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State Reconstructor for Real-Time Freeway Ramp Metering

Hirsh Majid, Hassane Abouaïssa, Daniel Jolly, Gildas Morvan

Abstract—The work presented in this paper aims to solve the problem of real-time ramp metering in the case of unavailable (not measured) traffic flow at an on-ramp or measurements provided by a faulty detector which lead to creation of unpredictable traffic states. A state reconstructor is then proposed and a systematic approach is developed for real time freeway ramp metering. The state reconstructor is developed using a second order sliding mode observer, while the control algorithm is based on the concept of differential flatness. Numerical simulations demonstrate the relevance of the provided algorithm for both state reconstructor and control of freeway networks.

Index Terms—Reconstructor, sliding mode, ramp metering.

I. INTRODUCTION

The main objective of real-time traffic control is to deal with a daily problem of congestion. Such problem is well identified as a source of continuous growth of the direct and indirect costs (drivers health and stress problems, pollution, increase in energy consumption, ... etc.) [1], [2]. The principle of traffic control is based on the use of a variety of adequate actions such as: dynamic speed limits using variable message signs, driving information and alerts, tidal control, dynamic route guidance and ramp metering.

Ramp metering or admissible control, represents the most efficient and direct way to solve congestion problems and to upgrade freeway traffic [3], [4]. This control action consists in regulating the ramp flow at the entrance of the freeway [1]. As stated in [5], ramp metering strategies are a valuable tool for an efficient traffic management that can be classified into: *reactive strategies* that aim to maintain the traffic conditions in freeway close to pre-specified set values using real-time measurements, and *proactive strategies*, aiming at specifying optimal traffic conditions for a whole freeway network based on the demand and the model prediction over a time horizon [2].

Ramp metering strategies can be implemented locally (*isolated ramp metering*) in the vicinity of each ramp to calculate the corresponding ramp metering values. It can be implemented simultaneously (*coordinated ramp metering*) when the objective is to use the available traffic measurements from larger freeway sections.

However, the performance of any control strategy is closely related to the ability to have all needed information about the

traffic state¹. Indeed, traffic control problems usually require the knowledge of both the present and the future traffic behaviours. Such information is provided via a set of sensors already installed along the freeway infrastructure. The future traffic behaviour can be computed using a suitable traffic model.

The main objective of this paper is to deal with isolated ramp metering problems when no information about the traffic state variable at the merge segment is available. In other words, we assume that the measurements just downstream of the controlled ramp are provided by a faulty detector. Having the state reconstructor, a systematic approach for real time isolated ramp metering is provided in order to maintain the traffic conditions at a desired optimal operating level.

The paper is organized as follow:

- Section II, recalls the used macroscopic model and provides the basis of the state reconstructor scheme.
- Section III, presents the freeway ramp metering algorithm based on the concept of differential flatness.
- Section IV, provides some numerical simulations and demonstrates the relevance of the proposed approach.
- The conclusion, section V, summarizes the main obtained results and lists some promising perspectives and further researches.

II. TRAFFIC FLOW MODELING AND STATE RECONSTRUCTOR

Several studies have demonstrated the relevance of macroscopic models for freeway traffic modeling, supervision and control [7], [8] and [9].

A. Macroscopic models

The first macroscopic model was developed by Lighthill and Whitham [10] and Richard [11] which is based on the conservation law:

$$\frac{\partial}{\partial t}\rho(x,t) + \frac{\partial}{\partial x}q(x,t) = g(x,t) \quad (1)$$

where, $\rho(x,t)$, $q(x,t)$ and $g(x,t)$ are, the traffic density in vehicles/km/lanes, the traffic volume in vehicles/h, the ramp generation term in vehicles/h/km, respectively.

Equation (1) is supplied by $q(x,t) = \rho(x,t)v(x,t)\beta$.

$v(x,t)$ is the mean speed and β , the number of lanes. The mean speed $v(x,t)$ and the traffic density $\rho(x,t)$ are related by

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¹Traffic density (or occupancy) constitutes the basis state variable of the most available traffic flow models and plays an important role for both traffic flow modeling and traffic control [6].

Fig. 1. Fundamental diagram

a so called "fundamental diagram" $v(x,t) = V_e(\rho(x,t))$. One of the most used form of such function is due to May [8]

$$V_e(\rho(x,t)) = v_f \exp\left(-\frac{1}{a} \left(\frac{\rho}{\rho_c}\right)^a\right) \quad (2)$$

where, a is a parameter, v_f is the free-flow speed and, ρ_c represents the critical density. Such fundamental diagram allows to identify the free flow zone and congested one (See Fig. 1).

As the first order macroscopic model is based on a static relationship between the main traffic variables, it presents several drawback such as the inability to describe the dynamic of the traffic behavior. In this context, the second order models seem to be more adapted. One of these models is the Payne's model [12]. In this work, we use the model proposed by [13], (See also [14]) which is discretized in time and space. Consider then the following freeway segment divided into n links (Fig. 2).

The studied freeway segment is then described by the following ordinary differential equations for each link i [15]

$$\dot{\rho}_i = \frac{1}{L_i} (q_{i-1} - \alpha \rho_i v_{i+1} - (1 - \alpha) \rho_{i+1} v_{i+2} + r_i(t) - s_i(t)) \quad (3)$$

$$\dot{v}_i = \frac{1}{\tau} [V_e(\rho_i) - v_i(t)] + \frac{\eta}{L_i} v_i(t) [v_{i-1} - v_{i+1}(t)] - \frac{v}{L_i} (\rho_{i+1} - \rho_i) - \frac{\delta}{L_i} (r_i(t) - s_i(t)) v_i \quad (4)$$

where, $r_i(t)$ and $s_i(t)$ are the on-ramp and off-ramp flows, respectively. τ , η , v and δ are model parameters. L_i is the length of the link i .

B. State reconstructor

All current freeway traffic control and monitoring algorithms require an accurate knowledge of traffic states along the studied segments [6]. From the macroscopic point of view, the main variables of interest are the traffic density in vehicules/km/lane and the average speed in km/h [16]. These important traffic data are usually obtained via a set of sensors, for example, such as inductive loop detector, camera video.

Nevertheless, The high cost of installation and maintenance of a large number of sensors or the occurrence of sensor's failures, led many researchers to develop traffic control methods

Fig. 3. Structure of the super twisting algorithm

and algorithms based on state estimation techniques in order to reduce the number of sensors or to reconstruct the missing state information.

Several methods were proposed, in [6], a state estimator was developed using Extended Kalman filters [17], other works use the well known Luenberger observer [18].

In [15], a first order sliding mode observer has been proposed in order to design a state observer. Such robust *variable structure observer* (see [19] and [20], for more explanation about Sliding mode control and observation) suffers from the high frequency chattering phenomenon generally present near the sliding surface. In this paper, a second order sliding mode observer is proposed in order to overcome this problem.

1) *Short overview on the observers:* The sliding mode technique has been used for the observer's design in many applications. Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x, \dots, x_{n-1}) \\ y = x_1 \end{cases} \quad (5)$$

The sliding mode differentiator of order 2 (super twisting algorithm) is given in its general form by [21] (figure 3).

$$\sum_{obs} = \left\{ \begin{array}{l} y(e_1) = y + \lambda_1 |e_1|^{1/2} \text{sign}(e_1) \\ \dot{y} = \pi_1 \text{sign}(e_1) \end{array} \right\}, \lambda_1, \pi_1 > 0$$

Where, λ_1 and π_1 are positive tuning parameters of the differentiator whose the output is y . Applying the exact differentiator to system (5) when $n=2$, one obtains:

$$\dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 |x_1 - \hat{x}_1|^{1/2} \text{sign}(x_1 - \hat{x}_1) \quad (6)$$

$$\dot{\hat{x}}_2 = f_q(x_1, \hat{x}_2) + \pi_1 \text{sign}(x_1 - \hat{x}_1) \quad (7)$$

$f_q(x_1, \hat{x}_2) = \text{equation}(6)$. Then, the estimated value of $\hat{x}_2$ can be found from equation (7). For more details see ([22], [23], [24], [25], [26], [27], [28] and [29]).

2) *Traffic state estimation:* Consider the following freeway segments (Fig. 4): Using the relationship between the three aggregated variables (flow q , density ρ ² and mean speed v),

$$q_i(x,t) = \rho_i(x,t) v_i(x,t) \quad (8)$$

²loop-detector (sensor) measures directly the occupancy, which refers to the percentage of time a detector is occupied by vehicles. It can be transformed to density by $TO = 100(L_b + L_v)\rho$ [30], where: L_b = length of the loop-detector, L_v = length of the vehicle, ρ = density and TO = occupation rate.

Fig. 2. Freeway example

Fig. 4. Freeway segments example

The output vector is: $y = [y_1 \ y_2]^T = [\rho_2 \ v_2]^T$

As long as the outgoing traffic speed satisfies the positivity condition: $v_1, v_2 = v_s \geq V_{min} > 0$, the studied freeway section (see figure 4) can be described by the following equations:

$$\dot{\rho}_1 = \frac{1}{L_1}(q_e + r - \alpha \rho_1 v_2 - (1 - \alpha) \rho_2 v_s) \quad (9)$$

$$\dot{\rho}_2 = \frac{1}{L_2}(\alpha \rho_1 v_2 - (1 - \alpha) \rho_2 v_s - q_s) = \dot{\rho} \quad (10)$$

$$\begin{aligned} \dot{v}_1 &= \frac{1}{\tau}(V_e(\rho_1) - v_1) + \frac{\eta}{L_1} v_1 (v_e - v_2) - \\ &\quad \frac{v}{L_1}(\rho_2 - \rho_1) - \frac{\delta}{L_1} r v_1 \end{aligned} \quad (11)$$

$$\dot{v}_2 = \frac{1}{\tau}(V_e(\rho_2) - v_2) + \frac{\eta}{L_2} v_2 (v_1 - v_s) = \dot{v} \quad (12)$$

In expressions (9)-(12), ρ_2 and v_2 are assumed to be known. $V_e(\rho_i)$ are given thanks to the May fundamental diagram (2).

It should be mentioned that q_s and v_s form the output measurements variables which are equal to q_2 and v_2 respectively, measured on sensor St.3. q_e and v_e (measured on sensor St.1) are external measured inputs (see figure 4). Assuming that sensor St.2 is broken down. We try to estimate the state variables ρ_1 and v_1 which are essential for the control system from the measured states on sensor St.3 :

Consider $\hat{\rho}$ and $\hat{v}$ are the estimated value of ρ and v respectively. Then, the observer errors are defined as: $e_\rho = \rho - \hat{\rho}$ and $e_v = v - \hat{v}$.

The proposed observer (called super-twisting) has the form

$$\dot{\hat{\rho}}_1 = \frac{1}{L_1}(q_e + r - \alpha \hat{\rho}_1 v_2 - (1 - \alpha) \hat{\rho}_2 v_s) + z_{\rho 1} \quad (13)$$

Where $z_{\rho 1}$ and $z_{\rho 2}$ are the output injection of the form:

$$\dot{\hat{\rho}}_2 = \hat{\rho}_1 + z_{\rho 2} \quad (14)$$

$$z_{\rho 1} = \pi \text{sign}(\rho_2 - \hat{\rho}_2) \quad (15)$$

$$z_{\rho 2} = \lambda |\rho_2 - \hat{\rho}_2|^{\frac{1}{2}} \text{sign}(\rho_2 - \hat{\rho}_2) \quad (16)$$

For the speed estimation $\hat{v}_1$, the observer reads:

$$\begin{aligned} \dot{\hat{v}}_1 &= \frac{1}{\tau}(V(\hat{\rho}_1) - \hat{v}_1) + \frac{\eta}{L_1} \hat{v}_1 (v_e - v_2) - \\ &\quad \frac{v}{L_1}(\rho_2 - \hat{\rho}_1) - \frac{\delta}{L_1} r \hat{v}_1 + z_{v 1} \end{aligned} \quad (17)$$

The output injection are of the form:

$$\dot{\hat{v}}_2 = \hat{v}_1 + z_{v 2} \quad (18)$$

$$z_{v 1} = \pi \text{sign}(v_2 - \hat{v}_2) \quad (19)$$

$$z_{v 2} = \lambda |v_2 - \hat{v}_2|^{\frac{1}{2}} \text{sign}(v_2 - \hat{v}_2) \quad (20)$$

III. REAL-TIME RAMP METERING

The ramp metering algorithm is based on the differentially flat systems concept, (see [31], [32], [33] and [34] for more explanations about flat systems).

Notice that the studied system (9)-(12) is flat with $F = \rho_2$ its flat output (see [35], for the first application of flatness concept in the traffic flow control). It means that all the system variables can be parametrized in term of this flat output and a number of its time-derivative. The state variable ρ_1 reads then

$$\rho_1 = L_2 \frac{\dot{F}}{\alpha v_2} + \frac{1 - \alpha}{\alpha} F + \frac{1}{\alpha v_2} q_s \quad (21)$$

From the expressions above, the control variable $r(t)$ can be parametrized in term of the flat output and its time derivatives up to two

$$r(t) = \frac{L_1 L_2}{\alpha v_2} \ddot{F} + \left[L_2 - \frac{L_1 L_2 \dot{v}_2}{\alpha v_2^2} + L_1 \frac{1 - \alpha}{\alpha} \right] \dot{F}$$

TABLE I
PARAMETERS OF THE MODEL

Parameter	Value	Parameter	Value
Free-flow speed	105km/h	η	0.42
Jam density	180veh/km	v	$5.8E-2 * \frac{1000^3}{3600^2} / \frac{m^3}{sec^2 \cdot veh}$
Critical density	35.86 veh/km	a	1.66
α	0.77	τ	$7.8E-3 * 3600 \text{ sec}$
L_1, L_2	500m	δ	0.17 m/veh

$$+ \frac{L_1}{\alpha v_2} \dot{q}_s + \left[1 - \frac{L_1 \dot{v}_2}{\alpha v_2^2} \right] q_s + 2(1 - \alpha) v_2 F - q_e \quad (22)$$

Given a desired reference trajectory for the traffic density, represented by F^* , a linearized control law achieving an exponential asymptotic tracking of the trajectory is given by the following expression:

$$r(t) = \frac{L_1 L_2}{\alpha v_2} \Gamma + \left[L_2 - \frac{L_1 L_2 \dot{v}_2}{\alpha v_2^2} + L_1 \frac{1 - \alpha}{\alpha} \right] \dot{F} + \frac{L_1}{\alpha v_2} \dot{q}_s + \left[1 - \frac{L_1 \dot{v}_2}{\alpha v_2^2} \right] q_s + 2(1 - \alpha) v_2 F - q_e \quad (23)$$

$$\Gamma = \ddot{F}^* - \lambda_2 (\dot{F} - \dot{F}^*) - \lambda_1 (F - F^*) \quad (24)$$

$\ddot{F} = \Gamma$ is an auxiliary control input. The set of parameters $\{K_1, K_2\}$, is chosen so that the roots of the closed loop characteristic polynomial, in the complex variable s : $p(s) = s^2 + K_2 s + K_1$ be a Hurwitz polynomial. Clearly, the tracking error, $e = F - F^*$, ($F = \rho_2$) satisfies

$$\ddot{e} + K_2 \dot{e} + K_1 e = 0 \quad (25)$$

A suitable choice of the design parameters is obtained by matching the characteristic polynomial, $p(s)$, to a desired characteristic polynomial $p_d(s)$, with pre-specified root locations.

IV. NUMERICAL SIMULATIONS

For numerical simulations, consider the same example of the stretch depicted in Fig. 4. We suppose that the sensor on St.2 is broken down. Thus, in order to the control system do not disrupt, we try to estimate the state variables (ρ_1 and v_1), which are essential for the control algorithm, from the measured state variables (ρ_2 and v_2) on St.3. The traffic demand (veh/h/lane) is represented in Fig. 5. For simulation, the model parameters are given in table (I).

In the no-control case, Fig.6 and Fig. 7, show the densities and speed evolutions. Notice that in this case, the congestion appears for about 40 min. Without additive noise, we obtain a good estimation for both the traffic density and the mean speed of the first link.

Fig. 8 and Fig. 9 show the effectiveness of the proposed controller. Indeed, the proposed algorithm ensures convergence to the desired value. Notice also the good estimation results for the main state variables.

Obviously, the implementation of the ramp metering leads to a queue formation as depicted in Fig. 10.

In the presence of additive noise, Fig.12 and Fig. 13 show a good robustness of the identification method as well as the

Fig. 5. Traffic demand

Fig. 6. Traffic densities evolution: no-control case

Fig. 7. Mean speed evolution: no-control case

Fig. 8. Traffic densities evolution: control case

Fig. 9. Mean speed evolution: control case

Fig. 10. Queue length evolution

control algorithm to keep the trajectory around the desired value. In table (II), the standard error occur in the estimated values of ρ and v , in the case of without additive noise and with the presence of additive noise, are given.

V. CONCLUSION

The work presented in this paper exploits the relevance of high order sliding mode observer for state estimation. The proposed super twisting algorithm seems to be very efficient for the case where any information about the state of the controlled ramp is available. On the other hand, the application

Fig. 11. Control variable

Fig. 12. Densities evolution with additive noise

Fig. 13. Speed evolution with additive noise

of the concept of differential flatness for real time ramp metering is based on the system inversion concept, which do not requires integration of any differential equations. Notice that, the flat output is defined by the state of the downstream segment. This means that in addition of on-ramp metering of the merge section, we are able to control all the downstream ones. This fact leads to propose in future works a coordinated ramp metering algorithm which takes measurement of all studied freeway network.

REFERENCES

- [1] A. Kostialos and M. Papageorgiou. Motorway network traffic control systems. *European Journal of Operational Research*, vol. 152:pages: 321–333, 2004.
- [2] E. Smaragdis M. Papageorgiou and E. Kosmatopoulos. A flow maximizing adaptive local ramp metering strategy. *Transportation Research Part B*, vol. 38:pages: 251–270, 2004.

TABLE II
ERROR NORM

		No control	Control
Without additive noise	v_1	0.131	0.147
	ρ_1	0.01	0.01
With additive noise	v_1	0.158	0.166
	ρ_1	0.01	0.01

(variance= 0.001, mean=0)

- [3] M. Papageorgiou C. Diakaki V. Dinoulou A. Kostialos and Y. Wang. Review of road traffic control strategies. *IEEE Transaction of Intelligent Transportation Systems*, vol. 91, N.12:pages: 2043–2067, 2003.
- [4] M. Papageorgiou. *Traffic Control*. Handbook of Transportation Science ch.8, 2001.
- [5] M. Papageorgiou and A. Kostialos. Freeway ramp metering: An overview. In *IEEE Intelligent Transportation Systems Conference, Dearborn (MI) USA*, pages 228–239, 2000.
- [6] M. Papageorgiou and G. Vigos. Relating time-occupancy measurements to space-occupancy and link vehicle-count. *Transportation Research Part C*, vol.16:Pages: 1–17, 2008.
- [7] S. Mammar(sous la direction de). *Systmes de transport intelligents: Modlisation, information et contrle*. Hermès-Lavoisier, 2007.
- [8] A. D. May. *Traffic Flow Fundamentals*. Prentice Hall, Englewood Cliffs, NJ, 1990.
- [9] M. Papageorgiou. Some remarks on macroscopic traffic flow modelling. *Transport. Research*, vol. t. 32:Pages: 323–329, 1998.
- [10] M. J. Lighthill and G.B. Whitham. On kinematic waves ii: A theory of traffic flow in long crowded roads. *Proc. Royal Soc. London*, vol. A-229:Pages: 317–345, 1955.
- [11] P.I. Richards. Shock waves on the highway. *Operations Research*, vol.4:Pages: 42–51, 1956.
- [12] H. J. Payne. Models of freeway traffic and control. In *In G. A. Bekey, editor Mathematical Models of Public Systems, volume 1 of Simulation Council Proceedings Series, pages 51–61, La Jolia, California*, 1971.
- [13] M. Cremer and K. Putensen. An approach for traffic state estimation using in-vehicle information. *IFAC Transportation Systems, Tianjin, PRC*, pages pages: 597–602, 1994.
- [14] M. Papageorgiou. Application of automatic control concepts to traffic flow modeling and control. *Springer-Verlag*, vol. 50, 1983.
- [15] Rashid R. Kohan. *Robust State Estimation and Control of Highway Traffic Systems*. PhD thesis, Electrical and Computer Engineering/ University of Toronto, 2001.
- [16] S. Mammar S. Mammar and M. Netto. Coordinated ramp metering via second order sliding mode control. In *9th IEEE Intelligent Transportation Systems Conf., Toronto*, pages 261–266, 2006.
- [17] Xiaotian Sun Laura Munoz and Roberto Horowitz. Highway traffic state estimation using improved mixture kalman filters for effective ramp metering control. In *Conference on Decision and Control*, 2003.
- [18] David G. Luenberger. An introduction to observers. *IEEE Transactions on Automatic Control*, vol. AC-16, NO.6:Pages: 596–602, 1971.
- [19] V. Utkin. Sliding modes in control and optimization. *Springer, Berlin*, 1992.
- [20] A. Isidori C.H. Moog and A De Luca. A sufficient condition for full linearization via dynamic state feedback. In *In proceedings 25th. IEEE conf. Decision Control*, pages 203–208, 1986.
- [21] L. Fridman A Levant. Sliding modes of higher order as a natural phenomenon in control theory. Technical report, In *Robust control via variable structure and Lyapunov techniques*. Lecture notes in control and information science 217. F. Garofalo, L. Glielmo Ed. Springer verlag London, pages 107–133, 1996.
- [22] Jorge Rivera Luis Garcia Christian Mora 0Juan J. Raygoza and Susana Ortega. Super-twisting sliding mode in motion control systems, sliding mode control. *Prof. Andrzej Bartoszewicz (Ed.)*, pages ISBN: 978–953–307–162–6, InTech., 2011.
- [23] Jorge Davila Leonid Fridman and Arie Levant. Second-order sliding-mode observer for mechanical systems. *IEEE Transaction on Automatic Control*, VOL. 50, NO.11:1785–1789, 2005.
- [24] N.K. M’sirdi A. Rabhi L. Fridman J. Davila and Y. Delanne. Second order sliding mode observer for estimation of velocities, wheel sleep, radius and stiffness.
- [25] I. Salgado I. Chairez J. Moreno L. Fridman A. Poznyak. Generalized super-twisting observer for nonlinear systems. In *Preprints of the 18th IFAC World Congress Milano (Italy) August 28- September 2*, 2011.
- [26] A. Rabhi N.K. M’Sirdi M. Ouladsine and L. Fridman. Estimation of road profile using second order sliding mode observer.
- [27] H. Saadaoui M. Djemai N. Manamanni and K. Bemsour. Super twisting algorithm observer for a class of switched chaotic systems.
- [28] T. Floquet and J. P. Barbot. Super twisting algorithm based step-by-step sliding mode observers for nonlinear systems with unknown inputs. *International Journal of Systems Science*, VOL. 38, No.10:Pages: 803–815, 2007.
- [29] N. K. M’Sirdi A. Rabhi L. Fridman, J. Davila, and Y. Delanne. Second order sliding-mode observer for estimation of vehicle dynamic parameters. *Int. J. Vehicle Design*, Vol. 48, Nos. 3/4:190–207, 2008.
- [30] Christine Buisson and Jean-Baptiste Lesort. *Comprendre le trafic routier, Mthodes et Calculs*. Certu, 2010.
- [31] M. Fliess J. Lvine P. Martin and P.Rouchon. Flatness and defect of non-linear systems: Introductory theory and examples. *Int. J. Control*, vol. 61:Pages:1327–1361, 1995.
- [32] V. Manish. Higher order sliding mode control of differentially flat systems. Master’s thesis, Indian Institute of Technology, Bombay, Mumbai, 2004.
- [33] J. Rudolph. Flatness based control of distributed parameter systems. *Shaker Verlag*, 2003.
- [34] H. Sira-Ramirez and S.K. Agrawal. Differentially flat systems. *Marcel Dekker*, 2004.
- [35] H. Abouassa V. Iordanova and D. Jolly. Flatness-based control of traffic flow. In *In Proc. 9th IEEE Intelligent Transportation Systems Conf. Toronto*, pages 1060–1065, 2006.