Mechanically Generated Heat

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Definition

Mechanically generated heat can be defined as heat produced by mechanical processes, involving motion and deformation of matter. In this entry, we will mostly focus on heat generated by deformation in elasto-viscoplastic solids as well as by frictional contact. These heat sources play a significant role in many industrial processes, such as metal forming and friction welding.

Overview

Temperature can have a significant effect on mechanical behavior of materials such as metals or polymers. Conversely, mechanical activity can be at the origin of heat generation (or absorption), contributing to thermal transfers within the material. Such thermomechanical coupling effects originate from various micro-mechanical processes. They can be classified in two broad categories: entropic effects and dissipative effects. Entropic effects are linked to variations of the mechanical entropy, hence the name. They are reversible, in the sense that if a given

evolution of the mechanical state produces heat, through a decrease of mechanical entropy, then the reverse evolution will absorb heat, through an increase in mechanical entropy. A typical example is thermoelasticity: compression of a thermoelastic material will generate heat, while mechanical expansion will be associated to heat absorption. Note that if temperature gradients are generated by entropic effects, some dissipation will occur by heat conduction. This is why thermoelastic effects can slightly dampen mechanical free vibrations, for example. But the thermoelastic coupling effect is in itself reversible. On the other hand, dissipative effects are linked to micro-mechanical irreversible processes, such as dislocation glide through obstacles, formation of micro-cracks, nucleation, growth, and coalescence of cavities (in a word, damage). They are thus irreversible by nature and always generate heat. A combination of entropic and dissipative effects is generally observed in metals, although dissipative effects rapidly become dominant once a significant amount of plastic strain or damage has developed (and excluding phase changes). For example, thermoelastic effects in metals can typically lead to variations in temperature of the order of 1 °C, while large viscoplastic strains can cause an increase of temperature of 10s to 100s degrees Celsius, as illustrated below. In addition to heat generated in the bulk of materials by such micro-mechanisms, boundary effects such as friction can significantly contribute as well to heat generation in mechanical processes.

Thermodynamic Description

The different sources of thermomechanical coupling appear clearly from a rigorous thermodynamic description. Consider a local state approach, where we describe the thermomechanical state of a material point by the strain tensor $\mathbf{E}$ and the absolute temperature T. In addition, irreversible micro-mechanisms are described by a set of internal variables $\boldsymbol{\xi}$ (the exact nature – scalar, vector, or tensor – of these internal variables depends on specific behaviors considered and does not need to be detailed here). Let us then introduce the Helmholtz free energy

$$W(\mathbf{E}, T, \boldsymbol{\xi}) = U - TS \tag{1}$$

where U is the internal energy and S the entropy (all quantities per unit volume). The free energy function defines the quantities thermodynamically conjugate to state variables (stress tensor $\mathbf{S}$, entropy S, and internal forces $\mathbf{X}$):

$$\mathbf{S} = \partial_{\mathbf{E}} W(\mathbf{E}, T, \boldsymbol{\xi}) \tag{2}$$

$$S = -\partial_T W(\mathbf{E}, T, \boldsymbol{\xi}) \tag{3}$$

$$\mathbf{X} = -\partial_{\boldsymbol{\xi}} W(\mathbf{E}, T, \boldsymbol{\xi}) \tag{4}$$

Conservation of energy is expressed in local form by the following balance equation:

$$\dot{U} = \mathbf{S} : \dot{\mathbf{E}} + Q_e - \operatorname{div} \mathbf{q} \tag{5}$$

where Q_e is the external heat supply and $\mathbf{q}$ the heat flux. Using the above relations, it can be rewritten as follows:

$$T\dot{S} = \mathbf{X} \cdot \dot{\boldsymbol{\xi}} + Q_e - \operatorname{div} \mathbf{q} \tag{6}$$

which is the heat equation in entropy form. Using relation (3), the rate of entropy can itself be expressed as

$$\begin{aligned} \dot{S} = & -\partial^2_{T\mathbf{E}} W(\mathbf{E}, T, \boldsymbol{\xi}) : \dot{\mathbf{E}} \\ & - \partial^2_{TT} W(\mathbf{E}, T, \boldsymbol{\xi})\, \dot{T} - \partial^2_{T\boldsymbol{\xi}} W(\mathbf{E}, T, \boldsymbol{\xi}) \cdot \dot{\boldsymbol{\xi}} \end{aligned} \tag{7}$$

Defining the heat capacity at constant strain (and constant internal variables) by

$$\rho c_\epsilon = -T \partial^2_{TT} W(\mathbf{E}, T, \boldsymbol{\xi}) \tag{8}$$

the classical heat equation is then recovered:

$$\rho c_\epsilon \dot{T} = Q_r + Q_i + Q_e - \operatorname{div} \mathbf{q} \tag{9}$$

where Q_r denotes source terms related to reversible (entropic) effects

$$Q_r = T\,\partial_T \mathbf{S}(\mathbf{E}, T, \boldsymbol{\xi}) : \dot{\mathbf{E}} - T\,\partial_T \mathbf{X}(\mathbf{E}, T, \boldsymbol{\xi}) \cdot \dot{\boldsymbol{\xi}} \tag{10}$$

while Q_i denotes source terms related to internal dissipation

$$Q_i = \mathbf{X}(\mathbf{E}, T, \boldsymbol{\xi}) \cdot \dot{\boldsymbol{\xi}} \tag{11}$$

The relative importance of these different heat source terms varies from case to case, as discussed below.

Entropic Effects

Thermal Softening

Considering the case of materials in the elastic range, one can see from relation (10) that entropic heat source terms will be generated by a temperature dependence of elastic constants. Indeed, if the stress–strain relation is given by

$$\mathbf{S}(\mathbf{E}, T) = \mathbf{C}(T) : \mathbf{E} \tag{12}$$

where $\mathbf{C}(T)$ is a fourth-order (temperature-dependent) elasticity tensor, then the entropic heat source term becomes

$$Q_r = T\,\mathbf{E} : \partial_T \mathbf{C} : \dot{\mathbf{E}} \tag{13}$$

Thus, in the case of thermal softening of elastic constants, increasing radial loadings (deformation-controlled) will be associated to heat absorption (i.e., temperature decrease under adiabatic conditions). Consider, for example, a sample under harmonic tension/compression cyclic loading: $e_{xx} = \epsilon_0 \sin \omega t$. This will generate a heat source term given by

$$Q_r^{(a)} = \frac{1}{2}\omega\,\epsilon_0^2\,T E'(T) \sin 2\omega t \tag{14}$$

where $E'(T)$ is the variation of Young modulus with temperature (with a typical value of $-6.6\ 10^{-2}$ GPa/K for an alloyed steel, in the range [−100 °C, 250 °C]).

Thermoelasticity

Consider now an isotropic thermoelastic material, for which the stress–strain relation is given by

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - (3\lambda + 2\mu)\alpha\theta\delta_{ij} \tag{15}$$

where λ and μ are Lamé coefficients, α the coefficient of thermal expansion, and $\theta = T - T_0$ the temperature change. The associated entropic source term is thus

$$Q_r = -3k\,\alpha T\ \dot{e}_{kk} \tag{16}$$

where $k = \lambda + 2/3\mu$ is the bulk modulus. Thus, volume expansion will generate cooling, while compression will generate heating. Considering again a sample under harmonic tension/compression, the heat source term due to thermoelasticity will then be given by

$$Q_r^{(b)} = -3k(1 - 2v)\,\alpha T\,\omega\epsilon_0 \cos \omega t \tag{17}$$

where we have neglected second-order effects (i.e., terms in α^2). The coefficient of thermal expansion of an alloyed steel is of the order of $15\ 10^{-6}$ m/(m.K). Table 1 compares values of heat source terms for an alloyed steel at room temperature ($T = 293$ K), with an axial strain amplitude $\epsilon_0 = 10^{-3}$. It shows that (for this material in this temperature range) thermoelastic effects are two orders of magnitude larger than thermal softening effects. Note also the doubling of frequency observed for the latter.

Mechanically Generated Heat, Table 1 Comparison of entropic heat source terms

Thermal softening	$Q_r^{(a)} = -9\,669\,\omega \sin 2\omega t$
Thermoelasticity	$Q_r^{(b)} = -922\,950\,\omega \cos \omega t$

All heat source terms in $\mathrm{W/m^{-3}}$

Dissipative Effects

Viscoplasticity

An important source of dissipation in metals is associated to (visco-)plastic deformations. Without going into the details of a particular plasticity model, the following basic assumptions generally hold:

- The strain tensor is split into an elastic and a plastic part: $\mathbf{E} = \mathbf{E}^e + \mathbf{E}^p$
- The free energy is also split into thermoelastic, plastic hardening, and thermal capacity parts:

$$W(\mathbf{E}, T, \boldsymbol{\xi}) = W^e(\mathbf{E}^e, T) + W^p(\mathbf{E}^p, \boldsymbol{\xi}) + W^t(T) \tag{18}$$

where $\boldsymbol{\xi}$ is the equivalent plastic strain:

$$\boldsymbol{\xi} = \int_0^t \left(\frac{2}{3} \dot{\mathbf{E}}^p : \dot{\mathbf{E}}^p \right)^{1/2} dt \tag{19}$$

The thermoelastic free energy W^e corresponds to energy stored through reversible crystal lattice distortion, while the plastic free energy W^p corresponds to energy stored through irreversible crystal lattice modifications (typically dislocation microstructures). Macroscopically, formation of these dislocation microstructures translates into kinematic hardening (dependence on $\mathbf{E}^p$ in W^p), but potentially also some amount of isotropic hardening (dependence on $\boldsymbol{\xi}$).

The stress tensor is now given by

$$\mathbf{S} = \partial_{\mathbf{E}} W = \partial_{\mathbf{E}^e} W^e(\mathbf{E}^e, T) \tag{20}$$

while the stress thermodynamically conjugate to the plastic strain tensor is given by

$$\mathbf{X}_{\mathbf{E}^p} = -\partial_{\mathbf{E}^p} W = \mathbf{S} - \mathbf{S}^c \tag{21}$$

where the backstress is defined by $\mathbf{S}^c = \partial_{\mathbf{E}^p} W^p$. This backstress describes kinematic hardening. Denoting by $\chi^p = \partial_{\boldsymbol{\xi}} W^p$ the scalar stress conjugate to the equivalent plastic strain, we can write the dissipative heat source as

$$Q_i = (\mathbf{S} - \mathbf{S}^c) : \dot{\mathbf{E}}^p - \chi^p \dot{\boldsymbol{\xi}} \tag{22}$$

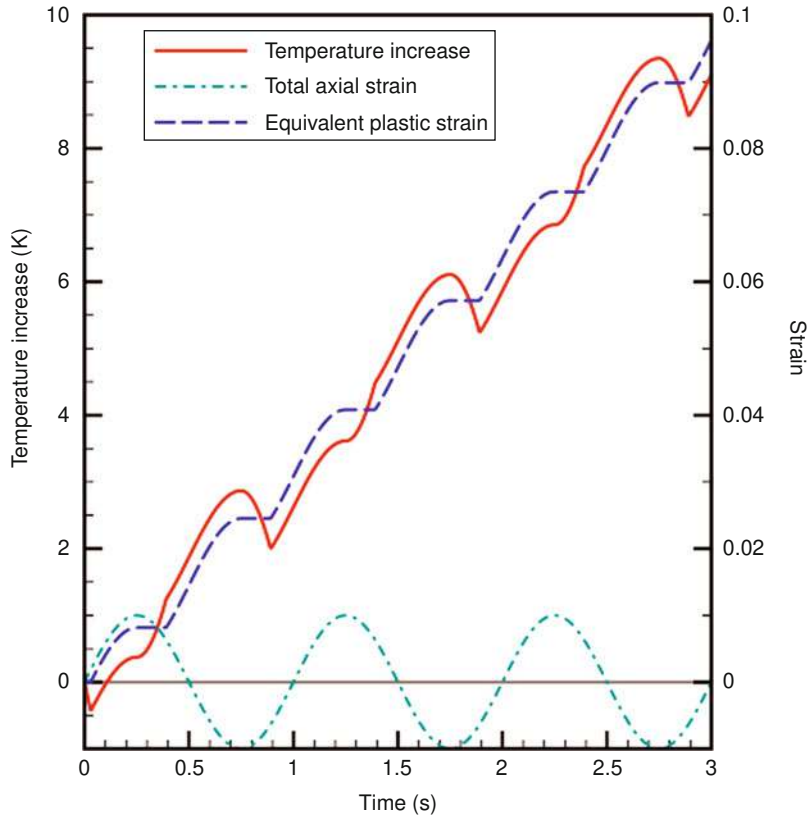
The dissipated heat power can alternatively be rewritten as a fraction β of the plastic power:

$$Q_i = \beta \, \mathbf{S} : \dot{\mathbf{E}}^p \tag{23}$$

A large volume of literature has been devoted to evaluating the actual fraction of plastic work transformed into heat. Seminal work on the topic dates back to Taylor and Quinney [7], with subsequent extensive reviews by Titchener and Bever [8] and Bever et al. [1]. The work of Taylor and Quinney is often referred to for justifying the choice of a constant coefficient β in (23), with values in the range [0.8, 1.0]. This is clearly in contradiction with (22), showing that the fraction of plastic power transformed into heat can in general be a function of strain, strain rate, and temperature. This is confirmed by experimental observations, as, for example, shown in Chrysochoos and Belmahjoub [2] or Macdougall [3]. These authors, as well as several others, measured fractions of plastic power transformed into heat varying between as low as 0.5 and maximal values close to 1.0. Lower values typically correspond to early stages of plastic deformation with increasing values as plastic strains develop and plastic storage mechanisms tend to saturate. Higher strain rates are typically associated to higher values of fraction β. As pointed out by Rittel [4], it is important to note that in their original work, Taylor and Quinney [7] actually measured fractions of plastic work, that is, energies integrated over the whole loading history:

$$\int_0^t Q_i \, dt = \beta_{\text{int}} \int_0^t \mathbf{S} : \dot{\mathbf{E}}^p \, dt \tag{24}$$

where β_{int} denotes an *integral* fraction of plastic work transformed into heat. Obviously, if β in (23) is constant, then $\beta_{\text{int}} \equiv \beta$. But in the general case, these correspond to different definitions.



Mechanically Generated Heat, Fig. 1 Temperature evolution for a perfectly plastic material under cyclic loading in adiabatic conditions

A rough estimate of the amplitude of plastic dissipation can be established as follows. Consider plastic flow under uniaxial loading at a constant yield stress of $\sigma_y = 400$ MPa (a common value for steel), with an axial plastic strain approximated by $e^p_{xx} \approx \epsilon^p_0 \sin \omega t$ (this expression obviously neglects elastic transition between tension and compression). Accounting for the isochoric nature of plastic flow, and assuming no plastic storage mechanisms ($\beta = 1$), the associated dissipated heat power will be given by

$$Q_i \approx \sqrt{\tfrac{3}{2}}\, \epsilon^p_0\, \sigma_y\, \omega |\cos \omega t| \qquad (25)$$

Considering a plastic strain amplitude of 1 %, we can then write $Q_i \approx 489\, 10^4\, \omega\, |\cos \omega t|$, which is about five times the amplitude of thermoelastic effects estimated here above (with elastic strains of an amplitude of 0.1 %). Note again that the dissipated heat power will always remain positive, contrarily to thermoelastic contributions. A more precise illustration or the relative importance of the various heat sources can be obtained by computing the thermomechanical response of an elastoplastic material point under uniaxial cyclic load in adiabatic conditions. Considering properties typical of steel (Young modulus $E = 217.5$ GPa, Poisson coefficient $\nu = 0.3$, yield stress $\sigma_y = 400$ MPa, no hardening, coefficient of thermal expansion $\alpha = 15\ 10^{-6}$ m/(m.K), volumetric heat capacity $\rho c_\epsilon = 3.925\, 10^6$ J/(K.m^3), initial temperature $T_0 = 293$ K) and a cyclic loading of amplitude $\epsilon_0 = 0.01$ at a frequency of 1 Hz, one obtains the results illustrated in Fig. 1. In this figure, we see that plastic dissipation leads to a monotonous temperature increase of about 10 K over three loading cycles, onto which is superposed a cyclic variation due to thermoelastic effects,

with an amplitude of about 1 K. Given the discussion above on the fraction of plastic power transformed into heat, these results represent an upper bound to temperature increase.

Few attempts have been made to explicitly model the evolution of factor β. One noticeable model is due to Zehnder [9], which postulates that factor β is directly related to strain hardening. Zehnder's model predicts a value of β starting relatively low and increasing with strain hardening, in agreement with many experimental observations. Rosakis et al. [5] have also studied the problem in a thermodynamic framework, proposing that factor β and its evolution can be characterized by a unique function of equivalent plastic strain $E(\boldsymbol{\xi})$, describing the stored energy of cold work. Yet, this last model may not be valid in presence of concurrent annealing effects (softening due to local heating), as illustrated in Stainier and Ortiz [6].

Finally, note that, although emphasis was given here to plastic storage mechanisms linked to hardening (kinematic or not), other phenomena can play an important role in this context. In particular, phase changes (e.g., martensitic transformations) could be included and have an effect of amplitude comparable to those linked to plastic hardening.

Frictional Contact

In many industrial processes, such as metal forming or friction welding, heat is generated not only in the bulk, by mechanisms linked to plastic deformation as discussed above, but also by friction on contact surfaces. Consider two material surfaces in contact, sliding with respect to each other at a relative velocity $\mathbf{V}_T$ (T stands for tangent). The frictional force $\mathbf{F}_T$ will act in the direction opposite to that of the relative velocity, and the dissipated energy rate (per unit surface) is given by

$$Q_f = -\mathbf{F}_T \cdot \mathbf{V}_T \tag{26}$$

The amount of heat generated then depends on the amplitude of the sliding velocity and of the frictional force.

Friction Models

Several friction models exist, yielding different predictions. The most common model is Coulomb friction, assuming that the friction force is proportional to the normal contact force F_N:

$$\mathbf{F}_T = -\mu F_N \frac{\mathbf{V}_T}{\|\mathbf{V}_T\|} \tag{27}$$

where the friction coefficient μ (unitless) depends on the specific pair of materials in contact (and also on temperature). If contact pressure is increased, the tangential force will at some point become so large that interlocking asperities (commonly considered as the main source of friction resistance) will reach their elastic limit in shear. This suggests to bound the amplitude of tangential force to a fraction of the shear yield stress τ_y of the material, leading to Tresca model:

$$\mathbf{F}_T = -k\,\tau_y \frac{\mathbf{V}_T}{\|\mathbf{V}_T\|} \tag{28}$$

where coefficient k accounts for geometrical factors. Under these assumptions, the generated heat rate is given by

$$Q_f = F_T \|\mathbf{V}_T\| \tag{29}$$

where F_T is the amplitude of the tangential force $\left(F_T = \min[\mu F_N, k\tau_y]\right)$. Dissipated power Q_f is then linear with respect to sliding velocity at the contact interface.

Alternatively, viscous friction assumes that the friction force is proportional to the relative sliding velocity:

$$\mathbf{F}_T = -\hat{\mu}\,\mathbf{V}_T \tag{30}$$

where $\hat{\mu}$ is a viscous friction coefficient (units, N.s/m^3). Dissipated power is then given by

$$Q_f = \hat{\mu}\|\mathbf{V}_T\|^2 \tag{31}$$

that is, the generated heat rate is proportional to the square of sliding velocity at the contact interface.

The choice of a friction model thus has a significant effect on the prediction of associated heat sources. Note that there are also more sophisticated friction models for lubricated contact, based on solutions to Reynolds equation.

Partition of Frictional Power

At the fine scale, friction can often be related to interactions between asperities of contacting surfaces. These asperities deform plastically, can break, leading to morphological changes at the microscopic level. Given the previous discussion on heat generated by plastic deformations, it would be legitimate to consider that only a fraction of frictional power is actually transformed into heat. Yet, by lack of experimental observations at that scale, most (not to say all) authors dealing with the topic assume that 100 % of frictional power is transformed into heat.

The distribution of heat generated by friction at a contact interface between the two bodies in contact is determined by a combination of many factors: morphology of asperities on each contact surface, presence of a lubricant and/or fragments (third body), etc. Macroscopic continuum mechanics models typically ignore these details, and it is then necessary to provide a rule defining the partition of heat generated by friction between the two bodies in contact. For more details, see entry on *Heat Conduction with Thermal Contact: Modeling and Analysis*.

References

1. Bever MB, Holt DL, Titchener AL (1973) The stored energy of cold work. Prog Mater Sci 17:5–177
2. Chrysochoos A, Belmahjoub F (1992) Thermographic analysis of thermomechanical couplings. Arch Mech 44(1):55–68
3. Macdougall D (2000) Determination of the plastic work converted to heat using radiometry. Exp Mech 40(3):298–306
4. Rittel D (1999) On the conversion of plastic work to heat during high strain rate deformation of glassy polymers. Mech Mater 31:131–139
5. Rosakis P, Rosakis AJ, Ravichandran G, Hodowany J (2000) A thermodynamic internal variable model for the partition of plastic work into heat and stored energy in metals. J Mech Phys Solids 48(3):581–607
6. Stainier L, Ortiz M (2010) Study and validation of a variational theory of thermomechanical coupling in finite viscoplasticity. Int J Solids Struct 47(5): 705–715
7. Taylor GI, Quinney H (1937) The latent heat remaining in a metal after cold working. Proc Royal Soc London A163:157–181
8. Titchener AL, Bever MB (1958) The stored energy of cold work. Prog Metal Phys 7:247–338
9. Zehnder AT (1991) A model for the heating due to plastic work. Mech Res Commun 18(1):23–28