Sensitivity analysis in dynamic WPT systems based on non-intrusive stochastic methods

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Abstract—**The analysis of coil pairs mutual inductance is of great interest in the characterization, design and optimization of dynamic Wireless Power Transfer (WPT) systems for automotive applications. The objective of this paper is to show the use of non-intrusive stochastic methods to build accurate predictors of the mutual inductance. These methods are based on a polynomial-Chaos-Kriging metamodeling approach, which enables an accurate sensitivity analysis at system level. This approach is here applied to study the most influential spatial parameters in a dynamic WPT system, given different trajectories of the vehicle during its motion.**

Index Terms—**Inductive power transfer, mutual inductance, non-intrusive stochastic methods, sensitivity analysis.**

I. Introduction

Dynamic battery-charging applications for Wireless Power Transfer (WPT) systems are crucial for the development of the overall mobility of electric vehicles [1]. The knowledge of the coupling between the receiving coil (RX) at the vehicle side and the transmitting coil (TX) at the ground side, is the key for maximizing the efficiency of the WPT system. In particular, the mutual inductance M is the key parameter influencing the overall performance of WPT systems, in terms of efficiency, power transfer, harmonic distortion, etc. In fact, the optimal design of such systems would require the knowledge of the mutual inductance in the variability range of the geometrical and physical parameters.

In real-world WPT systems, the coil pairs consist of complicated 3D geometries, including metallic and ferrite parts for the magnetic flux lines confinement. Analytical formulas of the mutual inductance are not easily computable, and its evaluation requires experimental measurements and/or numerical solutions of a magneto-quasi-static problem. Finite Element Method (FEM)-based analysis are in fact commonly adopted [2]. Recently, an analytical model of the mutual inductance of coil pairs in static WPT systems has been proposed in [3]. Similarly, in [4] an analytical model of the mutual inductance between coupled coils in dynamic WPT system has been presented. These models are obtained by using evolutionary algorithms. In both cases, several FEM simulations with different reciprocal positions between the coils are performed by means of a commercial 3D FEM solver (Ansys Maxwell) and an in-house solver (CARIDDI code, [5]).

Different geometrical, physical and/or spatial parameters can be taken into account for the identification of analytical behavioral models of the mutual inductance in WPT applications. In this paper, we aim to perform the sensitivity analysis of spatial parameters of interest for dynamic WPT systems. In particular, the goal is to show the usefulness of non-intrusive methods by combining polynomial chaos expansions with Kriging metamodels in assessing the sensitivity of the electromagnetic problem under discussion to the lateral and the longitudinal displacements of the RX coil with respect to the ground TX coils. Using the behavioral model derived in [6], it was possible to map the mutual inductance in a large set of coils reciprocal positions, so providing the results used in this work as the input for the proposed methodology. Specifically, they are here used to perform the Sobol index sensitivity analysis [7] at a low computation cost. Such tools, implemented by using the UQLab framework [8], have been successfully used in the past for the determination of specific absorption rate in biological tissues due to mobile phones at microwaves frequencies [9], [10]. The same goes for an automotive WPT system with a simplified 3D model, where Polynomial chaos and Kriging methods have been really efficient [11].

The case study presented in this paper refers to a real WPT system, realized by the Politecnico di Torino, Italy, and analyzed in the frame of the project "Metrology for Inductive Charging of Electric Vehicles" (MICEV) [12].

II. The mutual inductance model

A. The input model

Fig. 1 shows the reference geometry in the plane (y, z), where the vehicle trajectory lies. The nominal trajectory is given by the red arrow in Fig. 1, representing the RX coil moving along the y-axis, with no lateral displacement ($\Delta z = 0$). Any other trajectory can be represented by coordinates ($\Delta y, \Delta z$). In this paper, we consider the behavioral model given in [6], which describes the mutual inductance as a function of Δy and Δz, as given in (1):

$$\begin{aligned} M_{tot} &= M_{RX-TX1} + M_{RX-TX2} = \\ &= M_{tx1,bhv}(\Delta y, \Delta z) + M_{tx1,bhv}(\Delta y - 2\Delta y_{mid}, \Delta z) \end{aligned} \tag{1}$$

where $M_{tx1,bhv}$ is the analytical model for the $RX - TX1$ coil pair, and $2\Delta y_{mid} = 2.104\,\mathrm{m}$ is the longitudinal displacement between the center of the two TX coils. The inductance $M_{tx1,bhv}$, expressed in µH, is given by (2):

$$M_{tx1,bhv} = p_0 tanh[p_1(\Delta y^2 + p_2)] + p_3 atan(|p_4 \Delta y|^{p_5}) + p_6 \tag{2}$$

where the coefficients p_i (for $i = 0, .., 6$) are:

$$p_i = a_{i0} atan[a_{i1}(|\Delta z| - a_{i2})] + a_{i3} \tag{3}$$

with values of the fitting coefficients listed in Table I.

B. Polynomial-Chaos-Kriging metamodelling

Kriging is a stochastic interpolation algorithm that interpolates the local variations of the output $\mathbf{M}$ as a function of the neighboring experimental design points, whereas Polynomial-Chaos expansion approximates well the global behavior of $\mathbf{M}$. By combining the global and local approximation, a more accurate stochastic process can be achieved. Polynomial-Chaos-Kriging (PCK) is defined as a universal Kriging model the trend of which consists of a set of orthonormal polynomials. Given an input X of the parameters, the output $\mathrm{M}(X)$ can be estimated by (4):

$$\widehat{\mathrm{M}}(X) = \sum_{\alpha \in \mathcal{A}} y_\alpha \psi_\alpha(X) + \sigma^2 Z(X, \omega) \tag{4}$$

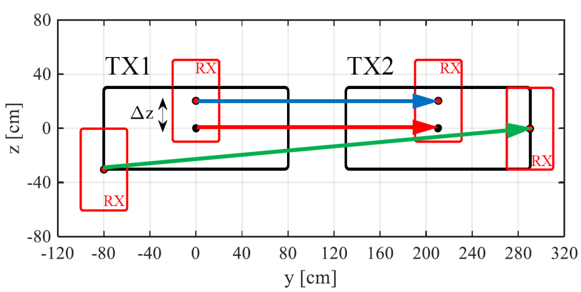

Fig. 1: Different trajectories of the RX coil moving along two TX coils. The nominal trajectory is represented by a red arrow.

TABLE I: Coefficient values for the model given in Eqs. (2)(3)

coefficient	a_{i0}	a_{i1}	a_{i2}	a_{i3}
p_0	$1.33 \cdot 10^1$	7.35	$1.90 \cdot 10^{-1}$	$-1.77 \cdot 10^1$
p_1	$1.36 \cdot 10^1$	$2.02 \cdot 10^1$	$2.57 \cdot 10^{-1}$	2.93
p_2	$-5.01 \cdot 10^{-2}$	8.40	$2.34 \cdot 10^{-1}$	$-4.84 \cdot 10^{-1}$
p_3	9.92	7.32	$1.87 \cdot 10^{-1}$	$-1.40 \cdot 10^1$
p_4	$1.20 \cdot 10^{-1}$	8.46	$2.63 \cdot 10^{-1}$	-1.50
p_5	1.08	7.28	$3.23 \cdot 10^{-1}$	-2.73
p_6	$-1.32 \cdot 10^1$	7.40	$1.89 \cdot 10^{-1}$	$1.79 \cdot 10^1$

where $\sum_{\alpha \in \mathcal{A}} y_\alpha \psi_\alpha(X)$ is a weighted sum of orthonormal polynomials describing the trend of the PCK model, σ^2 and $Z(X, \omega)$ denote the variance and the zero mean, unit variance, stationary Gaussian process, respectively. Hence, PCK can be interpreted as a universal Kriging model with a specific trend.

Consistency of the metamodel: let's consider a set $\{(X_1, \mathrm{M}_1), \ldots, (X_n, \mathrm{M}_n)\}$ of n datapoints, where X_i and M_i for $i = 1, .., n$ are the input and their corresponding outputs. Using this set, one can build a metamodel $\widehat{\mathrm{M}}(X)$ with PCK. The accuracy of the metamodel is calculated using the mean Leave-One-Out error (LOO), given in (5):

$$LOO = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\widehat{\mathrm{M}}_{/i}(X_i) - \mathrm{M}_i}{\mathrm{M}_i} \right)^2 \tag{5}$$

where $\widehat{\mathrm{M}}_{/i}$ is the mean predictor trained using all (X, Y) but (X_i, M_i). The LOO allows to evaluate the consistency of the metamodel. If the LOO is close to 1, the metamodel is highly modified if one datapoint is missing.

Accuracy of the metamodel: if one aims to build a metamodel $\widehat{\mathrm{M}}_k(X)$ using a subset of k datapoints out of the aforementioned n datapoints, the accuracy of the predictor on the $(n - k)$ remaining points $\{(X_1, \mathrm{M}_1), \ldots, (X_{n-k}, \mathrm{M}_{n-k})\}$ can be calculated using the Out-of-Sample-Error (OSE), given in (6):

$$OSE = \frac{1}{n-k} \sum_{i=1}^{n-k} \left(\frac{\widehat{\mathrm{M}}_k(X_i) - \mathrm{M}_i}{\mathrm{M}_i} \right)^2 \tag{6}$$

If the OSE for k datapoints is extremely small, it means that, at the non-sampled points, there is almost no difference between the predictor and the real value.

III. Case studies and Numerical results

A. Training datasets

To train our metamodels, three different datasets have been considered, referred to different trajectories of the vehicle during its motion (see Fig. 1):

- dataset 1, given by trajectories parallel to the nominal one, with only lateral displacement (blue arrow);
- dataset 2, given by a trajectory that moves diagonally with respect to the nominal one (green arrow);
- dataset 3, given by various trajectories for any possible misalignment of the coils: we sampled different points in the plane (y, z) to build our metamodel.

B. Dynamic charging applications with one TX coil

The aforementioned metamodel has been first run on a coil pair system made by only one TX coil. The mutual inductance values for the pair RX-TX1 against the longitudinal displacement Δy and the lateral misalignment Δz for any trajectory is shown in Fig. 2. It can be observed that the mutual inductance is also reaching its maximum in this area. The points sampled by our algorithm are shown in Fig. 3: a higher number of samples fall in the range $\Delta z \in [40, 40]$cm and $\Delta y \in [80, 80]$cm, whereas less samples can be found outside these ranges. The three computed metamodels (one for each dataset) have been used to perform the sensitivity analysis, whose results are given in Table II. First, the three metamodels are extremely consistent with themselves ($LOO < 10^{-5}$), which ensures three independent but accurate sensitivity analysis indices. Then, the three indices are almost giving the same result for the trajectories: the longitudinal displacement Δy is the most relevant parameter. Indeed, the lateral displacement Δz realizes only a minor shift when the vehicle is moving along the charging lane.

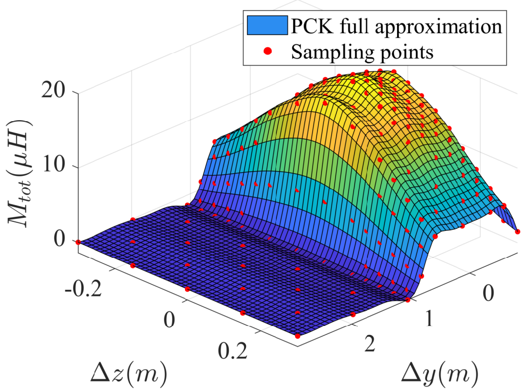

Fig. 2: Mutual inductance values for a single pair RX-TX against the longitudinal displacement Δy and the lateral misalignment Δz for any trajectory.

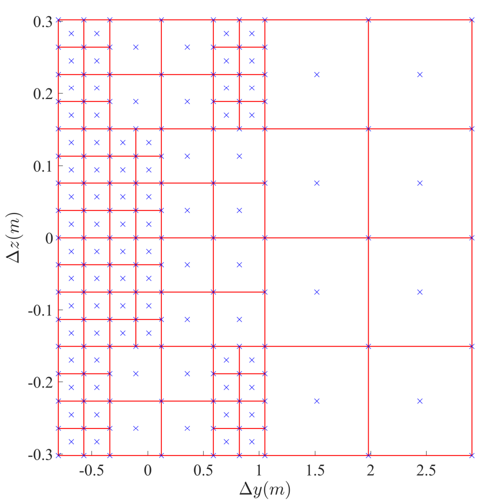

Fig. 3: Parameter domains and relevant samples used to build the metamodel for the mutual inductance for a single pair RX-TX against the longitudinal displacement Δy and the lateral misalignment Δz for any trajectory.

C. Dynamic charging applications with two TX coils

For two consecutive TX coils, as shown in Fig. 1, we consider the total mutual inductance M_{tot}. The area of

TABLE II: Sobol index analysis of the mutual inductance for a single pair RX-TX, against Δy and Δz.

Trajectory	Total $S_{\Delta y}$	Total $S_{\Delta z}$	LOO
dataset 1 (parallel trajectory)	0.943	0.132	$7.69 \cdot 10^{-6}$
dataset 2 (diagonal trajectory)	0.935	0.133	$1.26 \cdot 10^{-6}$
dataset 3 (any trajectory)	0.935	0.135	$1.56 \cdot 10^{-5}$

interest ranges from the position where the RX coil is on the top of the TX1 coil to that where it is on the top of the TX2 coil, which corresponds to $\Delta z \in [40, 40]$cm and $\Delta y \in [80, 80] \cup [130, 290]$cm. From Figs. 4 and 5 it can be seen that our algorithm sampled more points in these domains for various trajectories, and that the mutual inductance is also reaching its maximum in this area. As the spacing between the two TX coils is big enough, their areas of effect are not overlapped. As a consequence, the maximum value of the mutual inductance is the same one achieved for the model with only one TX coil. The three computed metamodels (one for each dataset) have been used to perform the sensitivity analysis, whose results are given in Table III. The three metamodels are still consistent with themselves ($LOO < 10^{-5}$), but the three analysis indices are not giving the same results. For a parallel trajectory, the RX coil is only seeing the effect of the second coil at the end of the trajectory which is not affecting the mutual inductance compared to the previous analysis. Conversely, for a diagonal trajectory and for any other trajectory, the RX coil cannot avoid the effect of the second TX coil. Therefore, the longitudinal displacement is not the most important parameter anymore. This means that a car, moving forward over a series of TX coils but not along a trajectory parallel to the nominal one, realizes a mutual inductance that is now much more dependent on its lateral misalignment. However, given the TX coil dimensions and the motion of the car along the y-axis, the longitudinal displacement remains the most influential parameter ($S_{\Delta y} > S_{\Delta z}$), as suggested by the results listed in Table III.

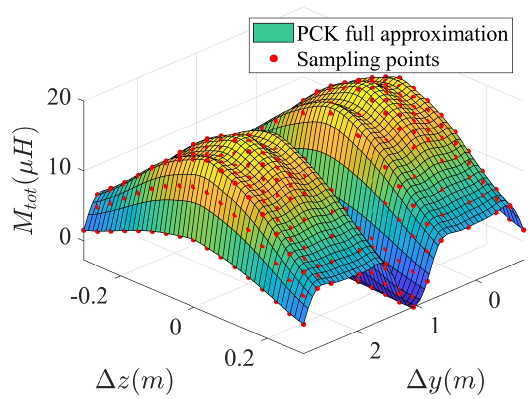

Fig. 4: Mutual inductance values for a single RX coil and two TX coils against the longitudinal displacement Δy and the lateral misalignment Δz for any trajectory.

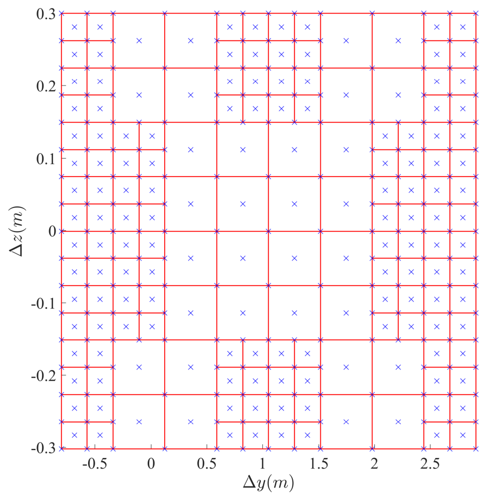

Fig. 5: Parameter domains and relevant samples used to build the metamodel for the mutual inductance for a single RX coil and two TX coils against the longitudinal displacement Δy and the lateral misalignment Δz for any trajectory.

TABLE III: Sobol index analysis of total mutual inductance over two TX coils, against Δy and Δz for various trajectories.

Trajectory	Total $S_{\Delta y}$	Total $S_{\Delta z}$	LOO
dataset 1 (parallel trajectory)	0.908	0.166	$1.60 \cdot 10^{-6}$
dataset 2 (diagonal trajectory)	0.656	0.375	$1.27 \cdot 10^{-6}$
dataset 3 (any trajectory)	0.684	0.349	$5.34 \cdot 10^{-6}$

D. Best sensitivity analysis

Three different metamodels have been computed so far, giving us three different sensitivity analysis. To find which metamodel is the best at predicting the behavior of the total mutual inductance, we tried to predict the values from the dataset 1 using the values of the dataset 2 and the other way around, while predicting both datasets with the metamodel build with any trajectory. By evaluating the OSE, we understand that the first two datasets are only able to predict the total mutual inductance behavior within their own validity range for the parameters values, as suggested by the results listed in Table IV. On the contrary, the third metamodel ensures quite low OSE values for both datasets, and can predict values of the mutual inductance in a wider range for the input parameters. As expected the metamodel for any trajectory is providing the best metamodel to work with, and its sensitivity analysis is the one to be taken into account.

TABLE IV: OSE on the values from datasets 1 and 2, against the metamodel predictors built with various trajectories.

Metamodel	dataset 1	dataset 2
dataset 1 (parallel trajectory)	$1.77 \cdot 10^{-2}$	2.28
dataset 2 (diagonal trajectory)	1.09	$5.29 \cdot 10^{-4}$
dataset 3 (any trajectory)	$3.04 \cdot 10^{-4}$	$4.20 \cdot 10^{-5}$

Conclusions

By using a Polynomial-Chaos-Kriging algorithm, we computed an accurate and consistent estimator for the mutual inductance in Wireless Power Transfer (WPT) system for dynamic charging applications. Using this predictor, an accurate sensitivity analysis has been performed to examine the effects of different spatial parameters at a low computation cost. Even if the effect of lateral misalignment between the coil pair on the resulting mutual inductance cannot be totally neglected for the design of WPT systems, the longitudinal displacement remains the most influential parameter in dynamic charging applications, which has to be taken into account in WPT systems analysis and design.

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