Rendezvous of Nonholonomic Robots via Output-Feedback Control Under Time-Varying Delays

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***Abstract*— We address the problem of making nonholonomic vehicles, with second-order dynamics and interconnected over a bidirectional network, converge to a formation centered at a non-prespecified point on the plane with a non-prespecified common orientation. We assume that only the Cartesian position of the center of mass of each vehicle and its orientation are available for measurement, but not the velocities. In addition, we assume that the interconnections are affected by time-varying delays. Our control method consists in designing a set of second-order systems that are interconnected with the robots' dynamics through virtual springs and transmit their own coordinates to achieve consensus. This and the virtual elastic couplings with the vehicles make the latter achieve consensus too. To the best of our knowledge, output feedback consensus control of underactuated nonholonomic vehicles has been little studied, all the less in the presence of delays.**

***Index Terms*— Rendezvous, consensus, autonomous vehicles, persistency of excitation, output feedback, differential-drive mobile robots.**

I. Introduction

For first and second-order integrators the leaderless consensus problem, which consists in the state variables of all agents converging to a common value, is well-studied and solved under many different scenarios [1]. The solution to this problem is more complex if one considers the agents' dynamics [2], [3], network constraints, such as communication delays [4], unavailability of velocity measurements [5], or nonholonomic constraints that restrict the system's motion [6]. For autonomous vehicles, which, in contrast to mathematical *models*, do occupy a physical space, the leaderless consensus problem consists in making all robots converge to a *rendezvous* point while forming a pattern with an unknown center. This is done by specifying an offset position from the unknown center for each robot. It may be required that either only the positions [6], [7] or only the orientations [8] achieve a common equilibrium point, or that both positions and orientations converge to a common value [9].

Rendezvous control is useful, e.g., if a group of robots must converge to postures that form a desired geometric pattern in order to subsequently maneuver as a whole [10]. This is a typical two-stage formation problem. In the first, a rendezvous algorithm is required for the stabilization of the agents [11]–[13] and in the second a formation-tracking controller is employed [14]. From a systems viewpoint, rendezvous control of nonholonomic vehicles is inherently a set-point stabilization problem. In that regard, it presents the same technical difficulties as the stabilization of a single robot. In particular, that nonholonomic systems are not stabilizable via time-invariant smooth feedback [15], but either via discontinuous time-invariant control [12] or time-varying smooth feedback [16], [17].

In this paper we solve the rendezvous problem for force-controlled nonholonomic systems unequipped of velocity sensors and interconnected over a network with bidirectional interconnections affected by time-varying delays. Delays are common in network control systems [18]. Not relying on velocity measurements is desirable since these measurements are often contaminated by noise and velocity sensors may be unreliable [19]. Measurement delays and lack of velocity measurements have been addressed in the literature on consensus of nonholonomic systems, but not simultaneously. For instance, [19]–[21] address control problems without velocity measurements, and delays are considered in [20], but they all concern leader-follower formation tracking control; output-feedback consensus is addressed in [22] and [23], but delays are not considered. In [24] a controller achieving consensus formation in the presence of smooth time-varying delays is proposed, but the controller uses state feedback.

The design of the controller that we propose in this paper is inspired by the previous works [5] and [24]. The control approach consists in designing a group of independent second-order systems to achieve output consensus among themselves and, then, steer the plants to output consensus through a virtual mechanical coupling. The underlying idea is reminiscent of that used in [5], for flexible-joint manipulators, but in contrast to the latter, we do not use a high-gain nonlinear observer. As in [24], our controller relies on persistency of excitation to overcome the difficulties imposed by the nonholonomicity on set-point stabilization, but it is not a certainty-equivalence modification of the controller in that reference.

The remainder of this paper is organized as follows. In the next section we describe the nonholonomic second-order dynamic model and lay our main assumptions. In Sections III and IV we present some preliminary, but original, results on state- and output-feedback consensus control that are useful to explain and put our main results in perspective. The latter are presented in Section V. In Section VI we provide some realistic simulations using the Gazebo-ROS environment and we provide concluding remarks in Section VII.

II. Model and problem formulation

We consider a group of N autonomous nonholonomic second-order vehicles modeled by the equations:

$$\text{angular motion} \quad \begin{cases} \dot{\theta}_i &= \omega_i & \text{(1a)} \\ \dot{\omega}_i &= u_{\omega i}, & \text{(1b)} \end{cases}$$

$$\text{linear motion} \quad \begin{cases} \dot{z}_i &= \varphi(\theta_i) v_i, & \text{(2a)} \\ \dot{v}_i &= u_{vi}, \qquad i \in \{1, 2 \dots N\} & \text{(2b)} \end{cases}$$

where

$$\varphi(\theta_i) := [\cos(\theta_i) \quad \sin(\theta_i)]^\top, \tag{3}$$

$z_i = [x_i \ y_i]^\top \in \mathbb{R}^2$ denotes the Cartesian coordinates of the ith

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vehicle on the plane, $\theta_i \in \mathbb{R}$ denotes its orientation, and

$$u_{vi} := \frac{1}{rm}[\tau_{i1} + \tau_{i2}], \quad u_{\omega i} := \frac{2R}{Ir}[\tau_{i1} - \tau_{i2}]$$

are the control inputs, which are defined in function of the wheel torques τ_{i1} and τ_{i2}, the robot inertia I, the mass m, the wheel radius r, and the wheel axle length R.

Remark 1: For the purpose of analysis, the angles are defined as real variables, but this may entail undesired unwinding. In practice, $\theta_i \in (-\pi, \pi]$. This is considered in Section VI. •

It is required that the vehicles meet in formation around a *non*-predefined rendezvous point on the plane, denoted $z_c := (x_c, y_c)$, and acquire a *non*-predefined common orientation, denoted θ_c, modulo a *given* offset $\delta_i = [\delta_{xi} \ \delta_{yi}]^\top$, with $i \leq N$, which determines the position of the ith vehicle relative to the unknown center of the formation. In other words, defining, $\bar{z}_i := z_i - \delta_i$ (correspondingly, $\bar{x}_i := x_i - \delta_{xi}$ and $\bar{y}_i := y_i - \delta_{yi}$) the control goal is to make

$$\lim_{t\to\infty} v_i(t) = 0, \quad \lim_{t\to\infty} \bar{z}_i(t) = z_c, \tag{4}$$

$$\lim_{t\to\infty} \omega_i(t) = 0, \quad \lim_{t\to\infty} \theta_i(t) = \theta_c \qquad \forall\, i \leq N. \tag{5}$$

This is a leaderless consensus problem. That is, neither the coordinates (x_c, y_c) nor the angle θ_c are determined a priori as a reference. They depend on the initial postures, on the systems' *nonlinear* dynamics, and on network features. The rendezvous problem has been successfully solved under different conditions, but the originality of this paper resides in considering the realistic scenario determined by the following assumptions simultaneously.

First, owing to the fact that velocity measurements are often corrupted by noise and sensor defects we pose the following hypothesis.

Assumption 1: Only the coordinates (z_i, θ_i) are measured.

Second, we assume that a WiFi communication network is available over which the ith robot communicates with a set of neighbors, which we denote by $\mathcal{N}_i$. It is naturally assumed that once a communication is set between two robots i and $j \in \mathcal{N}_i$, the flow of information is bidirectional and is never lost. Whence the following.

Assumption 2: The network may be modeled using an interconnection graph that is undirected, static, and connected.

Remark 2: In graph theory, a graph is undirected if the nodes exchange information in both direction, it is static if the interconnection is constant, and it is connected if any node is reachable from any other node. The latter is a necessary condition to achieve consensus [1]. •

On the other hand, because the robots communicate through a WiFi network, the communication between the robots i and j is affected by non-constant time-delays. More precisely, we consider the following.

Assumption 3: The communication from the jth to the ith robot is subject to a variable time-delay denoted $T_{ji}(t)$. It is assumed that the function $t \mapsto T_{ji}(t)$ is bounded, has bounded time-derivatives, up to the second, and the upper-bound, denoted $\overline{T}_{ji}$, is known.

Assumption 3, which is imposed only for technical reasons imposed by the method of formal analysis, carries certain conservatism in the supposition that the delays are differentiable. Indeed, time-delays over WiFi networks or the Internet may rather be of a non-smooth nature [9], [18]. In Section VI we provide realistic simulations in which Assumption 3 is violated. Yet, the formal solution to the rendezvous problem defined above under Assumptions 1, 2, and under discontinuous time-varying delays remains an open problem.

III. Control Architecture: State-Feedback Case

An essential feature of the model (1)–(2) is that it consists of *two* coupled second-order systems driven by independent control inputs. One system determines the linear motion and the other the angular one. Each of the latter being a second-order mechanical system, the control design starts by devising a consensus controller for (1) and (2) separately. To that end, we revisit a controller from the literature, but we provide an original analysis of robust stability that serves as design-basis for our dynamic output-feedback controller, presented in Section V.

A. Robust consensus control of second-order systems

The consensus problem for systems with dynamics

$$\ddot{\vartheta}_i = u_i \quad i \leq N,\ u_i \in \mathbb{R} \tag{6}$$

(that is steering $\vartheta_i \to \vartheta_c$, $\dot{\vartheta}_i \to 0$, and $\ddot{\vartheta}_i \to 0$ with ϑ_c constant and not imposed *a priori*) is now well understood in various settings. For instance, it is well known (see [1]) that if the systems modeled by (6) communicate over a network modeled by an undirected, static, and connected graph, the distributed control law, of proportional-derivative (PD) type,

$$u_i = -d_i\dot{\vartheta}_i - p_i \sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_i - \vartheta_j); \quad d_i,\ p_i > 0, \tag{7}$$

where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise, solves the consensus problem. There are many reported ways to show this. For further development we provide here a simple and original proof based on Lyapunov's direct method. Let $\vartheta := [\vartheta_1 \ \cdots \ \vartheta_N]^\top$ and

$$\tilde{\vartheta} := \vartheta - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^\top\vartheta, \quad \mathbf{1}_N := [1 \cdots 1]^\top. \tag{8}$$

Indeed, $\tilde{\vartheta}$ denotes a vector whose ith element corresponds to the difference between ϑ_i and the average of all states, i.e., $\vartheta_c := (1/N)\mathbf{1}_N^\top\vartheta$. In addition, under Assumption 2, ϑ_c corresponds to the consensus equilibrium point. Now, to abbreviate the notation, we also define

$$\Pi := I - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^\top.$$

Note that $\Pi = \Pi^\top$ and $\|\Pi\| \leq 1$, where $\|\Pi\|$ corresponds to the induced norm of Π, and $\tilde{\vartheta} = \Pi\vartheta$.

Next, we introduce the Laplacian matrix, $L := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, where

$$\ell_{ij} = \begin{cases} \sum\limits_{k\in\mathcal{N}_i} a_{ik} & i = j \\ -a_{ij} & i \neq j. \end{cases} \tag{9}$$

By construction, $L\mathbf{1}_N = 0$ and, after Assumption 2, L is symmetric, it has a unique zero-eigenvalue, and all of its other eigenvalues are strictly positive. Thus, $\text{rank}(L) = N - 1$. Also, the last term on the right-hand side of Equation (7) satisfies

$$\text{col}\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_i - \vartheta_j)\Big] = L\tilde{\vartheta}, \tag{10}$$

where $\text{col}[(\cdot)_i]$ denotes a column vector of N elements $(\cdot)_i$. Indeed, by the definition of the Laplacian, we have

$$\text{col}\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_i - \vartheta_j)\Big] = L\Big[\vartheta - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^\top\vartheta\Big] + \frac{1}{N}L\mathbf{1}_N\mathbf{1}_N^\top\vartheta.$$

However, $L\mathbf{1}_N = 0$, so the right hand side of the equation above equals to $L\Pi\vartheta$, which corresponds to $L\tilde{\vartheta}$, by definition. These identities are useful to write the closed-loop system (6)–(7) in the multi-variable form

$$\ddot{\vartheta} = -D\dot{\vartheta} - PL\tilde{\vartheta}, \tag{11}$$

where $P := \text{diag}[p_i]$ and $D := \text{diag}[d_i]$, and to see that the Lyapunov function

$$V_1(\tilde{\vartheta}, \dot{\vartheta}) := \frac{1}{2}\Big[\tilde{\vartheta}^\top L\tilde{\vartheta} + \dot{\vartheta}^\top P^{-1}\dot{\vartheta}\Big] \tag{12}$$

is positive definite, even if L is rank deficient. Indeed, the term $\tilde{\vartheta}^\top L\tilde{\vartheta} \geq \lambda_2(L)|\tilde{\vartheta}|$, where $\lambda_2(L) > 0$ corresponds to the second eigenvalue of L (that is, the smallest positive eigenvalue), not for *any* $\tilde{\vartheta} \in \mathbb{R}^N$, but for $\tilde{\vartheta}$ as defined in (8). Now, evaluating the total derivative of V_1 along the trajectories of (11) and using $L\dot{\tilde{\vartheta}} = L\dot{\vartheta}$ (again, this holds because $L\mathbf{1}_N = 0$) we see that

$$\dot{V}_1(\tilde{\vartheta}, \dot{\vartheta}) = \dot{\vartheta}^\top P^{-1} D\, \dot{\vartheta}. \tag{13}$$

Global asymptotic stability of the consensus manifold $\{(\tilde{\vartheta}, \dot{\vartheta}) = (0, 0)\}$ may be ascertained from (13) by invoking Barbashin-Krasovskĭi's theorem (also, but wrongly, known as LaSalle's theorem). As a matter of fact, since the system is linear time-invariant, it is also globally exponentially stable and robust to external perturbations.

To see this more clearly, using V_1 it is possible to construct a simple strict Lyapunov function. This is useful to assess the robustness of system (6) in closed loop with the consensus control law defined in (7) in terms of input-to-state stability. Let

$$V_2(\tilde{\vartheta}, \dot{\vartheta}) := V_1(\tilde{\vartheta}, \dot{\vartheta}) + \varepsilon \tilde{\vartheta}^\top P^{-1} \dot{\vartheta}, \quad \varepsilon \in (0, 1). \tag{14}$$

In view of the properties of V_1 it is clear that V_2 also is positive definite and radially unbounded, but only for all $\tilde{\vartheta}$ as defined in (8) and for sufficiently small values of $\varepsilon \in (0, 1)$. The total derivative of V_2 along the closed-loop trajectories yields

$$\dot{V}_2(\tilde{\vartheta}, \dot{\vartheta}) = \dot{V}_1 + \varepsilon \Big[\dot{\vartheta}^\top \Pi P^{-1} \dot{\vartheta} - \tilde{\vartheta}^\top \Pi P^{-1} D \dot{\vartheta} - \tilde{\vartheta}^\top L \tilde{\vartheta} \Big], \tag{15}$$

which, in view of (13) and the fact that $\|\Pi\| \leq 1$, implies that

$$\dot{V}_2(\tilde{\vartheta}, \dot{\vartheta}) \leq -c_1 \frac{d_m}{p_M} |\dot{\vartheta}|^2 - \varepsilon c_2 |\tilde{\vartheta}|^2 \tag{16}$$

where d_m and p_M are the smallest and largest coefficients of D and P respectively, $c_1 := 1 - \varepsilon[\frac{1}{\lambda} + \frac{1}{d_m}]$ and $c_2 := \ell_2 - \lambda \frac{d_m}{p_M}$ are positive for appropriate values of λ and $\varepsilon \in (0, 1)$ and any $\ell_2 := \lambda_2(L) > 0$.

Remark 3: Consider, now, the systems $\ddot{\vartheta}_i = u_i + \alpha_i$ where α_i is a bounded external disturbance. Then, the previous computations lead to the inequality

$$\dot{V}_2(\tilde{\vartheta}, \dot{\vartheta}) \leq -c_1 \frac{d_m}{p_M} |\dot{\vartheta}|^2 - \varepsilon c_2 |\tilde{\vartheta}|^2 + \dot{\vartheta}^\top \alpha, \tag{17}$$

with $\alpha := [\alpha_1 \cdots \alpha_N]^\top$. It follows that the map $\alpha \mapsto \dot{\vartheta}$ is state-strictly passive [25] and, also, the closed-loop system is input-to-state stable with respect to the input α. •

From the previous analysis, we conclude that for the angular-motion subsystem (1) the controller

$$u_{\omega i} = -d_{\omega i}\omega_i - p_{\omega i} \sum_{j \in \mathcal{N}_i} a_{ij}(\theta_i - \theta_j) + \alpha_i, \tag{18}$$

where $d_{\omega i}$ and $p_{\omega i} > 0$, ensures global asymptotic stability of the consensus manifold $\{\omega_i = 0 \ \wedge \ \theta_i = \theta_j\}$ if $\alpha_i \equiv 0$ and the closed-loop system is input-to-state stable with respect to α_i —cf. [24].

Remark 4: The previous computations hold with obvious changes in the notation for the angular-motion dynamics $\ddot{\theta}_i = u_{\omega i}$, which is equivalent to (1). This is used farther below. •

B. On consensus in the linear motion

After the developments in Section III-A and with the purpose of designing two independent controllers for the angular and linear motion, it appears appealing to use the following control law for the subsystem (2). Let

$$u_{vi} = -d_{vi}v_i - p_{vi}\varphi(\theta_i)^\top \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j) \tag{19}$$

and let us replace the state variable θ_i with an arbitrary trajectory $\theta_i(t)$ which, for the time being we assume to be bounded and to have a bounded derivative $\omega_i(t)$, for all $t \geq 0$ and all $i \leq N$ (this technical assumption is relaxed later). Thus, the closed-loop linear-motion dynamics, formed by Eqs. (2) and (19), may be regarded as a *time-varying* subsystem, decoupled from the angular motion dynamics —cf. [25, p. 657], [26]. That is,

$$\dot{\bar{z}}_i = \varphi(\theta_i(t))v_i, \tag{20a}$$

$$\dot{v}_i = -d_{vi}v_i - p_{vi}\varphi(\theta_i(t))^\top \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j). \tag{20b}$$

Next, akin to V_1 in (12), we define the Lyapunov function

$$V_3(v, \bar{z}) := \frac{1}{2} \sum_{i \leq N} \Big[\frac{1}{p_{vi}} v_i^2 + \frac{1}{2} \sum_{j \in \mathcal{N}_i} a_{ij} |\bar{z}_i - \bar{z}_j|^2 \Big], \tag{21}$$

where $v := [v_1 \cdots v_N]^\top$ and $\bar{z} := [\bar{z}_1 \cdots \bar{z}_N]^\top$ —cf. (10). This function is positive definite and radially unbounded in the velocities v_i and the consensus errors. The total derivative of V_3 along the closed-loop trajectories of (20) yields

$$\dot{V}_3(v, \bar{z}) = -v^\top D_v P_v^{-1} v, \tag{22}$$

where $P_v := \text{diag}[p_{vi}]$ and $D_v := \text{diag}[d_{vi}]$.

Now, the system in (20) being non-autonomous, Barbashin-Krasovskĭi's theorem does not apply, but we may use Barbălat's Lemma [25] to conclude (after integrating on both sides of (22)) that $v_i \to 0$ and $\dot{v}_i \to 0$. In turn, from (20b), we see that

$$\lim_{t \to \infty} \varphi(\theta_i(t))^\top \sum_{j \in \mathcal{N}_i} a_{ij} \big(\bar{z}_i(t) - \bar{z}_j(t) \big) = 0.$$

This expression, however, does not imply that the consensus objective is reached. Indeed, note that the set of equilibria of the system in (20) corresponds to points belonging to the set

$$\mathcal{U} := \Big\{ v_i = 0 \quad \wedge \quad \varphi(\theta_i)^\top \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j) = 0 \Big\},$$

which admits points such that $\bar{z}_i \neq \bar{z}_j \in \mathbb{R}^2$ because $\text{rank}\, \varphi(\theta) = 1$. This means that if orientation consensus is reached and, for instance, $\theta_i(t) \to 0$ then $\bar{x}_i \to x_c$, but $\bar{y}_i \not\to y_c$ —see Eq. (3).

Remark 5: This shows that the consensus problem for nonholonomic systems cannot be treated as that for ordinary second-order systems —cf. [27]. •

To ensure consensus it is necessary that the set of equilibria correspond to the set $\mathcal{U} \cap \mathcal{U}^\perp$, where

$$\mathcal{U}^\perp := \Big\{ v_i = 0 \quad \wedge \quad \varphi(\theta_i)^{\perp\top} \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j) = 0 \Big\},$$

where $\varphi(\theta_i)^\perp := [-\sin(\theta_i) \quad \cos(\theta_i)]^\top$ is the annihilator of $\varphi(\theta_i)$ hence, $\varphi(\theta_i)^{\perp\top}\varphi(\theta_i) = \varphi(\theta_i)^\top \varphi(\theta_i)^\perp = 0$.

Roughly speaking, the controller must "pull" out non-equivalently-equal-to-zero trajectories that may remain "trapped" in $\mathcal{U}$ and away from $\mathcal{U}^\perp$. To that end, we endow the angular-motion controller with a term that incorporates an external function of time (smooth and bounded) and acts as a perturbation to the angular-motion closed-loop dynamics. This "perturbation" is designed to *persist* as long as

$$\varphi(\theta_i)^{\perp\top} \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j) \neq 0.$$

More precisely, let ψ_i, $\dot{\psi}_i$, and $\ddot{\psi}_i$ be bounded (belong to $\mathcal{L}_\infty$) let $\dot{\psi}_i$ be persistently exciting, i.e., let there exist T and $\mu > 0$ such that

$$\int_t^{t+T} \psi_i(s)^2 ds \geq \mu \quad \forall\, t \geq 0. \tag{23}$$

Then, for the control law in (18), we define

$$\alpha_i(t, \theta_i, \bar{z}_i) := k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top}(\bar{z}_i - \bar{z}_j), \quad k_{\alpha i} > 0. \tag{24}$$

Thus, while α_i injects excitation into the system, which ensures that the position consensus errors converge, it acts as a bounded (hence harmless) perturbation on the angular-motion dynamics —cf. (18). Indeed, the state-feedback controller defined by (18), (19), and (24) ensures full consensus, in position and orientation, for the closed-loop system, even in the presence of delays; this is shown in [24].

IV. Control architecture: output-feedback case

As in the case where state feedback is available, the output-feedback control design relies on the dichotomy of the system's dynamics (1)–(2). Let us consider, first, the angular-motion dynamics, (1). Note that, expressed as $\ddot{\theta}_i = u_{\omega i}$, this system corresponds to an elementary Newtonian force-balance equation with unitary inertia. The problem at hand still is to synchronize the angular positions θ_i for N such systems, but since ω_i is not available, we cannot use the control law in (18) —with $\alpha_i \equiv 0$—. Yet, it appears reasonable to conjecture that the objective $\theta_i \to \theta_j$ for all i, $j \leq N$ may be achieved by coupling the subsystems $\ddot{\theta}_i = u_{\omega i}$, via torsional springs, to virtual second-order oscillators for which the states are available and are *synchronized by design* —see Fig. 1 for an illustration.

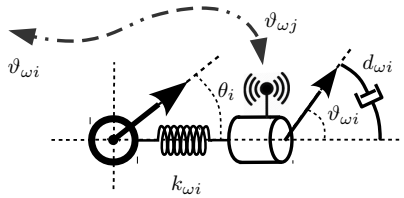

Fig. 1: Schematic representation of coupled mass-spring-damper systems: angular motion. It is the controller state variable, $\vartheta_{\omega i}$ that is transmitted to neighboring robots and, correspondingly, $\vartheta_{\omega j}$ is received from neighbors in the set $\mathcal{N}_i$.

More precisely, consider the dynamic system

$$\ddot{\vartheta}_{\omega i} + d_{\omega i}\dot{\vartheta}_{\omega i} + p_{\omega i}\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_{\omega i} - \vartheta_{\omega j}) = \nu_{\omega i} \qquad (25)$$

where $\nu_{\omega i}$ is an external input to be defined, the state $\vartheta_{\omega i} \in \mathbb{R}$, and $d_{\omega i}$, $p_{\omega i} > 0$.

As we showed in Section III-A, for (25) consensus is achieved, that is, there exists a real constant $\vartheta_{\omega c}$, such that $\vartheta_{\omega i} \to \vartheta_{\omega c}$, $\dot{\vartheta}_{\omega i} \to 0$, for all $i \leq N$, provided that $d_{\omega i}$, $p_{\omega i} > 0$, and $\nu_{\omega i} = 0$. On the other hand, the system in (25) defines a passive map $\nu_{\omega i} \mapsto \dot{\vartheta}_{\omega i}$. Furthermore, the system (1b) also defines a passive map, $u_{\omega i} \mapsto \omega_i$. Hence, it appears natural to hinge the systems (25) and (1) by setting

$$\nu_{\omega i} := -u_{\omega i}, \quad u_{\omega i} := -k_{\omega i}(\theta_i - \vartheta_{\omega i}), \quad k_{\omega i} > 0. \qquad (26)$$

That is, the coupling $-k_{\omega i}(\theta_i - \vartheta_{\omega i})$ may be interpreted as the force exerted by a torsional spring that hinges the (angular) positions of the two subsystems —again, see Fig. 1. Therefore, consensus among the angular positions θ_i is achieved indirectly by imposing consensus on the *dynamic controllers'* variables $\vartheta_{\omega i}$. Consensus among the plants' variables θ_i is achieved in view of the virtual mechanical coupling in (26). As a matter of fact, the control law in (26) is inspired by how joint flexibility in robot manipulators is modeled —cf. [28], [29] and the fact that consensus in the link positions may be achieved by applying a consensus control law on the actuator dynamics —cf. [5]. Then, we have the following original statement on output-feedback consensus control of second-order systems $\ddot{\theta}_i = u_i$.

Proposition 1 (Output feedback orientation consensus): Consider a group of differential-drive robots, each with dynamic model (1), in closed loop with the dynamic controller defined by (25), (26) and under Assumptions 1 and 2. Then, for any initial conditions $(\theta_{i\circ}, \omega_{i\circ}, \vartheta_{\omega i\circ}, \dot{\vartheta}_{\omega i\circ}) \in \mathbb{R}^4$ there exist constants θ_c and $\vartheta_c \in \mathbb{R}$ such that, for all i and $j \leq N$,

$$\lim_{t\to\infty}\theta_i(t) = \lim_{t\to\infty}\theta_j(t) = \theta_c, \qquad \lim_{t\to\infty}\omega_i(t) = 0,$$
$$\lim_{t\to\infty}\vartheta_{\omega i}(t) = \lim_{t\to\infty}\vartheta_{\omega j}(t) = \vartheta_c, \qquad \lim_{t\to\infty}\dot{\vartheta}_{\omega i}(t) = 0.$$

□

Proof: Consider the function

$$W_3(\dot{\vartheta}_\omega, \vartheta_\omega, \theta, \omega) := W_1(\vartheta_\omega, \dot{\vartheta}_\omega) + W_2(\theta, \omega, \vartheta_\omega),$$

where $\vartheta_\omega := [\vartheta_{\omega 1} \cdots \vartheta_{\omega N}]^\top$,

$$W_1(\vartheta_\omega, \dot{\vartheta}_\omega) := \frac{1}{2}\sum_{i\leq N}\Big[\frac{\dot{\vartheta}_{\omega i}^2}{p_{\omega i}} + \frac{1}{2}\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_{\omega i} - \vartheta_{\omega j})^2\Big], \qquad (27)$$

$$W_2(\theta, \omega, \vartheta_\omega) := \frac{1}{2}\sum_{i\leq N}\Big[\frac{\omega_i^2}{p_{\omega i}} + k_{\omega i}(\theta_i - \vartheta_{\omega i})^2\Big]. \qquad (28)$$

The function W_2 corresponds to the total energy of the mass-spring (closed-loop) system $\ddot{\theta}_i = -k_{\omega i}(\theta_i - \vartheta_{\omega i})$; the first term is the kinetic energy and the second the potential energy "stored" in the torsional spring of stiffness $k_{\omega i}$. Akin to V_3 in (21) and V_1 in (12), the function W_3 is positive definite and radially unbounded in the consensus errors and the velocities. The total derivative of W_3 along the closed-loop trajectories yields

$$\dot{W}_3(\dot{\vartheta}_\omega, \vartheta_\omega, \theta, \omega) = -\frac{1}{2}\sum_{i\leq N}\frac{d_{\omega i}}{p_{\omega i}}\dot{\vartheta}_{\omega i}^2. \qquad (29)$$

Then, the system being autonomous, we may invoke Barbashin-Krasovskĭi's theorem. First, we see that $\dot{W}_3 = 0$ if and only if $\dot{\vartheta}_{\omega i} = 0$. This implies that $\ddot{\vartheta}_{\omega i} = 0$ and $\vartheta_{\omega i} = \text{const}$ for all $i \leq N$. From (25) and $\nu_{\omega i} := k_{\omega i}(\theta_i - \vartheta_{\omega i})$ we conclude that $\theta_i = \text{const}$, i.e., $\omega_i = \dot{\omega}_i = 0$. In turn, from $\dot{\omega}_i = -k_{\omega i}(\theta_i - \vartheta_{\omega i}) = -\nu_{\omega i} = 0$ and (25) we obtain

$$\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_{\omega i} - \vartheta_{\omega j}) = 0 \quad \text{and} \quad \theta_i = \vartheta_{\omega i}\ \forall\, i, j \leq N.$$

After Assumption 2, it follows that the only solution to these equations is $\theta_i = \vartheta_{\omega i} = \vartheta_c$ for all i, $j \leq N$. ■

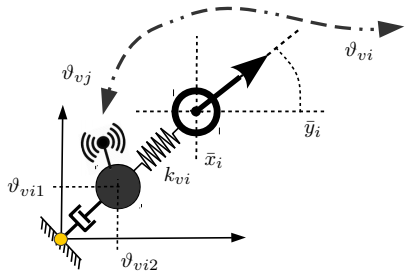

Fig. 2: Schematic representation of coupled mass-spring-damper systems: linear motion. The controller state variable ϑ_{vi} is sent to neighboring robots and, correspondingly, ϑ_{vj} is received from $\mathcal{N}_i$ neighbors.

Next, to steer the Cartesian positions $\bar{z}_i$ to a consensual point, we also use a second-order dynamic controller that is reminiscent of the equation (6) in closed loop with the control (7), and an added virtual-spring coupling term, $-k_{vi}(\vartheta_{vi} - \bar{z}_i)$. That is, let

$$\ddot{\vartheta}_{vi} + d_{vi}\dot{\vartheta}_{vi} + p_{vi}\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_{vi} - \vartheta_{vj}) = -k_{vi}(\vartheta_{vi} - \bar{z}_i), \qquad (30)$$

where $\vartheta_{vi} \in \mathbb{R}^2$ and $\dot{\vartheta}_{vi}$ are controller's state variables, and all control gains d_{vi}, p_{vi} and k_{vi} are positive.

Then, the dynamical system (30) is coupled to the double (nonholonomic) integrator (2). In contrast to the case of the angular motion, however, for the linear motion the control input u_{vi} must incorporate the change of coordinates defined by φ. Therefore, we define

$$u_{vi} := -\varphi(\theta_i)^\top k_{vi}(\bar{z}_i - \vartheta_{vi}), \quad k_{vi} > 0 \tag{31}$$

—cf. Eq. (26).

Thus, the controller (25)–(26) achieves consensus for the angular-motion dynamics (1) via output feedback while the controller (30)–(31) steers the linear-motion dynamics to consensus provided that $\varphi(\theta_i(t))$ is persistently exciting. To that end, as in Section III-B, we "perturb" the control law in (26) with an additional term α_i that is designed to *persist* as long as so do the synchronization errors e_{zi}.

V. Output Feedback Control under Delays

Based on (30) and (31), the controller for the linear-motion dynamics (2), in the presence of measurement delays, is given by

$$\begin{aligned} u_{vi} &= -k_{vi}\varphi(\theta_i)^\top(\bar{z}_i - \vartheta_{vi}), && \text{(32a)}\\ \ddot{\vartheta}_{vi} &= -d_{vi}\dot{\vartheta}_{vi} - k_{vi}(\vartheta_{vi} - \bar{z}_i) - p_{vi}e_{vi}, && \text{(32b)}\end{aligned}$$

where the linear position errors are given by

$$e_{vi} := \sum_{j\in\mathcal{N}_i} a_{ij}\big(\vartheta_{vi} - \vartheta_{vj}(t - T_{ji}(t))\big). \tag{33}$$

On the other hand, for the angular motion dynamics, we introduce

$$\begin{aligned} u_{\omega i} &= -k_{\omega i}(\theta_i - \vartheta_{\omega i}) + \alpha_i(t,\theta_i,e_{vi}), && \text{(34a)}\\ \ddot{\vartheta}_{\omega i} &= -d_{\omega i}\dot{\vartheta}_{\omega i} - k_{\omega i}(\vartheta_{\omega i} - \theta_i) - p_{\omega i}e_{\omega i}, && \text{(34b)}\end{aligned}$$

where

$$e_{\omega i} := \sum_{j\in\mathcal{N}_i} a_{ij}\big(\vartheta_{\omega i} - \vartheta_{\omega j}(t - T_{ji}(t))\big). \tag{35}$$

All constant parameters are defined as above. In addition, in order to be used with an output-feedback controller, the function α_i is redefined —cf. Eq. (24)— as

$$\alpha_i(t,\theta_i,e_{vi}) := k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top}(\vartheta_{vi} - \bar{z}_i), \tag{36}$$

where $k_{\alpha i} > 0$, ψ_i is twice differentiable, bounded, with bounded derivatives and $\dot{\psi}_i$ is persistently exciting —cf. Section III-B. That is, α_i in (34a) fulfills the same role as explained above. Then, our main statement is the following.

Proposition 2 (Main result): Consider the system (1)–(3), under Assumptions 1–3, in closed-loop with (32)-(36). Then, for any initial conditions $(\theta_{io}, \omega_{io}, \bar{z}_{io}, v_{io}, \vartheta_{vio}, \dot{\vartheta}_{vio}, \vartheta_{\omega io}, \dot{\vartheta}_{\omega io}) \in \mathbb{R}^{11}$, there exist constants $\theta_c \in \mathbb{R}$ and $z_c \in \mathbb{R}^2$ such that (4) and (5) hold, if

$$d_{vi} > \frac{1}{2}\,p_{vi}\sum_{j\in\mathcal{N}_i} a_{ij}\Big[\beta_i + \frac{\overline{T}_{ji}^2}{\beta_j}\Big] \tag{37}$$

$$d_{\omega i} > \frac{1}{2}\,p_{\omega i}\sum_{j\in\mathcal{N}_i} a_{ij}\Big[\varepsilon_i + \frac{\overline{T}_{ji}^2}{\varepsilon_j}\Big] \tag{38}$$

for some β_i, $\varepsilon_i > 0$, for all $i \le N$. □

The conditions (37) and (38) impose bounds on the controller's damping gains, depending on the bounds on the delays (and their derivatives). Note that these conditions are completely distributed; a different bound is required for each vehicle individually.

The arguments behind the statement of Proposition 2 rely on the observed dichotomy of the model (1)-(2) and the separate control designs for the linear- and angular-motion dynamics. Indeed, the resulting closed-loop equations have an underlaying cascaded structure:

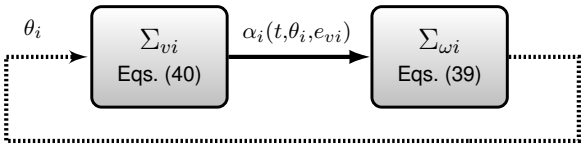

Fig. 3: Schematic representation of the closed-loop system . Even though the systems are feedback interconnected , they may be regarded as in cascade [26], whence the feedback represented by a dashed arrow.

$$\Sigma_{\omega i}: \begin{cases} \dot{\theta}_i = \omega_i & \text{(39a)}\\ \dot{\omega}_i = -k_{\omega i}(\theta_i - \vartheta_{\omega i}) + \alpha_i(t,\theta_i,e_{vi}) & \text{(39b)}\\ \ddot{\vartheta}_{\omega i} = -d_{\omega i}\dot{\vartheta}_{\omega i} - k_{\omega i}(\vartheta_{\omega i} - \theta_i) - p_{\omega i}e_{\omega i} & \text{(39c)}\end{cases}$$

$$\Sigma_{vi}: \begin{cases} \dot{\bar{z}}_i = \varphi(\theta_i)v_i & \text{(40a)}\\ \dot{v}_i = -k_{vi}\varphi(\theta_i)^\top(\bar{z}_i - \vartheta_{vi}) & \text{(40b)}\\ \ddot{\vartheta}_{vi} = -d_{vi}\dot{\vartheta}_{vi} - k_{vi}(\vartheta_{vi} - \bar{z}_i) - p_{vi}e_{vi} & \text{(40c)}\end{cases}$$

As illustrated in Fig. 3, for each robot, the closed-loop equations consist of two dynamical feedback-interconnected systems. However, as explained in Section III-B, by replacing the state variables θ_i with fixed, but arbitrary, trajectories $\theta_i(t)$ in (40a) and (40b), these systems may be considered as if interconnected in cascade. Then, the analysis of $\Sigma_{\omega i}$ and Σ_{vi} may be carried out using arguments for such systems [26]. In a nutshell, one needs to establish that: 1) all trajectories are bounded; 2) for $\Sigma_{\omega i}$ with $\alpha_i \equiv 0$ the consensus errors converge to zero —cf. Proposition 1; 3) For Σ_{vi} the consensus errors converge, under the persistently exciting effect of α_i. A detailed proof is provided in [30], but for completeness we provide a sketch of proof in the Appendix.

VI. Simulation Results

We used the simulator Gazebo-ROS and the Robot Operating System (ROS) interface to evaluate the performance of our controller in a scenario that reproduces as closely as possible that of a laboratory experimental benchmark. We employed the model of a PIONEER 3-DX wheeled robot [31], available in Gazebo's library. For simplicity, it is assumed that all the robots have the same inertial and geometrical parameters given by $m = 5.64$ kg, $I = 3.115$ kg·m^2, $r = 0.09$ m and $R = 0.157$ m. It must be underlined that for this robot the center of mass is not located on the axis joining the two wheels'. Consequently, in this case, the Coriolis terms $\frac{r_i}{3}\omega_i^2$ and $-\frac{r_i m_i}{3I_i}\omega_i v_i$ appear on the left-hand side of Eqs. (1b) and (2b) respectively. Akin to an actual experimentation set-up, these constitute dynamic effects not considered in the model for which the controller is designed.

The six PIONEER 3D-X robots communicate over the undirected connected graph like the one illustrated in Fig. 4, below.

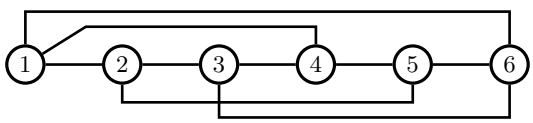

Fig. 4: Communication topology: undirected connected graph

Then, to emulate the time-varying delays $T_{ji}(t)$, which are different for each pair of robots, we use randomly generated signals following a normal distribution with mean $\mu = 0.3$, variance $\sigma^2 = 0.0003$ and a sample time of 10 ms —see Fig. 5 for the illustration of one of such delays. Such time delay (non-smooth but piece-wise continuous) does not satisfy Assumption 3 since its time-derivative is bounded only almost everywhere (that is, except at the points of discontinuity). However, it is considered in the simulations since it is closer to what is encountered in a real-world set-up.

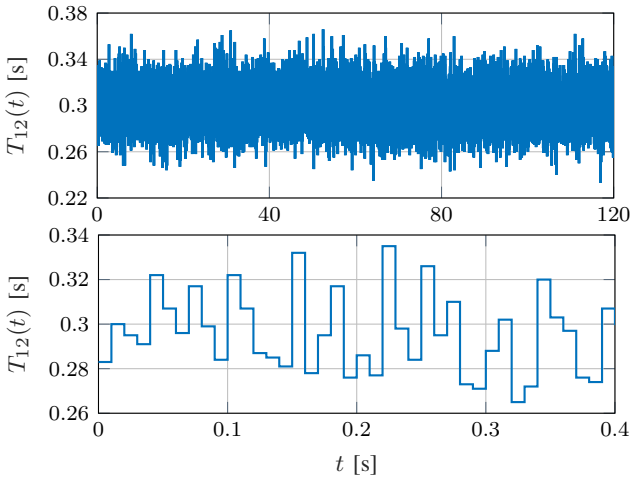

Fig. 5: Variable delay between the robot 1 and the received information from neighbor 2.

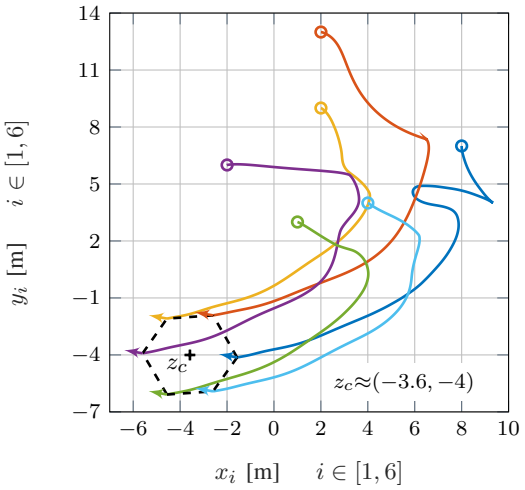

Fig. 6: Paths followed by the PIONEER 3D-X robots up to full formation consensus —Gazebo-ROS simulation. A hexagonal formation is achieved with coinciding orientations (illustrated by arrows).

The initial postures of the robots are given in the 2nd-4th columns of Table I, below.

TABLE I: Initial conditions and ofsets

Index	x_i **[m]**	y_i **[m]**	θ_i **[rad]**	δ_{xi} **[m]**	δ_{yi} **[m]**
1	8	7	1.57	2	0
2	2	13	0.0	1	2
3	2	9	-0.39	-1	2
4	-2	6	0.39	-2	0
5	1	3	-0.39	-1	-2
6	4	4	-0.39	1	-2

The desired formation at rendezvous corresponds to a hexagon determined by desired offsets $\delta_i = (\delta_{xi}, \delta_{yi})$ with respect the unknown center of the formation. These constants are presented in the last two columns of Table I.

The control gains were set to $k_{vi} = 1$, $k_{\omega i} = 2$, $d_{vi} = 3$, $p_{vi} = 0.4$, $d_{\omega i} = 2$, $p_{\omega i} = 0.1$, for all $i \in [1, 6]$. These values correspond to magnitudes compatible with the emulated physics of the PIONEER 3D-X robots in Gazebo-ROS and are chosen so that the poles of the 2nd-order system $\ddot{x} = -d_{(\cdot)}\dot{x} - p_{(\cdot)}x$ have negative real parts and the system have an over-damped step-response. The functions α_i were taken as in (36) with $k_{\alpha i} = 0.4$ and using the following multi-periodic function (any persistently-exciting function applies):

$$\psi_i(t) = 2.5 + \sin(2\pi t) + 0.3\cos(6\pi t) - 0.5\sin(8\pi t) \\ -0.1\cos(10\pi t) + \sin(\pi t) \quad \forall\, i \leq 6. \tag{41}$$

The robots appear to achieve consensus, i.e., to meet at a non-predefined rendezvous point in hexagonal formation and with common non-predefined orientation —see Fig. 6. The center of the formation is located at $(-3.6, -4)$ and the consensual orientations settle at $\theta_c \approx -2.932$ rad. Note that the center of the formation and the common orientation do not correspond to the average of the vehicles' initial conditions. Hence, the simulations illustrate that for networks of nonholonomic vehicles, the initial conditions do not determine the consensus point, as is the case of linear systems interconnected over static undirected connected graphs.

In addition, for the purpose of graphic illustration, following [3], we define the following synchronization errors, as the difference between each robot's variables and the corresponding average:

$$e_{zi} := \bar{z}_i - \frac{1}{N}\sum_{j\in\mathcal{N}_i} \bar{z}_j, \quad e_{\theta i} := \theta_i - \frac{1}{N}\sum_{j\in\mathcal{N}_i} \theta_j, \tag{42}$$

That is, the limits in (4) and (5) hold if the error trajectories $e_{zi}(t)$ and $e_{\theta i}(t)$ as defined above converge to zero, but the errors in (42) do not correspond to variables actually used by the controller nor measured for that matter. These errors are illustrated in Fig. 7 below.

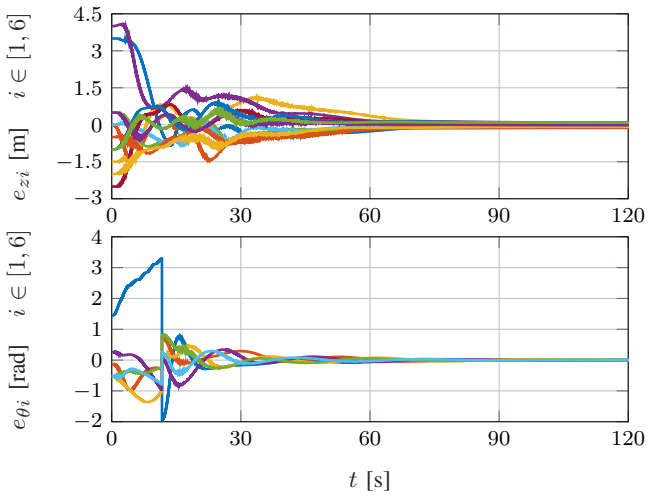

Fig. 7: Consensus errors (top: position errors, bottom: orientation errors)

From the top plot in Fig. 7 one can appreciate that the position synchronization errors e_{zi} tend to a steady state-error —a keen observer will notice that the hexagon in Fig. 6 is actually not quite so. The reason is that in the Gazebo-ROS simulation, after a transient, the amplitude of the input torques becomes considerably small in absolute value —see Fig. 8 below. Now, the presence of a steady-state error and the persistency-of-excitation effect in the controller maintain the input torques oscillating (periodically in this case due to the choice of $\psi_i(t)$ in (41)), but, *physically*, they result insufficient to overcome the robots' inertia and friction forces that oppose their forward and angular motions. It seems fitting to say that in numerical simulations using Matlab, hence without considering the same physical phenomena, it may be appreciated that the synchronization errors tend to zero asymptotically and so do the control torques —see [30]. Also, we emphasize that in the Gazebo-ROS environment θ_i is defined in $(-\pi, \pi]$ to avoid unwinding, whence the apparent discontinuity appreciated in the bottom plot of Fig. 7.

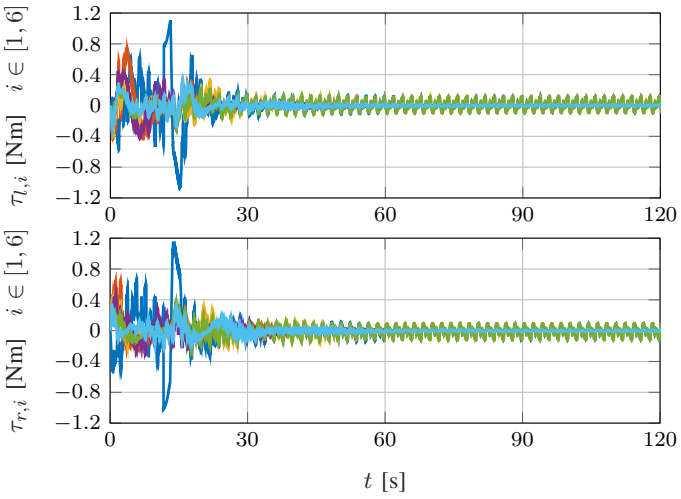

Fig. 8: Input torques — Gazebo-ROS simulation.

VII. Conclusions

The dynamic output-feedback controller for rendezvous of differential-drive robots that we propose has the neat physical interpretation of a second-order mechanical system itself and performs well even in the presence of discontinuous time-varying delays. Some readers may see a resemblance of our angular-motion controller with an observer, since $\vartheta_{\omega i} \to \theta_i$. However, note that one does not necessarily have that $\dot{\vartheta}_{\omega i} \to \omega_i$. This being said, there remains the question of to what extent our control strategy may serve as a (partial) state estimator in other output-feedback control problems.

Even though the assumptions that our main results rely on are somewhat realistic, there are other hypotheses whose relaxing needs further study. For instance, the study of multiagent nonholonomic vehicles interconnected over directed graphs is a significant problem under investigation. So is the formal analysis of the controller without assuming differentiability of the time-varying delays.

Also, the numerical tests using the Gazebo-ROS simulator clearly show the effects of the nonlinearities in the consensus. This is an intriguing aspect to investigate further, notably by extending our main statements to hold for nonholonomic systems modeled by second-order systems that include Coriolis terms.

Appendix: Sketch of Proof of Proposition 2

First, we analyze the linear-motion dynamics, Σ_v.

Claim 1 (Boundedness): For the system Σ_{vi} in (40), the following holds true: $\dot{\vartheta}_{vi} \in \mathcal{L}_2$ whereas v_i, $\dot{\vartheta}_{vi}$, $|\vartheta_{vi} - \bar{z}_i|$, and $|\vartheta_{vi} - \vartheta_{vj}|$, all belong to $\mathcal{L}_\infty$ (are bounded). Furthermore, also $e_{vi} \in \mathcal{L}_\infty$ and, consequently, $\ddot{\vartheta}_{vi} \in \mathcal{L}_\infty$ too. Finally, $\dot{\vartheta}_{vi} \to 0$.

The proof of Claim 1 relies on Barbălat's Lemma and the energy-based Lyapunov-Krasovskĭi functional

$$\mathcal{V} := \sum_{i\le N}\left[\frac{1}{p_{vi}}\mathcal{E}_i + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}|\vartheta_{vi}-\vartheta_{vj}|^2 + \Upsilon_i^v\right] \tag{43a}$$

$$\mathcal{E}_i := \frac{1}{2}\left[v_i^2 + |\dot{\vartheta}_{vi}|^2 + k_{vi}|\vartheta_{vi}-\bar{z}_i|^2\right] \tag{43b}$$

$$\Upsilon_i^v := \frac{1}{2\beta_i}\sum_{j\in\mathcal{N}_i} a_{ij}\overline{T}_{ji}\int_{-\overline{T}_{ji}}^{0}\int_{t+\eta}^{t}|\dot{\vartheta}_{vj}(\sigma)|^2 d\sigma d\eta, \tag{43c}$$

where $\beta_i > 0$ is introduced in the statement of Proposition 2. The function $\mathcal{E}_i$ is reminiscent of an energy function; the first two quadratic-in-velocities terms on the right-hand side may be assimilated to kinetic energy terms while the third corresponds to a potential-energy term associated to the springs with stiffness k_{vi} —see Fig. 2. The second term in the definition of $\mathcal{V}$ is quadratic in the consensus errors and it is equivalent to $\tilde{\vartheta}_{vi}^\top L\tilde{\vartheta}_{vi}$ with $\tilde{\vartheta}_{vi}$ defined as in (7) —cf. V_1 in (12). Finally, because the closed-loop system is in the form of a set of functional differential equations, we use a so-called Lyapunov-Krasovskĭi functional term Υ_i^v. Now, the total derivative of $\mathcal{V}$ along the trajectories of (40) yields

$$\dot{\mathcal{V}} \le -\sum_{i\le N}\left[\frac{d_{vi}}{p_{vi}} - \sum_{j\in\mathcal{N}_i} a_{ij}\left[\frac{\beta_i}{2} + \frac{\overline{T}_{ji}^2}{2\beta_j}\right]\right]|\dot{\vartheta}_{vi}|^2. \tag{44}$$

Note that the factor of $|\dot{\vartheta}_{vi}|^2$ for any $i \le N$ is positive under condition (37). The claim follows from integrating along trajectories on both sides of (44) and invoking Barbălat's Lemma and [32, Theorem 5].

Next, we analyze the trajectories of Σ_ω subject to $\alpha_i \equiv 0$.

Claim 2: Under the condition that $\alpha_i \equiv 0$ the trajectories of $\Sigma_{\omega i}$ satisfy the following: $\dot{\vartheta}_{\omega i} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ whereas ω_i, $[\vartheta_{\omega i} - \theta_i]$, $[\vartheta_{\omega i} - \vartheta_{\omega j}]$, and $e_{\omega i} \in \mathcal{L}_\infty$. Finally, $\dot{\vartheta}_{\omega i} \to 0$ asymptotically.

The proof of Claim 2 relies on the Lyapunov-Krasovskĭi functional

$$\mathcal{W} := \sum_{i\le N}\left[\frac{1}{p_{\omega i}}\mathcal{H}_i + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}(\vartheta_{\omega i}-\vartheta_{\omega j})^2 + \Upsilon_i^\omega\right] \tag{45a}$$

$$\mathcal{H}_i := \frac{1}{2}\left[\omega_i^2 + \dot{\vartheta}_{\omega i}^2 + k_{\omega i}(\vartheta_{\omega i}-\theta_i)^2\right], \tag{45b}$$

where $\varepsilon_i > 0$ —cf. Eq. (43c) and Υ_i^ω is defined as in (43c), replacing β_i with ε_i and ϑ_{vj} with $\vartheta_{\omega j}$. Its derivative along the trajectories of $\Sigma_{\omega i}$ with $\alpha_i \equiv 0$ satisfies

$$\dot{\mathcal{W}} \le -\sum_{i\le N}\left[\frac{d_{\omega i}}{p_{\omega i}} - \sum_{j\in\mathcal{N}_i} a_{ij}\left[\frac{\varepsilon_i}{2} + \frac{\overline{T}_{ji}^2}{2\varepsilon_j}\right]\right]\dot{\vartheta}_{\omega i}^2, \tag{46}$$

in which the factor of $\dot{\vartheta}_{\omega i}^2$ is positive in view of the condition (38). The first part of the Claim follows by integrating on both sides of the inequality in (46). Then, a simple inspection of (39) reveals that $\dot{\omega}_i$ and $\ddot{\vartheta}_{\omega i}$ are also bounded (belong to $\mathcal{L}_\infty$), so after Barbălat's Lemma it follows that $\dot{\vartheta}_{\omega i} \to 0$. In addition, by successive derivations it also follows that $\theta_i \to \theta_c$ and $[\theta_i - \vartheta_{\omega i}] \to 0$.

Now, the previous statements regarding Σ_ω hold under the restriction that $\alpha_i \equiv 0$. If this is not the case, we remark that since, ψ_i, $\vartheta_{vi} - \bar{z}_i$ and $\varphi^\perp$ are bounded along all trajectories, so is α_i — see Eq. (36). This and the fact that Σ_ω is a marginally stable linear time-varying system with uniformly bounded time-delays, imply that $\dot{\omega}_i, \omega_i, \dot{\vartheta}_{\omega i}, \ddot{\vartheta}_{\omega i} \in \mathcal{L}_\infty$ and so $[\theta_i - \vartheta_{\omega i}]$ and $e_{\omega i}$ are also bounded —cf. [33, Proposition 3]. Moreover, the limits in (5) follow from the marginal stability of $\Sigma_{\omega i}$ with $\alpha_i \equiv 0$ and, once more, invoking [33, Proposition 3], provided that $\alpha_i \to 0$. In turn, the latter follows from the following statement that holds under the condition of persistency of excitation (23) and whose proof is provided farther below.

Claim 3: The term $(\bar{z}_i - \vartheta_{vi})$ converges to zero asymptotically.

Now we establish the first limit in (4). We argue as follows. Since φ is uniformly bounded, from (40a) and (40b) we have that $\dot{\bar{z}}_i \in \mathcal{L}_\infty$ and $\dot{v}_i \in \mathcal{L}_\infty$. Moreover, from Assumption 3 (in view of the boundedness of $\dot{T}_{ji}$ and $\ddot{T}_{ji}$), and the fact that $\ddot{\vartheta}_{vi}, \dot{\vartheta}_{vi}, \dot{\bar{z}}_i \in \mathcal{L}_\infty$ it holds that $\vartheta_{vi}^{(3)}$ and $\vartheta_{vi}^{(4)}$ are bounded too, so by Barbălat's Lemma, $\ddot{\vartheta}_{vi} \to 0$ and $\vartheta_{vi}^{(3)} \to 0$. From this it is concluded that $\dot{\bar{z}}_i \to 0$. Hence $\lim_{t\to\infty} v_i(t) = 0$, which corresponds to the first limit in (4).

Now, the second limit in (4) is implied by $\dot{\vartheta}_{vi}, \ddot{\vartheta}_{vi}$, and $[\bar{z}_i - \vartheta_{vi}]$ converging to zero (for the latter see Claim 3 above). Indeed, if $\ddot{\vartheta}_{vi}$, $\dot{\vartheta}_{vi}$, and $\bar{z}_i - \vartheta_{vi} \to 0$, from the first equation in (40), it follows that also e_{vi} converges to zero and this in turn implies that the second limit in (4) holds. This comes from the fact that $e_{vi} = 0$ and $\dot{\vartheta}_{vi} = 0$

imply that $(L \otimes I_2)\vartheta_v = 0$ which, in view of the properties of L, implies the existence of $\bar{z}_c \in \mathbb{R}^2$ such that $\vartheta_v = 1_N \otimes \bar{z}_c$, or $\vartheta_{vi} = \bar{z}_c$ for all $i \leq N$. Hence, from the third equation in (40), we see that if $\ddot{\vartheta}_{vi}$, $\dot{\vartheta}_{vi}$, and $e_{vi} \to 0$, then $\lim_{t\to\infty} \bar{z}_i(t) = \lim_{t\to\infty} \vartheta_{vi}(t)$ and $\lim_{t\to\infty} \bar{z}_i(t) = \bar{z}_c$. The statement follows.

Proof of Claim 3: From the second equation in (40) it follows that, since $\dot{v}_i$, $\ddot{v}_i$, $\dot{\vartheta}_{vi}$, and $v_i \to 0$,

$$\varphi(\theta_i)^\top(\bar{z}_i - \vartheta_{vi}) \to 0, \quad \omega_i \varphi(\theta_i)^{\perp\top}(\bar{z}_i - \vartheta_{vi}) \to 0. \tag{47}$$

On the other hand, the solutions of the equation

$$\varphi(\theta_i)^\top(\bar{z}_i - \vartheta_{vi}) = 0 \tag{48}$$

are of the form $(\bar{z}_i - \vartheta_{vi}) = c_1\varphi(\theta_i)^\perp$ with $c_1 \in \mathbb{R}$ while the solutions of the equation

$$\omega_i \varphi(\theta_i)^{\perp\top}(\bar{z}_i - \vartheta_{vi}) = 0 \tag{49}$$

are of the form $(\bar{z}_i - \vartheta_{vi}) = c_2\omega_i\varphi(\theta_i)$ with $c_2 \in \mathbb{R}$. Therefore, (48) and (49) hold together if and only if $c\omega_i\varphi(\theta_i) = \varphi(\theta_i)^\perp$ with $c := c_1/c_2$. In turn, the latter may hold only if either $c = 0$ or $\omega_i = 0$. Now, if $c = 0$ then $(\bar{z}_i - \vartheta_{vi}) = 0$. Thus, (47) imply that either $(\bar{z}_i - \vartheta_{vi}) \to 0$, which is to be showed, or $\omega_i \to 0$. In the latter case $\lim_{t\to\infty} \int_0^t \dot{\omega}(s)ds = -\omega_{i\circ} < \infty$ and since $\ddot{\omega}_i \in \mathcal{L}_\infty$ we obtain, from Barbălat's Lemma, that $\dot{\omega}_i \to 0$. From a similar argument we conclude that $\ddot{\omega}_i \to 0$. Next, we show that $\omega_i, \dot{\omega}_i, \ddot{\omega}_i \to 0$ and $\dot{\psi}_i \not\to 0$ —see (36)— imply together that $(\bar{z}_i - \vartheta_{vi}) \to 0$, so the proof ends. To that aim, we recall that the total derivative of $\mathcal{W}$ in (45a) along the trajectories of Σ_ω satisfies $\dot{\mathcal{W}} \leq -\sum_{i\leq N}\left[\lambda_{\omega i}\dot{\vartheta}_{\omega i}^2 - \frac{1}{p_{\omega i}}\alpha_i\omega_i\right] =: -\Psi$. Now, integrating on both sides of the inequality $\Psi \leq -\dot{\mathcal{W}}$, along the system's trajectories, and using the boundedness of $\mathcal{W}$, we see that $\Psi \in \mathcal{L}_1$. Furthermore, all the terms defining $\dot{\Psi}$ depend, also, on bounded functions of time, so $\dot{\Psi} \in \mathcal{L}_\infty$. It follows, after Theorem 5 in [32], that $\Psi \to 0$. Since $\alpha_i \in \mathcal{L}_\infty$ and $\omega_i \to 0$ it follows, in turn, that $\dot{\vartheta}_{\omega i} \to 0$. Therefore, since ω_i, $\dot{\omega}_i$, $\ddot{\omega}_i$, $\dot{\vartheta}_{\omega i}$, $\dot{\vartheta}_{vi} \to 0$, all the terms in the definition of $\ddot{\omega}_i$ tend to zero but in view of (47) and $\dot{\psi}_i \not\to 0$ we necessarily have $(\vartheta_{vi} - \bar{z}_i) \to 0$. •

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