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FORC signatures and switching-field distributions of dipolar coupled nanowire-based hysterons

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Abstract

Analysis of first-order reversal curves (FORCs) is a powerful tool to probe irreversible switching events in nanomagnet assemblies. As in essence switching events are related to the intrinsic properties of the constituents and their interactions, resulting FORC diagrams contain much information that can be crosslinked and complex to deconvolute. In order to quantify the relevant parameters that drive the FORC diagrams of arrays of perpendicularly magnetized nanomagnets, we present step-by-step simulations of assemblies of hysterons to determine the specific signatures related to different known inputs. While we explored the consequences of dipolar interactions using either mean field or magnetostatic approaches, we completed by taking the hysteron switching-field distribution (SFD) as either normal or log-normal. We demonstrated that the transition between FORC diagrams composed of an isolated interaction field distribution (IFD) and a wishbone shape operates *via* the SFD deviation, $\sigma_{H_{sw}}$, in the presence of a weakly dispersed interaction field. In the presence of a magnetostatic interaction field, the IFD profile is peaked and a coercive field distribution (CFD) sums to the IFD as $\sigma_{H_{sw}}$ increases. A transition between IFD+CFD and wishbone shapes is clearly demonstrated as a function of the interaction field deviation $\sigma_{H_{int}}$. In addition, we demonstrate that whatever the considered cases, $\sigma_{H_{sw}}$ can be quantitatively extracted from the FORC diagrams within an error inferior to 10%. These findings are of interest for dipolar coupled perpendicularly magnetized nanomagnets, as in assemblies of magnetic nanowires and nanopillars, as well as bit patterned media.

Introduction

The extraction of the individual behaviour of entities based on the measurements of their collection is a fundamental problem encountered in many fields such as biology, sociology, ethology, physics, etc. Concerning hysteretic processes, Mayergoyz introduced an approach for the determination of the weight function of a set of hysteresis operators [1], typically based on the Preisach model of hysterons describing magnetic hysteresis [2]. Experimentally, this approach resides in the measurement of first-order reversal curves (FORCs) [3,4], which consist in saturating a magnetic sample in the positive direction, then decreasing the magnetic field H up to a field H_r , in order to measure the magnetization M of the corresponding FORC between H_r and the positive saturation. Typically, between one and two hundreds of H_r values are chosen to fill the major hysteresis loop (MHL) to compute the FORC diagram using the second order mixed derivative :

$$\rho_{FORC}(H, H_r) = -\frac{1}{2} \frac{\partial^2 M(H, H_r)}{\partial H_r \partial H}, (H \geq H_r) \quad (\text{Eq. 1})$$

FORC analysis is now widely used in several laboratories because of its implementation simplicity [3,5]. Focusing on magnetic systems, FORC technique has been used to investigate diverse systems such as rock and mineral magnetism [3,6,7,8], magnetic media [9,10,11,12], GMR spin valves [13], magnetic tunnel junctions [14], thin films [15,16,17,18], magnetic multilayers with perpendicular magnetic anisotropy [19,20,21,22,23,24], magnetic dot/pillar assemblies [25,26,27], permanent magnets [28,29], exchange spring magnets [30], and nanowire assemblies [31,32,33,34,35,36,37,38,39,40,41,42], since isolated small diameter nanowires behave as basic hysterons. However, the understanding of the resulting FORC diagrams is not straightforward as it does not strictly represent the Preisach distribution. Therefore, simulations are mandatory to qualitatively and quantitatively read the relevant information [5,35,43,44,45,46,47,48,49].

One of the key features of a nanomagnet assembly is the switching field distribution (SFD). It is the figure of merit of recording media as it drives magnetic stability and achievable recording density. In a dense assembly where dipolar interactions are important, the SFD extraction represents a complex problem. Different methods have been proposed [50,51], each one introducing inherent assumptions. The FORC approach, with its ability to map switching events, interactions, multiphase systems, microscopic behaviour, etc, would be a powerful tool in quantifying the SFD [52,53]. The drawback of such a multitude of attainable properties is that crosslinked behaviours can alter its quantification.

Here, we present systematic simulations of assemblies of nanowire-based hysterons in order to determine the specific FORC signatures related to: (i) dipolar interactions, using both mean field and magnetostatic approaches, with tuned amplitude and deviation; (ii) hysteron SFD, for both normal and log-normal distributions, also varying amplitude and deviation. First, we consider the case of identical hysterons (part A) interacting *via* (i) a mean demagnetizing dipolar field, and (ii) a calculated magnetostatic interaction field. Normal and log-normal switching field distributions (SFD) are then introduced in both cases (part B). The effect of the SFD standard deviation is investigated in both in the presence of a mean or magnetostatic interaction field, as well as those arising from the interaction field amplitude and deviation. Finally, we demonstrate that the SFD deviation can be extracted from the FORC diagrams with an error inferior to 10%, and this, whatever the SFD shape and interaction field regime.

Model

Assemblies of N hysterons were considered with determined switching-field value $\langle H_{sw} \rangle$ and standard deviation $\sigma_{H_{sw}}$. All thermal effects and exchange interactions between hysterons were neglected. The modelled systems correspond to 2D nanowire arrays with an applied magnetic field

perpendicular to the assembly plane (*i.e.*, along the nanowire axis, z). As a consequence, the nanowires are only magnetized either up or down, yielding an interaction field remaining along the nanowire axis as well.

For a mean interaction field, it is always opposite to the averaged magnetization M_z . Therefore, we considered a mean interaction field of demagnetizing type (moving Preisach model [54]):

$$\mu_0 H_{int,zz} = \alpha_m \frac{M_z}{M_S} \quad (\text{Eq. 2})$$

where μ_0 is the Bohr magneton, M_S the saturation magnetization, and with $\alpha_m < 0$ (α_m expressed here in T). Since the resulting mean interaction field is uniform, all $N = 1000$ hysterons are submitted to the same value, *i.e.* there is no dispersion ($\sigma_{Hint} = 0$).

On the other side, for a magnetostatic dipolar field, the hysterons were taken as spatially localized and referring to nanowires of diameter D and length L . Calculations were performed considering a planar triangular assembly with lattice vectors $\vec{a} = \begin{pmatrix} d+D \\ 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} (d+D)/2 \\ (d+D)\sqrt{3}/2 \end{pmatrix}$, with d the edge to edge inter-hysteron distance. Unless stated, the dimensions used in the simulations are $D = 7$ nm, $L = 60$ nm, and $d = 6$ nm, with $N = 1681$ hysterons. The interaction field was calculated using the magnetostatic field between neighbouring nanowires [55]:

$$\mu_0 H_{int,zz} = \mu_0 D^2 L M_S \sum_{i \neq j} \left(\frac{1}{(\rho_{ij}^2 + L^2)^{3/2}} \right) \quad (\text{Eq. 3})$$

where ρ_{ij} is the distance between two i and j nanowire-based hysterons. The sum was calculated over all hysterons and updated after every switching event. Such a calculated interaction field allows considering the case of an inhomogeneous interaction field. In particular, it scales with the number of considered hysterons. Since the interaction field is higher for hysterons located at the assembly centre than at its periphery, modulating the hysteron number, *i.e.* the size of the assembly, introduces a deviation σ_{Hint} of the interaction field.

Whatever the considered interaction field, it sums to the applied external magnetic field H_{ext} , yielding a total field acting on each hysteron $H_{tot} = H_{ext} + H_{int}$ (Eq. 4). As the applied field changes quasistatically, the hysterons are flipped one by one if $H_{tot} > H_{sw}$, then $H_{int,zz}$ is updated and all new possible switchings are examined, until the equilibrium magnetization is reached [43].

For each specific case described below, 80 FORCs were calculated. The FORC diagrams were computed (Eq. 1) and plotted using the interaction field axis $H_{int} = -\frac{H+H_r}{2}$ and the hysteron half-width axis $H_c = \frac{H-H_r}{2}$ (Supplementary Material, part B) and normalized with respect to the maximum value ρ_{FORC}^{max} of the corresponding diagram. Negative ρ_{FORC} values were not considered here.

Results

A. Identical hysterons

Starting from the simplest but unrealistic case, we first consider identical hysterons, *i.e.* hysterons with identical switching field H_{sw} . In this case, the intrinsic switching field of the N hysterons is set to $\mu_0 H_{sw} = 300$ mT, with no switching field distribution ($\sigma_{H_{sw}} = 0$). The effect of a mean interaction field is first presented, followed by a magnetostatic field, both as previously described.

1. Mean interaction field

The resulting major hysteresis loop (MHL, inset of Fig. 1a)) is clearly sheared due to demagnetizing interaction. Its differential susceptibility, dM/dH , remains constant during the magnetization reversal, with $(d(M/M_s)/d(\mu_0 H))^{-1} = \alpha_m$. The resulting FORC signature is a linear distribution of ρ_{FORC} strictly parallel to the H_{int} axis (Fig. 1(a)) [56]. It originates from the H_{int} variation of (Eq. 2) during the major hysteresis loop, with $-\alpha_m \leq \mu_0 H_{int} \leq \alpha_m$. As a consequence, this FORC feature is usually referred as the interaction field distribution (IFD). The profile along the IFD is constant in height and extrema correspond to $\mu_0 H_{int}^{sat} = \pm \alpha_m$ (Fig. 1(b)). Moreover, the crossing point between ρ_{FORC} and H_c axis, is $H_c^{MHL} = H_{sw}$ in this case.

Figure 1: Identical hysterons ($N=1000$ hysterons, $\mu_0 H_{sw} = 300$ mT, $\sigma_{H_{sw}} = 0$) submitted to a mean interaction field ($\alpha_m = -200$ mT). a) FORC diagram calculated from the simulated curves shown in inset. b) Profile along the H_{int} axis of the interaction field distribution (IFD).

2. Magnetostatic interaction field

Contrary to the mean interaction field that is homogeneous, using a magnetostatic interaction field yields a MHL differential susceptibility that is not constant during the magnetization reversal (inset of Fig. 2(a)). The FORC diagram still exhibits an IFD, but with an amplitude that strongly varies (Fig. 2(b)). This characteristic can be considered as the FORC signature of hysterons interacting in an inhomogeneous interaction field, in opposition to the homogeneous interaction regime exhibiting IFD flat profile (Fig. 1(b)). Numerous FORC measurements performed on nanowire assemblies display such a signature [31,33,42,56,57,58,59,60,61,62,63,64,65]. Previous simulations of hysteron assemblies in the moving Preisach model indicated that IFD profiles flatten when increasing $|\alpha_m|/\sigma_{int}$ [66], in agreement with the data reported here. As for the mean interaction field, the crossing point between ρ_{FORC} and H_c axis, is $H_c^{MHL} = H_{sw}$.

Figure 2 : Identical hysterons ($N = 961$ hysterons, $\mu_0 H_{sw} = 300$ mT, $\sigma_{H_{sw}} = 0$) submitted to a magnetostatic interaction field. a) FORC diagram calculated from the simulated curves shown in inset. b) IFD profile along the H_{int} axis.

B. Dispersed hysterons

1. Without interaction

Before analysing the complex cases of hysteron assemblies presenting switching field distribution (SFD), it is important to understand the behaviour modifications caused without interaction. Introducing a SFD into an assembly of non-interacting hysterons organizes the switching events from the less coercive to the most coercive hysteron. For both normal and log-normal SFD, the resulting FORC signature is a ρ_{FORC} distribution aligned along the H_c axis (Fig. 3). As expected, the associated ρ_{FORC} profiles along the H_c axis are found identical to the normal and log-normal SFD introduced into the simulations. Fits of the ρ_{FORC} profile give $H_c^{mean} = 297$ mT and $\sigma_{H_{sw}}^{FORC} = 51$ mT for the normal SFD (Fig. 3(b)). For the log-normal SFD (Fig. 3(d)), H_c^{MHL} is equal to the fitted median value $H_c^{median} = 299$ mT, and $\sigma_{H_{sw}}^{FORC} = 110$ mT. These values are consistent with the introduced ones $< \mu_0 H_{sw} > = 300$ mT and $\sigma_{H_{sw}} = 50$ mT (normal) and $\sigma_{H_{sw}} = 110$ mT (log-normal).

Figure 3: Dispersed non-interacting hysterons ($N = 1000$ hysterons, $< \mu_0 H_{sw} > = 300$ mT) with (a)-(b) a normal SFD ($\sigma_{H_{sw}} = 50$ mT) and (c)-(d) log-normal SFD ($\sigma_{H_{sw}} = 110$ mT). (a)-(c) FORC diagrams calculated from the simulated curves shown in inset. (b)-(d) Profiles of ρ_{FORC} along the H_c axis and corresponding fits.

2. Interactions and the switching field distribution

When assemblies of moderately dispersed hysterons are interacting, the resulting FORC diagrams are similar to the identical hysteron cases. Indeed, for both mean (Fig. 4(a)-(b)) and magnetostatic interaction fields (Fig. 4(c)-(d)), the linear IFD persists with a width always equals to $2H_{int}^{sat}$. Additionally, the ρ_{FORC} profiles along H_{int} remain flat and peaked for mean and magnetostatic interaction fields, respectively. In all cases, increasing $\langle H_{sw} \rangle$ only shifts the IFD along the H_c axis (See Supplementary Material, part A, Figs. S1-S2). However, despite these similarities, the hysteron SFD induces modification in the FORC diagrams. Since the interaction field nature alters differently the FORC results arising from dispersed hysterons, the impact of varying the hysterons non-uniformity is investigated separately.

Figure 4 : FORC diagrams calculated for assemblies of hysterons interacting in a (a)-(b) **mean interaction field** or (c)-(d) **magnetostatic interaction field** for (a)-(c) normal and (b)-(d) log-normal switching field distributions. $\langle \mu_0 H_{sw} \rangle = 500$ mT, $\sigma_{H_{sw}} = 10$ mT, $\alpha_m = -500$ mT ($\mu_0 H_{int}^{sat} = \pm 500$ mT). FORC diagrams calculated with various $\langle \mu_0 H_{sw} \rangle$ values are presented in Figs. S1 and S2 of Supplementary Material.

a. Mean interaction field

When increasing $\sigma_{H_{sw}}$, *i.e.* nanowires less uniform, Figure 5 (normal SFD) and S3 (log-normal SFD, Supplementary Material) evidence the progressive transition occurring between an IFD FORC diagram at low $\sigma_{H_{sw}}$ and to a FORC diagram generally called “wishbone” [26] at large $\sigma_{H_{sw}}$. This is coherent with previous simulations where the wishbone shape was simulated in assemblies of distributed hysterons with mean field interactions [26]. Indeed, the wishbone shape is commonly encountered in assemblies of magnetic pillars or nanowires, as well as in thin films with perpendicular anisotropy [9][10][26][41][42][67].

Figure 5 : (a)-(e) FORC diagrams calculated for assemblies of hysterons presenting a **normal SFD** and interacting in a **mean interaction field**. $\langle \mu_0 H_{sw} \rangle = 200$ mT, $10 \leq \sigma_{H_{sw}} \leq 60$ mT, $\alpha_m = -500$ mT ($\mu_0 H_{int}^{sat} = \pm 500$ mT). Profiles of (f) ρ_{FORC} along H_c axis and (g) along H_{int} axis for different $\sigma_{H_{sw}}$ (curves are normalized with respect to the ρ_{FORC} value at $H_{int} = 0$). Corresponding FORC diagrams calculated for a log-normal SFD are presented in Figs. S3 of Supplementary Material.

This specific shape, shown in more details in Figure 6, is the signature of a dispersed assembly interacting in a mean demagnetizing field. The AB branch of Fig. 6 is the IFD previously described, for which the profiles remain flat (see Figs. 5(g) and S3(g)), as expected for a mean interaction field. However, it can be seen that it is no longer parallel to the H_{int} axis as soon as $\sigma_{H_{sw}}$ increases, as frequently observed experimentally [9][26][41][42][67]. On the other side, the AC branch, which could only be evidenced in the presence of a SFD, contains the last switching events that occur just before reaching saturation in a given FORC. As explained in details in part B of Supplementary Material, the most important aspect is that the switching events producing the AC branch do not correspond to intrinsic switching values of hysterons, but to hysteron widths artificially enhanced by interactions.

Figure 6 : Larger view of the wishbone shape obtained for a (a) normal (Fig. 5(e)) and (b) log-normal (Fig. S3(e)) SFD. $\langle \mu_0 H_{sw} \rangle = 200$ mT, $\sigma_{H_{sw}} = 60$ mT, $\alpha_m = -500$ mT). Insets are the corresponding simulated FORCs.

Quantitative data such as the SFD deviation can be extracted from these FORC diagrams. Prosaically, based on the FORC diagram analysis of dispersed non-interacting hysterons (Fig. 3), profiles of ρ_{FORC} along H_c axis (Fig. 5(f)) could be fitted to extract both the mean and deviation values, $H_c^{\rho_{FORC}(H_c)} (A_0$

point, Fig. 6) and $\sigma_{fit}^{\rho_{FORC}(H_c)}$, respectively. Such values, reported in Table 1 (normal SFD, Fig. 5) and Table S1 (log-normal SFD, Fig. S3), clearly differ from the $\langle H_{sw} \rangle$ and $\sigma_{H_{sw}}$ values introduced in the simulations. Therefore, even though they follow the same tendency, it indicates that a $\rho_{FORC}(H_c)$ profile does not represent the switching field distribution. In particular, one can note than $\langle H_c^{\rho_{FORC}(H_c)} \rangle$ decreases with $\sigma_{H_{sw}}$ due to the progressive tilt of the IFD.

$\langle H_{sw} \rangle$ (mT)	$\sigma_{H_{sw}}$ (mT)	$H_c^{\rho_{FORC}(H_c)}$ (mT)	$\sigma_{fit}^{\rho_{FORC}(H_c)}$ (mT)	H_c^{mean} (mT)	$\sigma_{H_{sw}}^{FORC}$ (mT)
200	10	191	19.1 (91%)	204 (2%)	10.8 (8%)
200	15	184	20.6 (37%)	196 (2%)	15.9 (6%)
200	30	146	22.5 (25%)	202 (1%)	29.6 (1.3%)
200	45	132	23.4(48%)	205 (2.5%)	45.9 (2%)
200	60	125	36.5(39%)	205 (2.5%)	58.2 (3%)

Table 1: Main properties extracted from FORC diagrams calculated for hysteron assemblies of presenting a **normal SFD** and interacting in a **mean interaction field (Fig. 5)**.

$\langle H_{sw} \rangle$ and $\sigma_{H_{sw}}$ are the mean switching field and deviation values of the SFD introduced in simulations, $H_c^{\rho_{FORC}(H_c)}$ and $\sigma_{fit}^{\rho_{FORC}(H_c)}$ are the mean and deviation values deduced from H_c axis profile fitting (Fig. 5(f)). H_c^{mean} is the H_c value read on FORC diagrams at point B . $\sigma_{H_{sw}}^{FORC}$ is the SFD deviation deduced from the FORC diagram and Eq. 5. The discrepancies with respect to the values introduced in simulations are indicated in parentheses.

However, the SFD parameters can be successfully estimated exploiting the A , B and C point coordinates in the (H_c, H_{int}) plane [5,68] : $A: (H_c^{min}; |H_{int,FORC}^{sat}|)$, $B: (H_c^{mean}; H_c^{mean} + H_c^{min} - |H_{int,FORC}^{sat}|)$, $C: (H_c^{max} + |H_{int,FORC}^{sat}|; 0)$. As outlined by A. Stancu *et al.* [5], estimation of the SFD deviation $\sigma_{H_{sw}}$ of a normal SFD can be obtained using $\sigma_{H_{sw}}^{FORC} = \frac{\langle H_c^{mean} \rangle - H_c^{min}}{3}$ (Eq. 5), as 99.7% of variables in a normal distribution are contained in a $\pm 3\sigma$ width from average value.

For a log-normal distribution, the A , B , C point coordinates become : $A: (H_c^{min}; |H_{int,FORC}^{sat}|)$, $B: (H_c^{median}; H_c^{median} + H_c^{min} - |H_{int,FORC}^{sat}|)$, $C: (H_c^{max} + |H_{int,FORC}^{sat}|; 0)$. $\sigma_{H_{sw}}^{FORC}$ can be estimated following the same approach of identifying the $(\langle H_c^{mean} \rangle - H_c^{min})$ interval as a function of the log-normal variance to deduce $\sigma_{H_{sw}}^{FORC}$ as equals to the variance square root (see Supplementary Material, part C). Tables 1 and S1 show that the A , B , C points are good indicators to retrieve the hysteron intrinsic properties: the H_c^{mean} (normal) and H_c^{median} (log-normal) values extracted at point B corresponds to $\langle H_{sw} \rangle$ and H_c^{MHL} , while $\sigma_{H_{sw}}^{FORC}$, estimated from H_c^{mean} (normal) or H_c^{median} (log-normal) and H_c^{min} (at point A), yields the adequate value of $\sigma_{H_{sw}}$ with errors less than 10% for both normal and log-normal SFDs.

b. Magnetostatic interaction field

In the case of a magnetostatic interaction field, (Fig. 7), the switching field distribution induces an additional branch parallel to the H_c axis where $\rho_{FORC} \gtrsim 0$, sometimes referred as the coercive field distribution (CFD) [5]. Indeed, for both normal (Fig. 7) and log-normal (Fig. S4) SFDs, FORC diagram are composed of an IFD and such a CFD. This last distribution is enhanced with $\sigma_{H_{sw}}$: ρ_{FORC} increases into the AA_0C triangle, and particularly along the A_0C segment. This can be understood following the wishbone shape description (part B of Supplementary Material): in the presence of a SFD, the hysteron width (H_c^i) can be artificially increased due to interaction. In the presence of a mean interaction field, the AC segment arises from hysterons whose switchings $up \rightarrow down$ (just before the i^{th} FORC) and $down \rightarrow up$ (during the i^{th} FORC) occur at different $|M|$ values as a result of the total field scanning of the SFD. Therefore, it generates enhancement of both bias (H_u^i) and width (H_c^i) of the hysteron (Eq. S3 and S4 of Supplementary Material). On the opposite, in the presence of a magnetostatic interaction field, the AC segment is almost missing, while the strong presence of the CFD on the A_0C segment reveals mainly width (H_c^i) enhancement in this case. It arises from the fact that both switchings of the i^{th} hysteron ($up \rightarrow down$ and $down \rightarrow up$) happen for nearly equal $|M|$ values, leading to $H_u^i = 0$ (Eq. S1 of Supp. Mat.), which is a direct consequence of the local magnetostatic interaction field being nonuniform. Hence, switching events are guided not only by the hysteron SFD, but are also strongly affected by the local environment, *i.e.* the H_{int} local value. Such a point is addressed in more details in Sect. 3b, introducing the deviation of the interaction field, $\sigma_{H_{int}}$. Nonetheless, this dependence on the interaction field local value explains why the IFD profiles (Fig. 7(g) and S4(g)) keep the peaked feature associated to the presence of a nonuniform interaction field, without clear distinction between the simulated hysteron distributions.

Figure 7 : (a)-(e) FORC diagrams calculated for assemblies of hysterons presenting a **normal SFD** and interacting in a **magnetostatic interaction field** $\langle \mu_0 H_{sw} \rangle = 300$ mT, $25 \leq \sigma_{H_{sw}} \leq 85$ mT, $\langle \mu_0 H_{int}^{sat} \rangle \approx \pm 500$ mT. (f) Profiles of ρ_{FORC} along H_c axis for different $\sigma_{H_{sw}}$. (g) Profiles of ρ_{FORC} along the IFD.

Following the analysis previously detailed for a mean interaction field, one can observe in Tables 2 (normal SFD) and S2 (log-normal SFD) that the resulting $\rho_{FORC}(H_c)$ profiles in the presence of a magnetostatic interaction field also do not directly represent the switching field distributions. Despite following a similar tendency, both $\langle H_c^{\rho_{FORC}(H_c)} \rangle \neq \langle H_{sw} \rangle$ and $\sigma_{fit}^{\rho_{FORC}(H_c)} \neq \sigma_{H_{sw}}$. Similar

as observed for a mean interaction field, $\langle H_c^{\rho_{FORC}(H_c)} \rangle$ decreases with $\sigma_{H_{sw}}$ due to the IFD progressive tilting. However, A, B, C points remain reliable indicators for quantitative extraction of the hysteron intrinsic distribution, since H_c^{mean} (taken at point B) corresponds to $\langle H_{sw} \rangle$ and H_c^{MHL} , while $\sigma_{H_{sw}}^{FORC} = \sigma_{H_{sw}}$. These conclusions are valid for both normal (Table 2) and log-normal SFDs (Table S2).

$\langle H_{sw} \rangle$ (mT)	$\sigma_{H_{sw}}$ (mT)	$H_c^{\rho_{FORC}(H_c)}$ (mT)	$\sigma_{fit}^{\rho_{FORC}(H_c)}$ (mT)	H_c^{mean} (mT)	$\sigma_{H_{sw}}^{FORC}$ (mT)
300	25	278	39.2 (57%)	302 (0.7%)	27.0 (8%)
300	40	261	40.2 (0.5%)	303 (1%)	40.4 (1%)
300	55	249	44.9 (18%)	291 (3%)	53.4 (3%)
300	70	238	48.6 (31%)	298 (0.7%)	70.5 (0.7%)
300	85	217	58.9 (31%)	309 (3%)	88.0 (3.5%)

Table 2: Main properties (Cf Table 1) extracted from FORC diagrams calculated for hysteron assemblies presenting a **normal SFD** and interacting in a **magnetostatic interaction field (Fig. 7)**. $\sigma_{H_{sw}}^{FORC}$ is the SFD deviation deduced from the FORC diagram and Eq. 5.

3. Interaction field influence

Previous section reveals that the degree of homogeneity of the interaction field impacts the FORC signature. A mean interaction field leads to constant IFD profiles and wishbone shapes for non-negligible SFD deviations. Introducing realistic homogeneities, a magnetostatic interaction field yields to peaked IFD profiles and IFD+CFD FORC signatures for dispersed assemblies. In order to link these two interaction regimes, we focus in this part on the influence of the interaction field, particularly in modulating the interaction field deviation $\sigma_{H_{int}}$ in the magnetostatic interaction regime.

a. Mean interaction field

For an assembly of hysterons presenting a large deviation ($\sigma_{H_{sw}}/H_{sw} = 30\%$) of normal (Fig. 8) or log-normal (Fig. S5) SFD, the wishbone shape persists whatever the amplitude of H_{int} . As interaction increases, point C moves toward higher H_c (Fig. 8(f)), Fig. S5(f)). Such a behaviour is coherent with the description given in Sect. 2a : the wishbone shape, and particularly the point C , originate from the enhancement of hysteron widths in the presence of a mean interaction field and SFD. As a result, concerned hysterons move toward higher H_c values in the FORC diagram for increasing H_{int} . The ρ_{FORC} profiles along the IFD evidence the H_{int} evolution with an almost constant amplitude (except at point A), which constitutes the signature of a mean interaction field. For both types of SFD, extraction of quantitative information at points A and B gives the expected H_c^{mean} and $\sigma_{H_{sw}}^{FORC}$ values, with errors less than 3% and 8%, respectively (not shown).

Figure 8: (a)-(e) FORC diagrams calculated for assemblies of hysterons presenting a **normal SFD** and interacting in a **mean interaction field**. $\langle \mu_0 H_{sw} \rangle = 200 \text{ mT}$, $\sigma_{H_{sw}} = 60 \text{ mT}$, $-600 \leq \alpha_m \leq -200 \text{ mT}$. (f) $\rho_{FORC}(H_c)$ and (g) ρ_{FORC} profiles along IFD for different H_{int} .

b. Magnetostatic interaction field

In the present case, the modulation of H_{int} is realized by varying the distance between hysterons (Eq. 3). For simplicity, we focus on the case of a normal SFD, with $\sigma_{H_{sw}}/H_{sw} = 20\%$. For simulations containing a constant number of hysterons, the assembly size consequently evolves and adds a modulation on $\sigma_{H_{int}}$, as small assemblies undergo important edge effects: the hysterons at the periphery present reduced numbers of neighbours and thus experience a smaller interaction field than hysterons located at the assembly centre. Consequently, the increase of $\langle H_{int} \rangle$ at low inter-hysteron distances is concomitant with a $\sigma_{H_{int}}$ increase.

Figure 9 indicates that FORC diagrams change from a wishbone shape at large inter-hysteron distances to IFD+CFD at low distances. In other words, when $\sigma_{H_{int}}$ is minimum, at large distances, the interaction regime tends toward a mean field interaction regime for which the FORC signature is a wishbone, whereas at low distances, the interaction regime tends toward an inhomogeneous interaction regime for which the FORC signature is an IFD+CFD. ρ_{FORC} profiles along IFD (Fig. 9(g)) illustrate this transition in the interaction regime with peaked profiles for low distances and more constant ones for larger distances. Additionally, $\rho_{FORC}(H_c)$ profiles of Fig. 9(f) show that point C translates to higher H_c values at low distances, as a consequence of the $\langle H_{int} \rangle$ increase. Quantitative analysis continues to demonstrate that $\rho_{FORC}(H_c)$ is not directly representative of the SFD, whereas the correct values of $\sigma_{H_{sw}}$ are extracted exploiting A and B points (Table 3).

Figure 9: (a)-(e) FORC diagrams calculated for hysteron assemblies presenting a **normal SFD** with inter-hysteron distances $4 \leq d \leq 12$ nm ($D = 7$ nm) and interacting via a **magnetostatic interaction** field. $N = 1681$ hysterons, $\langle \mu_0 H_{sw} \rangle = 300$ mT, $\sigma_{H_{sw}} = 60$ mT. (f) $\rho_{FORC}(H_C)$ and (g) ρ_{FORC} profiles along the IFD for the different inter-hysteron distances.

$\langle H_{sw} \rangle$ (mT)	$\sigma_{H_{sw}}$ (mT)	d (nm)	$H_C^{\rho_{FORC}(H_C)}$ (mT)	$\sigma_{fit}^{\rho_{FORC}(H_C)}$ (mT)	H_C^{mean} (mT)	$\sigma_{H_{sw}}^{FORC}$ (mT)
300	60	12	267	68.2 (13%)	302 (0.7%)	59.6 (0.6%)
300	60	10	263	62.4 (4%)	300 (0%)	61.6 (2.7%)
300	60	8	253	54.1 (14%)	306 (2%)	58.6 (2.3%)
300	60	6	253	54.4 (9%)	296 (1.3%)	58.7 (2.2%)
300	60	4	242	48.4 (19%)	302 (0.7%)	58.3 (2.8%)

Table 3: Main properties (Cf Table 1) extracted from FORC diagrams of Fig. 9, calculated for assemblies of hysterons with edge-to-edge inter-hysteron distances $4 \leq d \leq 12$ nm ($D = 7$ nm) in a **magnetostatic interaction field**.

Data of Fig. 9 indicate that a transition between wishbone and IFD+CFD shapes is obtained when both $\langle H_{int} \rangle$ and $\sigma_{H_{int}}$ increase. In order to modulate $\sigma_{H_{int}}$ independently, the size of the hysteron assembly can be modified by varying the inter-hysteron distance d , but keeping $\langle H_{int} \rangle$ constant by adjusting accordingly the diameter D of the nanowire-based hysteron (Eq. 3). Therefore, $\sigma_{H_{int}}$ is calculated as the mean deviation of the averaged sum of interaction field to the one of each

hysteron: $\sigma_{H_{int}} = \sum_{i=1}^N \frac{\sqrt{(\langle H_{int} \rangle - H_{int}^i)^2}}{N}$, with $\langle H_{int} \rangle = \frac{1}{N} \sum_{i=1}^N H_{int}^i$ and H_{int}^i the interaction field of the i^{th} hysteron calculated using Eq. 3. Note that $\sigma_{H_{int}}$ is here tuned by varying the assembly size, but it can also be done alternatively introducing dispersions in D , L or d in Eq. 3, which are the real origins of $\sigma_{H_{int}}$ in practice.

In order to compare small vs large inter-hysteron distances d , the diameter D is adjusted to yield the same calculated $\langle H_{int} \rangle$ value and this whatever the magnetization state (Fig. 10(a) for three $\langle H_{int} \rangle$ values). Corresponding $\sigma_{H_{int}}$ is plotted as a function of applied field in Fig. 10(b). Figure 11 plots the corresponding FORC diagrams to compare small vs large inter-hysteron distances which give the same $\langle H_{int} \rangle$. At low inter-hysteron distances, *i.e.* for small assemblies, the saturation value of $\sigma_{H_{int}}$ in Fig. 10(b) reaches highest values as a result of edge effects. Indeed, for large inter-hysteron

distances, saturation values of $\sigma_{H_{int}}$ are comparatively lower. An important contrast is that $\sigma_{H_{int}}$ is constant during the magnetization reversal for large inter-hysteron distances ($d = 50$ nm), which lead to wishbone FORC diagrams (Figs. 11 (d)-(e)-(f)). It illustrates the fact that in this case, the switching of hysterons is not guided by the interaction field, but solely by the SFD. Indeed, if the demagnetizing interaction field would drive the magnetization reversal, areas with large local interaction fields would be favoured and consecutive switchings would lead to a local decrease of H_{int} : $\sigma_{H_{int}}$ should strongly vary to reach a minimum at H_c . This behaviour is observed in Fig. 10(b) for small inter-hysteron distances, which present IFD+CFD FORC diagrams (Figs. 11 (a)-(b)-(c)). In that case, $\sigma_{H_{int}}$ is minimum in the demagnetized state, at H_c , and maximum at saturation, in agreement with previous results [5]. Therefore, in the case of a highly dispersed interaction field, the reversal is guided by the non-uniformity of the interaction field, whereas for less dispersed interactions, the reversal tends to be guided by the SFD.

Figure 10 : a) Interaction field averaged on the whole hysterons and b) standard deviation of the interaction field $\sigma_{H_{int}}$, plotted as a function of applied field. c) ρ_{FORC} profiles along the IFD for inter-hysteron distance $d = 50$ nm (FORC diagrams of Fig. 11(d)-(e)-(f)). d) ρ_{FORC} profiles along the IFD for diameter of nanowire-hysteron $D = 7$ nm (FORC diagrams of Fig. 11(a)-(b)-(c)). H_{int}^{sat} corresponds to the averaged saturation value of H_{int} over the assembly and is obtained from Fig. 10(a).

Figure 11: (a)-(c) FORC diagrams calculated for hysteron assemblies with edge-to-edge inter-hysteron distances $4 \leq d \leq 6$ nm and diameter $D = 7$ nm. (d)-(f) FORC diagrams calculated for hysteron assemblies with inter-hysteron distances $d = 50$ nm. The diameter D of the nanowires whose hysterons referred to has been modulated within $35.8 \leq D \leq 57$ nm to obtain $\langle H_{int} \rangle$ identical to the FORC diagrams (a)-(c). $N = 1681$ hysterons, $\langle \mu_0 H_{sw} \rangle = 300$ mT, $\sigma_{H_{sw}} = 60$ mT. H_{int}^{sat} values are obtained from Fig. 10(a) and are here located on FORC diagrams for illustration.

The IFD profiles are almost constant for large inter-hysteron distances (Fig. 10(c)) whereas they are sharpened for small ones (Fig. 10(d)). As seen in previous parts, this difference has been ascribed to mean-field and local interaction field, respectively. Assemblies with large inter-hysteron distances, exhibiting low $\sigma_{H_{int}}$ at saturation, behave as in a mean-field regime and a wishbone type FORC diagram is obtained, provided that a SFD is present. On the other side, low distance assemblies, exhibiting high $\sigma_{H_{int}}$ at saturation, behave as in a local interaction field regime and the FORC signature is IFD+CFD. Here we can conclude that, among constant parameters $(H_{int}, H_{sw}, \sigma_{H_{sw}})$, $\sigma_{H_{int}}$ is an important

parameter that drives the FORC signature. Nevertheless, the quantitative determination of $\sigma_{H_{int}}$ from the FORC diagram is not accessible.

Discussion

The simulations presented here allow to distinguish three major FORC signatures that can be encountered in assemblies of perpendicularly magnetized nanomagnets described by rectangular hysterons in the presence of demagnetizing dipolar interactions: linear (or IFD), wishbone, and IFD+CFD.

First, the linear FORC is obtained for hysteron assemblies presenting no or extremely small switching-field distributions. This signature consists in the interaction field distribution (IFD) entirely due to the sheared hysteresis loop in the presence of demagnetizing dipolar interactions. IFD is parallel to the H_{int} axis and the profile is constant for a mean interaction field, whereas it is peaked for a magnetostatic interaction field with a maximum at $H_{int} = 0$. This feature can constitute a criterion to distinguish the interaction regime. For both regime of interactions, the IFD width is $2H_{int}^{sat}$, thus H_{int}^{sat} is easily accessible typically at point *A*. The coercive field, *i.e.* the median switching field, can be extracted at point *B* because its abscissa is always $H_c^{median} = \langle H_{sw} \rangle$. Modulations of $\langle H_{sw} \rangle$ or amplitude of H_{int} do not affect the FORC signature and only shift the ρ_{FORC} distributions. If a (low) SFD is present, the $\rho_{FORC}(H_c)$ profile does not quantitatively represent it. Nevertheless, it can be successfully addressed by $\sigma_{H_{sw}}^{FORC}$ determined from H_c^{min} and H_c^{median} extracted at points *A* and *B*, respectively, and this whatever the SFD shape.

Secondly, the wishbone FORC is clearly obtained for hysteron assemblies presenting a switching-field distribution and where a uniform interaction field, *i.e.* mean-field, dominates. The *AC* branch of the wishbone shape originates from the most coercive hysterons of a given FORC : they undergo a bias H_u proportional to the f_1 proportion of *up* hysterons, together with an artificial increase of their width $2H_c$ proportionally to the f_2 proportion of *down* hysterons. IFD occurs due to the sheared hysteresis loop and accounts for the bias field underwent by the less coercive hysterons. As a consequence, the tilt between IFD and H_{int} axis increases with the SFD deviation $\sigma_{H_{sw}} \cdot H_c^{median} = \langle H_{sw} \rangle$ and H_{int}^{sat} can be read at point *B* and point *A* respectively. Whereas $\rho_{FORC}(H_c)$ profile does not quantitatively account for SFD, $\sigma_{H_{sw}}$ can be also obtained using abscissa of points *A* and *B*.

Finally, the IFD+CFD FORC signature is mainly composed of the IFD, which is still due to the MHL shearing. Similarly to the precedent cases, the interaction field value at saturation H_{int}^{sat} can also be obtained at point *A*, while point *B* indicates $H_c^{median} = \langle H_{sw} \rangle$. Since CFD is being mostly aligned with H_c axis (*i.e.* $H_{int} = 0$), it concerns symmetrical or quasi-symmetrical hysterons : it occurs for assemblies of hysterons interacting *via* a local interaction field with a significant deviation $\sigma_{H_{int}}$. For large deviations, $\sigma_{H_{int}}$ evolves strongly during the magnetization reversal, the non-uniform interaction regime gives peaked $\rho_{FORC}(IFD)$ profiles with maximum at $H_{int} = 0$ and the FORC diagram is IFD+CFD. Conversely, for small deviations, $\sigma_{H_{int}}$ does not strongly change with M , the interaction field tends to be homogenous with constant ρ_{FORC} profiles along IFD and the FORC diagram is a wishbone. Whatever the IFD+CFD or wishbone shape, deviation of the switching field distribution can be quantified with error less than 10% using abscissa of points *A* and *B*, provided that the studied system corresponds to isolated coupled elements with only demagnetizing dipolar interactions.

From the experimental point of view, as quantitative data are contained in points *A*, *B* and *C*, it is crucial to accurately locate these points on the FORC diagrams. Point *A* and *B* can be determined

moving horizontal lines parallel to the H axis and intercepting extremal values of the IFD where $\rho_{FORC} \neq 0$ (Fig. 6). Point C , in principle horizontally aligned with point B , is located on the H_c axis where ρ_{FORC} reaches the highest H_c value. In order to try to access the SFD shape, a deshearing process [9] can be done if the FORC diagram reveals that the interaction regime is uniform, as attested by the IFD profile and/or the presence of a wishbone shape. In that case, H_{int}^{sat} determined at point A can be used to evaluate α_m and plotting the FORCs as a function of H_{tot} instead of H_{ext} using Eq. 4. SFD can then be obtained differentiating the MHL or using $\rho_{FORC}(H_c)$ profile. Nevertheless, deshearing is non trivial in practice, as it can often lead to overskewing due to the inherent assumptions associated with such an approach [69].

Conclusion

Main signatures of FORC diagrams encountered in 2D arrays of perpendicularly magnetized nanomagnets in the presence of demagnetizing dipolar interactions are reproduced and analysed with the sake to propose their quantified reading. For systems satisfying the present model, interaction regime together with its amplitude, as well as mean coercivity and SFD deviation, can be obtained, even if the SFD shape in itself cannot be determined. Next step is to overcome some of the assumptions, for example in introducing a magnetizing coupling between magnets, such as exchange interaction or magnetizing dipolar interactions in assemblies where aligned easy axes lie in the assembly plane. In addition to the extraction of extrinsic and intrinsic parameters, FORC is also an exciting tool to probe fingerprints related to the reversal processes such as distributions related to nucleation/annihilation of magnetic domain in thin films [18][19] or vortex in dots [25]. Such behaviours cannot be addressed using the hysteron-based approach and thus require FORC modelling with the use of micromagnetic simulations, which are up to now time-consuming and limits a broad scanning of parameters.

Supplementary Material

See supplementary material for supplementary figures and tables, description of the origin of the FORC diagram wishbone shape, and extraction of the standard deviation of a log-normal distribution from the FORC diagram.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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