on the speed of sound and we will address attenuation
90 in a future communication. Results are first presented for a suspension representative of magmas to
91 illustrate how its composition and the characteristics of the perturbation (frequency and propagation
92 direction) affect the speed of sound. We then apply the model to magmas having different chemical
93 compositions representative of arc magmatism to highlight the key features of the propagation of sound
94 in magmas. Finally, we compare the results of our model with other relationships proposed or
95 employed by authors to estimate the speed of sound in magmas.

96

97 **II Method**

98 To present the model, we first introduce the physical model and assumptions about the initial
99 conditions (section II.1). Then, we introduce the conservation equations describing the dynamics of the
100 phases (section II.2). These equations are similar to those reported in the literature (e.g. Harker &
101 Temple 1988, Atkinson & Kytömaa 1992, Evans & Attenborough 1997) and include a few
102 modifications to account for the presence of three phases and the dynamic viscosity of the melt. The
103 details of the derivation of these equations are presented in supplementary material 1 (online). In
104 section II.3 we present the relationships controlling the transfer momentum between the phases and
105 within the liquid and solid phases. Section II.4 details the relationship we used to account for the
106 transfer of heat within the liquid phase and between the carrier and suspended phases. Section II.5
107 describes briefly the calculation of the speed of sound using the coupled phase theory. We employed
108 the same method as presented and employed by several authors to calculate the velocity of a
109 compressional wave from the(e.g. Harker & Temple 1988, Atkinson & Kytömaa 1992, Evans &
110 Attenborough 1997). We detail the calculation of the speed of sound in supplementary information 2
111 (online). In section II.6, we derive the bounds of the speed of sound in magma using an alternative
112 approach considering an isotropic compression of an isolated volume of magma. Finally, we present the
113 initial composition of the magmas and how the evolution of their physical properties change as a
114 function of temperature are computed (section II.7).

115

116 **II.1 Physical model**

117 We consider an elementary volume of a suspension composed of solid particles and gas bubbles
118 in a viscous liquid (Fig. 1). Both discrete phases are represented with monodisperse spheres. For the
119 initial conditions, we consider all the constituents of the suspension to be static and in thermodynamic

equilibrium. The thermodynamic properties of each phase (e.g. bulk modulus or heat capacity) are considered uniform within the elementary volume and constant with pressure and temperatures. The presence of mass transfer or chemical reactions between the phases is neglected.

A plane and monochromatic compressional wave propagates in the direction x with an angle θ from the horizontal with respect to the gravitational acceleration (Fig. 1). The geometry of the acoustic perturbation allows us to express the conservation equations describing the dynamics of each phase in one dimension aligned with the propagation direction x ($\partial y = \partial z = 0$). The wave has a small amplitude and a frequency in the range of the acoustic signals recorded in nature (0.001-1000 Hz) for which its wavelength is much larger than the diameters of the particles and bubbles. Therefore, the scattering of the acoustic wave resulting from resonance effects in the discrete solids and bubbles may be neglected (Atkinson & Kytömaa 1992).

131

132 **II.2 Conservation equations**

The propagation of an acoustic perturbation in the suspension is governed by the conservation equations of each phase. Neglecting the transfer of mass between the phases and chemical reactions, the conservation of mass of the liquid phase reads:

$$136 \quad \partial_t (\phi_l \rho_l) + \partial_x (\phi_l \rho_l u) = 0 \quad , \quad (1)$$

where ρ_l is the density of the liquid phase, ϕ_l is the volume fraction of liquid in the suspension, and u is the liquid velocity in the direction x ($u = u_x$, the velocities in the other directions are null because we consider a plane wave and the suspension to be initially static). Similarly, for the solid and gas phases, mass conservations are:

$$141 \quad \partial_t (\phi_s \rho_s) + \partial_x (\phi_s \rho_s v) = 0 \quad , \quad (2)$$

$$\partial_t (\phi_g \rho_g) + \partial_x (\phi_g \rho_g w) = 0 \quad , \quad (3)$$

where ρ_s is the density of the solids, ϕ_s is the volume fraction of solids, v is the velocity of the solids in the direction x , ρ_g is the gas density, ϕ_g is the volume fraction of gas, and w is the velocity of the gas bubbles.

The rate of change of momentum of the liquid equals the sum of the applied force and may be expressed as (see supplementary material 1, online):

$$\phi_l \rho_l (\partial_t (u) + u \partial_x (u)) + \phi_l \partial_x (P) + I_{ll} + I_{ls} + I_{lg} + \phi_l \rho_l g \sin \theta = 0 \quad , \quad (4)$$

where P is the pressure, I_{ll} is the rate of momentum exchange among the liquid, I_{ls} is the rate of momentum exchange between the liquid and solid phases, I_{lg} is the rate of momentum exchange between the liquid and gas phases, and g is the gravitational acceleration. The exchange of momentum within the liquid, I_{ll} , is equal to the divergence of the viscous stress tensor and indicates the rate at which the viscous stress propagates in the liquid. The two other terms, I_{ls} and I_{lg} , expresses the exchanges of momentum between the carrier and discrete phases through the drag forces when they experienced relative velocities. The conservation of momentum in the solid phases is:

$$\phi_s \rho_s (\partial_t (v) + v \partial_x (v)) + \phi_s \partial_x (P) + I_{ss} - I_{ls} + \phi_s \rho_s g \sin \theta = 0 \quad , \quad (5)$$

where I_{ss} represents the transfer of momentum between close solids. Here this term corresponds to the lubrication forces relating to the squeezing of the interstitial liquid located between two grazing particles (see next section for details about this term). The momentum conservation in the gas phase reads:

$$\phi_g \rho_g (\partial_t (w) + w \partial_x (w)) + \phi_g \partial_x (P) - I_{lg} + \phi_g \rho_g g \sin \theta = 0 \quad . \quad (6)$$

The conservation of energy in the carrier liquid expressed as a function of the temperature reads:

$$\begin{aligned} 164 \quad & \phi_l \rho_l C_{Pl} (\partial_t (T_l) + u \partial_x (T_l)) - \phi_l T_l \alpha_l (\partial_t (P) + u \partial_x (P)) + 2 \phi_l \rho_l g u \sin \theta \\ & - \bar{\bar{\sigma}}_{xx} \partial_x (u) + H_{ll} + H_{ls} + H_{lg} = 0 \end{aligned} \quad (7)$$

165 where T_l is the temperature of the liquid phase, C_{Pl} is the specific heat capacity at a constant pressure of
 166 the liquid, α_l is the coefficient of thermal expansion of the liquid, $\bar{\bar{\sigma}}$ is the liquid viscous stress tensor,
 167 H_{ll} is the rate of heat diffusion within the liquid phase by conduction, H_{ls} is the rate of heat exchange
 168 between the carrier liquid and discrete solids, and H_{lg} is the rate of heat exchange between the liquid
 169 and gas bubbles. The two terms H_{ls} and H_{lg} are the total heat flux over the interfaces between the carrier
 170 and discrete phases and depend on the temperature difference between constituents. Similarly, in the
 171 solid and gas phases, the conservation of energy is:

$$172 \quad \phi_s \rho_s C_{Ps} (\partial_t (T_s) + v \partial_x (T_s)) - \phi_s T_s \alpha_s (\partial_t (P) + v \partial_x (P)) + 2 \phi_s \rho_s g v \sin \theta - H_{ls} = 0 \quad , \quad (8)$$

$$173 \quad \phi_g \rho_g C_{Pg} (\partial_t (T_g) + w \partial_x (T_g)) - \phi_g T_g \alpha_g (\partial_t (P) + w \partial_x (P)) + 2 \phi_g \rho_g g w \sin \theta - H_{lg} = 0 \quad , (9)$$

174 where T_s is the temperature of the solids, T_g is the temperature of the gas, C_{Ps} is the specific heat
 175 capacity at a constant pressure of the particles, C_{Pg} is the specific heat capacity of the gas, α_s is the
 176 coefficient of thermal expansion of the solid particles, and α_g is the coefficient of thermal expansion of
 177 the gas.

178 The state equations link the variation of the density of the phases to the evolution of their
 179 temperatures and pressure:

$$180 \quad d \rho_l - \frac{\rho_l}{K_l} dP + \alpha_l \rho_l dT_l = 0 \quad , \quad (10)$$

$$181 \quad d \rho_s - \frac{\rho_s}{K_s} dP + \alpha_s \rho_s dT_s = 0 \quad , \quad (11)$$

$$182 \quad d \rho_g - \frac{\rho_g}{K_g} dP + \alpha_g \rho_g dT_g = 0 \quad , \quad (12)$$

183 where K_l is the bulk modulus of the liquid phase (inverse of the coefficient of isothermal
 184 compressibility), K_s is the bulk modulus of the solids, and K_g is the bulk modulus of the gas. The last
 185 conservation equation ensures that the sum of the volume fraction of all the phases is always equal to
 186 one. In differential form, it reads:

$$187 \quad \partial_t(\phi_l) + \partial_t(\phi_s) + \partial_t(\phi_g) + u \partial_x(\phi_l) + v \partial_x(\phi_s) + w \partial_x(\phi_g) = 0 \quad . \quad (13)$$

188 **II.3 Interphase exchanges of momentum**

189 The exchange of momentum within the liquid phase, I_l , is equal to the divergence of the viscous
 190 stress tensor, $\bar{\sigma}$, which depends on the dynamic shear viscosity, η , and volume viscosity, λ , of the
 191 liquid ($I_l = \phi \nabla \cdot [\eta \bar{\epsilon} + \lambda \text{tr}(\bar{\epsilon}) \bar{I}]$, where $\bar{\epsilon}$ is the strain rate tensor and $\bar{I}$ is the unit tensors). For a
 192 magmatic melt in relaxed conditions (low-frequency perturbations) Dingwell & Webb (1989), showed
 193 that $\lambda = \eta/3$. Since the velocity of the liquid in the directions y and z are null, the rate of momentum
 194 exchange in the liquid can be calculated as:

$$195 \quad I_l = \frac{7}{3} \phi_l \eta \partial_x^2(u) \quad . \quad (14)$$

196 While contacts are neglected, crystals can exchange momentum through lubrication forces
 197 (Marzougui *et al.* 2015, Bergantz *et al.* 2017, Carrara *et al.* 2019). Lubrication refers to the
 198 hydrodynamic forces resulting from the resistance of the liquid located in the gap between two
 199 neighboring particles to their relative motions. These forces influence the duration of the initiation and
 200 closure of motion of the solid phase (Carrara *et al.* 2019). The propagation of a wave in a suspension
 201 can be viewed as a “transient steady-state” in which the relative motions between neighboring solids
 202 are repetitively initiated and dissipated. To derive an expression of the rate of momentum exchange
 203 between the solids, I_{ss} , we consider a suspension where spherical particles are regularly organized
 204 forming a hexagonal close-packed lattice (Fig. 2). In this configuration, the distances between the

205 particles are identical and minimized such that the influence of lubrication is maximized. The solid
 206 lattice is oriented along the direction of propagation of the wave (x) such that it can be represented as
 207 three layers of particles orthogonal to the direction x (Fig. 2). The total lubrication force between two
 208 neighboring particles (here labeled as i and k) including both normal and tangential components can be
 209 expressed as (Marzougui *et al.* 2015, Carrara *et al.* 2019):

$$210 \quad F_{lub}(k, i) = \frac{3 \eta A}{\rho_s d_s^2} (v_k - v_i) \quad , \quad (15)$$

211 where A is a geometrical parameter indicating the relative importance between normal and tangential
 212 lubrication forces that depends on the distance between the surface of the particles and on the grazing
 213 angle β (Carrara *et al.* 2019):

$$214 \quad A = \frac{3 \cos \beta}{2 j} - \ln(j) \sin \beta \quad , \quad (16)$$

215 where j is the ratio between the distance separating the surface of the neighboring particles and their
 216 radius. Both the incidence angle and distance between the surface of the particles can be deduced from
 217 geometrical arguments since the solid lattice is regular. For a compressional wave $\beta=(2/3)^{1/2}$ and j is
 218 related to the volume fraction of solids by (Atkinson & Kytömaa 1992):

$$219 \quad j = 1 - \left(\frac{\phi_s}{\phi_{s \max}} \right)^{\frac{1}{3}} \quad , \quad (17)$$

220 where $\phi_{s \max}$ is the maximum volume fraction at which the solids start to touch each other. For a
 221 hexagonal close-packed lattice, $\phi_{s \max}=0.64$. Summing all the lubrication interaction experienced by the
 222 particle located on the second layer in Fig. 2 gives:

$$223 \quad F_{lub}^{tot}(2) = \frac{9 \eta A}{\rho_s d_s^2} (v_1 + v_3 - 2 v_2) \quad , \quad (18)$$

224 where v_1 , v_2 , and v_3 are the velocity of the particles in the layer 1, 2, and 3 in Fig. 2, respectively. The
 225 sum of the solid velocities on the right-hand side of Eq. (18) may be approximated with the second-
 226 order derivative in space of the solid velocity:

$$227 \quad \frac{\partial^2 v_x}{\partial x^2} \simeq \frac{(v_1 + v_3 - 2 v_2)}{\Delta_x^2} , \quad (19)$$

228 where Δ_x is the distance in the direction x separating two successive layers of solids in Fig. 2, which
 229 can be calculated as:

$$230 \quad \Delta_x = \sqrt{\frac{2}{3}} d_s \left(\frac{j}{2} + 1 \right) . \quad (20)$$

231 Inserting Eqs. (19–20) into Eq. (18) gives the following expression for the exchange of momentum
 232 between the solids:

$$233 \quad I_{ss} = F_{lub} \simeq \frac{6 \eta A \left(\frac{j}{2} + 1 \right)^2}{\rho_s} \frac{\partial^2 v}{\partial x^2} . \quad (21)$$

234 The liquid exchanges momentum with the other phases (particles and bubbles) because of their
 235 relative motions. The transfers of momentum between the carrier and discrete phases include both
 236 steady (drag) and unsteady (added mass and Basset forces) contributions. Because of the viscosity of
 237 magmatic melts, the frequency range considered here is well below those at which unsteady forces
 238 become significant compared to the steady contribution (Gumerov *et al.* 1988, Atkinson & Kytömaa
 239 1992). Therefore, the rate of momentum exchange between the liquid and solid phases can be reduced
 240 to the steady term (Gidaspow 1994):

$$241 \quad I_{ls} = \beta_{ls} (u - v) , \quad (22)$$

242 and the rate of momentum exchange between liquid and bubbles is:

$$243 \quad I_{lg} = \beta_{lg} (u - w) , \quad (23)$$

244 where β_{ls} is the coefficient of momentum exchange between the liquid and solids and β_{lg} is the
 245 coefficient of momentum exchange between the liquid and gas phases. To compute these two
 246 coefficients, several empirical correlations exist in the literature (e.g. Ergun 1952, Wen & Yu 1966,
 247 Syamlal et al. 1993, Gidaspow 1994, Benyahia et al. 2006). Here, we combine a Stokes drag law for
 248 high porosity (high liquid volume fraction) and an Ergun relationship at lower liquid volume fraction.
 249 Because of the high viscosity of the liquid phase, the Ergun drag law may be reduced to the Carman-
 250 Kozeny relationship because the inertial term becomes negligible:

$$251 \quad \beta_{ls} = \begin{cases} \frac{18 \eta \phi_s}{d_s^2} & \text{if } \phi_l > \frac{25}{28} \\ \frac{150 \phi_s^2 \eta}{\phi_l d_s^2} & \text{if } \phi_l \leq \frac{25}{28} \end{cases}, \quad (24)$$

$$252 \quad \beta_{lg} = \begin{cases} \frac{18 \eta \phi_g}{d_g^2} & \text{if } \phi_l > \frac{25}{28} \\ \frac{150 \phi_g^2 \eta}{\phi_l d_g^2} & \text{if } \phi_l \leq \frac{25}{28} \end{cases}, \quad (25)$$

253 where d_s is the diameter of the solid particles and d_g is the diameter of the gas bubbles. The drag laws
 254 we used here are similar to the one proposed by Gidaspow (1994), but uses the Stokes drag instead of
 255 the Wen-Yu drag correlation at high porosity such that the drag forces are linearly dependent on the
 256 relative velocities between the phases, which is suitable for the coupled phase theory.

257

258 **II.4 Interphase exchanges of heat**

259 The amount of heat transferred within the carrier liquid by conduction is calculated using
 260 Fourier's law:

$$261 \quad H_{ll} = \phi_l k_l \partial_x^2 (T_l), \quad (26)$$

262 where k_l is the heat conductivity of the liquid.

263 The rate of heat exchange between the carrier and discrete phases are expressed by:

264
$$H_{ls} = \gamma_{ls} (T_l - T_s) \quad , \quad (27)$$

265 and,

266
$$H_{lg} = \gamma_{lg} (T_l - T_g) \quad , \quad (28)$$

267 where γ_{ls} is the coefficient of heat transfer between the fluid and solids, and γ_{lg} is the coefficient of heat
268 exchange between the fluid and gas. In the absence of mass transfer between the phases, the
269 coefficients of heat transfer can be estimated as (Syamlal *et al.* 1993):

270
$$\gamma_{ls} = \frac{6 k_l \phi_s Nu}{d_s^2} \quad , \quad (29)$$

271 and,

272
$$\gamma_{lg} = \frac{6 k_l \phi_g Nu}{d_g^2} \quad , \quad (30)$$

273 where Nu is the Nusselt number. To estimate Nu , we used the empirical correlation proposed by Gunn
274 (1978), which depends on both the porosity and relative velocity between the phases. Since in our case
275 the relative velocities are very small because of the small amplitude of the perturbation and the
276 viscosity of the liquid, Nu may be expressed as a function of ϕ_l only:

277
$$Nu = (7 - 10 \phi_l + 5 \phi_l^2) \quad . \quad (31)$$

278

279 **II.5 Coupled phase model**

280 To compute the speed of sound from Eqs. (1-13), we employed the coupled phase theory (e.g.
281 Harker & Temple 1988, Atkinson & Kytömaa 1992, Evans & Attenborough 1997, Valier-Brasier et al.
282 2015; see supplementary material 2, online, for details about the method and equations), which consists

in imposing a small and monochromatic perturbation to all the variables that oscillate during the propagation of the acoustic perturbation ($\rho_l, \rho_s, \rho_g, u, v, w, T_l, T_s, T_g, \phi_l, \phi_s, \phi_g, P$) by using wave-like solutions (here for the density of the liquid phase):

$$\rho_l = \rho_l^0 + \bar{\rho}_l e^{i(kx - \omega t)} , \quad (32)$$

where ρ_l^0 is the static fluid density, $\bar{\rho}_l$ is the amplitude of the perturbation of the fluid density at the source, and $i^2 = -1$. Note that since all the phases are static and in thermal equilibrium before the perturbation, $u_0 = v_0 = w_0 = 0$ and $T_l^0 = T_s^0 = T_g^0 = T^0$. The exponential term in Eq. (32) expresses the spatial and temporal variations of the liquid density and depends on ω , the angular frequency ($\omega = 2\pi f$, f is the frequency of the perturbation), and k the complex wavenumber defined as:

$$k = \frac{\omega}{c} + i \alpha , \quad (33)$$

where c is the speed of sound and α is the associated intrinsic attenuation coefficient. After the introduction of the oscillating variables and linearization (the products of two small oscillations are neglected), the set of equations can be expressed as a matrix equation (see supplementary information 1 for details about the matrix equation):

$$\mathbf{M} [\bar{\rho}_l, \bar{\rho}_s, \bar{\rho}_g, \bar{\phi}_l, \bar{\phi}_s, \bar{\phi}_g, \bar{T}_l, \bar{T}_s, \bar{T}_g, \bar{u}, \bar{v}, \bar{w}, P]^T = 0 , \quad (34)$$

where $\mathbf{M}$ is a coefficients matrix containing k as unique unknown. To ensure the equality in Eq (34), the nontrivial solution (a perturbation exists) imposes that $\mathbf{M}$ can be inverted and thus:

$$\det(\mathbf{M}) = 0 . \quad (35)$$

The speed of sound and associated attenuation coefficient at a given frequency can be found from the wavenumber, k , that is physically meaningful and that satisfies Eq (35).

303

304 II.6 Bounds of the sound speed in magmas

305 The speed of sound in a suspension depends on the variation of its density and volume during
 306 an adiabatic compression or dilatation (Temkin 1998). Let's consider an elementary volume, V ,
 307 containing a constant mass of solids and gas bubbles suspended in a viscous liquid. By neglecting the
 308 relative motions between the phases, the suspension can be approximated as a homogeneous material
 309 having a bulk density, ρ^* , defined as (Brennen 2005):

$$310 \quad \rho^* = \phi_l \rho_l + \phi_s \rho_s + \phi_g \rho_g \quad . \quad (36)$$

311 The total net change of the elementary volume, dV , can be written as the sum of the net changes of the
 312 volume of the three phases:

$$313 \quad dV = dV_l + dV_s + dV_g \quad , \quad (37)$$

314 where dV_l is the net change of the volume of liquid, dV_s is the net change of the volume of solid, and
 315 dV_g is the net change of the volume of gas. Neglecting phase transformations, Eq. (37) may be
 316 expressed as:

$$317 \quad \frac{d\rho^*}{\rho^*} = \frac{\phi_l}{\rho_l} d\rho_l + \frac{\phi_s}{\rho_s} d\rho_s + \frac{\phi_g}{\rho_g} d\rho_g \quad , \quad (38)$$

318 where $d\rho^*$ is the net change in the bulk density of the suspension, $d\rho_l$ is the net change of the liquid
 319 density, $d\rho_s$ is the net change of the density of the solids, and $d\rho_g$ is the net change of the density of the
 320 gas bubbles. The evolution of the density of each phase depends on the change in pressure (here we
 321 consider that the pressure is the same in all the phases) and in its temperature as (here for a phase i):

$$322 \quad d\rho_i = \left(\frac{\partial \rho_i}{\partial P} \right)_T dP + \left(\frac{\partial \rho_i}{\partial T_i} \right)_P dT_i \quad , \quad (39)$$

323 where $(\partial \rho_i / \partial P)_T$ is the variation of the density of a constituent with the pressure at a constant
 324 temperature, and $(\partial \rho_i / \partial T_i)_P$ is the derivative of its density with respect to its temperature at a constant
 325 pressure. Introducing Eq. (39) into (38) yields:

$$\frac{d\rho^*}{\rho^*} = \frac{1}{K^*} dP + \frac{\phi_l}{\rho_l} \left(\frac{\partial \rho_l}{\partial T_l} \right)_P dT_l + \frac{\phi_s}{\rho_s} \left(\frac{\partial \rho_s}{\partial T_s} \right)_P dT_s + \frac{\phi_g}{\rho_g} \left(\frac{\partial \rho_g}{\partial T_g} \right)_P dT_g, \quad (40)$$

where K^* is the bulk modulus characterizing the suspension and defined as:

$$\frac{1}{K^*} = \frac{\phi_l}{K_l} + \frac{\phi_s}{K_s} + \frac{\phi_g}{K_g}, \quad (41)$$

with $K_l/\rho_l = (\partial P / \partial \rho_l)_T$, $K_s/\rho_s = (\partial P / \partial \rho_s)_T$, and $K_g/\rho_g = (\partial P / \partial \rho_g)_T$. When the net changes in temperature of the phases are neglected, the isothermal speed of sound is:

$$c^{-2} = \frac{d\rho^*}{dP} = \frac{\rho^*}{K^*}. \quad (42)$$

When considering temperature variations, the speed of sound depends on the evolution of the bulk density of the suspension with pressure at constant entropy (Temkin 2000, Rienstra & Hirschberg 2004). The net change of entropy, dS , in each phase may be expressed as a function of the net change in its temperature and pressure (here for a phase i):

$$T_i dS_i = \left(\frac{\partial S_i}{\partial T_i} \right)_P dT_i + \left(\frac{\partial S_i}{\partial P} \right)_T dP. \quad (43)$$

Considering an isentropic transformation, Eq. (43) can be expressed as:

$$\left(\frac{\partial T_i}{\partial P} \right)_S = - \left(\frac{\partial T_i}{\partial S_i} \right)_P \left(\frac{\partial S_i}{\partial P} \right)_T. \quad (44)$$

Because the magma constituents have different thermodynamic properties, the net changes in temperature of the phases for the same net change in pressure are not equal. Two end-member scenarios may be considered as a function of the perturbation frequency and characteristic times for the phases to reach thermal equilibrium, τ . When $f \gg \tau^{-1}$, the heat exchanges between the phases may be neglected such that $dT_l \neq dT_s \neq dT_g$. Inserting Eq. (44) into Eq. (40) and considering the relationships

344 $dT_i = (\partial T_i / \partial P)_S dP$, $(\partial T_i / \partial S_i)_P = T_i / C_{Pi}$, $(\partial S_i / \partial P)_T = -\alpha_i / \rho_i$, and $(\partial \rho_i / \partial T)_P = -\alpha_i \rho_i$, the speed of sound at
 345 thermal disequilibrium reads:

$$346 \quad c^{-2} = \frac{d\rho^*}{dP} = \rho^* \left(\frac{1}{K^*} - \frac{\phi_l \alpha_l^2 T^0}{C_{Pl} \rho_f} - \frac{\phi_s \alpha_s^2 T^0}{C_{Ps} \rho_s} - \frac{\phi_g \alpha_g^2 T^0}{C_{Pg} \rho_g} \right). \quad (45)$$

347 When $f \ll \tau^{-1}$, the rates of heat exchanges between the phases are efficient such that the phases
 348 may be considered in thermal equilibrium during the propagation of the perturbation. In adiabatic
 349 conditions, the total change of temperature in the suspension at equilibrium, dT^* , may be calculated as:

$$350 \quad dT^* = \frac{\phi_l \rho_l C_{Pl} dT_l + \phi_s \rho_s C_{Ps} dT_s + \phi_g \rho_g C_{Pg} dT_g}{\phi_l \rho_l C_{Pl} + \phi_s \rho_s C_{Ps} + \phi_g \rho_g C_{Pg}}. \quad (46)$$

351 Considering $dT_l = dT_s = dT_g = dT^*$ into Eq. (40) and inserting Eqs. (44) and (46) gives the following
 352 relationship for the speed of sound at thermal equilibrium:

$$353 \quad c^{-2} = \frac{d\rho^*}{dP} = \frac{\rho^*}{K^*} - \frac{\alpha^{*2} T^0}{C_P^*}, \quad (47)$$

354 where α^* is the bulk coefficient of thermal expansion defined as:

$$355 \quad \alpha^* = \phi_l \alpha_l + \phi_s \alpha_s + \phi_g \alpha_g, \quad (48)$$

356 and C_P^* is the specific bulk heat capacity at constant pressure calculated as the mass average (:

$$357 \quad C_P^* = \frac{\phi_l \rho_l C_{Pl} + \phi_s \rho_s C_{Ps} + \phi_g \rho_g C_{Pg}}{\rho^*}. \quad (49)$$

358 The characteristic time at which the transition between the two regimes occurs depends on the
 359 rate at which the heat is exchanged between the phases. Two characteristic times may be calculated
 360 since both the gas and solids are suspended in the liquid. To estimate these critical frequencies, we start
 361 by considering a static suspension of gas bubbles in a liquid. The evolution of the difference in
 362 temperature between the two phases resulting only from the heat exchanged between them may be
 363 approximated as:

$$\partial_t (T_l - T_g) + \tau_g^{-1} (T_l - T_g) = 0 \quad . \quad (50)$$

where τ_g is the characteristic time to equilibrate the temperature of the liquid and the gas phases given by:

$$\tau_g^{-1} = \gamma_{lg} \left(\frac{1}{\phi_l \rho_l C_{Pl}} + \frac{1}{\phi_g \rho_g C_{Pg}} \right) \quad . \quad (51)$$

Similarly, the characteristic time to equilibrate the temperature of the phases in a suspension of solids in a liquid, τ_s , is:

$$\tau_s^{-1} = \gamma_{ls} \left(\frac{1}{\phi_l \rho_l C_{Pl}} + \frac{1}{\phi_s \rho_s C_{Ps}} \right) \quad . \quad (52)$$

371

372 **II.7 Magma under considerations**

373 We considered three different magmas representative of compositions that may be encountered in
 374 arc magmatism (basalt, andesite, and dacite) and simulated their adiabatic cooling and crystallization
 375 using the software MELTS (Ghiorso 2004) at a pressure of 150 MPa using the Ni-NiO oxygen buffer
 376 for the andesite and dacite, and the quartz-fayalite-magnetite buffer for the basalt. The initial
 377 compositions were taken from Dufek & Bachmann (2010) and Martel et al. (1999) (see Table 1). We
 378 set the initial amounts of dissolved water in the magmas to ~3.5wt% to ensure that water vapor starts
 379 exsolving once the mass fraction in liquid is ~70 wt% (Duan 2014). We used the thermodynamic
 380 properties of each phase computed during the cooling simulations to estimate the speed of sound in the
 381 magmas as a function of their temperature. All the simulations were stopped when the crystallinity of
 382 the magmas reached the maximum packing fraction ($\phi_l = 0.36$).

383

384 III Results

385 III.1 The speed of sound in 3 phases suspensions

386 To illustrate how the material properties and characteristics of the perturbation affect the speed of
387 sound in magmas, we define reference conditions relevant to magmas (Table 2) and vary selected
388 parameters independently from each other. Figure 3A displays the evolution of the velocity of a
389 compressional wave as a function of the volume fraction in liquid, solids, and gas bubbles when $f=0.01$
390 Hz. It shows that the speed of sound decreases rapidly once a small volume fraction of volatiles is
391 exsolved. When $\phi_g > 0.05$, the solid volume fraction has a negligible influence on the compressional
392 wave velocity compared to that in the presence of gas.

393 Figure 3B displays the speed of sound in the same suspension as in Fig. 3A when $f=100$ Hz.
394 Results show the same dependence of the wave velocity on ϕ_g . The amplitude of the decrease of the
395 speed of sound for the same volume fraction of gas is, however, slightly lower as illustrated by the shift
396 of the position of the isocontour of $c = 500 \text{ m s}^{-1}$. The frequency of the perturbation changes both the
397 minimum ($c \approx 457 \text{ m s}^{-1}$ when $f = 0.01 \text{ Hz}$ and $c \approx 482 \text{ m s}^{-1}$ when $f = 100 \text{ Hz}$) and maximum ($c \approx 3117$
398 m s^{-1} when $f = 0.01 \text{ Hz}$ and $c \approx 3150 \text{ m s}^{-1}$ when $f = 100 \text{ Hz}$) velocities computed by the model. On the
399 contrary, when $\phi_l = 1$ the speed of sound is the same at the two frequencies (2582 m s^{-1}).

400 To further investigate the influence of the perturbation frequency on sound speed in a magma, we
401 set the volume fractions of its constituents to $\phi_l = 0.65$, $\phi_s = 0.3$, and $\phi_g = 0.05$ and calculate the
402 dispersion curve of the acoustic waves (Fig. 4). Results show that the speed of sound increases non-
403 linearly with the frequency and that three plateaux can be identified. The lowest plateau at $c \approx 1020 \text{ m}$
404 s^{-1} occurs when $f < 0.01 \text{ Hz}$. The second velocity plateau at $c \approx 1025 \text{ m s}^{-1}$ occurs when $0.1 \text{ Hz} < f < 1$
405 Hz whereas the third and fastest one at $c \approx 1090 \text{ m s}^{-1}$ is reached when $f > 100 \text{ Hz}$. As illustrated in Fig.
406 4, the uppermost plateau occurs when $f \gg \max(\tau_s^{-1}, \tau_g^{-1})$ and corresponds to the thermal disequilibrium

407 bound of the speed of sound predicted by Eq. (38). The lowest plateau is found when $f \gg \min(\tau_s^{-1}, \tau_g^{-1})$
408 and corresponds to thermal equilibrium bound of the speed of sound given by Eq. (40). The isothermal
409 bound (Eq. 35) underestimate the speed of sound at all frequencies.

410 Figure 5 displays the evolution of the wave velocity as a function of the propagation angle, θ , and
411 frequency of the perturbation. When $f \geq 0.1$ Hz, the velocity of the wave is lower when the wave
412 propagates upward than when it propagates downward. When $f < 0.1$ Hz, the speed of sound show a
413 complex dependence on the propagation angle. The maximum velocity is computed when $\theta = \pi/2$
414 whereas the minimum sound speed occurs when the wave propagates downward with a propagation
415 angle of $\sim 30^\circ$ from the horizontal. The propagation angle influences sound speed because of the terms
416 involving the gravitational acceleration in Eqs. (4–9). In momentum conservation, these terms express
417 the contribution of the change in the density of the phases to the gravitational force. In the energy
418 conservations, the term involving the gravitational acceleration express the rate at which the potential
419 energy change as a function of the velocity of the phase along the vertical direction. The amplitude of
420 the differences in the wave velocity as a function of the propagation angle ($< 1\%$) is negligible
421 compared to the influence of the volume fraction of the constituents (Fig. 3) and frequency of the
422 perturbation (Fig. 4).

423 Figure 5A displays the difference between the speed of sound computed when lubrication is
424 accounted for and when it is neglected as a function of the ratio $\phi_s/\phi_{s \max}$ and perturbation frequency.
425 The difference when the influence of lubrication forces increases with the ratio $\phi_s/\phi_{s \max}$ and with wave
426 frequency. When $f < 10^3$ Hz, neglecting lubrication forces results in a negligible underestimation of the
427 speed of sound ($< 10^{-4}$ m s $^{-1}$).

428

429

430 III.2 Application to magmas

431 Figure 6A-E displays the evolution of the phase assemblages and thermodynamic properties of the
432 three magmas computed by the cooling simulations and averaged over the phases with Eq. (29), (34),
433 (41), and (42) (see supplementary material 3, online, for details on the thermodynamic properties of the
434 constituents). The thermodynamic properties show sharp changes once the water vapor is exsolved. The
435 bulk moduli of the magmas, in particular, drop by almost one order of magnitude once a small fraction
436 of water is exsolved (Fig. 6D), resulting in a sharp decrease of the sound speed (Fig 6F). In the absence
437 of gas, the influence of the frequency of the perturbation is weak enough for the lower (Eq. 40) and
438 upper (Eq. 38) bounds of the sound speed to be almost equal. On the contrary, the two bounds show
439 significant differences when a gas phase is present (Fig. 6F). The amplitude of the difference between
440 the two bounds increases with the volume fraction of gas and decreases with the temperature. In the
441 final phases assemblages, the gas volume fractions are $\phi_g \approx 0.15$ in the basalt (Fig 6A), $\phi_g \approx 0.1$ in the
442 andesite (Fig. 6B), and $\phi_g \approx 0.05$ in the dacite (Fig. 6C). These values translate into amplitude
443 differences between the upper and lower bounds of $\sim 150 \text{ m s}^{-1}$ in the basalt, $\sim 200 \text{ m s}^{-1}$ in the andesite,
444 and $\sim 250 \text{ m s}^{-1}$ in the dacite (Fig 6F).

445

446

447 IV. Discussion

448 IV.1 Predicting the speed of sound in magmas

449 Several relationships have been proposed to estimate the speed of sound in two-phase
450 suspensions (e.g. Kuster & Toksöz 1974, Kieffer 1977, Berryman 1980, Harker & Temple 1988,
451 Commander & Prosperetti 1989, Atkinson & Kytömaa 1992), and employed for bubbly magmas (e.g.
452 Chouet 1996, Kumagai & Chouet 2000, Morrissey & Chouet 2001, Neuberg & O’Gorman 2002,
453 Karlstrom & Dunham 2016). To compare all these models with the results of Eq. (28) based on the
454 coupled phase theory, we considered a suspension of bubbles of an ideal gas suspended in water (see
455 Table 3 for thermodynamic properties of the phases). Figure 6A displays the comparison of the speed
456 of sound estimated with the different models for $10^{-3} \leq f \leq 10^3$ Hz. The models neglecting the evolution
457 of the temperature of the phases (Kuster & Toksöz 1974, Berryman 1980, Harker & Temple 1988,
458 Atkinson & Kytömaa 1992, Neuberg & O’Gorman 2002, Karlstrom & Dunham 2016) give the same
459 results as Eq. (35) and underestimate the speed of sound and do not capture its dispersion (Fig. 6A).
460 When a material is compressed or decompressed, the temperature of its constituents changes
461 accordingly, inducing their thermal expansions, which oppose the change of volume resulting from the
462 change in pressure. As a result, neglecting the thermal effects results in overestimating the ratio
463 $(d\rho^*/dP)$, and in turn, underestimating the speed of sound. In magmas and at low frequency, the
464 difference between the isothermal and isentropic speed of sounds is, however, small when a gas phase
465 is present (Fig. 6F). Neglecting the evolution of the temperature of the phases is thus an acceptable
466 assumption for bubbly magmas at low frequencies ($f < \sim 10^{-2}$ Hz in Fig. 4) given the uncertainty on the
467 thermodynamic properties of the constituents. At higher frequencies, the isothermal assumption results
468 in a large underestimation of the speed of sound in a bubbly magma (of ~ 200 m s⁻¹ in andesite with
469 $\phi_g \approx 0.10$; Fig 5F). On the contrary, in the absence of exsolved volatiles, the isothermal assumption

470 results in a significant underestimation of the speed of sound compared with the isentropic case at any
471 frequency (of $\sim 50 \text{ m s}^{-1}$ in Fig. 6F).

472 Other relationships account for the thermal effects during the propagation of an acoustic wave.
473 The model proposed by Kieffer (1977) predicts sound speeds between the upper and lower bounds but
474 does not capture the dispersion of the sound (Fig. 7A). In this model, while the temperature changes are
475 accounted for in the gas phase, they are neglected in the liquid. As a result, the compression and
476 decompression are isentropic in the gas while isothermal in the liquid, which explains why this model
477 predicts speed of sounds between those predicted with Eqs. (42) and (45). The lack of sound dispersion
478 results from the absence of heat exchange between the phases in the model. The dispersion of sound is
479 particularly important in bubbly magmas (Fig. 3 and Fig. 6F). In general, $\alpha_g/(\rho_g C_{pg}) \gg \alpha_l/(\rho_l C_{pl})$. As a
480 result, for the same net change in pressure $|dT_l| < |dT^*| < |dT_g|$. Since $\alpha_g \gg \alpha_l$ the amplitude of the thermal
481 expansion of the gas bubbles increases significantly out of thermal equilibrium, which in turn amplifies
482 the resistance of bulk material to compression and decompression and increases the speed of sound
483 (Temkin 2000). This effect results in the two increases of the speed of sound with frequency observed
484 in Fig. 4. The first increase occurs when the solids become out of thermal equilibrium with the liquid.
485 This velocity jump may be ignored in crystal-bearing magmas (Fig 6F) because the coefficients of
486 thermal expansion of the melt and crystals are smalls (see supplementary material 3, online).

487 The relationship proposed by Commander & Prosperetti (1989) and employed by Chouet
488 (1996), Kumagai & Chouet (2000) and Morrissey & Chouet (2001) accounts for the exchange of heat
489 from the bubbles to the liquid and captures the increase of the speed of sound at the same range of
490 frequencies as Eq. (28) (Fig. 6A). The evolution of the temperature in the liquid is, however, neglected
491 in this model. Consequently, it predicts sound speeds slower than Eq. (28) for all the frequencies. This
492 model also considers the dynamics of the interface between the gas bubbles and surrounding liquid, a

phenomenon not accounted for in our model. The dynamics of the bubbles are expected to cause a sudden increase in sound speed at a resonance frequency (Commander & Prosperetti 1989, Chouet 1996), which is not captured in Eq. (28) (Fig 7B). In magmas, the resonance frequency of bubbles is on the order of the kHz (Chouet 1996). This is above the frequencies usually used in geophysics and can be ignored for most applications.

It should be noted that another increase in the speed of sound is expected for each discrete phase at higher frequencies than explored herein (Temkin 2000). These velocity jumps result from the translational relative motions between the liquid and the discrete phases. As for heat, the constituents of the suspension exchange momentum during the propagation of an acoustic perturbation because of their relative motions. The rates of momentum transfer in the suspension depend on the coefficients of momentum exchange given by Eqs. (16-17). The evolution of the relative velocity between the carrier and suspended phases is (here between the liquid and the gas):

$$\partial_t (u - w) + \nu_{lg} (u - w) = 0 \quad (46)$$

where ν_{lg} is the critical frequency above which translational effect cannot be neglected. It is defined as the inverse of the characteristic time needed for the relative velocity between the gas and liquid phases to vanish:

$$\nu_{lg} = \beta_{lg} \left(\frac{1}{\phi_l \rho_l} + \frac{1}{\phi_g \rho_g} \right) \quad (47)$$

Similarly, the critical frequency for translational relative motion between the solids and liquid, ν_{ls} , is:

$$\nu_{ls} = \beta_{ls} \left(\frac{1}{\phi_l \rho_l} + \frac{1}{\phi_s \rho_s} \right) \quad (48)$$

When $f > \max(\nu_{ls}, \nu_{lg})$ the magnitude of the relative velocity between the carrier liquid and discrete phases becomes significant and cannot be ignored when computing the speed of sound in the

suspension. At these frequencies, the assumption of homogeneity required to express Eq. (29) is violated such that Eq. (38) cannot be employed to estimate the velocity of compressional waves. The relative motions between the phases cause the increase in the speed of sound predicted by Eq. (28) observed in Fig. 6B. The critical frequency for translational effects is inversely proportional to the dynamic viscosity of the liquid phases. For magmas, ν_{lg} and ν_{ls} are far above (>1 MHz) the maximum frequencies considered here and used in geophysics. The influence of the translational relative motions on the speed of sound can thus be safely neglected in magmas for most applications.

521

522 **IV.2 Limit of validity of the model**

In a suspension, the initiation of interactions between the discrete particles marks the onset of rigidity and the transition from liquid-like to solid-like elastic body. In our model, while contacts are neglected, we accounted for the exchange of momentum between neighboring crystals through lubrication forces. The initiation of lubricated interactions between neighboring particles has been suggested to result in an increase in the speed of sound, marking the onset of rigidity in the suspensions (Esquivel-Sirvent *et al.* 1995). Their experiment was performed at frequencies of the order of MHz, which are far above the one considered in our model. For the same solid volume fraction, Eq (21) shows that I_{ss} varies as a function of $\eta k^2/\rho_s$. For magmas and perturbations in the range of frequency used in geophysics, $\sim 10^{-9} < \eta k^2/\rho_s < \sim 10^1$, such that the influence of lubrication on the speed of sound is negligible (Fig. 5B). As a result, the initiation of contact between the crystals is associated with the onset of rigidity in magmas and is expected to result in the sharp increase of the velocity of the compressional waves and the emergence of shear waves (Caricchi *et al.* 2008). The volume fraction at which a continuous contact network form (at random loose packing) in a magma depends on the sizes, shapes, orientations, and roughnesses of the crystals such that it can be significantly larger than

537 $\phi \approx 0.36$, which corresponds to the minimum random close packing calculated for frictionless and
 538 monodisperse spheres (Bergantz et al. 2017). The difference between random loose and close packings
 539 is expressed in the coordination numbers (average number of particle-particle contacts per particle),
 540 which is larger at random close packing than at random loose packing. The transition between the two
 541 packings occurs as a consequence of the reorganization of the crystal network due to contact sliding
 542 and particle non-affine motions. The increase in the coordination number raises the rigidity of the
 543 suspension and the speed of sound. Thus the rigidity modulus is expected to increase progressively
 544 with the decreases of ϕ_f between the random loose and close packings. Since the influence of the
 545 contact between the solids is beyond the scope of our model, the initiation of a fragile contact network
 546 (Bergantz *et al.* 2017) between the crystals at magma/mush transition represents the limit of its
 547 applicability. Contacts between solids are implicitly accounted for in the effective medium theory (e.g.
 548 Kuster & Toksöz 1974, Berryman 1980) and the Hertz-Mindlin contact theory. In dense suspensions
 549 where solids are in cohesionless contact, the bulk and rigidity moduli also depend on the confining
 550 pressure and amplitude of the perturbation, which affects the non-affine motions of the grains, contact
 551 slidings, and shear dilatancy (e.g. Makse et al. 2004, Brum et al. 2019). Such phenomena may induce
 552 either strengthening or weakening of the rigidity of the suspension because of the change in the contact
 553 network (Van den Wildenberg *et al.* 2013). The effective medium theory is not able to account for the
 554 relaxations associated with changes in the contact network and usually overestimates the shear modulus
 555 (Makse *et al.* 2004). As a result, the applicability of methods based on the effective medium theory to
 556 compute the speed of sound between the random loose and random close packing is uncertain given the
 557 proneness of the crystal network to structural reordering and non-affine motions.

558 In addition to the absence of contact between particles, we made assumptions when deriving the
 559 conservation equations that may affect sound speed. We neglected the mass transfers associated with

560 the precipitation or melting of crystals and the growth, dissolution, or nucleation of gas bubbles. The
 561 importance of the mass transfers on the acoustic properties of a suspension depends on the rates of
 562 mass exchange between the constituents (Fuster & Montel 2015). In magmas, the exsolution or
 563 dissolution of the volatiles depends on the changes in their solubility in the melt phase, which is mainly
 564 controlled by the pressure changes. The nucleation of bubbles is expected to occur during a short
 565 period (Toramaru 1995) and requires a large supersaturation pressure (> 5 Mpa) even in the presence of
 566 crystals (Hurwitz & Navon 1994, Shea 2017). In our model, we considered small perturbations and the
 567 magma being initially at thermodynamic equilibrium (no steady mass or heat transfers). Thereby, the
 568 small amplitudes of the perturbations in pressure are not expected to trigger the nucleation of bubbles,
 569 which requires large amplitude waves (Rothery *et al.* 2007). The rate of the exchange of mass between
 570 the dissolved and exsolved volatiles is controlled by the bubbles sizes and the diffusion coefficient of
 571 the volatiles species in the melt phase (Toramaru 1995). The competition between diffusion and the rate
 572 of pressure change can be measured by the ratio of the diffusive time scale over the decompression
 573 time scale (Lensky *et al.* 2004). The diffusivity coefficient of water (the most common volatile in
 574 magma) is low (between $\sim 10^{-13}$ and $\sim 10^{-10}$ $\text{m}^2 \text{s}^{-1}$; Zhang & Behrens 2000) so that mass transfer between
 575 the melt and gas phases is negligible (i.e. the diffusive ratio is > 1) when changes in pressure are faster
 576 than ~ 0.01 – 0.0001 Hz for bubbles of 10 – 100 μm in radius, respectively. Similarly, the rates of the
 577 precipitation or melting of the crystals in magmas are small (e.g. $\sim 10^{-13}$ and 10^{-12} m s^{-1} for crystal
 578 growth; Hawkesworth *et al.* 2004) so that we do not expect the mass transfer between the liquid and
 579 solid phases to have a significant influence on the velocity of a compressional wave in magma.

580 Heat and mass exchanges between the phases are controlled by the coupling terms (β_{ls} , β_{lg} , γ_{ls} ,
 581 and γ_{lg}), which are based on empirical correlations. The choice for the correlations employed to
 582 calculate the coefficients of momentum exchange, β_{ls} and β_{lg} , influences the two critical frequencies, ν_{ls}

583 and v_{lg} , at which the translational relative velocities between the suspended and carrier phases start to
 584 significantly influence sound speed (when $f > 10^5$ Hz in Fig. 8B). Here we considered the creeping and
 585 steady flow of the liquid around the discrete phases because of the dynamic viscosity of the melt
 586 allowing us to neglect inertial and unsteady terms. We employed a Stokes law for high porosity ($\phi_l >$
 587 ~ 0.893) as usually used in the coupled phase theory (e.g. Harker & Temple 1988, Atkinson & Kytömaa
 588 1992, Evans & Attenborough 1997). For lower porosity ($\phi_l < \sim 0.893$), we used a Kozeny-Carman
 589 relationship instead of the Stokes law to account for the influence of the presence of the surrounding
 590 crystals and the associated decrease in permeability. The maximum difference between the momentum
 591 exchanges coefficients predicted by the Stokes and Kozeny-Carman relationships occurs at the
 592 maximum packing fraction and reaches ~ 1 order of magnitude. Therefore, account for the Stokes law
 593 instead of the Kozeny-Carman law would result in a decrease of the critical frequencies, β_{ls} and β_{lg} , of
 594 ~ 1 order of magnitude at maximum. The two critical frequencies, β_{ls} and β_{lg} , calculated considering a
 595 Stokes law remain above the range of frequency considered (10^{-3} – 10^3 Hz). As a result, in the range of
 596 the frequency considered here, the choice of the law for the exchange of momentum has a negligible
 597 influence on the calculated speed of sound in magma.

598 Similarly, the choice of the empirical relationship employed to predict Nusselt number in the
 599 coefficient of heat transfer between the phases, γ_{ls} and γ_{lg} , impacts the critical frequencies at which the
 600 transition between thermal equilibrium and disequilibrium regimes occurs. We used the correlation
 601 proposed by Gunn (1978) to obtain an expression depending on the porosity of the suspension. This
 602 expression also accounts for the influence of the relative velocity between the carrier and suspended
 603 phases. We showed that within the range of frequency considered, the relative velocity between the
 604 constituents is negligible. As a result, the influence of the relative motion between the constituents on
 605 the rates of heat exchange is weak and can be neglected. Furthermore, other empirical relationships

606 predicting the Nusselt number exist (e.g. Ranz, 1952; Li & Mason, 2000) but often reduce to $Nu = 2$ in
607 the absence of relative flow between the constituents, which is the same Nu as predicted by Eq. (31)
608 when $\phi_l=1$. At the minimum porosity ($\phi_l=0.36$), Eq. (31) predicts $Nu = \sim 4$. As a result, employing
609 another empirical correlation to calculate the Nusselt number will tend to decrease the two critical
610 frequencies, τ_g^{-1} and τ_g^{-1} , by a factor of 2 at maximum. The two theoretical maximum and minimum
611 wave velocities (Eqs. 45 and 47) remain unchanged since the correlation of the Nusselt number only
612 affects the rate of the heat exchanges but not the equilibrium temperature predicted by Eq. (46).

613

614 **IV.3 Implication in volcanology:**

615 Our results have implications for the interpretation of seismic signals recorded around
616 volcanoes. Long Period (LP) signals are thought to result from the radiation from the acoustic
617 excitation of the magma located in cracks (e.g. Chouet 1986; Kumagai & Chouet 2000) or in the
618 volcanic conduit (e.g. Jousset *et al.* 2003; Jousset *et al.* 2004) with the surrounding solid rock. The
619 velocity of compression waves in the magma located in the cracks or conduit affects the resonance
620 frequency, radiation attenuation, and delay between two successive LP events. The presence of
621 exsolved volatiles in magma induces that compressional waves propagate faster at high frequency
622 (>100 Hz) than low frequency (< 1 Hz). Consequently, the resonance frequency of a crack filled with
623 bubbly magma will be higher, and the radiation attenuation lower, at a high frequency than at a low
624 one. In the context of a volcanic conduit, the higher velocity of the wave at high frequency translates to
625 the decrease of the delay between two successive LP events and an increase in their frequency.

626 The detection of exsolved volatiles and estimation of their volume fraction in magma is
627 important to assess volcanic hazards. Tomography images of seismic waves velocity are an interesting
628 tool to map compositional changes in magma reservoirs. The speed of sound in magma depends on the

629 crystal and exsolved volatiles content. Crystals increase the velocity of compressional waves but
630 weakly affect the dispersion of sound in the magma. The presence of gas bubble greatly decrease and
631 induces the dispersion of sound. The difference between the speed of sound at high and low frequencies
632 is proportional to the gas volume fraction. Consequently, the comparison of tomography images
633 computed for different frequencies at low and high frequencies might help in highlighting the presence
634 of gas and in mapping magma reservoirs.

635 **V. Conclusion**

636 We developed an analytical model to estimate the speed of sound in magmas consisting of a
637 suspension of solids and/or gas bubbles in a viscous liquid. Our model shows that the velocity of
638 compressional waves in a magma varies non-linearly with frequency between two asymptotic bounds.
639 These two bounds correspond to the speed of sound when all the constituents of the magma are in
640 thermal equilibrium (lower bound) and when the heat exchanges between the phases are neglected
641 (upper bound). We then simulated the cooling of three magmas representative of the diversity of
642 compositions that may be encountered in arc magmatism and applied our model to calculate the speed
643 of sound. Results show that the presence of gas in a magma yields a sharp decrease in the velocity of
644 sound and enhances significantly its dispersion. We found that the exchanges of heat between the
645 constituents may be neglected in crystal-bearing magmas, but that they cannot be ignored once a gas
646 phase is present. Finally, we compared the speed of sound predicted by our model to the results of other
647 relationships usually used by authors for magmas in the range 10^{-3} - 10^3 Hz. The difference between our
648 modeled of the velocity of compressional acoustic waves and literature values results from the
649 simplifications and assumptions made when considering the evolution of the temperature of the phases
650 and the heat exchanges. These differences typically range from 0.5 to 8% and are largest at frequencies
651 >10 Hz.

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658

659 **Data Availability:**

660 All the data and code related to this manuscript are available in the supplementary materials (online).

661

662 **References:**

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Figure 1: Schematic representation of a suspension of solid particles and gas bubbles in a viscous liquid. The scheme represents a cross-section of an elementary volume perpendicular to the north direction. The liquid phase is represented in gray. The black and white disks correspond to the solid particles and gas bubbles, respectively. The three gray axes (east, north, and vertical) indicate the orientation with respect to the gravitational acceleration vector. The black orientation axes (x , y , and z) indicate the coordinate system used to express the conservation equations, in which the direction x is aligned in the direction of the propagation of the wave. The sinusoid represents the plane acoustic perturbation propagating along the direction x with an angle θ (positive clockwise) from the east direction.

674 **Figure 2:** Conceptual configuration of the crystals used to derive the rate of momentum exchange
 675 between the solids. The scheme represents one target particles located in the layer 2 and its six closest
 676 neighbors located in the layer 1 and 3. The color of the boundary of each particle depends on the layer
 677 in which it is located (green for layer 1, red for layer 2, and blue for layer 3). Each arrow indicates the
 678 velocity vector of the corresponding particles. The grazing angle β is represented in purple.

Figure 3: Speed of sound as a function of the volume fraction in liquid, solids, and gas when $f=0.01$ Hz (A) and $f=100$ Hz (B). The background color depends on the velocity of compressional waves. The white dashed curves indicate isocontours of the sound speed. The black dash line indicates the theoretical limit of the validity of the model at $\phi_l = 0.36$. The material properties of the constituents and the incidence angle are indicated in Table 2. The black cross indicates the volume fractions used to compute the dispersion curve in Fig.3.

688 **Figure 4:** Dispersion curve of a magmatic suspension. The volume fraction of the constituents are $\phi_l =$
 689 0.6, $\phi_s = 0.35$, and $\phi_g = 0.05$. The solid black curve indicates the results obtained with Eq. (35). The
 690 black, red, and blue dashed lines indicate the isothermal speed of sound (Eq. 42), the isentropic speed
 691 of sound at thermal equilibrium (Eq. 47), and isentropic speed of sound out of thermal equilibrium (Eq.
 692 45), respectively. The black and blue vertical dotted lines indicate the critical frequencies above which
 693 the solid and gas bubbles are not in thermal equilibrium with the surrounding liquid, respectively (Eqs.
 694 40 and 41). The material properties of the constituents and the incidence angle are indicated in Table 2.

695

696

697 **Figure 5:** Evolution of the compression wave velocity as a function of the incidence angle and
 698 frequency of the perturbation. The radial axis indicates the difference in % between the velocity of the
 699 compressional wave at θ and $\theta=0$. The blue, red, green, and purple curves correspond to frequencies of
 700 0.01Hz, 0.1 Hz, 1Hz, and 10Hz, respectively. The volume fraction of the constituents are $\phi_l = 0.65$, $\phi_s =$
 701 0.3 and $\phi_g = 0.05$. The material properties of the constituents are indicated in Table 2.

702

704 **Figure 6:** Influence of the lubrication forces on the speed of sound in a suspension of solid particles in
 705 a viscous fluid. **[A]** Difference between the speed of sound computed with Eq. (35) considering and
 706 neglecting lubrication forces. The physical properties of the liquid and solids are the ones indicated in
 707 Table 2. **[B]** Evolution of the difference between the speed of sound computed with Eq. (35)
 708 considering (c_{lub}) and neglecting (c_{nolub}) lubrication forces as a function of $\eta k^2/\rho_s$. The shaded area
 709 indicates the area covered by magmas.

Figure 7: Evolution of the phase assemblage, thermodynamic properties, and speed of sound of the magmas during the cooling simulations. **[A]**, **[B]**, and **[C]** are the phases assemblage computed during the simulation of the cooling of the basalt (A), andesite (B), and dacite (C). **[D]** evolution of the bulk densities and bulk moduli of the magmas. **[E]** Evolution of the bulk coefficient of thermal expansion and bulk heat capacity at constant pressure as a function of temperature. **[F]** Evolution of the speed of sound in the magmas. The solid, dashed and dashed-dotted curves indicate the thermal disequilibrium, thermal equilibrium, and isothermal bounds, respectively. The shaded area corresponds to the range of velocity that may be computed with Eq. (35) at different frequencies.

Figure 8: [A] Comparison of sound speeds predicted by various model in a suspension of gas bubbles in water for $10^{-3} \leq f \leq 10^3$ Hz. [B] Comparison of the speed of sound computed with Eq. (35) and with Eq. (41) in Commander & Prosperetti (1988) for $10^3 \leq f \leq 10^6$. The properties of the fluid are the same as in (A). The density of the water and gas were calculated following Eq. (8) and (9) in Kieffer (1977). The bulk modulus and coefficient of thermal expansion, $K_g = P$, $\alpha_g = 1/T$. The heat capacity of the gas is calculated as $C_{Pg} = (\gamma/(\gamma-1))R/M_g$, where γ is the heat capacity ratio ($\gamma=1.4$), R is the ideal gas constant, and M_g is the molar mass of the gas.

726 **Table 1:** Initial chemical composition, pressure, and temperature of the magmas.

Composition wt%	Basalt	Andesite	Dacite
SiO ₂	48.108	59.736	66.013
TiO ₂	0.970	0.469	0.440
Al ₂ O ₃	16.883	17.341	15.263
Fe ₂ O ₃	1.755	1.277	0.663
FeO	8.279	4.765	2.031
MnO	0.174	0.176	0.069
MgO	5.925	2.239	0.908
CaO	10.396	6.100	2.910
Na ₂ O	2.657	3.451	3.691
K ₂ O	1.193	1.026	4.004
P ₂ O ₅	0.214	0.000	0.186
H ₂ O	3.447	3.4213	3.435
T start (°C)	1100	1050	950
T stop (°C)	960	700	715
P (Mpa)	150	150	150
fO ₂ buffer	QFM	Ni-NiO	Ni-NiO
Source	Dufek & Bachmann (2010)	Martel et al. (1999)	Dufek & Bachmann (2010)

727

728 **Table 2:** Reference physical properties used to explore the influence of the composition of the
729 suspension and characteristics of the perturbation on the speed of sound. They correspond to an
730 approximate total pressure of 150 MPa.

Variable	Reference value
T	1000 °C
ρ_l^0	2500 kg m ⁻³
ρ_s^0	3000 kg m ⁻³
ρ_g^0	350 kg m ⁻³
K_l	15 GPa
K_s	50 GPa
K_g	150 MPa
η	1000 Pa s
C_{Pl}	1300 J kg ⁻¹ K ⁻¹
C_{Ps}	1200 J kg ⁻¹ K ⁻¹
C_{Pg}	3750 J kg ⁻¹ K ⁻¹
α_l	10 ⁻⁴ K ⁻¹
α_s	10 ⁻⁶ K ⁻¹
α_g	10 ⁻³ K ⁻¹
k_l	1 W m ⁻¹ K ⁻¹
d_s	5 mm
d_g	0.5 mm
g	-9.81 m s ⁻²
θ	0°

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732

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734

735 **Table 3:** List of the thermodynamical properties, or expressions employed to estimate them, used to
736 compare the different models in Fig. 6. The surface tension at the interfaces between the bubbles and
737 liquid is neglected. The thermodynamic properties and relationships are taken from Kieffer (1977).

Parameters	Expression or value
ρ_l^0	1000 kg m ⁻³
ρ_g^0	$\rho_g^0 = \left(\frac{P}{G_{air}} \right)^{\frac{1}{\gamma}}$
ρ_g^{ref}	690 kg m ⁻³
G_{air}	$G_{air} = \frac{TR}{M_g \rho_g^{ref \gamma - 1}}$
T	20 °C
P	500 bars
γ	1.4
d_g	0.5 mm
η	10 Pa s
Mg	29.98 10 ⁻³ kg mol ⁻¹
C_{pl}	1300 J kg ⁻¹ K ⁻¹
C_{Pg}	$C_{Pg} = \frac{R}{M_g} \left(1 - \frac{1}{\gamma} \right)$
K_f	1 GPa
K_g	$K_g = P$
α_l	1 10 ⁻⁴ K ⁻¹
α_g	$\alpha_g = 1/T$

738

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