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n-Cyclic Refined Neutrosophic Groups

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Abstract

This paper introduces for the first time the concept of n-cyclic refined neutrosophic group as a direct application of the concept of n-cyclic refined neutrosophic set. Also, it discusses some of its elementary properties such as AH-subgroups, kernels, and direct products.

Keywords: n-cyclic refined neutrosophic group, AH-subgroup, AH-homomorphism

1. Introduction

Neutrosophy is a philosophical concept founded by Smarandache to generalize the fuzzy logic [1,32]. Neutrosophic sets were very applicative in many areas of science such as topology [2,27,60,61,63], decision making [21,35], applied mathematics [3,4,20,25,28,57,64], and pure mathematics [9,10,19,39,43,44,47,50,51,52].

Neutrosophic sets were used in the study of algebraic structures such as modules [6], rings [12], groups [13], and matrices [6,53,62]. In the literature, we find three interesting generalizations of neutrosophic sets, where refined neutrosophic sets [13] n-refined neutrosophic sets [15,50], and n-cyclic refined neutrosophic sets [31]. These kinds lead us to many interesting generalizations of classical algebraic structures such as n-refined neutrosophic modules [5], n-refined spaces and matrices [8,46,54], refined neutrosophic ring [23,48,49], and n-cyclic refined neutrosophic modules [31].

In this work, we use the concept of n-cyclic refined neutrosophic set to defined n-cyclic refined neutrosophic groups. These groups will be studied carefully, and many elementary properties will be discussed through this paper, especially AH-subgroups, kernels, and homomorphisms.

2. Preliminaries

Definition 2.1: [31]

Let $(R, +, \times)$ be a ring and $I_k; 1 \leq k \leq n$ be n indeterminacies. We define $R_n(I) = \{a_0 + a_1I + \dots + a_nI_n; a_i \in R\}$ to be n-cyclic refined neutrosophic ring.

Operations on $R_n(I)$ are defined as:

$$\sum_{i=0}^n x_i I_i + \sum_{i=0}^n y_i I_i = \sum_{i=0}^n (x_i + y_i) I_i, \quad \sum_{i=0}^n x_i I_i \times \sum_{i=0}^n y_i I_i = \sum_{i,j=0}^n (x_i \times y_j) I_i I_j = \sum_{i,j=0}^n (x_i \times y_j) I_{(i+j \bmod n)}$$

Where $\times$ is the multiplication on the ring R.

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It is obvious that $R_n(I)$ is a ring in the algebraic ordinary concept.

Definition 2.2: [31]

Let $R_n(I)$ be an n-cyclic refined neutrosophic ring, it is called commutative if $xy = yx$ for all $x, y \in R_n(I)$. If there is $1 \in R_n(I)$ such that $1.x = x.1 = x$, then it is called an n-cyclic refined neutrosophic ring with unity.

Definition 2.3: [31]

Let $R_n(I)$ be a commutative n-cyclic refined neutrosophic ring and $P: R_n(I) \rightarrow R_n(I)$ is a function defined as $P(x) = \sum_{i=0}^m a_i x^i$ such that $a_i \in R_n(I)$, we call P an n-cyclic refined neutrosophic polynomial on $R_n(I)$.

We denote by $R_n(I)[x]$ to be the ring of n-cyclic refined neutrosophic polynomials over $R_n(I)$.

Since $R_n(I)$ is a classical ring, then $R_n(I)[x]$ is a classical ring.

Definition 2.4 : [31]

Let $(M, +, \cdot)$ be a module over the ring R, we say that $M_n(I) = M + MI_1 + \dots + MI_n = \{x_0 + x_1 I_1 + \dots + x_n I_n; x_i \in M\}$ is a weak n-cyclic refined neutrosophic module over the ring R. Elements of $M_n(I)$ are called n-cyclic refined neutrosophic vectors, elements of R are called scalars.

If we take scalars from the n-cyclic refined neutrosophic ring $R_n(I)$, we say that $M_n(I)$ is a strong n-cyclic refined neutrosophic module over the n-cyclic refined neutrosophic ring $R_n(I)$. Elements of $M_n(I)$ are called n-refined neutrosophic scalars.

Remark 2.5: [31]

Addition on $M_n(I)$ is defined as:

$$\sum_{i=0}^n a_i I_i + \sum_{i=0}^n b_i I_i = \sum_{i=0}^n (a_i + b_i) I_i.$$

Multiplication by a scalar $m \in R$ is defined as:

$$m. \sum_{i=0}^n a_i I_i = \sum_{i=0}^n (m. a_i) I_i.$$

Multiplication by an n-cyclic refined neutrosophic scalar $m = \sum_{i=0}^n m_i I_i \in R_n(I)$ is defined as:

$$\sum_{i=0}^n m_i I_i . \sum_{i=0}^n a_i I_i = \sum_{i=0}^n (m_i . a_i) I_i I_j.$$

Where $a_i \in M, m_i \in R, I_i I_j = I_{(i+j \bmod n)}$.

3. Main Concepts and discussion

Definition 3.1:

Let I be the neutrosophic element, which refers to indeterminacy, we define n-cyclic refining system of I by the set $L = \{I^1 = I, I^2, \dots, I^{n-1}\}$, where $I^i \neq I^j$ for all $i \neq j$ and $i, j < n$.

We define a binary operation on L as follows:

$I^j . I^i = I^{i+j \pmod n}$. It is clear that L has a structure of the subset $(Z_n / \{0\}, +)$, where Z_n is the additive group of integers modulo n.

Definition 3.2:

Let $(G, *)$ be a group, and I is the neutrosophic element with property $I^n = I$; $n \geq 2$, with

$I^i \neq I^j$ for all $i \neq j$ and $i, j < n$. We call $M(G) = G \cup GI \cup GI^2 \cup \dots \cup GI^{n-1}$ an n -cyclic refined neutrosophic group.

It is easy to see that 2-cyclic refined neutrosophic group is the classical neutrosophic group.

Remark 3.3:

The sets GI^k are groups under the binary operation $(xI^k)(yI^k) = (xy)I^k$; $k < n$ with identity I^k and each one of them must be isomorphic to G .

Definition 3.4:

Let $M(G)$ be an n -cyclic refined neutrosophic group, H be a subset of $M(G)$. We call H an AH-subgroup if $H = H_0 \cup H_1I \cup H_2I^2 \cup \dots \cup H_{n-1}I^{n-1}$, where H_i is a subgroup of G for all i .

We call H an AHS-subgroup if $H_0 = H_1 = \dots = H_{n-1}$.

We call H an AH-normal if H_i is normal subgroup of G for all i .

We call H an AHS-normal if it is AHS-subgroup and AH-normal.

Definition 3.5 :

Let $M(G)$, $M(H)$ be two n , m -cyclic refined neutrosophic groups respectively, $f: M(G) \rightarrow M(H)$ be a map, we say that f is an AH-homomorphism if it is a homomorphism between G , H i.e. $f(xy) = f(x)f(y) \forall x, y \in G$ and $f(I^k) = (I')^k$ such I' is the neutrosophic element of H .

We define $AH - Ker(f) = Ker f_G \cup Ker f_G I \cup \dots \cup Ker f_G I^{n-1}$, we regard that $AH - Ker(f)$ is an AHS-normal subgroup of $M(G)$

We say that f is an isomorphism if it is a correspondence one-to-one homomorphism.

If $H = H_0 \cup H_1I \cup \dots \cup H_{n-1}I^{n-1}$ and $K = K_0 \cup K_1I \cup \dots \cup K_{n-1}I^{n-1}$ are two AH-subgroups of $M(G)$. We say that they are isomorphic if $H_i \cong K_i$ for all i .

Definition 3.6 :

Let H , K be two AH-subgroups of $M(G)$. We define

$$HK = H_0K_0 \cup H_1K_1I \cup \dots \cup H_{n-1}K_{n-1}I^{n-1}.$$

Definition 3.7 :

Let $M(G)$ be an n -cyclic refined neutrosophic group, $H = H_0 \cup H_1I \cup H_2I^2 \cup \dots \cup H_{n-1}I^{n-1}$ be an AH-normal subgroup of $M(G)$. We define the corresponding AH-factor as $M(G)/H = (G/H_0) \cup (G/H_1)I \cup \dots \cup (G/H_{n-1})I^{n-1}$.

Definition 3.8 :

Let $M(G)$ be an n -cyclic refined neutrosophic group. We define the AH-center of $M(G)$ by

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$$Z(M(G)) = Z(G) \cup Z(G)I \cup \dots \cup Z(G)I^{n-1}.$$

It is easy to see that $Z(M(G))$ is an AHS-normal subgroup of $M(G)$.

Definition 3.9 :

Let $M(G)$ be an n -cyclic refined neutrosophic group. We say that $M(G)$ is abelian if G is abelian, i.e. $M(G) = Z(M(G))$.

$M(G)$ is said to be cyclic if G is cyclic.

Theorem 3.10 :

Let $M(G)$ be an n -cyclic refined neutrosophic group, then

- (a) If H is an AH-normal subgroup and $M(G)$ is abelian, then $M(G)/H$ is abelian.
- (b) If $M(G)$ is finite and H is an AHS-subgroup, then $O(H)$ divides $O(M(G)) = n O(G)$.
- (c) If H is an AH-normal subgroup and $M(G)$ is cyclic then $M(G)/H$ is cyclic.

Proof:

- (a) Since $M(G)/H = (G/H_0) \cup (G/H_1)I \cup \dots \cup (G/H_{n-1})I^{n-1}$ and G/H_i is abelian for all i , then $M(G)/H$ is abelian.
- (b) We have that $O(H) = n O(H_0)$ and $O(H_0)$ divides the order of G then $O(H)$ divides $O(M(G)) = n O(G)$.
- (c) Since G/H_i is cyclic for all i then $M(G)/H$ is cyclic.

Theorem 3.11 :

Let $M(G)$ be an n -cyclic refined neutrosophic group and H, K be two AH-subgroups, then

- (a) $H \cap K$ is an AH-subgroup.
- (b) If H, K are AHS-subgroups, then $H \cap K$ is an AHS-subgroup.
- (c) If H, K are AH-normal subgroups, then $H \cap K$ and HK are AH-normal subgroups.
- (d) If H, K are AHS-normal subgroups, then $H \cap K$ and HK are AHS-normal subgroups.

Proof : Suppose that $H = H_0 \cup H_1I \cup \dots \cup H_{n-1}I^{n-1}$ and $K = K_0 \cup K_1I \cup \dots \cup K_{n-1}I^{n-1}$ then $H \cap K = (H_0 \cap K_0) \cup (H_1 \cap K_1)I \cup \dots \cup (H_{n-1} \cap K_{n-1})I^{n-1}$ by this argument we can easily find that the proof holds.

Theorem 3.12 :

Let $M(G), M(H)$ be two n, m -cyclic refined neutrosophic groups respectively, and $f: M(G) \rightarrow M(H)$ be a homomorphism, then

- (a) $n \geq m$.
- (b) If K is an AH-subgroup of $M(G)$ then $f(K)$ is an AH-subgroup of $M(H)$.
- (c) If K is an AHS-subgroup of $M(G)$ then $f(K)$ is an AHS-subgroup of $M(H)$.

(d) If K is an AH-normal subgroup of $M(G)$ then $f(K)$ is an AH-normal subgroup of $f(M(G))$.

(e) If K is an AHS-normal subgroup of $M(G)$ then $f(K)$ is an AHS-normal subgroup of $f(M(G))$.

(f) $M(G)/\text{Ker}f \cong f(M(G))$.

Proof :

(a) Suppose that $n < m$ then $I' = f(I) = f(I^n) = (I')^n$ and this is a contradiction thus $n \geq m$.

(b) Suppose that $K = K_0 \cup K_1 I \cup \dots \cup K_{n-1} I^{n-1}$, then $f(K) = f(K_0) \cup f(K_1) I \cup \dots \cup f(K_{n-1}) I^{n-1}$ with subgroups $f(K_i)$ for all i of $M(H)$, so that $f(K)$ is an AH-subgroup of $M(H)$.

(c) It is obvious that if $K_i = K_j$, then $f(K_i) \cong f(K_j)$, thus $f(K)$ is an AHS-subgroup of $M(H)$.

(d), (e) hold directly from (b) and (c) and from the fact that if K_i is normal, then $f(K_i)$ is normal.

(f) From the definition, we find $M(G)/\text{Ker}f = (G/\text{Ker}f_G) \cup (G/\text{Ker}f_G)I \cup \dots \cup (G/\text{Ker}f_G)I^{n-1}$; but $G/\text{Ker}f_G \cong f(G)$, thus $M(G)/\text{Ker}f \cong f(G) \cup f(G)I \cup \dots \cup f(G)I^{n-1} = f(M(G))$.

Theorem 3.13 :

Let $M(G)$ be an n -cyclic refined neutrosophic group and H, K be two AH-normal subgroups with $K \leq H$, then $(M(G)/K)/(H/K) \cong M(G)/H$.

Proof :

Suppose that $H = H_0 \cup H_1 I \cup \dots \cup H_{n-1} I^{n-1}$ and $K = K_0 \cup K_1 I \cup \dots \cup K_{n-1} I^{n-1}$ with $K \leq H$, then $(M(G)/K)/(H/K) = ((G/K_0) \cup \dots \cup (G/K_{n-1}) I^{n-1}) / ((H_0/K_0) \cup \dots \cup (H_{n-1}/K_{n-1}) I^{n-1}) \cong G/K_0 / (H_0/K_0) \cup \dots \cup (G/K_{n-1}) / (H_{n-1}/K_{n-1}) I^{n-1} \cong G/H_0 \cup (G/H_1) I \cup \dots \cup (G/H_{n-1}) I^{n-1} = M(G)/H$.

Theorem 3.14 :

Let $M(G)$ be an n -cyclic refined neutrosophic group, and H is an AH-normal subgroup, then for each AH-subgroup T of $M(G)/H$ there is an AH-subgroup of $M(G)$ contains H .

Proof : It can be proved as the classical case.

Definition 3.15 :

Let $M(G), M(H)$ be two n -cyclic refined neutrosophic groups,

we define $M(G) \times M(H) = (G \times H) \cup (G \times H) I I' \cup \dots \cup (G \times H) I^n (I')^n$ with $(I I')^k = I^k (I')^k$ for all k , it is clear that $M(G) \times M(H)$ is an n -generalized neutrosophic group with neutrosophic element $I I'$.

Theorem 3.16 :

Let $M(G), M(H)$ be two n -cyclic refined neutrosophic groups, then

(a) If $M(G), M(H)$ are abelian then $M(G) \times M(H)$ is abelian.

(b) If T, S are two AH-subgroups of $M(G), M(H)$ respectively, then $T \times S$ is an AH-subgroup of $M(G) \times M(H)$.

(c) If T, S are two AH-normal subgroups of $M(G)$, $M(H)$ respectively, then $T \times S$ is an AH-normal subgroup of $M(G) \times M(H)$.

(d) If T, S are two AHS-subgroups of $M(G)$, $M(H)$ respectively, then $T \times S$ is an AHS-subgroup of $M(G) \times M(H)$.

(e) If T, S are two AHS-normal subgroups of $M(G)$, $M(H)$ respectively then $T \times S$ is an AHS-normal subgroup of $M(G) \times M(H)$.

Proof :

(a) It is clear since $G \times H$ is abelian.

(b) Assume that $T = T_0 \cup \dots \cup T_{n-1}I^{n-1}$ and $S = S_0 \cup \dots \cup S_{n-1}(I')^{n-1}$, then

$T \times S = (T_0 \times S_0) \cup \dots \cup (T_{n-1} \times S_{n-1})(II')^{n-1}$, we can regard that $T_i \times S_i$ is a subgroup of $G \times H$, so $T \times S$ is an AH-subgroup.

(c) It holds directly from (b).

(d) If $S_j \cong T_i$ and $S_j \cong S_i$ then $T_i \times S_i \cong T_j \times S_j$ and then $T \times S$ is an AHS-subgroup.

(e) It holds directly from (d) and (c).

Theorem 3.17 :

Let $M(G)$, $M(H)$ be two n -cyclic refined neutrosophic groups and T, S be two AH-normal subgroups of $M(G)$, $M(H)$ respectively, then

$$M(G) \times M(H)/T \times S \cong M(G)/T \times M(H)/S.$$

Proof :

Suppose that $T = T_0 \cup \dots \cup T_{n-1}I^{n-1}$ and $S = S_0 \cup \dots \cup S_{n-1}(I')^{n-1}$, then

$$\begin{aligned} M(G)/T \times M(H)/S &= (G/T_0 \times H/S_0) \cup (G/T_1 \times H/S_1)II' \cup \dots \cup (G/T_{n-1} \times H/S_{n-1})(II')^{n-1} \\ &\cong G \times H/T_0 \times S_0 \times \dots \times (G \times H/T_{n-1} \times S_{n-1})(II')^{n-1} = M(G) \times M(H)/T \times S. \end{aligned}$$

Example 3.18:

Consider the additive group $(R^*, .)$ and the integer $n = 3$. The corresponding 3-cyclic refined neutrosophic group is

$$M(R) = \{a, bI, cI^2; a, b, c \in R^*\}.$$

1) We know that $(Q^*, .)$ is a subgroup of $(R^*, .)$, hence $L = Q^* \cup Q^*I \cup Q^*I^2 = \{x, yI, zI^2; x, y, z \in Q^*\}$ is a 3-cyclic refined AHS-subgroup.

2) The corresponding AH-factor is $M(R)/_L = R^*/_{Q^*} \cup R^*/_{Q^*}I \cup R^*/_{Q^*}I^2$.

Example 3.19:

Consider the following two groups $G = (Z, +)$, $H = (2Z, +)$, and the integer $n = 4$, the corresponding 4-cyclic refined neutrosophic groups are $M(G) = \{a, bI, cI^2, dI^3; a, b, c, d \in Z\}$, $M(H) = \{2a, 2bI, 2cI^2, 2dI^3; a, b, c, d \in Z\}$,

1) $g: G \rightarrow H; g(x) = 4x$ is a group homomorphism. $\text{Ker}(g) = \{0\}$.

2) The corresponding AH-homomorphism is $f: M(G) \rightarrow M(H); f(x) = 4x$, and $f(xI^l) = 4xI^l; 1 \leq l \leq 3$.

3) The corresponding AH-kernel is $AH - \text{Ker}(f) = \text{Ker}(g) \cup \text{Ker}(g)I \cup \text{Ker}(g)I^2 \cup \text{Ker}(g)I^3 = \{0, 0 + I, 0 + I^2, 0 + I^3\}$

Conclusion

In this paper, we have defined for the first time the concept of n -cyclic refined neutrosophic group as a new application of n -cyclic refined neutrosophic sets. We have discussed some of their elementary properties such as AH-subgroups, AH-kernels and direct products.

As a future research directions, we aim to study the n -cyclic refined neutrosophic semi groups and loops.

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