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Diffusivity, Kinetic Energy Dissipation, and Closure Theories for the Poleward Eddy Heat Flux

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ABSTRACT

Diffusive eddy closure theory for estimating the poleward heat flux is reexamined and tested in the context of a two-layer homogeneous model. Consideration of the inverse energy cascade induced by baroclinic turbulence on the β plane leads to an expression for diffusivity in terms of the kinetic energy dissipation and the β effect. A key step in the closure is the identification of this diffusivity with that for potential vorticity in the lower of the two layers in the model. This assumption is then combined with an exact expression relating the diffusivity to the baroclinic energy generation and the mean vertical shear. The theory is closed by identifying the kinetic energy dissipation entering the inverse cascade argument with the baroclinic energy production. It is found that the first constraint in isolation based on inverse cascade arguments between the diffusivity of lower-layer potential vorticity and the kinetic energy dissipation is robust and accurate, whereas the final theory relating diffusivity to vertical shear remains useful but has somewhat degraded accuracy and is more sensitive to model parameters, such as numerical resolution and small-scale dissipation. In the limit of large supercriticality, this theory reduces to that of Held and Larichev. However, it is much more accurate in reproducing numerical results from a two-layer homogeneous model on a β plane for the moderate supercriticalities that are typical of model atmospheres. The problems involved in generalizing this result to models with more layers on the vertical or with a continuous stratification are discussed.

1. Introduction

The theory for baroclinic eddy heat fluxes attributed to Held and Larichev (1996, hereafter HL) can be recast in a form in which one simultaneously satisfies two constraints between the eddy diffusivity and the rate of baroclinic energy production. The first of these constraints arises from the assumption that the energy-containing eddies stem from an inverse energy cascade that is halted by the β effect, as discussed originally by Rhines (1975). From this assumption, one can relate the eddy length and velocity scales to ε_d , the rate per unit of mass at which kinetic energy is flowing through the inverse cascade and being dissipated at large horizontal scales by surface friction. From these length and velocity scales, one can estimate a diffusivity from their product:

$$\mathcal{D} \propto \varepsilon_d^{3/5} \beta^{-4/5}. \quad (1)$$

This is the unique way of constructing a kinematic dif-

fusivity, given only ε_d and β , and this scaling underlies the theory of HL [see their Eqs. (26) and (32)]. Among others, Maltrud and Vallis (1991), Panetta (1993), Spall (2000), and Danilov and Gurarie (2002) evaluate aspects of the predictions of eddy length and velocity scales following the Rhines argument. Smith et al. (2002) discuss scaling arguments of this type for the diffusion of a passive tracer by two-dimensional turbulence.

The second half of the theory of HL consists in relating ε_d with the production of available potential energy. In a quasigeostrophic system in which baroclinic production is the only significant eddy source, eddy energy is created through downgradient eddy heat fluxes and is typically dissipated at large scales through friction (vertical turbulent diffusion in surface boundary layers). In the Boussinesq approximation, for example, the baroclinic production is

$$\varepsilon_p = - \left[\overline{v'b'} \frac{\partial B / \partial y}{\partial B / \partial z} \right]. \quad (2)$$

Here, the climate is assumed to be zonally symmetric, B is the zonal mean buoyancy, $\overline{v'b'}$ is the eddy horizontal buoyancy flux, and the brackets denote an av-

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erage over latitude and height. If small-scale dissipation is negligible, the production of available potential energy is equal to the energy dissipation at large scales.

To close the theory, one needs to relate the buoyancy flux to the diffusivity in (1). In the two-layer model of HL, one will obtain different answers, in general, if one chooses to diffuse buoyancy, lower-layer potential vorticity, or upper-layer potential vorticity with the value of the diffusivity set by (1). However, in the limit of strong supercriticality, in which the vertical shear is much larger than the minimum shear required for instability, the distinction between diffusing buoyancy and potential vorticity disappears in this two-layer model. We return to this point below as it is a key ingredient in the modification proposed here to the theory of HL. Putting this question aside for the moment, and setting $v'b' = -\mathcal{D}\partial B/\partial y$, we write

$$\varepsilon_p = \left[\mathcal{D} \frac{(\partial B/\partial y)^2}{\partial B/\partial z} \right] = \left[\frac{\mathcal{D}}{\mathcal{T}^2} \right], \quad (3)$$

where $\mathcal{T}^{-1}$ is an inverse timescale proportional to the local Eady growth rate $f\text{Ri}^{-1/2}$. If one can now replace $\mathcal{D}$ and $\mathcal{T}$ with appropriately weighted global means, equate the resulting value of $\mathcal{D}$ with that in (1), and equate ε_p and ε_d , one obtains the result in HL:

$$\mathcal{D} \propto \frac{1}{\beta^2 \mathcal{T}^3}. \quad (4)$$

Recently, Barry et al. (2002) have examined the scaling properties of the eddy heat transport for a set of general circulation experiments by varying external parameters, and their results support the connection between diffusivity and ε in the more realistic context of atmospheric general circulation models. However, they argue that their results are not well fit by (4), although it is implicit that (1) is well satisfied. There are several reasons why the theory of HL might break down when applied to comprehensive GCMs. Some of these can be addressed in the context of dry quasigeostrophic dynamics, and others relate especially to the extra complexities introduced by moisture. This paper is devoted to some of the dry quasigeostrophic issues. In particular we reexamine the dry two-layer, homogeneous model of HL and test the different parts of the theory that leads to (4). We provide a modified heuristic theory that fits the numerical results much more closely. This modified theory is of particular relevance when the *supercriticality* of the flow is relatively weak. In the process we make connections with some results of Pavan and Held (1996) for horizontally inhomogeneous quasigeostrophic flows. The impact of moist thermodynamics on these issues will be addressed in a further study.

Through the different sections of this paper, we discuss and assess the different steps leading to (4). We use the numerical setting of a two-layer model but believe that similar issues arise in other models. Indeed,

we briefly refer to the three-layer model of Held and O'Brien (1992) to help make this point. The numerical model is presented in the appendix. In section 2, we examine the arguments leading to (1) and test its accuracy. In section 3, we discuss the relation between the energy production and dissipation and also how to relate our present theory to the results of Barry et al. (2002).

2. Relation between diffusivity and cascading eddies

a. Inverse cascade scalings

The relation (1) is tightly linked to the properties of the barotropic inverse cascade of energy on the β plane. Given assumptions about this inverse cascade, several relations exist between the characteristic eddy velocity scale V and length scale k_0^{-1} . The first of these assumes that there exists an inertial range with the universal $C\varepsilon_d^{2/3}k^{-5/3}$ form. This assumption means that the cascade is localized in the spectral sense. From this, just as in the case of the inertial range in three-dimensional turbulence, one can relate the rate at which energy is flowing through the inverse cascade to V and k_0 :

$$\varepsilon_d \propto V^3 k_0. \quad (5)$$

Here, ε_d is the dissipation of barotropic energy by the large-scale drag. See, for example, Smith and Vallis (2002) for a discussion of this relation in the two-dimensional context.

We can use this relation to define a length scale k_B^{-1} such that

$$k_B = \varepsilon_d/V^3. \quad (6)$$

The second assumption is that the eddies are characterized by the Rhines scale k_R^{-1} , that is,

$$k_R = (\beta/V)^{1/2}, \quad (7)$$

where V is the rms velocity. Rhines (1975; see also Panetta 1993) introduced this scale to predict the stopping scale of β -plane turbulence. However, as noted originally by Rhines and as discussed by Danilov and Gurarie (2002) and Smith et al. (2002), the β effect alone cannot arrest the cascade completely; less efficient energy transfer to zonal flows of larger and larger meridional scale continues until arrested by frictional drag. If we want to identify V and k_R with scales relevant for meridional mixing, we need to assume that it is the point of transition from a more or less isotropic cascade to the anisotropic interaction with jets that is the determining characteristic of the flow, and not the larger jet scales that evolve if the damping is weak enough. See Smith et al. (2002) for support for this picture.

One can introduce another scale involving β and ε_d :

$$k_\beta = \beta^{3/5} \varepsilon_d^{-1/5}. \quad (8)$$

If k_B and k_R are proportional to each other, then they

are also proportional to k_β . Also, assuming that ε_d and β are the only relevant quantities and by a dimensional analysis, one finds that the three scales are proportional. Vallis and Maltrud (1993) argue that (8) has advantages over (7) in predicting the stopping scale for barotropic turbulence as it is a Galilean invariant. However, the advantage of (7) is that it gives information on the energy content of the flow. This can be seen in the context of 2D turbulence when one varies bottom drag keeping fixed ε (see, e.g., Smith et al. 2002).

Assuming that the eddy scales (6), (7), and (8) are the same, one obtains immediately a characteristic diffusivity Vk_0^{-1} defined by (1). The differences between these scales can be examined in Fig. 1. This figure shows results from our 256^2 and 1024^2 simulations (see the appendix) and results from the experiments of HL, as a function of supercriticality ξ (shear divided by the critical shear required for inviscid instability). For clarity, the scales have been multiplied by appropriate constants so that they equal the Rhines scale k_R^{-1} at $\xi = 4$. Also, ε_d has been evaluated as the dissipation of barotropic energy by Ekman friction. One can see that the different scales match each other for $\xi < 5$. For strongly supercritical flows ($\xi > 5$), the two Rhines scales k_R and k_β remain very similar, but k_β strongly overestimates the other scales in the HL experiments and underestimates them in our 1024^2 runs. The actual relationship between these scales could involve the properties of the inverse cascade, such as the presence of inertial range and the effect of large-scale dissipation. The equality between k_R and k_β contrasts with the one-layer β -plane case studied by Vallis and Maltrud (1993), who found important differences between these scales. In the present model, we can understand the similarity between k_R and k_β as follows. If one assumes

$$k_R/k_\beta = \xi^x,$$

one finds

$$k_R/k_\beta = \xi^{-x/5}.$$

Thus, even if one finds a large difference between k_R and k_β , the difference between the two Rhines scales k_R and k_β will remain small.

To assess further if a theory for eddy scales can be expressed only in terms of ε_d and β , we can examine how the velocity scale V scales as a function of ε_d and β . From $k_R = k_\beta$, one would have

$$V \propto \beta^{-1/5} \varepsilon_d^{2/5}. \quad (9)$$

As can be seen in Fig. 1d, this works indeed very well for all of the experiments, even when k_β departs from k_R . Assuming as before that $k_R/k_\beta = \xi^x$, we find

$$V = \varepsilon_d^{2/5} \beta^{-1/5} \xi^{2x/5}.$$

This relation shows that the velocity scaling is only weakly sensitive to the departure of k_β from k_R . Such a departure is expected to occur if the model does not

produce the Kolmogorov spectral shape, that is, if it does not have a well-defined inverse energy cascade.

The barotropic energy spectra from our two-layer model are presented on Fig. 2. The spectra are steeper than $k^{-5/3}$. This is presumably related to the fact that the barotropic energy production (i.e., the baroclinic to barotropic energy transfer) is not localized in spectral space but is distributed over a wide range of scales, as was shown by HL. It could also result from nonlocality of energy transfers, and/or large-scale anisotropic structures (e.g., zonal jets). This helps explain why k_β behaves differently from the other scales.

These spectra help one to understand some important differences between the different sets of simulations. In our 256^2 case, the friction is not large enough to prevent the inverse cascade from reaching the largest scale when the supercriticality is large. So the barotropic kinetic energy peaks at the domain scale (see Fig. 2a). Because of this piling up of energy at the domain scale, the eddies are more energetic than predicted: $V/(\beta^{-1/5} \varepsilon_d^{2/5})$ increases as ξ increases (see inset in Fig. 1d). On the other hand, in the 1024^2 case, bottom friction is large enough that the stopping scale is not set anymore by β alone: as can be seen in Fig. 2b, the eddy length scale is no longer set by the Rhines scale. Both β and drag are important in this case. Presumably, the velocity scale is no longer set by ε_d and β and should also depend on friction. This behavior is consistent with the physical fields as we find that the eddies are almost isotropic in this case (implying that β is not the essential parameter). As a result, the energy-containing eddies have less energy for a given energy generation rate than predicted: $V/(\beta^{-1/5} \varepsilon_d^{2/5})$ decreases as ξ increases (inset in Fig. 1d). These results show that, for a given barotropic energy injection rate, the energy of the eddies may vary substantially, depending on the efficiency of the drag in arresting the cascade. Danilov and Gurarie (2001) show that for barotropic f -plane turbulence, large-scale drag can be responsible for non-universal properties of the inverse cascade. Here, another source of nonuniversality is the broad distribution of the barotropic energy generation. In consequence, the precise equality of all three scales defined above may be difficult to realize.

b. Diffusivity scaling

Introducing a mixing scale k_d^{-1} such that

$$k_d = V/\mathcal{D}, \quad (10)$$

and equating it with k_β and k_R , we obtain

$$\mathcal{D} = c\beta^{-4/5} \varepsilon_d^{3/5}. \quad (11)$$

We could also derive this relation from $\mathcal{D} = Vk_0^{-1}$, using the length scale $k_0 = k_\beta$ and the velocity scale defined by (9).

One now needs to equate the diffusivity $\mathcal{D}$ either with the diffusivity of buoyancy $\mathcal{D}_b = -[v'b']/\partial_y B$ or of

FIG. 2. Spectra of barotropic kinetic energy for different values of supercriticality (a) for our 256^2 simulations and (b) for our 1024^2 simulations. The upper curves correspond to larger supercriticality. The stars correspond to the Rhines scale $k_R = (\beta/2V)^{1/2}$, where V is the barotropic velocity rms. The figure insets are the compensated energy spectra (i.e., multiplied by $k^{-5/3}$). The circles still correspond to k_R .

As discussed in numerous studies (see, e.g., Tréguier et al. 1997; Marshall et al. 1999), it seems more appropriate to diffuse a quantity conserved following the geostrophic flow (when adiabatic and inviscid), such as PV rather than buoyancy. The choice of the vertical level to use for diffusing PV is less clear cut. In the two-layer model, in the presence of the β effect, typical eddy phase speeds are such that $U - c$ is smaller in the lower than the upper layer, so the lower layer more nearly serves as the steering level for these eddies. As a result, meridional particle displacements are larger for a given streamfunction perturbation in the lower layer, and the eddies can be expected to mix PV more efficiently. The upper layer is characterized by coherent zonal jets in which large values of $U - c$ and PV gradients tend to restrict the large-scale meridional transport. The lower layer flow creates mixing that is more plausibly described by simple diffusion.

Using the lower-layer PV diffusivity for $\mathcal{D}$ in (1) does work quite well for all the simulations, as can be seen on Fig. 3. Actual and predicted diffusivities have been nondimensioned by $U\lambda$, where λ is the deformation radius, and we multiplied each set of predictions by a constant chosen to best fit with the numerical results. The value of the constant is provided in the caption. As evident in the figure, there is a clear range of values where this scaling for the lower-layer diffusivity works quite well, and this agreement is improved for the 1024^2 simulations. The fact that this theory works so well may be partly due to the fact that, if there is some departure from the scaling expected for a pure inertial range, that is, $k_R/k_B = \xi^x$, this gives a diffusivity $\mathcal{D} \approx \beta^{-4/5} \varepsilon_d^{3/5} \xi^{3x/5}$. Thus, there is a rather weak dependence on discrepancies of this type in the length scales.

We have also observed that the result is not very

sensitive to the exact definition of ε . In the above we have used the energy lost to surface friction from the barotropic mode, but an equally good fit is obtained if one uses the total energy dissipation by surface friction (in the baroclinic as well as the barotropic mode). Replacing ε_d with the energy production ε_p , which balances energy lost through both large-scale drag and small-scale mixing, does not alter the fit either (not shown). The $3/5$ power law in the expression for $\mathcal{D}$ makes the result rather insensitive to these types of differences.

If one instead plots the diffusivity for the upper-layer PV against $\beta^{-4/5} \varepsilon_d^{3/5}$, the resulting fit is clearly not as good, as shown in Fig. 3b. It does appear to be appropriate, in this two-layer model, to associate the lower-layer diffusivity of PV, and not the upper-layer diffusivity, with the energy throughflow in this way.

Concerning the lower-layer diffusivity scaling, we observe some departure at both ends of the range of supercriticality. For small ξ , the diffusivity increases with decreasing ξ despite the fact that energy decreases (not shown). We relate this behavior to the effect of bottom friction and frictional instability that seem to increase the PV fluxes, as we find that a passive tracer advected by the bottom layer has a smaller diffusivity than does PV. For large ξ , the simulations present different behaviors: the 256^2 simulations show that the diffusivity is larger than its predicted value, whereas the opposite is true for the 1024^2 case. The causes of this discrepancy relate to Ekman friction and numerical resolution: in the 256^2 case, we have seen that friction is inefficient in arresting the inverse cascade, so that eddies feel the size of the domain at large ξ and are more energetic. This leads to a diffusivity larger than predicted. On the other hand, in the 1024^2 case, bottom friction is quite efficient

FIG. 4. Potential vorticity diffusivities as a function of prediction (11) using the three-layer model of Held and O'Brien (1992). The PV diffusivities were shifted to separate each layer.

Tréguier et al. 1997; Smith and Vallis 2002). We can hope to write

$$\mathcal{D}(z) = \gamma(z)\mathcal{D}_0, \quad (12)$$

where γ is the distribution of the diffusivity in the vertical, $\gamma(z_0) = 1$ for some z_0 at which the scaling described above works best, and $\mathcal{D}_0$ satisfies

$$\mathcal{D}_0 = c\beta^{-4/5}\varepsilon_d^{3/5}.$$

Because of zero total PV flux, one has the extra constraint

$$[v'q'] = \mathcal{D}_0 \int \gamma(z) \partial_z Q(z) dz = 0,$$

but it is only in the two-layer model that this constraint, together with the expression above for the diffusivity in a particular layer, fully determines the PV fluxes [without using (12)].

3. Energy production

a. Relation between diffusivity and supercriticality

To close the theory in the two-layer model, one has to find another relation between ε_d and $\mathcal{D}$. The generation of eddy kinetic energy plus available potential energy per unit mass can be written as

$$\varepsilon_p = -\frac{1}{H_1 + H_2} \sum_{k=1}^2 H_k U_k [v'q']_k. \quad (13)$$

This expression reduces to the two-layer version of (2) in the horizontally homogeneous case (in which there is no transfer of eddy kinetic to mean kinetic energy). In a statistically steady state, the production of baro-

clinic energy, ε_p , is exactly compensated by the dissipation of baroclinic and barotropic energy by surface friction and small-scale mixing. Given the insensitivity of our tests of the relation (1) to the precise definition of ε_d , we proceed by identifying ε_p with ε_d in (1). In this picture, we assume that all the energy produced baroclinically cascades barotropically to large scales. Then we have

$$\varepsilon_d \approx \varepsilon_p = \mathcal{U}[v'q']_2 = \mathcal{U}\beta\mathcal{D}_2(\xi - 1), \quad (14)$$

where $\mathcal{U} \equiv (U_1 - U_2)/2$ so that $\xi = \mathcal{U}/(\beta\lambda^2)$. This results in the following expression for the diffusivity:

$$\mathcal{D}_2 \propto \mathcal{U}\lambda\xi^2(1 - \xi^{-1})^{3/2} = \beta^{-2}\mathcal{U}^3\lambda^{-3}(1 - \xi^{-1})^{3/2}, \quad (15)$$

which we can write in the form (4) if we set

$$\mathcal{T} \equiv (1 - \xi^{-1})^{-1/2}\lambda/\mathcal{U}.$$

The identification of $\mathcal{D}$ with $\mathcal{D}_2$ has the immediate advantage that the resulting expression for $\mathcal{D}$ tends to zero as $\xi \rightarrow 1$. The expression of HL results by taking the limit $\xi \rightarrow \infty$.

In a comparison of homogeneous and channel quasigeostrophic models, Pavan and Held (1996) fit the lower-layer PV fluxes in the homogeneous model with a diffusive closure and then use this diffusivity to obtain good fits to numerical integrations for inhomogeneous baroclinically unstable jets. *The expression that they obtained is precisely (15)*, although they were unaware of the argument of the preceding paragraph.

Given the good agreement with (1) whether we use ε_d or ε_p , and that (14) is exact for our two-layer model, we might expect the Pavan and Held formula (15) to work equally well. However, it is informative, in this regard, to consider the implications of assuming a small departure in the $\mathcal{D}$ - ε scaling. More precisely, suppose that, for fixed β ,

$$\mathcal{D} \propto \varepsilon^{3/5+\alpha}, \quad (16)$$

where α is a small number. Using (11), we find

$$\mathcal{D} \propto \xi^\mu(\xi - 1)^\mu; \quad \mu = \frac{3/5 + \alpha}{2/5 - \alpha}. \quad (17)$$

Assuming α to be small, the scaling exponent is $3/2 + 25\alpha/4$. Thus, a small imprecision in α is amplified nearly sixfold in the expression for $\mathcal{D}(\xi)$. The implication is that it is much more demanding to obtain an accurate theory for heat flux, or diffusivity, as a function of the mean gradients than a theory for diffusivity as a function of ε .

The numerical results are displayed in Fig. 5a for our new simulations and for those tabulated in HL. For the 256^2 simulations, the scaling works only marginally and the differences between the various versions of the model are larger than in Fig. 3a, as anticipated from the argument of the preceding paragraph. The 1024^2 simulations show a better fit with the scaling provided by (15), especially if we discard the larger values of ξ for which we know that the surface drag is stopping the cascade before β can play its role. Additionally, the fit

another relation between ε , the lower-layer PV diffusivity and the mean PV gradients that one can couple with (1) to determine the energy throughput and the diffusivity separately. In a model with more than two levels in the vertical, this argument is no longer closed, as one needs a theory for the vertical distribution of the PV flux. See Held and O'Brien (1992), Smith and Vallis (2002), and Tréguier et al. (1997) for a discussion of this issue, which we consider an unsolved problem.

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APPENDIX

Numerical Model

We have reexamined integrations similar to those in HL, partly as a point of departure for the moist model described in Lapeyre and Held (2003, unpublished manuscript). We take the standard horizontally homogeneous model on the β plane with a zonally symmetric climate and layers of equal mean depths, as in HL. The mean zonal velocity in each layer is $U_1 = U$ and $U_2 = -U$ where the subscripts 1 and 2 refer respectively to the top and bottom layers of fluid. Each layer has equal mean depth $H_1 = H_2 = H$. The supercriticality $\xi \equiv (U_1 - U_2)/(\beta\lambda^2)$ (where λ is the radius of deformation) must be larger than unity for instability.

We examined different sets of simulations, using both 256^2 resolution as in HL, but also the higher resolution of 1024^2 . The current model uses an exponential filter to dissipate small scales [see Smith et al. (2002) for a description of the scheme], rather than the ∇^8 damping used in HL. This modifies the energetics of the system as we now find typically that less than 3% of the energy input rate ε_p is dissipated by small-scale diffusion, leaving 97% to be destroyed by Ekman friction (the analogous figures were only 30% and 70% in HL). As a consequence our 256^2 and 1024^2 simulations are substantially more energetic than in HL, holding other parameters fixed. Additionally, we vary the supercriticality ξ in our 256^2 simulations by varying λ , rather than β as in HL, motivated by the desire to test if the dependence of the dry model results on static stability can be used to help understand our companion moist simulations. The largest wavelength in the domain ranges from $20\pi\lambda$ for the smaller supercriticality to $34\pi\lambda$ for the largest. This leaves more room for the direct cascade of enstrophy compared to the simulations of HL where the largest scale was chosen to be $100\pi\lambda$. For our 1024^2

simulations, we chose $100\pi\lambda$ for the largest scale as in HL, but we used a larger bottom friction $r = 0.32 U\lambda^{-1}$ instead of $r = 0.16 U\lambda^{-1}$ for the 256^2 simulations here and in HL. In summary, our new simulations are less viscous than those in HL, and, in conjunction with HL, provide some information on the robustness of the scaling to the size of the domain and to the strength of surface friction. The range of supercriticality ξ we use in these different simulations is between 1.1 and 14 in order to push the limits of the theory to the weakly unstable limit.

REFERENCES

- Barry, L., G. C. Craig, and J. Thuburn, 2002: Poleward heat transport by the atmospheric heat engine. *Nature*, **415**, 774–777.
- Danibu, S., and D. Gurarie, 2001: Nonuniversal features of forced two-dimensional turbulence in the energy range. *Phys. Rev.*, **63E**, R020203.
- , and —, 2002: Rhines scale and spectra of the beta-plane turbulence with bottom drag. *Phys. Rev.*, **65E**, R067301.
- Haynes, P. H., J. F. Scinocca, and M. D. Greenslade, 2001: Formation and maintenance of the extratropical tropopause by baroclinic eddies. *Geophys. Res. Lett.*, **28**, 4179–4182.
- Held, I. M., and E. O'Brien, 1992: Quasigeostrophic turbulence in a three-layer model: Effects of vertical structure in the mean shear. *J. Atmos. Sci.*, **49**, 1861–1870.
- , and V. D. Larichev, 1996: A scaling theory for horizontally homogeneous, baroclinically unstable flow on a beta plane. *J. Atmos. Sci.*, **53**, 946–952.
- Lorenz, E. N., 1967: *The Nature and Theory of the General Circulation of the Atmosphere*. WMO, 161 pp.
- Maltrud, M. E., and G. K. Vallis, 1991: Energy spectra and coherent structures in forced two-dimensional and beta-plane turbulence. *J. Fluid Mech.*, **228**, 321–342.
- Marshall, D. P., R. G. Williams, and M.-M. Lee, 1999: The relation between eddy-induced transport and isopycnal gradients of potential vorticity. *J. Phys. Oceanogr.*, **29**, 1571–1578.
- Panetta, R. L., 1993: Zonal jets in wide baroclinically unstable regions: Persistence and scale selection. *J. Atmos. Sci.*, **50**, 2073–2106.
- Pavan, V., and I. M. Held, 1996: The diffusive approximation for eddy fluxes in baroclinically unstable jets. *J. Atmos. Sci.*, **53**, 1262–1272.
- Rhines, P. B., 1975: Waves and turbulence on a β -plane. *J. Fluid Mech.*, **69**, 417–443.
- , 1979: Geostrophic turbulence. *Annu. Rev. Fluid Mech.*, **11**, 404–441.
- Salmon, R. S., 1980: Baroclinic instability and geostrophic turbulence. *Geophys. Astrophys. Fluid Dyn.*, **15**, 167–211.
- Smith, K. S., and G. K. Vallis, 2002: The scales and equilibration of midocean eddies: Forced-dissipative flows. *J. Phys. Oceanogr.*, **32**, 1699–1721.
- , G. Bocaletti, C. C. Henning, I. N. Marinov, C. Y. Tam, I. M. Held, and G. K. Vallis, 2002: Turbulent diffusion in the geostrophic inverse cascade. *J. Fluid Mech.*, **469**, 14–47.
- Spall, M. A., 2000: Generation of strong mesoscale eddies by weak ocean gyres. *J. Mar. Res.*, **58**, 97–116.
- Tréguier, A. M., I. M. Held, and V. D. Larichev, 1997: Parameterization of quasigeostrophic eddies in primitive equation ocean models. *J. Phys. Oceanogr.*, **27**, 567–580.
- Vallis, G. K., 1988: Numerical studies of eddy transport properties in eddy-resolving and parameterized models. *Quart. J. Roy. Meteor. Soc.*, **114**, 183–204.
- , and M. E. Maltrud, 1993: Generation of mean flows and jets on a beta plane and over topography. *J. Phys. Oceanogr.*, **23**, 1346–1362.