On theories of everything : are physicists actually ”lost in math”?

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Abstract Even though the march towards unification in physics began in the 17th century with the birth of mathematical physics, the notion of "Theory Of Everything" (TOE) only appeared around the 1980s. A TOE was supposed to unify all the fundamental interactions of nature: electromagnetism, weak interaction, strong interaction and gravitation. However, faced with the limits of most of the attempts (grand unification theory or GUT, string theory, loop quantum gravity, causal fermions systems, causal sets, Garrett Lisi's E8, causal dynamical triangulation, knot theory, ER = EPR ...), some wonder if we should not change the method, especially as logical or philosophical arguments (essential incompleteness of powerful theories, absence of fundamental laws, impossibility of embrassing "everything", essential infinitude of the universe, limited precision of calculations ...) could dissuade from seeking to build a definitive physical theory. More than anything, what is often disputed is the deductive nature of theories and the overuse of mathematics. Against these defeatist opinions, this article tries to rehabilitate the current approach of physicists to which we owe in fact many victories.

Key words. Theories of everything, String Theory, GUTs, Standard Model, E8, Garrett Lisi, Sabine Hossenfelder.

1 Introduction

The unification of scientific theories has always been one of the major goals of scientific research, whose presence in history can be seen for a very long time. In a conference at the French ENS-sciences in Paris, Frank Wilzeck (see [Wilzeck 15]), co-winner of the 2004 Nobel Prize in physics with H. David Politzer and David Gross[1], recalled that we have attended several "success stories", bringing together successively: space and number, geometry and algebra (Descartes); celestial and terrestrial law (Galileo, Newton); mechanics and ray optics (Hamilton); electricity, magnetism and optics (Maxwell); space and time (Einstein, Minkowski); wave and Particle (Einstein, de Broglie); reasoning and calculation (Boole, Turing), etc.

Some epistemologists, like the French philosopher G. Bachelard (see [Bachelard 34]), had also hailed this progressive generalization of theories, inducing a transition from the local to the global which relativizes negation: Euclid's geometry is only a special case of non Euclidean geometry (when we add the axiom of parallels), Newton's physics may be found from Einstein's when the Lorentz transformation is reduced to the Galileo transformation, i.e. when the v/c ratio tends towards zero, etc. Commenting on the work that this philosopher had devoted to the theory of relativity (see [Parrochia 14]), I had stressed that this vision of physical theories, nested one inside the other like nesting dolls, anticipated the future of science in the twentieth century. But I had mentioned also that it risked nevertheless having reached certain limits today, as if we had ignored this Cartesian warning, aiming at all deductive physics: as it becomes difficult to link very abstract principles to distant empirical facts and that deduction can sometimes follow several paths, experiments are then necessary to decide between them[2]. Of course it is still necessary to be able to practice the experiments in question, and this is the problem with which we are confronted today, especially in the domain of particle physics.

In this domain, basic for physics, particular theories - it's no secret - are supposed to move towards their unification. But this one, however, is not free and must not only assume that certain energy levels can be reached, but that principles and experimental facts could be well connected. As we know, electroweak unification occurs at around 100 GeV, which is not very considerable, but grand unification is predicted to occur at 10^{15} or 10^{16} GeV, and unification of the GUT force with

[1] For their discovery of asymptotic freedom in the theory of strong interaction

[2] "...it is necessary also to confess that the power of nature is so ample and vast, and these principles so simple and general, that I have hardly observed a single particular effect which I cannot at once recognize as capable of being deduced in many different modes from the principles, and that my greatest difficulty usually is to discover in which of these modes the effect is dependent upon them; for out of this difficulty cannot otherwise extricate myself than by again seeking certain experiments, which may be such that their result is not the same, if it is in the one of these modes at we must explain it, as it would be if it were to be explained in the other" (see [Descartes 37], part VI).

gravity is expected at the Planck energy, roughly 10^{19} GeV. Several Grand Unified Theories (GUTs) have been proposed to unify electromagnetism and the weak and strong forces, but the simplest GUTs have been experimentally ruled out and none of the particles the remaining theories predicted were found at the LHC. As the small, "curled up" extra dimensions of string theory (one of the most convincing attempt, until today, to solve the problem) can be compactified in an enormous number of different ways (one estimate is 10^{500}), each of which leading to different properties for the low-energy particles and forces, a so vast landscape is rather disappointing. Perhaps the time has come to take stock? This is what I propose to do here, even if it means recalling some stages in the development of physics that are well known to the oldest among us.

2 A glance at history of particle physics

Particle physics has been around for about a century. The first elementary particle to be discovered was the electron, identified in 1897 by J.J. Thomson. In 1911, Rutherford, for his part, demonstrated the existence of the atom's nucleus – in the case of hydrogen, a simple proton. In 1932, Chadwick discovered the neutron, and so, one understood the structure of the atom (a central nucleus comprising protons and, except for ordinary hydrogen, neutrons, surrounded by orbiting electrons). But other elementary particles not found in ordinary atoms immediately began to appear.

In 1928 the relativistic quantum theory of P. A. M. Dirac hypothesized the existence of a positively charged electron, or positron, which is the antiparticle of the electron. Dirac theory led to the discovery of this antiparticle, first detected in 1932, and so revealed the existence of a hitherto unknown universe: the world of antimatter. About the same time, difficulties in explaining β decay led to Pauli prediction of the neutrino in 1930, and by 1934, the existence of this particle was firmly established in theory (although it was not actually detected until 1956). Another particle was also added to the list: the photon, which had been first suggested by Einstein in 1905 as part of his quantum theory of the photoelectric effect.

The next particles discovered were related to attempts to explain the strong interactions, or strong nuclear force binding nucleons (protons and neutrons) together in an atomic nucleus. In 1935, Hideki Yukawa suggested that a meson (a charged particle with a mass intermediate between those of the electron and the proton) might be exchanged between nucleons[3]. The meson emitted by one nucleon would be absorbed by another nucleon; this would produce a strong force between the nucleons, analogous to the force produced by the exchange of photons between charged particles interacting through the electromagnetic force. (It is now known, of course, that the strong force is mediated by the gluon.) The next year, a particle of approximately the required mass (about 200 times that of the electron) was discovered and named the μ meson, or muon. However, its behavior did not conform to that of the theoretical particle. In 1947 the particle predicted by Yukawa was finally discovered and named the π meson, or pion.

Both the muon and the pion were first observed in cosmic rays. Further studies of cosmic rays turned up more particles. By the 1950s, these elementary particles were also being observed in the laboratory as a result of particle collisions produced by particle accelerators.

These ones had generated many new particles and scattering resonances. The masses and spins (intrinsic forms of angular momentum) of these particles were measured, and the patterns of allowed and forbidden decays were observed. These discoveries brought new information about the strong nuclear force, at much higher energies than in previous experiments. They revived the attempt to build a theory of the strong interactions : a summary of this history may be found at the end of the book by Roland Omnès (see [Omnès 70], 427-430).

As Jeffrey E. Mandula recalled, at that time the only successful theory of elementary particle interactions known was quantum electrodynamics. But it appeared that this theory could not be a natural model from which to start : trying to describe the strong interactions as a quantum field theory was not straightforward because the existence of a host of obstacles (for example, there were no reliable methods of calculation in strongly coupled field theory, and no more method for choosing a set of fundamental fields).

In such a context, it was better to try to exploit general properties of relativistic quantum field theories, whose results could be true properties of the strong interactions. One approach was the use of dispersion relations. Supplemented by quite reasonable simplifying assumptions, they gave good descriptions of many aspects of high energy scattering, such as the electromagnetic form factors of nucleons. The construction of a complete theory of the strong interactions was not immediate, but the ideas developed in this context have had a powerful and continuing effect on

[3] After Tomonaga, the new idea of the meson was already taking shape in Yukawa's mind in 1934 (see [Tomonaga 97], 112).

elementary particle theory.

«A particularly successful approach was to look for symmetry principles to organize the data. Of course, symmetries alone could not give a complete description of strong interactions, but it was plausible that one could discover the symmetries of the theory underlying the resonances and stable particles before having found the true theory. Furthermore, discovering the symmetries of the theory could be a major step in finding the theory itself» (see [Mandula 15a]).

So, contrary to what Sabine Hossenfelder has claimed (see [Hossenfelder 18]), although some physicists may be sensitive to beauty, is it not because of a fascination for it that they have embarked on the search for larger and larger groups of symmetries. It was simply the only rational way to continue to do physics and to do it wisely and economically, by planning the experiments to be carried out rather than launching headlong into an all-out exploration whose benefits could prove to be most uncertain.

Since its first use by Hermann Weyl[4] in 1918 (see [Weyl 18]), to build his unified theory of electromagnetism and gravitation – theory which failed, like that of Yang and Mills in 1954, but today credited to have been the first theory with a local gauge symmetry[5] (see [Marrani 15b], 35-37) –, the importance of symmetry has continued to grow in elementary particle physics and, more generally, in science (see [Rosen 95]; [Cohen-Tannoudji 99]).

In particular, symmetry groups have been successfully applied in nuclear physics. The main result is that if G is a group, an elementary particle may be described as a unitary irreducible representation of the group G, which is then by definition the group of symmetries of the particle[6]. A state of a particle is simply a vector of

[4]As we know, though Weyl was the first to explicit the method, numerous others had drawn attention to the same idea. Among the physicists in question, London, Fock, Schrödinger and Dirac (see [O'Raifeartaigh 97] and [Vizgin 94], chapters 3 and 6)

[5]A gauge theory is a type of field theory in which the Lagrangian is invariant under local transformations from certain Lie groups. A local (or internal) symmetry (in contrast with space-time symmetry) is symmetry of some physical quantity (observable, tensor, lagrangian...), which depends on the point of the base manifold. Yang-Mills theories (as, we will see, the Standard Model of particle physics) are typically local gauge symmetries. Bosonic fields, like the photon or the gluon, induce a force in addition to requiring conservation laws.

[6]Let G be a group and V a n-dimensional vector space. It is possible to associate with any element of G a linear operator $\Gamma(G)$ acting on the vectors of V. The set of matrices $M(G)$ linked to these operators constitutes a *representation* of the group. The vector space being able to be arbitrary, the representations can consist of matrices of any order. We therefore have an infinity of possible representations of which a non-mathematician will not a priori see the benefit. But in

the chosen representation space. Of course, this definition of an elementary particle depends on the fixed group of symmetries. But group theory allows us to classify known particles and to predict the existence of other ones.

For example, one thus understood that quantum electrodynamics was definitively associated with the unitary group U(1). Soon after, the existence of an internal symmetry relating the nuclei of isotopes of different elements, but with the same atomic number (what we call now "isotopic spin") was seen to be a symmetry of the strong interactions. As Mandula resumes, "resonances with different charges but (almost) identical mass could be grouped into multiplets that formed representations of the isospin group SU(2), and their allowed decays followed the patterns expected from group representation theory" (see [Mandula 15a]).

After the introduction of a new quantum number called "strangeness", that had to be conserved in strong interaction decays (see [Nakano 53]; [Gell-Mann 56]), it appeared that particles in the same isotopic spin multiplet had the same value of this new quantum number, that is, strangeness commuted with isotopic spin.

Along the 1960s, there was a great expansion of the use of symmetry groups in particle physics. One observed, in particular, that resonances with similar masses but different charge, isospin and strangeness could be collected into multiplets forming representations of a larger group, SU(3) ([Gell-Mann 61]). This was called the "unitary symmetry" group at the time, and is called the "flavor" SU(3) group today. The lightest bosonic particles and resonances formed two 8-dimensional representations of SU(3), one consisting of pseudoscalar mesons and the other of vector mesons. The lightest spin 1/2 baryons also formed an 8-dimensional representation while the lightest spin 3/2 baryons formed a 10-dimensional representation. The assumption that the interactions violating unitary symmetry transformed in a specific simple way under the symmetry group led to many relations among particle masses that were accurately obeyed ([Coleman 61]; [Gell-Mann 61]; [Okubo 62]).

This prompted Gell-Mann to propose what we call now the Standard Model (SM).

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reality, for a given group, all these representations can be broken down into a number of so-called *irreducible representations*, constituting a well-defined characteristic of the group and allowing the applications of group theory in physics.

3 The Standard Model

The standard model assumes the existence of five fermionic fields, of Weyl spinors with two components Q_L and E_L with left chirality and of quarks u_R, d_R and e_R with right chirality. These fields exist in three different versions, called "generations" or "families". They are subject to a gauged interaction governed by the product of groups $\mathrm{SO}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Only the quarks Q_L, u_R and d_r are sensitive to the $\mathrm{SU}(3)_C$ color force mediated by the eight G_μ^a gauge vectors, while only the left chiral fields Q_L and E_L interact via the left force $\mathrm{SU}(2)_L$ (called "weak isospin") under the influence of the three W_μ^a bosons. All these fields have a different hypercharge y under the group $\mathrm{U}(1)_Y$, force propagated by the single boson of gauge B_μ. The E_L and e_R fields are grouped together under the name of leptons. The theory also stipulates the existence of a complex scalar field, the Higgs boson, only loaded under $\mathrm{SU}(2)_L \times U(1)_Y$. We can see in Table 1 the fundamental fields of the standard model of particle physics.

Fields	Symmetry groups			
	$SO(1,3)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_L	$(\frac{1}{2},0)$	**3**	**2**	$\frac{1}{6}$
u_R	$(0,\frac{1}{2})$	**3**	**1**	$\frac{2}{3}$
d_R	$(0,\frac{1}{2})$	**3**	**1**	$-\frac{1}{3}$
E_L	$(\frac{1}{2},0)$	**1**	**2**	$-\frac{1}{2}$
e_R	$(0,\frac{1}{2})$	**1**	**1**	-1
ϕ	$(0,0)$	**1**	**2**	$\frac{1}{2}$
B_μ	$(\frac{1}{2},\frac{1}{2})$	**1**	**1**	0
W_μ	$(\frac{1}{2},\frac{1}{2})$	**1**	**3**	0
G_μ	$(\frac{1}{2},\frac{1}{2})$	**8**	**1**	0

Table 1: The Standard Model

As every one can see, the standard model is a gauge theory based on a non-simple gauge group in which the different factors play very distinct roles. Since the group has three factors, the theory depends a priori on three independent coupling constants.

4 The march towards a unitary theory and its interruption

As Zuber (see [Zuber 13]) – among others – observed, "the standard model is both remarkably verified and unsatisfactory". Apart from the presence of massive neutrinos, which we are now convinced of and which requires small Lagrangian amendments, no significant disagreement has so far been observed between the experimental results and model predictions. However, there are some unsatisfactory aspects of the model. For example, there exist many standard parameters: the number deemed excessive (around twenty) of free parameters in the model, the lack of "naturalness" in the way in which certain terms have to be extremely finely adjusted; the question of the Brout-Englert-Higgs mechanism[7] which seems to be confirmed by the discovery of the Higgs boson, but which some physicists consider as a construction ad hoc, etc.

So have come some attempts to improve the standard model. This can be done by merging the 3 groups of gauge within a larger group of a "grand-unified" theory (GUT) or by postulating a "supersymmetry", i.e. the existence of supersymmetric partners for all known particles, or again by developing the new paradigm of superstring theories.

As we know, none of these models actually works very well and the lack of results in the LHC does not allow to validate or contradict any of them. Since 2012, when the existence of the Higgs boson was confirmed, physics has entered the most serious crisis it has ever gone through.

4.1 Grand Unified Theories or GUTs

As it is known, the three gauge couplings do not quite meet when extrapolated using the SM model expression. However, the unification works quite well in some extensions. Indeed, the observation that the three coupling constants g_1, g_2, g_3 appear from their values measured at current energy converge under the effect of the renormalization group towards a value common to an energy of around 10^{15} or 10^{16}

[7]The Brout-Englert-Higgs mechanism (or "Higgs mechanism," for short) is the mechanism which is supposed to give mass not only to weak particles, but also to electrons, quarks, and other fundamental particles. The more strongly a particle interacts with the Higgs field, the more massive it is.

GeV was a strong incentive in the direction of great unification (see Fig. 2).

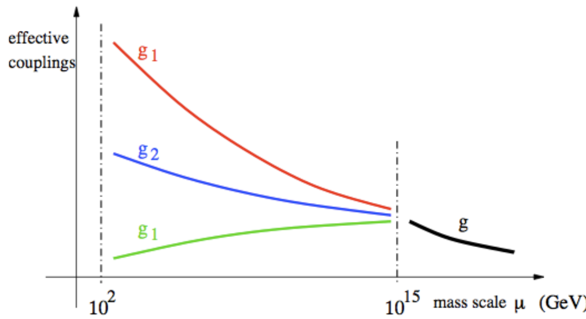

Figure 1: Schematic evolutions of the 3 effective couplings of the standard model and that of the grand-unified theory (from [Zuber 13])

The grand-unified theory which results from it must not only be a theory of gauge provided with only one coupling if the group of unification G is simple, but also to be able to predict the content in fields and particles of matter according to the representations of SU(3) $\times$ SU(2) $\times$ U(1) from representations of group G. Now the question becomes: which group to choose? We know (see [Anglès 08]) that the main Lie groups form a characteristic lattice shown here in Fig. 3.

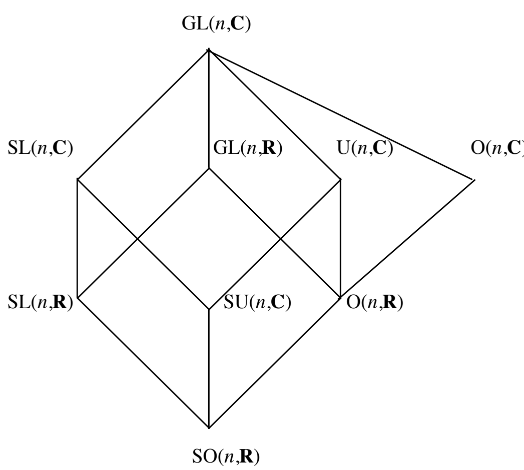

Figure 2: Lattice of Lie groups

As it is known, the inclusions of some Lie groups in small dimensions give also a useful diagram (see Fig. 4).

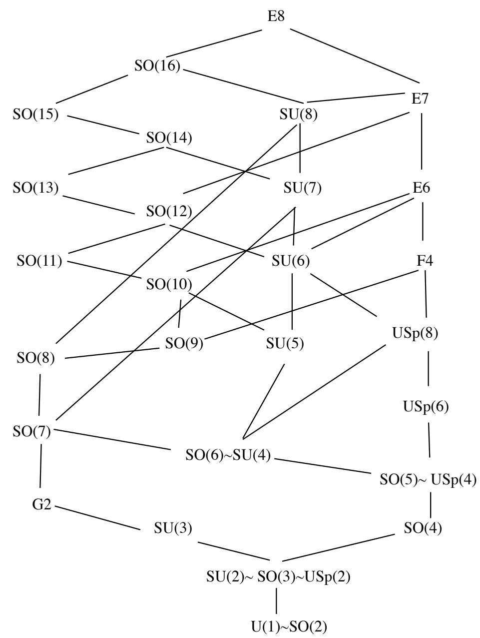

Figure 3: Inclusions of Lie groups in small dimensions

4.1.1 A first attempt with $\mathbf{SU(4)}_C\times$ $\mathbf{SU(2)}_L$ $\times$ $\mathbf{SU(2)}_R$

As Stuart Raby (see [Raby 08]) has recalled, one first tried to unify quarks and leptons into two irreducible representations of the group $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, i.e. the so-called Pati-Salam model (1974) (see [Pati 74]).

The $\mathrm{SU}(4)_C$ group extends QCD to include a fourth color associated with the leptons, so that, for example, the three colors of u quark would be related to the ν_e by the symmetry and a new interaction. The $\mathrm{SU}(4)_C$ symmetry was assumed to be spontaneously broken to $\mathrm{SU}(3) \times \mathrm{U}(1)_{B-L}$ at a sufficiently high scale, where $\mathrm{U}(1)_{B-L}$ is associated with baryon number (B) - lepton number (L). The electroweak $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R\times$ $\mathrm{U}(1)_{B-L}$ group is a left-right symmetric (parity conserving) version of the Standard Model, eventually broken to $\mathrm{SU}(2) \times \mathrm{U}(1)$. Extensions of the model involved extended electroweak groups but, as it is clear, the unification remains a partial one.

More precisely, the Pati-Salam field PS:

$$Q = (ql), Q^c = (q^c l^c)$$

where:

$$q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, \quad l^c = \begin{pmatrix} \nu^c \\ e^c \end{pmatrix}$$

transform as the irreductible representations $(4, 2, 1) \oplus (\bar{4}, 1, \bar{2})$ under PS, where, as we know, 4, 2, 1 represents spinors, 4 and $\bar{4}$ or 2 and $\bar{2}$, spinors of opposite chirality. One can check that baryon number minus lepton number acting on a 4 of SU(4) is given by:

$$B - L = \begin{pmatrix} \frac{1}{3} & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{3} & \\ & & & 1 \end{pmatrix}.$$

and similarly, electric charge is given by :

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B - L).$$

Charge is quantized since it is embedded in a non-abelian gauge group. One family is contained in two irreducible representations. Finally, if we require parity (L $\leftrightarrow$ R) then there are two independent gauge couplings.

What about the Higgs? The two Higgs doublets H_u, H_d are combined into one irreducible PS Higgs multiplet

$$\mathcal{H} = (H_d \ H_u)$$

transforming as a $(1, 2, \bar{2})$ under PS. Thus for one family, there is a unique renormalizable Yukawa coupling given by:

$$\lambda \mathcal{Q}^c \ \mathcal{H} \ \mathcal{Q},$$

giving the GUT relation:

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_\nu = \lambda.$$

Now Pati-Salam is not a grand unified gauge group. However, since SU(4) $\approx$ SO(6) and SU(2) $\times$ SU(2) $\approx$ SO(4) (where $\approx$ signifies a homomorphism), it is easy to see that PS $\approx$ SO(6) $\times$ SO(4) $\subset$ SO(10). In fact one family of quarks and leptons is contained in the spinor representation of SO(10), i.e.

$$SO(10) \to SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$16 \to (4,2,1) \oplus (\bar{4},1,\bar{2}).$$

Hence by going to SO(10) we have obtained quark-lepton unification (one family contained in one spinor representation) and gauge coupling unification (one gauge group).

4.1.2 The Georgi-Glashow SU(5) model

Before SO(10), an intermediate model was favored for some time by physicists to go beyond the standard model, the group SU(5). What is called the Georgi-Glashow SU(5) model (see [Georgi-Glashow 74]) was the first full unification of SU(3)× SU(2) × U(1) into a simple group.

The main reason for choosing SU(5) comes from the number of chiral fermions per generation. Each generation of Standard Model contains two flavors of quarks each coming in 3 colors, plus a lepton, and each of these 6+1 fields can have two chiralities, plus a supposed neutrino of zero mass and chiral. In total there are 15 chiral fermions per generation. Now, as the antiparticle of a right fermion is left, we can just reason on left fermions. So we are looking for a simple group G with a representation (reducible or irreducible) of dimension 15 which can group all the left fermions of each generation. The only candidate is ultimately the group SU(5) which has representations of dimension 15: the symmetric tensor representation, and sum representations of 5 (or $\bar{5}$) and 10 (or $\bar{10}$) spinors.

The SU(5) group of 5 × 5 unit matrices contains a subgroup SU(3) (3 × 3 submatrices of the upper left corner) and a subgroup SU(2) (blocks 2 × 2 in the lower right corner), which gives the correspondent generators of SU(3) × SU(2); the subgroup U(1) is generated by the diagonal matrix of zero trace $\mathrm{diag}(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2})$. It is clear that these three groups commute between them. We must then decompose all the fields (the representations 5, 10, 15 and 24) into representations of SU(3) × SU(2).

This shows that the representation 15 should be discarded and that the reduction representation $\bar{5} \oplus 10$ is the appropriate representation for fermion fields: $\bar{5}$ is broken down into representations $(\bar{3},1) \oplus (1,2)$ and contains $\bar{d}_L$ antiquarks and left leptons e_L and ν_e ; 10 breaks down into $(1,1) \oplus (3,2) \oplus (\bar{3},1)$ containing left lepton e_L^+, the singlet of SU(2) and SU(3), the two left quarks u_L, d_L which form a doublet of SU(2) and $\bar{u}_L$ antiquarks. Similarly, the 24 gauge fields incorporate the 8 gluon fields, the 3 + 1 vectors of the electroweak sector, plus 12 additional fields, which acquire a very large mass during the expected breaking of SU(5) → SU (3)× SU(2) × U(1).

The SU(5) $\rightarrow$ SU(3)$\times$ SU(2) $\times$ U(1) break should intervene at a great-unification energy of the order of 10^{15} or 10^{16} GeV, energy at which the couplings g_3, g_2, g_1 of SU(3), SU(2) and U(1) seem to converge (see Fig. 1). Since the infinitesimal generators are now rigidly linked within the simple group SU(5), we can connect the electric charge and coupling to the SU(2) gauge field and predict the Weinberg angle at the unification energy. But this one must obviously be renormalized between this energy and the energies of current physics.

A striking consequence of the quark-lepton unification within multiplets is the violation of respective conservations of the leptonic and baryonic numbers. In particular, the existence of interaction terms with one of the additional gauge fields allows the disintegration of the proton $p = duu \rightarrow d\bar{d}e+ = \pi^0 e^+$ and through other channels as well. It is necessary therefore, to carefully calculate whether the decay rate is compatible with experimental data on the proton lifetime (current limit $10^{32} \pm 1$ years)... which is not the case. It would still be necessary to show in which representation the Higgs bosonic fields are placed to allow breaking in two steps SU(5) $\rightarrow$ SU(3) $\times$ SU(2) $\times$ U(1) $\rightarrow$ SU(3) $\times$ U(1) at two extremely different scales.

In the end, the SU(5):

- incorporates by construction the generation structure of fermions;
- places leptons and quarks in the same representation and therefore explains the commensurability of their electrical loads and compensation for anomalies (see next section);
- reduces the number of parameters of the standard model and predicts the value of the Weinberg angle (at scale unification);

but conversely :

- it does not explain the reason for the three generations observed;
- it does not elucidate the question of "naturalness" of the "standard model" nor that related to the "hierarchy" (why is the ratio M_{GUT}/M_W so great?);
- finally, by default, it predicts effects such as proton decay at rates that seem incompatible with observations.

It is this last point which has led to abandon this unification scheme and to prefer supersymmetric paths. Before studying them, let us take a glance at another type of unification beyond SU(5), the one which uses larger groups.

4.2 Larger groups

As we can see in Fig. 3, SU(5) can be embedded into larger groups, such as SO(10) or E6.

4.2.1 More on SO(10)

As seen before, in SO(10) each fermion family transforms as an irreducible 16-dimensional representation ψ_{16}, which contains the reducible $\bar{5} \oplus 10$ of SU(5) as well as the (now required) SU(5)-singlet right-handed neutrino ν_L^c. SO(10) has an additional diagonal generator compared to the SM or SU(5) (i.e., it is rank 5). The breaking pattern SO(10) $\rightarrow$ SU(5) $\times$ U(1)$_\chi$ therefore allows for an additional neutral gauge boson, the Z$_\chi$, which could be almost as light as the SM gauge bosons, e.g., at the TeV scale. SO(10) has other symmetry breaking patterns, including the Pati-Salam group and flipped SU(5) (which involves an alternative identification of the particles in the 16-plet). Fermion masses can be generated by adding a 10-dimensional Higgs representation ϕ_{10}. The 10 decomposes as $5 \oplus \bar{5}$ under SU(5), which implies that ϕ_{10} actually contains two distinct Higgs doublets. These play the roles of the H and H† of SU(5), and can generate masses for the (u, ν) and (d, e), respectively. However, the SO(10) symmetry allows only a single Yukawa interaction (up to fermion family indices), of the form $\psi_{16}\psi_{16}\phi_{10}$, which leads to disastrous mass relations. More realistic models can be obtained by including additional Higgs multiplets, including high-dimensional ones such as 120 or 126. The 126 also allows couplings that can generate Majorana neutrino masses, such as a GUT-scale mass for ν_L^c, which leads to a small Majorana mass for the ν_L due to mixing (the seesaw model). SO(10) models are therefore frequently combined with family symmetries to generate detailed models of neutrino, quark, and charged lepton masses. However, large representations such as 126 are unlikely to emerge from an underlying superstring construction. An alternative is to replace them by higher-dimensional operators to generate fermion masses.

4.2.2 What about E6?

E6 is an even larger group which emerges from some superstring constructions. It is of rank 6, and contains the subgroup SO(10)$\times$ U(1)$_\psi$ (an alternative breaking is to SU(3)$_c\times$ SU(3)$_L\times$ SU(3)$_R$). Each fermion family is assigned to an irreducible 27-plet, which decomposes as 16+10+1 under SO(10). The 16 contains an SO(10) family, the

1 is an additional SM singlet which can break the U$(1)_\psi$ symmetry when it obtains a vacuum expectation value. The 10 contains new predicted exotic fermions:

$$10 = \begin{pmatrix} E^0 \\ E^- \end{pmatrix}_L + \begin{pmatrix} E^0 \\ E^- \end{pmatrix}_R + D_L + D_R,$$

which can also be given masses when the U$(1)_\psi$ is broken. The $(E^0, E^-)_{L,R}$ are color-singlet fields that transform as SU(2) doublets, and can be thought of as heavy leptons. Similarly, the $D_{L,R}$ are heavy down-type (charge -1/3) quarks. The two additional U(1) factors in E6 ($U(1)_\chi$ and $U(1)_\psi$)) and their associated exotic fermions are often used as examples of new physics that could possibly be present at the TeV scale or even lower, and are often considered outside of the original E6 context.

4.2.3 Garrett Lisi and E8

Some years ago, in 1982, Frampton and Kephart have proposed a unification theory based on E6, whose title was quite similar to the title of the famous Garrett Lisi's paper (see [Frampton 82]). As we know, since 2007, i.e. 25 years later, the popular physicist and surfer Antony Garrett Lisi(see [Lisi 07]) tried to persuade the scientific community that E8, the largest of the exceptional Lie groups, could be a good candidate to build a theory of everything – until now, in vain.

No matter the details of Lisi's construction, the crucial point is that the groups SU(3) × SU(2) × U(1) associated with the standard model must be embedded in E8 with the neither simple nor compact Lie group of gravity Spin$(3,1)_0$, isomorphic to SL(2, $\mathbb{C}$) = SL(2, $\mathbb{R}$) × SL(2, $\mathbb{R}$). So, as Distler showed, one would like to find an embedding of :

(1) $$G = \mathrm{SL}(2,\mathbb{C}) \times \mathrm{SU}(3)\times \mathrm{SU}(2)\times \mathrm{U}(1),$$

in a suitable noncompact real form of E8, such that one finds 3 copies of the representation :

(2) $$\begin{aligned} R =& \mathbf{2} \times [(3,2)_{1/6} + (\overline{3},1)_{-2/3} + (\overline{3},1)_{1/3} + (1,2)_{-1/2} + (1,1)_1] \\ &+\overline{\mathbf{2}} \times [(\overline{3},2)_{-1/6} + (3,1)_{2/3} + (3,1)_{-1/3} + (1,2)_{1/2} + (1,1)_{-1}] \end{aligned}$$

in the decomposition of the 248 of E8. Here SL(2,$\mathbb{C}$)= Spin$(3,1)_0$ is the connected part of the Lorentz Group, the "gauge group" in the MacDowell-Mansouri formulation of gravity.

Garret Lisi says that the embedding of G in E8 is supposed to proceed via the subgroup F4 $\times$ G2 $\subset$ E8. But, whether we take E8(8) or E8(-24) on one side, and consider F4(4) or F4(-20) on the other, it turns out that such an embedding is not possible.

Now if, rather than attempting to embed G in F4$\times$ G2, we just try to find some embedding of G in E8, we must recognize this is possible to do in quite a number of ways. However, for the split real form, E8(8), one cannot obtain even one copy of R. Of course it is possible to find an embedding, but it necessarily leads to a completely nonchiral “fermion” representation (and hence contains no copies of R).

A paper published together by Garibaldi and Distler concludes that "no proposed Theory of Everything constructed using subgroups of a real form E of E8 has a sufficient number of weight vectors in the -1-eigenspace to identify with all known fermions". But some of their theorems gives much more : "It shows that you cannot obtain a chiral gauge theory for any candidate subgroup of E, whether E is a real form or the complex form of E8". In particular, it is impossible to obtain even the 1-generation Standard Model in this fashion.

Thus, despite its many symmetries and its undeniable beauty, the E8 group is not enough to form the basis of the famous theory of everything in search of which everyone is[8].

4.3 The no-go theorems

One of the reasons for Lisi’s failure is the Coleman-Mandula theorem[9].

Soon after SU(6) was proposed, several papers explored the problems associated

[8]On some explicit errors of Lisi, see also [Rausch 09].

[9]Lisi and his supporters pretend that the CM-theorem does not apply in the case of E8 supersymmetry : «It is well known, from the no-go theorem of Coleman and Mandula, that when global symmetries of the S-matrix are concerned, such a unification cannot be accomplished without supersymmetry. However, this result does not contradict our unification program because a spacetime geometry that could be used to define the S-matrix only exists after the g symmetry has broken down to the direct sum, h. Before symmetry breaking, there is no metric and thus no S-matrix – a loophole allowing the unification of gravity and gauge fields, and this was not a recent result»([Lisi 10]). Lisi, Smolin and Speziale refer to some papers from Percacci (see [Percacci 84], [Percacci 91], [Percacci 08]). But if Percacci asserts that the Coleman-Mandula theorem "is sometimes misunderstood as forbidding any mixing between internal and spacetime invariances", he adds also "that in a large class of examples where the CM theorem cannot be applied, spacetime and internal symmetries still do not mix"(see [Percacci 08]).

with formulating SU(6) symmetry, and other hybrid symmetries, in a relativistic context (see [Mandula 15a]). And there were fundamental difficulties. The Coleman-Mandula theorem expressed clearly the reasons that hybrid symmetries could not be invariances of particle physics, and that the only possible Lie groups that can be symmetries of a relativistic particle theory are (locally) isomorphic to the direct product of the Poincaré group and an internal symmetry group.

As said above, the Coleman-Mandula theorem rests on the incompatibility of Poincaré invariance and the conservation of hybrid quantum numbers that involve spin. Because the result involves the interplay of relativistic scattering theory and group representation theory, the proof is quite convoluted. The logical structure of the argument is to begin with an arbitrary symmetry group generator and whittle its structure down to the sum of a translation, a pure Lorentz transformation, and an internal symmetry generator.

The Coleman-Mandula theorem deals only with symmetries expressed in terms of Lie groups, whose structure is described by the commutation relations between their generators. There are symmetries that cannot be so expressed, however. A class of such symmetries, called supersymmetries, involve transformations that change bosons into fermions and vice versa. These symmetries were discovered several years after the Coleman-Mandula theorem was proved.

Supersymmetries were first discovered in the context of string theory. Gervais and Sakita formulated an action for a theory with fermionic as well as bosonic variables, and observed that their action was invariant under set of transformations that interchanged the fermionic and bosonic world sheet fields (see [Gervais 71]). This was effectively a supersymmetry of a two dimensional field theory. A couple of years later, Wess and Zumino succeeded in extending the idea to four dimensional field theory (see [Wess 74a]). In a subsequent paper, they traced the reason that the Coleman-Mandula theorem does not apply to supersymmetries to the fact that the generators of supersymmetries are fermionic operators, and their structures are expressed by anticommutation relations (see [Wess 74b]).

Nonetheless, the possible supersymmetries are quite as restricted as ordinary symmetries. The restrictions on the possible supersymmetries were found by Haag, Łopuszański and Sohnius, and is given by the theorem that bears their names (see [Haag 75]). The proof of the Haag-Łopuszański-Sohnius theorem follows the same strategy as that of the Coleman-Mandula theorem. That is one begins with a completely general supersymmetry generator and, step by step, finds restrictions on the allowed generators and their anticommutators.

There are therefore certain limits to the progressive nesting of physical theories into one another, which the optimism of certain epistemologists like Bachelard or even already Poincaré judged a guarantee of the progress of scientific knowledge. In reality, this interlocking, over time, becomes more and more constrained and many pitfalls accompany this approach of progressive extension of physical theories. Nothing says that it can continue indefinitely.

4.4 E8 again as a regret?

Jackson recently remarked that the Lie groupe E8, of course, "is comfortably large enough to contain as a sub-group the Standard Model gauge group SU(3)$\times$ SU(2)$\times$U(1) together with the external Lorentz symmetry $SO^{+}(1,3)$, and hence on its own has the potential to be utilised by a theory seeking, beyond the ambition of a GUT, to unify the internal gauge forces together with gravity through a single symmetry group"(see [Jackson 17]). And though he recognized that the second and third generations of the 'fermion' states lack the appropriate external and internal symmetry properties other than through a 'graviweak' SO(8) $\subset$ E8 'triality' transformation, the impossibility of modifying this divergence with the standard model, being linked to the insufficient number of non-compact generators for any real form of E8, he maintains that E8 remains interesting for physical unification. Firstable, the fact that structures resembling the Standard Model can be identified for some exceptional Lie algebras, together with the observation that E8 (and already E7) are large enough to incorporate the external Lorentz group alongside the Standard Model gauge group, is "suggestive". A further exploration of all that and tentative connections with physics is seen in [Marrani 15b], in which 4-dimensional spacetime itself is proposed to emerge through fundamental interactions which in turn can be defined in terms of the structure of the E8 Lie algebra. Apart from its beauty, E8 would have many advantages (see [Jackson 17], 4-5).

However, we should not be too delusional on E8. In fact, this group had already been proposed as a unifying group in the 80s, when the preon model culminated. Chong Leong Ong, for example (see [Chong Leong 84]), has proposed some model of this kind. It is known that the standard SU(3) $\times$ SU(2) $\times$ U(1) gauge theory for the strong, weak, and electromagnetic interactions is renormalizable. When extended to incorporate the quarks, leptons, and the Higgs scalar fields, renormalizability is preserved and the characteristic mass scales brought out by renormalization procedure, and at which the gauge couplings diverge, appear at the infrared region.

For the standard formulation, there is an implicit assumption that quarks and leptons are elementary, or equivalently, that there is no critical mass scale (Λ_σ) in the ultraviolet region, around and beyond which the quarks and leptons will not be the proper dynamical degrees of freedom. Though there is no experimental evidence in direct conflict with this assumption, one may think that if Λ_σ does exist in nature, then $\Lambda_\sigma > 750$ Gev (from e^+e^- Bhabha scattering[10]), and we may even have $\Lambda_\sigma > 10^3$ Tev (from g-2 factor experiment on electron or muon at Fermilab), which proves that Λ_σ is much greater than the known masses of quarks and leptons.

Chong Long explores the implications of the plausible existence of Λ_σ in terms of supersymmetric nonlinear sigma model. The model assumes that the phase beyond Λ_σ, which isl called the *preonic phase*, possesses supersymmetry. Each nonlinear sigma model is characterized by an abstract manifold on which the spin-0 Bose fields take values, associated with an isometry group G and an isotropy group H. Chong Long shows that, among this class of abstract manifolds, only those with $G =$ E7, E8 can have an isotropy representation capable of accommodating three families of quarks and leptons. He also shows that when $G =$ E8 and $H =$ SO(10) $\times$ U(1), SO(10) $\times$ SU(2) $\times$ U(1), and SO(10) $\times$ SU(3) $\times$ U(1), the corresponding models can accommodate the three left handed families of quarks and leptons without incurring anomalies. Moreover, there is a right-handed, fourth family of quarks and leptons and the isotropy representations of the associated abstract manifolds are reducible. In the end, it is proved that there exists a unique choice of the ratio of resealings for which a Kahlerian manifold like E8 or E7 is Einsteinian. The problem with preonic idea is that it brings with it "major hurdles, which need to be overcome if the idea has to get off the ground. First we need to find a mechanism which would adequately protect the masses of composite quarks and leptons compared to their compositness scale. Second, one needs to understand family replications. Third, considering that all three families arer presumably made of the same type of constituents, bound by the same force, one need to understand why there is such a large hierarchy between the masses of the three families"(see [Pati 94], 377).

[10] As we know, in quantum electrodynamics, Bhabha scattering is the electron-positron scattering process:

$$e^+e^- \to e^+e^-,$$

an interaction to which two leading-order Feynman diagrams contribute: an annihilation process and a scattering process. Bhabha scattering is named after the Indian physicist Homi Jehangir Bhabha (1909-1966), the father of Indian nuclear program.

4.5 Supersymmetry

Another possibility is supersymmetry. Supersymmetry refers to possible relations between the spectrum and interactions of fermions (half-integer spin particles) and bosons (integer spin particles). It can be viewed as a space-time extension of the Poincare (Lorentz plus translational invariance) group, involving new anti-commuting dimensions. Under reasonable assumptions it is the unique extension of the usual Poincare and internal symmetries of field theory. Supersymmetry provides a possible route to unify gravity with the other interactions through superstring theory. If supersymmetry exists in nature it must be broken. For the connection with gravity it would suffice for the breaking scale to be very large. However, as mentioned earlier, there would be a number of advantages to a low breaking scale, e.g., a TeV, including the Higgs/hierarchy problem, gauge coupling unification, and the existence of a plausible dark matter candidate in some versions.

It is straightforward to construct supersymmetric versions of the standard model (the Minimal Supersymmetric Standard Model, ou MSSM), or of SU(5) and the larger grand unified groups (see [Dimopoulos 81], [Raby 08]). In each case, each particle (spin-0, 1/2 or 1) is accompanied by a predicted superpartner, which differs in spin by 1/2 unit. In addition to this doubling of the spectrum one must introduce two distinct Higgs doublets:

$$h_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$$

and

$$h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$$

as well as their spin 1/2 superpartners. Their SU(5) analogs are :

$$H_{ua} = \begin{pmatrix} \mathcal{H}_\alpha \\ h_u^+ \\ h_u^0 \end{pmatrix}, \quad H_d^a = \begin{pmatrix} \mathcal{H}^{c\alpha} \\ h_d^0 \\ h_d^- \end{pmatrix}$$

which transform as 5 and $\bar{5}$, respectively. h_u and h_d can have Yukawa couplings which generate masses for the (u, ν) and (d, e), respectively (similar to SO(10)). The second Higgs multiplets are needed because supersymmetry forbids couplings involving H†, as well as for anomaly cancellation.

One may also observe that the unification scale is higher in the supersymmetric case, $M_X \sim \times 10^{16}$ GeV rather than 10^{14-15} GeV. Since the lifetime for the proton to decay scales as M_X^4, this implies a much longer lifetime into modes such as $e^+\pi_0$, considerably longer than experimental limits. However, there are additional decay mechanisms involving the superpartners that scale as M_X^2 , leading to faster decays into different final states, such as $\bar{\nu}K^+$. The minimal versions of supersymmetric SU(5) and SO(10) are already excluded by the non-observation of such decays, while non-minimal versions should allow observable proton decay rates in future experiments. There are also (unrealistic) versions of low-scale supersymmetry in which new interactions of the superpartners would lead to rapid proton decay, with a rate that is not suppressed by powers of M_X.

4.6 Extra dimensions and strings

As far as we are aware, there are three dimensions of space and one of time. In particular, the space dimensions are large or infinite in size. However, it is possible that there are additional space dimensions that we cannot readily perceive, perhaps because they are compactified (curled up) into a tiny circle or other manifold, highly warped by gravitational effects, or because some dynamical principle causes us to be stuck in a limited domain of the new dimensions. Considerable theoretical activity has been directed towards such possibilities, e.g., in connection with the fermion or Higgs/hierarchy problems, or in connection with gravity (superstring theories require additional dimensions for a consistent formulation).

In orbifold GUTs the grand unification is present in a higher-dimensional space. The GUT symmetry may be broken by boundary conditions in the extra dimensions, so that our apparent four-dimensional world has a lower symmetry, e.g., of the SM or MSSM. Orbifold GUTs may retain the desirable features of grand unification, such as gauge coupling unification, third family Yukawa relations, etc., while avoiding such difficulties as the doublet-triplet problem, too rapid proton decay, and the need for large Higgs representations.

4.7 Superstring theories

Superstring theories incorporate quantum gravity, and are therefore more ambitious than the SM (Standard Model), MSSM (Minimal Supersymmetric Standard Model), or grand unification (GUTs). There are actually a large number of string theories, which may be thought of as different points in an enormous landscape of string vacua.

Many of these include underlying grand unification symmetries. They may compactify into an effective four-dimensional GUT, although it is difficult to generate the adjoint and other large Higgs multiplets introduced in many non-string motivated models. They may also lead to versions of orbifold GUTs, or compactify directly to the SM or MSSM, or to an extended version, with limited memory of the underlying GUT. Constructions may retain simple MSSM-type gauge unification, or the unification may be modified (and complicated) by the effects of new particles and/or by the string scale gauge coupling boundary conditions. The fermion families or the elements of the families may have different origins in the construction, breaking or modifying GUT Yukawa relations and possibly leading to family nonuniversal couplings to new neutral gauge bosons. Other classes of string theories usually do not involve a full underlying GUT, but they often descend to four dimensions using a Pati-Salam group.

5 Lost in math?

The quest for symmetry and breaks in symmetry, via the Lie groups, guided all of the physics of the second half of the 20th century. With the introduction of larger and larger symmetry groups, we therefore witnessed an inflation of mathematics in physics. The last positive result was the demonstration of the Higgs boson which, if the one we have seen in the LHC is confirmed to be the good candidate, completes the standard model. As it is well known, however, SM was not the last word in physics and the physicists moved soon after to the great unification theories (GUT) and to supersymmetry (SUSY). Alas, the LHC, despite the possibility of collisions at 13 or 14 Tev did not highlight the low energy decay particles that were expected as the residual proofs of these theories. So we end up with an inflation of theories devoid of experimental verifications, and, given the energies it would take to have a chance to prove them, without any possibility of reaching them - for a long time, and perhaps forever. What to do with all this stacked mathematics, many of which being probably irrelevant? If no possibility of verification is emerging, one can obviously wonder what may well be the interest of studying these rather fleeting states of matter.

The French mathematician René Thom, Fields medal 1958, was already worried about this situation in 1972: "The choice of the phenomena considered as interesting is undoubtedly largely arbitrary. The current Physics builds enormous machines to highlight states whose duration of life does not exceed 10^{-23} seconds[11]; it is probably not wrong to want, by the use of all the technical means available, to make an inventory of all the phenomena accessible to experiments We can nevertheless legitimately ask ourselves a question: a number of familiar phenomena (to the point that they no longer attract attention!) need however hard theory; for example, the cracks in an old wall, the form of a cloud, the fall of a dead leaf, the foam of a beer bock ... Who knows if a little more mathematical reflection on this kind of little phenomena would not be revealed, ultimately more profitable for the science?"(see [Thom 72], 26).

Should we abandon the quest for a unified theory? This was what seemed to suggest as early as 2013, that is to say long before Sabine Hossenfelder, Freeman Dyson and Ashutosh Jogalekar. The latter, in an article in *Scientific American* entitled "Why the search for a unified theory may turn out to be a pipe dream" (see [Jogalekar, 13]) reports that, although unification is a very old goal in physics, it is not certain that this quest can continue indefinitely. Very present from Maxwell in the 19th century, unification thinking pervaded the twentieth century. The great physicists of this century often believed themselves very close to an ultimate theory which would sound the end of physics. But, as we know, gravity remained intractable and its union with quantum theory never appeared. String theory remained itself impossible to test and the hope placed in it gradually eroded.

In the case of gravitation, there are in particular problems to detect the so called "gravitons", i.e. the particles that are thought to mediate the gravitational force. The extremely weak nature of the gravitational force suppose sensitive equipment to do that, as the famous LIGO (Laser Interferometer Gravitational Wave Observatory), which is using extremely sensitive interferometers to detect the minuscule shifts in space-time caused by the passage of a gravitational wave. The problem is that this phenomenon is very subtle. Freeman Dyson, who has tried to quantify this subtlety (see [Dyson 13]) demonstrated that this change might be so small that it would be swamped by "background" quantum fluctuations in space-time. After him, even an ideal LIGO detector could not detect a single graviton. "To detect a single graviton with a LIGO apparatus, the mirrors must be exactly so heavy that they will attract each other with irresistible force and collapse into a black hole. In other words, nature herself forbids us to observe a single graviton with this kind of apparatus".

[11] In the 2nd edition (see [Thom 77], 10), probably because the power of the accelerators has increased in the meantime, the value of this duration drops to 10^{-30} seconds.

If true, this limitation, of course, goes much beyond detecting discrete gravitons. It could mean that the world of gravity and the world of subatomic particles will forever stay separate from each other. According to Dyson,"it would imply that theories of quantum gravity are untestable and scientifically meaningless. The classical universe and the quantum universe could then live together in peaceful coexistence. No incompatibility between the two pictures could ever be demonstrated. Both pictures of the universe could be true, and the search for a unified theory could turn out to be an illusion".

This is not necessarily boring: maybe the universe is much more diverse than we think. But the lack of a unifying theory puts a definitive halt to the indefinite overlapping of scientific theories in which epistemologists of the last century still believed.

6 Conclusion : completely lost or simply disoriented, or none of this?

Speculating on the morale or state of mind of physicists may do not reflect a scientific attitude. Naturally, one will not prevent some of them from wondering : do they have drowned in mathematics that go far beyond physical reality?[12] Too confident in the virtues of symmetry and breaks in symmetry, have they granted too much to the common thread of group theory? Do they have to find another method? Or go back to pure and simple experimentation? In fact, these questions may be perfectly idle. Can anyone, in truth, experiment blindly, without the aid of some theory? Since Kant, no one can seriously think so. On the other hand, it is absolutely false to assume that a real physicist is ready to give in to aesthetics, even if he may, moreover, be sensitive to it. Abdus Salam tells the following anecdote about Dirac, who had always been struck by the beauty of the theory of special relativity, to the point that it seemed as he was making it a criterion of any theory[13]:

[12]Even Max Tegmark, who imagined at first that mathematics and physics coincide (the famous Mathematical Universe Hypothesis or MUH) (see [Tegmark 98]) seems to admit now (see [Tegmark 06]) that only Gödel-complete (fully decidable) mathematical structures have physical existence, which amounts to re-establishing a distinction between mathematics and physics.

[13]Dirac's famous phrase, quoted everywhere, repeated over and over (([Dalitz 87], 20; [McAlister 96], 16, [Kragh 16], chap. 9, [Hossenfelder 18], 37), and which states that "a physical law must possess mathematical beauty," is not taken from a publication. Dirac would have written it on the blackboard when he visited the University of Moscow in 1956 and was asked to

«I remember talking about supersymmetry in the presence of Dirac at the Miami conference in 1974. Dirac was sitting at the back of the room, and as usual he said nothing during my speech. I went over to him and said, "Professor Dirac, don't you think this is a nice theory? Does it not meet your criteria for being correct?" He conceded that "it was indeed a fine theory", but said also that "if supersymmetry was truly a symmetry of nature, these new fermions and bosons would have been found a long time ago!" I was extremely surprised because it seemed contrary to his own claims of the primacy of the beautiful. (It could be that, by intuition, he was in spite of everything right, we never know in our subject »(see [Salam 90]).

Einstein had the same type of reaction in front of the Kaluza-Klein theory: although he liked the idea of an invisible extra dimension, he doubted its relevance[14]. Probably he would doubt even more today the string theory, which is a resurrection of Kaluza-Klein idea, but where the number of extra dimensions is brought to 6 or 7. Surely he would take up his words of yesteryear: "we cannot yet say for the moment if the idea will be validated".

It is difficult to talk of "common sense" in physics. Everyone remembers Niels Bohr's word in front of Pauli: "we are all agreed that your theory is crazy. The question which divides us is whether it is crazy enough to have a chance of being correct"[15].

Should we argue for a well-tempered use of mathematics in physics? One of my mathematician colleagues, member of a research project evaluation committee in France, said to me recently: "when I see a project file where there is a lot of math, I tell myself that it must be that of a physicist!". Of course it is a joke, just like the formula – which we heard a lot a few years ago – according to which Fields medals became, in the course of days, "Quantum Fields medals".

In fact, mathematics are very useful in particle physics: surely not all mathematics; possibly not only group theory. Far from any inflation in the matter, if particle

write an inscription summarizing his basic view of physics. In other words, one of Hossenfolder's main arguments is based on a reported anecdote. We can then easily oppose another anecdote, that of Abdus Salam, who has the merit of coming from a great physicist.

[14]Kaluza had submitted his article to Einstein, but, due to his doubts, this publication was delayed for two years. In 1921 Kaluza finally received the answer : « Ich habe grossen Respekt vor der Schönheit und Kühnheit Ihres Gedankens.» ("I have great respect for the beauty and boldness of your thoughts"). Very impressed by Kaluza's work, Einstein recommended the article to appear in the *Prussian Academy of Sciences Proceedings*, but his doubts remained.

[15]Said to Wolfgang Pauli after his presentation of Heisenberg's and Pauli's nonlinear field theory of elementary particles, at Columbia University (1958), as reported by F.J. Dyson in [Dyson 58].

physicists manage to build experimentally testable theories of mathematical physics, there is probably much to be hoped for from the future of physics. Maybe we only have to be patient.

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