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Mid-term capacity redeployment of Rescue Centers under simultaneous requirements

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ABSTRACT

In this paper, we consider the problem of allocation of Emergency Vehicles (EVs) to Rescue Centers (RCs). The objective is to improve the operational response of the Fire and Rescue Services (SDIS) in France. More precisely, we focus on the mid-term management, and on the process of redeployment of EVs in RCs according to the evolution of requirements. At this level of decision, we do not have information on accidents. We only used the available information on the simultaneity of requirements. Based on the Hurwicz criterion, we develop a mathematical model and an iterative algorithm to solve it. The proposed approach takes into account both the uncertainty on the occurrence sequence of accidents, and the attitude towards optimization of the decision maker (DM). An illustration from the DM's point of view is presented. It shows that this uncertainty significantly impacts the deployment of EVs. Results show that the proposed approach has an efficient resolution time for real size problems.

KEYWORDS
Emergency medical service, Capacity deployment, Uncertainty, Hurwicz criterion

1. Introduction

In France, each department has a public service called the SDIS (Fire and Rescue Services). A department is a territorial division which is composed of several civil townships. Emergency, fire and rescue services provide assistance that is supplied by vehicles from fixed locations. The SDIS is made up of a fire-fighting unit (professional as well as voluntary), technical and administrative staff. Its main duties are rescue operations, preparation and prevention operations. It aims to prevent all risks to civil safety, such as accidents, damage and natural hazards; it prepares protective measures, organizes special equipment, fights any kind of fire and also offers emergency aid. In this paper, we focus on the Emergency Medical Service of the SDIS.

The SDIS splits each department into several sectors which are geographic zones. It is composed of Rescue Centers (RCs) where the vehicles are located. Each sector is

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linked to all RCs in the department by a priority list named the 'Dispatching Plan' (DP). The first RC of each DP, named the 'first call center', is normally called to cover the sector. The second one on the DP, named the 'second call center', is called when the first call center is unavailable and so on until the last RC. The DPs are generated automatically based on the time to reach a sector from an RC: the first center of the DP is the closest, then the second center is the second closest and so on.

When lives are at stake, the aim of the SDIS is to have a set of RCs that can intervene within 10 to 20 minutes for 80% of interventions (according to predefined zones). In this context, one important area of improvement is to determine the capacity (number of EVs) of each RC. This capacity depends on the requirement's distribution over the sectors. A report named SDACR (for instance, see SDACR31 (2012)), is produced every four years to analyse the requirements and discuss the redeployment of EVs. In order to determine this capacity, the SDIS has at its disposal only the simultaneous requirements of EVs in each sector. This information depends on the number of simultaneous accidents in a given DP sector. In other words, there may be one or more accidents in the DP sector, each of which requires at least one EV at the same time.

In Aboueljinane, Sahin, and Jemai (2013), the authors distinguish three levels of decision: long, middle and short term. The long term decisions consist of determining the covered region, the location and the capacity of the facilities. The mid-term decisions concern the deployment/redeployment decisions and the vehicles and resources assignment problems. The short term decisions aim to determine the dispatching rules, hospital destination or short term redeployment strategy. In our context, at the mid-term management level, where only the simultaneous requirements of EVs are available, it is mandatory to consider the uncertainty of the occurrence sequences. In order to investigate this uncertainty, we take into account the decision maker's attitude towards optimization using the Hurwicz criterion. This criterion considers the two extreme values of the objective function, namely the best-case and the worst-case. Those values depend on the best/worst-case occurrence sequences of the accidents. A decision support system is proposed to evaluate imprecise performances (i.e. the minimal and the maximal average times to reach the accidents).

2. Literature review

In the literature, a wide variety of models have been proposed for Emergency Medical Services (EMS) problems, which are derived to optimize the covering problem in a facility location. Previous relevant reviews include: Brotcorne, Laporte, and Semet (2003), Daskin and Dean (2005), Li, Zhao, Zhu, and Wyatt (2011), Farahani, Asgari, Heidari, Hosseininia, and Goh (2012), Hulshof, Kortbeek, Boucherie, Hans, and Bakker (2012) and Aboueljinane et al. (2013). Toregas, Swain, ReVelle, and Bergman (1971), C. S. ReVelle and Swain (1970) and Carbone (1974) were among the first to study the Location Set Covering Problem (LSCP). The main objective of these studies was to minimize the number of facilities (centers) to cover the set of demand points short of the indicated coverage. After that, the Maximal Covering Location Problem (MCLP) was introduced by Church and ReVelle (1974) to determine the number of facilities required to maximize the number of demand points covered by a limited fixed number of facilities. Recently, Akbari, Eiselt, and Pelot (2018) studied the MCLP for the location of Maritime Search and Rescue resources. Authors used

real data which are derived from the Search and Rescue Information Management system database. The resource centers are inshore and offshore stations. Each station has a maximum of one vessel of each category. Nevertheless in this paper, a station is not defined in the traditional sense of the term, but is 'a central location for a vessel that spends much of its time patrolling or performing other tasks at sea'.

To introduce capacity constraints, a Capacited Maximal Covering Location Problem (CMCLP) was proposed by Current and Storbeck (1988). These authors considered a fixed capacity level of the facility for each potential site. More recently, Yin and Mu (2012) proposed a Modular Capacited Maximal Covering Location Problem (MCMCLP) with different capacity levels (varied numbers of stationed emergency vehicles). However, these two models, do not specify which demand can be covered by which facility. To cope with this limitation, Backup Cover in Location Problems (BCLP) models (Hogan & ReVelle, 1986), Generalized MCLP (GMCLP) models (Berman & Krass, 2002) and Gradual Covering Location Problems (GCLP) models (Berman, Drezner, & Krass, 2010) have been developed. Nevertheless, these studies focus on the location of facilities, and the first center allocation under finite capacity without dispatching rules.

To take into account the availability of resources, probabilistic models have been proposed to determine the best location that maximizes, with a given probability, the population that can be satisfied by an available facility. Many of these models are derived from MCLP with scenario optimization (Chow & Regan, 2011; Haight & Fried, 2007; MacLellan & Martell, 1996; Serra & Marianov, 1998), the Maximum Expected Covering Location Problem (MEXCLP) introduced by Daskin (1983) and the Maximum Availability Location Problem (MALP) proposed by C. ReVelle and Hogan (1989). These models have also been combined with simulation models in order to evaluate and compare policies derived from optimal or near optimal solutions (McCormack & Coates, 2015). Otherwise, stochastic programming models with probabilistic constraints site have been developed. They aim to solve both the location of facilities (emergency service sites) and the allocation of emergency vehicles to each site (Beraldi, Bruni, & Conforti, 2004; Berman et al., 2010; Naoum-Sawaya & Elhedhli, 2013).

In these models, the following probabilities are assumed: (i) facilities may not be available, and (ii) requirements at given demand points are known. In our context the non-availability of EVs of the first call center is a consequence of a high simultaneity of accidents that required EVs from the second or the third call center, etc. Unlike the previous approaches, this paper explicitly proposes to integrate this cause into the model.

In recent studies, the dynamic aspect of the problem has been incorporated into EMS location models (Aringhieri, Bruni, Khodaparasti, & van Essen, 2017) considering the redeployment of capacities. This redeployment problem consists of relocating vehicles to deal with variation in demands. The research on the area of redeployment problem is recent (Aboueljinane et al., 2013; Aringhieri et al., 2017; Bélanger, Ruiz, & Soriano, 2012). In this context, two types of problems are considered: multi-period (relocation of the rescue teams to base) and dynamic redeployment problems (coverage of future requests given the set of idle vehicles). These redeployment problems focus on short term decisions. Therefore they are based on the occurrence of demand (accidents) at a given period. In contrast, in this study we are at an aggregate level and only the simultaneously of requirements is known.

Concerning the mid-term redeployment plans, a heuristic was proposed by Schmauch (2007) to affect EVs to RCs based on the simultaneous requirement of EVs of the first call center. Nevertheless, this approach does not take into account other RCs of the DP when estimating the maximal time needed to reach the accident. In fact, a center is often engaged in other sectors (i.e. a sector outside its own geographical zone). Thus, a sector may be frequently covered by the second or third call center. Later, Guillaume, Ben-Ammar, Thierry, and Chabriac (2015) generalize this heuristic to take into account this weakness. Nevertheless, the main limitation of that study is that does not include the impact of the occurrences' uncertainty. In this paper, we show that this impact is not negligible. To address it, we propose a two-stage model and an iterative algorithm to determine the best deployment plan.

3. Two-stage approach and features of the problem

In this section, we start by introducing the background of the two-stage programming approach. The second part is devoted to presenting the simultaneous requirement parameter that introduces uncertainty.

3.1. Two-stage programming: background

Two-stage linear programming problems often arise in the context of strategic planning models. We start by reminding the reader about the formal tools used to build our proposed two-stage approach.

Suppose that some parameters of the model are uncertain. Let the set, denoted by $\mathcal{S}$, contain all scenarios s of possible vectors of the uncertain parameters.

Firstly, we fix the decision variables $\boldsymbol{x} \in \mathbf{X}, \forall s \in \mathcal{S}$ (with cost function $F^1(x)$) to determine a partial solution in the first stage. Secondly, we fix the decision variables $\boldsymbol{y} \in \mathbf{Y}^{\boldsymbol{x}}$ (where $\mathbf{Y}^{\boldsymbol{x}}$ the domain of $\boldsymbol{y}$ for a $\boldsymbol{x}$ fixed) to complete the solution in the second stage (after the scenario came true). In this way, the problem can be formulated as follows (where G is an aggregation function).

$$\min_{\boldsymbol{x}\in\mathbf{X}}(F^1(x) + G(\min_{\boldsymbol{y}\in\mathbf{Y}^{\boldsymbol{x}}} F^2(\boldsymbol{y}, s)) \tag{1}$$

In our case, the redeployment capacity is determined at the first stage, and the EV available for accidents at the second stage. Moreover, the choice of aggregation function G depends on the knowledge of the scenarios. If probability distribution over the scenarios is available, a stochastic approach can be introduced and G becomes the expected value (Birge & Louveaux, 2011). Then, the problem can be written as follows:

$$\min_{\boldsymbol{x}\in\mathbf{X}}(F^1(x) + \mathbb{E}[\min_{\boldsymbol{y}\in\mathbf{Y}^{\boldsymbol{x}}} F^2(\boldsymbol{y}, s)]) \tag{2}$$

Otherwise, if the probability distribution is not available, another aggregation function from decision theory can be used. Normally, the min-max criterion is introduced at the second stage, which leads to the so-called two-stage robust optimization approach

(Bertsimas, Brown, & Caramanis, 2011; Kasperski & Zieliński, 2017). The min-max criterion focuses on the worst-case scenario, that is, the scenario that maximizes the objective function. This criterion gives a guarantee of the maximum value that the function can take knowing that there is uncertainty in the parameters. In this case the problem is:

$$\min_{\boldsymbol{x}\in\mathbf{X}}(F^1(x) + \max_{s\in\mathcal{S}} \min_{\boldsymbol{y}\in\mathbf{Y}^{\boldsymbol{x}}} F^2(\boldsymbol{y}, s)) \tag{3}$$

Nevertheless, this min-max criterion of the second stage can be seen as ultraconservative since it focuses only on the worst scenario. Thus a criterion which concentrates both on worst-case and best-case scenarios can lead a less conservative solution since it considers not only the worst possible value but also the best possible value. This criterion is the *Hurwicz criterion* (see e.g. Hurwicz (1951)). Using this criterion a solution that minimizes the parametric convex combination of the objective function of the best-case scenario and the worst-case scenario is sought. In this case, we solve the following problem:

$$\min_{\boldsymbol{x}\in\mathbf{X}}(F^1(x) + \alpha \min_{s\in\mathcal{S}} \min_{\boldsymbol{y}\in\mathbf{Y}^{\boldsymbol{x}}} F^2(\boldsymbol{y}, s) + (1-\alpha) \max_{s\in\mathcal{S}} \min_{\boldsymbol{y}\in\mathbf{Y}^{\boldsymbol{x}}} F^2(\boldsymbol{y}, s)) \tag{4}$$

where parameter of the convex combination, namely the *optimism-pessimism index* $\alpha \in [0, 1]$ is fixed and given. α is the weight of the objective function for the best-case scenario and $1 - \alpha$ is the the weight of the objective function for the worst-case scenario. Clearly, if $\alpha = 0$, we solve Problem (3); if $\alpha = 1$, we seek a plan which minimizes the total cost over the scenario set. Hence, $\alpha \in [0, 1]$ controls which of the two extremes is preferred, since we do not have probability distribution over the scenarios.

The Hurwicz criterion gives the possibility of modeling the attitude towards optimization of the decision maker. It was introduced in Problem (4). We remind readers that in this problem, we deal with the simultaneous requirement of EVs. In the next subsection, we explain this source of uncertainty.

3.2. Uncertainty on the sequences of the occurrence of accidents under the simultaneity of requirements

The primary objective of an Emergency Medical Service is to reach the accident site as soon as possible. In an unconstrained resource context, there would be enough vehicles at each RC to satisfy the demand of all the closest sectors. In our case, as the total number of EVs is fixed, this is not the case. In this constrained context, the DPs determine the order of the RCs to be contacted when an accident occurs. The first call center in a DP is the nearest RC to the accident site. Under resource constraints, it is possible that the first call center is not available to deal with the accident, hence the second call center, which is the second nearest RC to the accident, is called. This RC is the second RC of the DP for the sector where the accident occurs. In the same way, there is the third, then the fourth RC of the DP and so on. Figure 1 illustrates the DP assignment for 3 RCs and 3 sectors. it can be seen that Sector a has the DP: RC_1, RC_2, RC_3 since the closest RC is RC_1 then RC_2 and finally RC_3. Sector b has the DP: RC_2, RC_3, RC_1 and Sector c: RC_3, RC_2, RC_1. In this paper, the DPs are

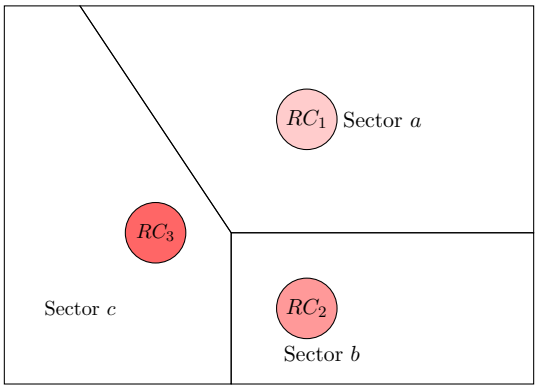


Figure 1. Illustration of DPs.

fixed and determined as follows: if the average time to reach a sector from RC_1 is less than the average time to reach the same sector from RC_2, then RC_2 follows RC_1 in the DP of this sector.

Moreover, the requirement of EVs in each RC depends on the number of accidents that occur simultaneously in the sectors that have the same DP. This simultaneity of requirements is due to the occurrence of one or more accidents that each need one or more EVs during the time that several EVs are occupied by other accidents. In our case, we are at the mid-term process. The data of past accidents is available and aggregated to estimate the simultaneity of requirements (SDACR31, 2012). This aggregation strategy facilitates the resolution of the problem and allows us to translate the historical data into a global requirement for each sector.

We note that this simultaneity of requirements does not fully define the occurrence of the accidents. Thus, there is a set of occurrences of accidents, denoted by $\mathcal{S}$, with a given simultaneous requirement. This simultaneity impacts the performance of the SDIS in terms of expected time to reach the scenes of the accidents as shown in the following example (see Ex.3.1).

Example 3.1. In this example, the total simultaneous requirements of EVs are equal to 6. Only 2 sequences of occurrences are considered (see Figure 2). We suppose that 4 accidents have occurred and each one requires 1 or 2 EV(s). In other words, 6 EVs are needed simultaneously to rescue the accident victims.

For these cases, two accidents (*Acc* 1 and *Acc* 2) have occurred in the same sector whose dispatching plan is DP_a. They require 2 and 1 EV(s), respectively. A third accident requires 2 EVs and has occurred in another sector whose dispatching plan is DP_b. A fourth accident requires 1 EV and has occurred in a sector whose dispatching plan is DP_c.

Thus, the simultaneity of requirements of DP_a, DP_b and DP_c are 3, 2 and 1, respectively. Nevertheless, the sequences of occurrences of these accidents with the same simultaneous requirements are different for the two cases. Moreover, considering the DPs, these sequences lead to different assignments to the RCs, as shown below (see Figure 2).

Let the following DPs define the level of the different RCs as call centers for the

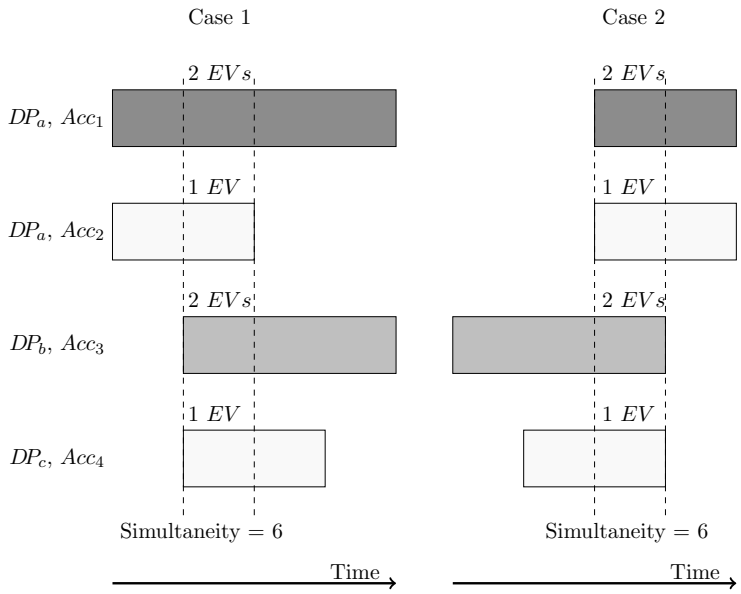


Figure 2. Simultaneity of 6.

different sectors (each DP is associated with a sector):

- $a = (RC_1; RC_2; RC_3)$
- $b = (RC_2; RC_3; RC_1)$
- $c = (RC_3; RC_2; RC_1)$

Moreover, each average time needed for an EV from the RC to reach the scene of the accident is given below:

- (RC_1, a) =4 min, (RC_1, b)=10 minutes and (RC_1, c)=9 min
- (RC_2, a) =6 min, (RC_2, b)=3 minutes and (RC_2, c)=6 min
- (RC_3, a) =12 min, (RC_3, b)=5 minutes and (RC_3, c)=2 min

Initially, 2 EVs are located in RC_1, 1 EV in RC_2 and 3 EVs in RC_3.

Two examples of occurrences of these accidents with the same simultaneity are considered.

- Case 1 (see Figure 2): the sequence of occurrences of accidents is Acc_1, Acc_2 then Acc_3 and Acc_4.
- Case 2 (see Figure 2): the sequence is Acc_3, Acc_4 then Acc_1 and Acc_2.

Using the DPs, sequence 1 gives the following results (see Figure 3), where the number in brackets following RC_i (resp. Acc_i) is the number of EV of RC_i (resp. the number of required EV of Acc_i)):

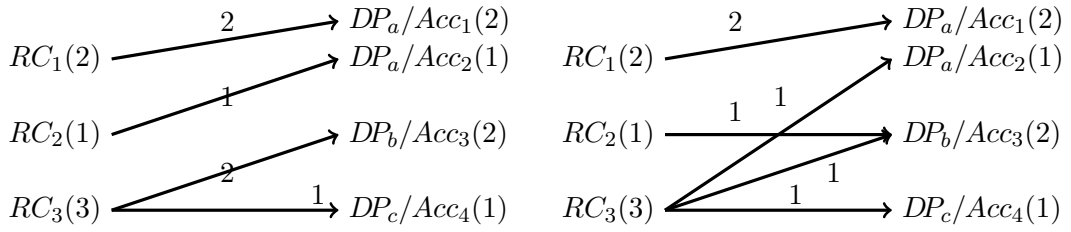
- 2 accidents corresponding to DP_a and DP_c are rescued by the first call center, RC_1 and RC_3, respectively with 2 and 1 EVs respectively,
- 2 accidents corresponding to DP_a and DP_b are rescued by the second call center, RC_2 and RC_3, respectively, with 1 and 2 EVs respectively.

Sequence 2 (see Figure 3) gives the following result:

- 3 accidents corresponding to DP_a, DP_b and DP_c are rescued by the first call center, RC_1, RC_2 and RC_3, respectively, with 2, 1 and 1 EVs respectively,
- 1 accident corresponding to DP_b is rescued by the second call center (RC_3) with 1 EV,
- 1 accident corresponding to DP_a is rescued by the third call center (RC_3) with 1 EV.

Then, the expected times to rescue the accidents are:

- $(4 * 2 + 6 + 5 * 2 + 5)/6 \approx 4.34$ minutes for sequence 1
- $(4 * 2 + 3 + 12 + 5 + 2)/6 = 5$ minutes for sequence 2.



(a) Sequence 1.

(b) Sequence 2.

Figure 3. Assignment of EVs to DPs.

Remark 1. The values of the number of EVs from RC_i allocated to DP_π (noted $h_{i,\pi}$) are linked to the sequence of occurrence of the accidents. In the example (see Figure 3), sequence 1 leads to the solution $h_{1,a} = 2$, $h_{2,a} = 1$, $h_{3,b} = 2$, $h_{3,c} = 1$ and the solution corresponding to sequence 2 is $h_{1,a} = 2$, $h_{3,a} = 1$, $h_{2,b} = 1$, $h_{3,b} = 1$, $h_{3,c} = 1$. In the next section, we will distinguish the number of EVs from RC_i allocated to DP_π for the best-case sequence and the worst-case sequence.

In the next section, we introduce our two-stage optimization approach based on simultaneous requirement of EVs.

4. Mid-term redeployment of capacity: a two-stage linear programming problem

Our problem, denoted $\mathcal{P}$, can be expressed as a two-stage linear programming problem. The capacity redeployment is determined at the first stage (before the sequence of accidents is known), and the EVs allocated to accidents at the second one (by using DP logic). Here, we use the Hurwicz criterion at the second stage. Consequently, the model takes into account the two extreme sequences of accidents i.e. the best and the worst. The best (resp. worst) sequence leads to the best (resp. worst) rescue time. Thus, we do not need to compute the best (resp. worst) sequence but only the related best (resp. worst) rescue time. To do so, we compute the best (resp. worst) affectation of EVs to DPs.

Let $\mathbb{C}$ be the set of RCs, $\text{card}(\mathbb{C}) = c$, Π the set of DPs. Each DP is denoted $\pi = (\pi(1), \pi(2), \cdots)$ where $\pi(1)$ is the first RC to be called (level 1), $\pi(2)$ the second RC to be called (level 2) if the first one has no available EV(s), and so on. For simplicity of notation, we use the notation $i \succ_\pi j$ when $\pi(a) = i$ and $\pi(a + 1) = j$. Moreover

we need to define another set noted $\mathbb{W}$ which contains all impossible transportation plans by respecting DP logic. Before giving the formal definition of this set, let's recall the logic of DP: when an accident occurs in a sector with DP_π, the first related RC which has enough available EVs is affected to the accident rescue. In other words, this means that if RC_i, where $i = \pi(n)$, is required to rescue an accident occurring in the sector that has the DP_π, $RC_j = \pi(m)$ such that $m < n$ does not have enough free EVs. Hence, an accident that occurs in a sector with a $DP_{\pi'}$ such that $i = \pi'(a)$ and $j = \pi'(b)$ with $a < b$ can not be rescued by RC_j, (Figure 4 illustrates the impossible case). In fact, RC_j has to rescue the accident occurring in π because RC_i cannot rescue it (see Ex. 4.1).

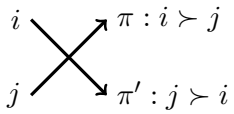


Figure 4. Impossible transportation plan.

So, the formal definition of $\mathbb{W}$ is as follows:

$$\mathbb{W} = \{(i, j, \pi, \pi') \colon \pi, \pi' \in \Pi, i, j \in \mathbb{C} \colon \pi \neq \pi', i \succ_\pi j, j \succ_{\pi'} i\}$$

Therefore, a vector (i, j, π, π') is in $\in \mathbb{W}$ means that we can not rescue a sector whose dispatching plan is DP_π with an EV from RC_j and can rescue a sector whose dispatching plan is $DP_{\pi'}$ with an EV from RC_i if i is before j in the DP_π and j before i the DP_π.

The input data are:

- $t_{i,\pi}$: the average time needed for an EV from RC_i to reach the scene of the accident. If RC_i is the first center called, $t_{i,\pi} = t_{\pi(1),\pi}$,
- DP_π: the number of simultaneously required EVs for DP_π,
- K_i^{init}: initial capacity of EVs,
- K_i^{min}: the minimal capacity of assignment of EVs to RC_i,
- K_i^{max}: the maximal capacity of assignment of EVs to RC_i,
- $T_{i,j}^{max}$: the maximal transferred EVs from i to j ,$\forall i, j \in \mathbb{C}$,
- T_i^{RC}: the maximal number of transferred EVs for an RC.

The decision variables are integers:

- K_i: the capacity of RC_i in terms of the number of EVs,
- $T_{i,j}$: the number of EVs tranfered from RC_i to RC_j,
- $h_{i,\pi}^b$: the number of EVs from RC_i allocated to DP_π for the best-case sequence of accident,
- $h_{i,\pi}^w$: the number of EVs from RC_i allocated to DP_π for the worst-case sequence of accident,
- $x_{i,\pi}$: a binary variable such that $x_{i,\pi} = 1$ if $h_{i,\pi}^w > 0$.

Without loss of generality, we consider that the simultaneous requirement is equal to the total number of EVs. If that is not the case, a virtual RC or sector can be added as proposed in Guillaume et al. (2015). Hence, all the EVs are used. The mathematical formulation of the problem with uncertainty on the sequences of accident is:

$$\mathcal{P}: \min_{\mathbf{K},\mathbf{T}} \quad \alpha \min_{\mathbf{h^b}} \sum_{\pi\in\Pi}\sum_{i\in\mathbb{C}} t_{i,\pi}h^b_{i,\pi} + (1-\alpha)\max_{\mathbf{h^w}} \sum_{\pi\in\Pi}\sum_{i\in\mathbb{C}} t_{i,\pi}h^w_{i,\pi} \tag{5}$$

s.t.

$$\sum_{i\in\mathbb{C}} K_i = \sum_{i\in\mathbb{C}} K_i^{init} \tag{5.a}$$

$$K_i^{init} + \sum_{j\in\{\mathbb{C}:j\neq i\}} T_{j,i} = K_i \qquad \forall i \in \mathbb{C} \tag{5.b}$$

$$T_{j,i} \leq T_{j,i}^{max} \qquad \forall i \in \mathbb{C}, j \in \mathbb{C} : i \neq j \tag{5.c}$$

$$\sum_{j\in\{\mathbb{C}:j\neq i\}} T_{i,j} \leq T_i^{RC} \qquad \forall i \in \mathbb{C} \tag{5.d}$$

$$T_{i,j} = -T_{j,i} \qquad \forall i \in \mathbb{C}, \forall j \in \mathbb{C} \tag{5.e}$$

$$K_i^{min} \leq K_i \leq K_i^{max} \qquad \forall i \in \mathbb{C} \tag{5.f}$$

$$\sum_{i\in\mathbb{C}} h^b_{i,\pi} = DP_\pi \qquad \forall \pi \in \Pi \tag{5.g}$$

$$\sum_{\pi\in\Pi} h^b_{i,\pi} = K_i \qquad \forall i \in \mathbb{C} \tag{5.h}$$

$$\sum_{i\in\mathbb{C}} h^w_{i,\pi} = DP_\pi \qquad \forall \pi \in \Pi \tag{5.j}$$

$$\sum_{\pi\in\Pi} h^w_{i,\pi} = K_i \qquad \forall i \in \mathbb{C} \tag{5.i}$$

$$h^w_{i,\pi} \leq x_{i,\pi}K_i \qquad \forall \pi \in \Pi, \forall i \in \mathbb{C} \tag{5.k}$$

$$x_{i,\pi} + x_{j,\pi'} \leq 1 \qquad \forall (i,j,\pi,\pi') \in \mathbb{W} \tag{5.l}$$

$$K_i \in \mathbb{N}, T_{j,i} \in \mathbb{Z}, x_{i,\pi} \in \{0,1\} \qquad i,j \in \mathbb{C}, \pi \in \Pi \tag{5.m}$$

The constraints from (5.a) to (5.f) are constraints of the first stage decision variables. More precisely, constraints (5.a) mean that the number of EVs is constant. Constraints (5.b) mean that the new capacity assignment is equal to the sum of the initial capacity and the transferred EVs. Constraints (5.c) express the maximal number of EVs which could be transferred between two RCs. Constraints (5.d) express the maximal number of EVs which could be transferred from an RC. Constraints (5.e) express the link between the number of EVs transferred from RC_i to RC_j and the number of EVs transferred from RC_j to RC_i. Constraints (5.f) express the capacity constraint for each RC.

The second stage contains two problems. In the first (resp. second) one we look for the minimal (resp. maximal) evaluation. The first problem is a minimization problem in which we focus on the EVs allocated to accidents for the best-case sequences of accidents (Constraints 5.g-5.h). The second problem is a maximization problem in which we focus on the EVs allocated to accidents for the worst-case sequences of accidents (Constraints 5.i-5.l on the model).

Constraints (5.h) and (5.j) express the impossibility of an RC rescuing more accidents than its capacity.

These two types of constraints define the Transportation Problem in which $h^b_{i,\pi}$

(resp. $h^w_{i,\pi}$) is the flow of EV from RC_i to DP_π for the best-case (resp. worst-case) sequence of accident occurrences.

The constraints (5.k) and (5.l) guarantee that the transportation plan that concerns the worst-case sequence of accidents satisfies the DP logic. From Proposition 1, these constraints can be omitted for the best-case sequence problem. In addition, they are not trivial, so the justification will be detailed in the next paragraph.

Proposition 1. The optimal solution of the following problem satisfies the DP constraints (Constraints 5.k-5.l):

$$\min_{\mathbf{h^b}} \sum_{\pi\in\Pi}\sum_{i\in\mathbb{C}} t_{i,\pi}h^b_{i,\pi}$$
$$s.t. \quad (5.a)-(5.i) \text{ and } (5.m).$$

Proof: Let suppose that a solution, which does not satisfy DP constraints, is optimal. Hence there are at least two variables h^{b*} such that $h^{b*}_{j,\pi} > 0$ and $h^{b*}_{i,\pi'} > 0$ with RC_i and RC_j, and DP_π and $DP_{\pi'}$ such that $\pi : i \succ j$ and $\pi' : j \succ i$. The cost of the solution to recuse π and π' is $t_{j,\pi}h^{b,*}_{j,\pi} + t_{i,\pi'}h^{b,*}_{i,\pi'} + K$ where K is the cost which does not depend on decision variables $h^{b,*}_{j,\pi}$ and $h^{b,*}_{i,\pi'}$. Let $h^{b,DP}$ be the solution that differs from h^{b*} only for $h^b_{i,\pi'}$, $h^b_{j,\pi}$, $h^b_{i,\pi}$ and $h^b_{j,\pi'}$, and which satisfies the flow constraints. We have two possible cases: $h^{b,*}_{j,\pi} \geq h^{b,*}_{i,\pi'}$ or $h^{b,*}_{j,\pi} \leq h^{b,*}_{i,\pi'}$. Both are symmetric. So, we only develop the first one. Therefore we build the following solution:

$$\begin{cases} h^{b,DP}_{i,\pi'} = h^{b,*}_{i,\pi'} - h^{b,*}_{i,\pi'} = 0 \\ h^{b,DP}_{i,\pi} = h^{b,*}_{i,\pi} + h^{b,*}_{i,\pi'} \\ h^{b,DP}_{j,\pi'} = h^{b,*}_{j,\pi'} + h^{b,*}_{i,\pi'} \\ h^{b,DP}_{j,\pi'} = h^{b,*}_{j,\pi} - h^{b,*}_{i,\pi'} \end{cases}$$

Since the rescue time $t_{i,\pi}$ satisfies the constraints $t_{i,\pi} < t_{j,\pi}$ and $t_{i,\pi'} > t_{j,\pi'}$, the cost of solution $h^{b,DP}$ is lower than h^{b*}. This is in contradiction with the assumption that h^{b*} is optimal. □

Example 4.1. Let $(1, 2, 3)$ be the RCs and (a, b, c) the DPs:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

We suppose that RCs 1, 2 and 3 rescue the sectors of DP a so $h^w_{1,a}$, $h^w_{2,a}$, $h^w_{3,a} > 0$. Hence, RC_1 and RC_2 do not have enough EVs to satisfy the requirement of DP a and EVs are needed from RC_3. It follows from this that the DP which has 2 before 1 can not be rescued by RC_1 if RC_2 does not have enough EVs to satisfy the requirement of this DP. So, $h^w_{1,b}$ is equal to 0. More over, the DP that has 3 before 1 and 2 cannot be rescued by RC_1 or RC_2 if RC_3 does not have enough EVs to satisfy the requirement of this DP. So, $h^w_{1,c}$, $h^w_{2,c}$ are equal to 0.

5. Resolution of the problem

In this section, an iterative algorithm is proposed to solve the problem. For simplicity of notation, let $F(\mathbf{h}^b) = \sum_{\pi\in\Pi}\sum_{i\in\mathbb{C}} t_{i,\pi}h^b_{i,\pi}$, $F(\mathbf{h}^w) = \sum_{\pi\in\Pi}\sum_{i\in\mathbb{C}} t_{i,\pi}h^w_{i,\pi}$, $\mathcal{K}$ be the set of $\mathbf{K}$ that meet the constraints (5.a)-(5.f), $\mathcal{H}^{\mathbf{K}}_b$ be the set of $\mathbf{h}^b$ that meet the constraints (5.g)-(5.h) and $\mathcal{H}^{\mathbf{K}}_w$ be the set of $\mathbf{h}^w$ that meet the constraints (5.i)-(5.l). Now we are able to reformulate the problem $\mathcal{P}$ as:

$$\begin{aligned}\mathcal{P}:\ \min_{\mathbf{K}\in\mathcal{K}}(\quad & \alpha \min_{\mathbf{h}^b\in\mathcal{H}^{\mathbf{K}}_b} F(\mathbf{h}^b) \\ & +(1-\alpha)\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(\mathbf{h}^w))\end{aligned} \tag{6}$$

5.1. MIP reformulation of problem $\mathcal{P}$

Unfortunately, the maximization part of the second stage may be considered as a MIP problem, hence it is not possible to use dual theorem directly to transform the maximization into a minimization as is usually done in robust optimization field. Therefore, it is necessary to modify the formulation of the maximization problem $\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(\mathbf{h}^w)$.

Let $\mathcal{X}$ be the set of vectors $\mathbf{x} \in \{\mathbf{0},\mathbf{1}\}^{|\mathbb{C}||\mathbf{\Pi}|}$ which satisfy the DP constraints: $x_{i,\pi^{'}} + x_{j,\pi} \leq 1, \forall(i,j,\pi,\pi^{'}) \in \mathbb{W}$. Moreover, Constraints (5.k) are taken into account through a penalty depending on $x_{i,\pi}$: $t_{i,\pi} \rightarrow t_{i,\pi} - M\cdot(1-x_{i,\pi})$. Then, the problem $\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(\mathbf{h}^w)$ can be reformulated as follows:

$$\begin{aligned}&\max_{\mathbf{x}\in\mathcal{X}}\max_{\mathbf{h^w}} \quad \sum_{\pi\in\Pi}\sum_{i\in\mathbb{C}}(t_{i,\pi} - M\cdot(1-x_{i,\pi}))h^w_{i,\pi} \\ &\text{s.t.} \\ &(a) \qquad \sum_{i\in\{\mathbb{C}\}} h^w_{i,\pi} = DP_\pi \qquad\qquad \forall\pi\in\Pi \\ &(b) \qquad \sum_{\pi\in\{\Pi\}} h^w_{i,\pi} = K_i \qquad\qquad \forall i\in\mathbb{C} \\ &(c) \qquad h^w_{i,\pi}\in\mathbb{N} \qquad\qquad \forall\pi\in\Pi,\forall i\in\mathbb{C}\end{aligned} \tag{7}$$

On one hand the problem $\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(\mathbf{h}^w)$ has at least one feasible solution that satisfies the DP constraints (see Proposition 1). On the other hand Problem (7) is a *Big M* relaxation of $\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(\mathbf{h}^w)$. Thus, the optimal solution of Problem (7) has the same optimal value and optimal solution than problem $\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(\mathbf{h}^w)$.

Let us focus on the second part of Problem (7) that consists on computing the maximal value of the objective function for a given vector $\mathbf{x}^* \in \mathcal{X}$. It can be seen that for a given vector $\mathbf{x}^*$, this problem is a transportation problem. Hence, the linear relaxation of Problem (7) is equivalent to the integer formulation due to the unimodularity of the matrix. Hence, we can apply the dual theorem to Problem (7). Therefore, Problem (7) is equivalent to Problem (8). We note:

- λ_π: the dual variable of Constraints (7.a)
- β_i: the dual variable of constraints (7.b)

$$
\begin{array}{ll}
 & \max_{\mathbf{x}\in\mathcal{X}} \min \sum_{\pi\in\Pi} \lambda_\pi D_\pi + \sum_{i\in\mathbb{C}} \beta_i K_i \\
\text{s.t.} & \\
(a) & (1 - x_{i,\pi}) \cdot M + \lambda_\pi + \beta_i \geq t_{i,\pi},\ \forall i \in \mathbb{C}, \forall \pi \in \Pi
\end{array}
\tag{8}
$$

To linearize the maximization over the set $\mathcal{X}$, we note $x_{q,i,\pi}$ the value of $x_{i,\pi}$ of the q^{th} element of $\mathcal{X}$. Hence, the dual variables λ_π and β_i become $\lambda_{\pi,q}$ and $\beta_{i,q}$ respectively, the dual variables for the q^{th} element of $\mathcal{X}$. We note W the maximal value over the set $\mathcal{X}$. So, by using $min-max$ theorem (Sion, 1958) and the linearization of the max function, Model (8) can be transformed into Model (9).

$$
\begin{array}{ll}
 & \min W \\
\text{s.t.} & \\
(a) & \sum_{\pi\in\Pi} \lambda_{\pi,q} D_\pi + \sum_{i\in\mathbb{C}} \beta_{i,q} K_i \leq W,\ \forall q = 1, ..., |\mathcal{X}| \\
(b) & (1 - x_{q,i,\pi}) \cdot M + \lambda_{\pi,q} + \beta_{i,q} \geq t_{i,\pi},\ \forall i \in \mathbb{C}, \forall \pi \in \Pi, \forall q = 1, ..., |\mathcal{X}|
\end{array}
\tag{9}
$$

Using Formulation (9), Problem (6) becomes:

$$
\begin{array}{ll}
\mathcal{P}: & \min \alpha F(h_b) + (1-\alpha) W \\
\text{s. t.} & \\
(a) & \sum_{\pi\in\Pi} \lambda_{\pi,q} D_\pi + \sum_{i\in\mathbb{C}} \beta_{i,q} K_i \leq W,\ \forall q = 1, ..., |\mathcal{X}| \\
(b) & (1 - x_{q,i,\pi}) \cdot M + \lambda_{\pi,q} + \beta_{i,q} \geq t_{i,\pi},\ \forall i \in \mathbb{C}, \forall \pi \in \Pi, \forall q = 1, ..., |\mathcal{X}| \\
 & \mathbf{h}_b \in \mathcal{H}_b^{\mathbf{K}} \\
 & \mathbf{K} \in \mathcal{K}
\end{array}
\tag{10}
$$

Proposition 5.1. *If $\alpha = 1$ then Problem (10) is a MIP with a polynomial number of constraints:*

$$
\begin{array}{ll}
 & \min F(h^b) \\
\text{s. t.} & \\
 & \boldsymbol{h}^b \in \mathcal{H}_b^{\boldsymbol{K}} \\
 & \boldsymbol{K} \in \mathcal{K}
\end{array}
\tag{11}
$$

and can be solved directly by an MIP solver.

Nevertheless, Problem (10) is not tractable in the current form (for $\alpha < 1$, see Proposition 5.1) since it contains concave constraints (10.a) and an exponential number of constraints due to $|\mathcal{X}|$. To deal with the first problem, we linearize the concave constraints using a binary variable:

- $y_{i,e}$: binary variable equal to 1 if the value K_i is below the $e^t h$ interval.
- $b_{i,q}$: the approximation of $\beta_{i,q} \cdot K_i$
- $\widehat{K}_{i,e}^{inf}$,$\widehat{K}_{i,e}$ and $\widehat{K}_{i,e}^{sup}$ are the lower, middle and upper values, respectively, of the e^{th} interval for the $RC_i \in \mathbb{C}$

Model (10) can be equivalent to Model (12) under the condition on $\widehat{K}_{i,e}^{inf}$,$\widehat{K}_{i,e}$.

Remark 2. Since $\mathbf{K} \in \mathbb{N}^{|\mathbb{C}|}$ Model (12) is equivalent to Model (10) when $\widehat{K}_{i,e} = a, a \in \{\mathbb{N} | K_i^{min} \leq a \leq K_i^{max}\}$ and $\widehat{K}_{i,e}^{inf} > a - 1$ and $\widehat{K}_{i,e}^{sup} < a - 1$.

Finally Problem $\mathcal{P}$ can be formulated as MIP with exponential number of constraints:

$$
\begin{array}{ll}
\mathcal{P}: & \min_{\mathbf{K},\mathbf{h}^b,\lambda,\beta} \alpha F(h^b) + (1-\alpha)W \\
\text{s. t.} & \\
(a) & \sum_{\pi\in\Pi} \lambda_{\pi,q} D_\pi + \sum_{i\in\mathbb{C}} b_{i,q} \leq W,\ \forall q = 1,...,|\mathcal{X}| \\
(b) & K_i \leq (1-y_{i,e})\cdot M + \widehat{K}^{sup}_{i,e},\ \forall e \in E, \forall i \in \mathbb{C} \\
(c) & K_i \geq (y_{i,e}-1)\cdot M + \widehat{K}^{inf}_{i,e},\ \forall e \in E, \forall i \in \mathbb{C} \\
(d) & \sum_{e\in E} y_{i,e} = 1,\ \forall e \in E \\
(e) & (y_{i,e}-1)\cdot M + \beta_{i,q}\cdot \widehat{K}_{i,e} \leq b_{i,q},\ \forall e \in E, \forall q = 1,...,|\mathcal{X}|, \forall i \in \mathbb{C} \\
(f) & (1-x_{q,i,\pi})\cdot M + \lambda_{\pi,q} + \beta_{i,q} \geq t_{i,\pi},\ \forall i \in \mathbb{C}, \forall \pi \in \Pi, \forall q = 1,...,|\mathcal{X}| \\
 & \mathbf{h}^b \in \mathcal{H}^{\mathbf{K}}_b \\
 & \mathbf{K} \in \mathcal{K}
\end{array}
\tag{12}
$$

5.2. Iterative algorithm

Since $|\mathcal{X}|$ is too big, we propose an iterative algorithm based on the constraints generation method to compute the optimal solution. Let $\mathcal{X}^{sub} \subseteq \mathcal{X}$, so that solving Problem (12) for the subset $\mathcal{X}^{sub}$ is a lower bound of Problem (12). Clearly, for a given capacity vector $\mathbf{K}^*$, $\alpha \cdot F^{min}(h^b, \mathbf{K}^*) + (1-\alpha)\cdot F^{max}(h^w, \mathbf{K}^*)$ is an upper bound of Problem (12) (with $F^{min}(h^b, \mathbf{K}^*)$ and $F^{max}(h^w, \mathbf{K}^*)$ respectively the optimal values of models $\min_{\mathbf{h}^b\in\mathcal{H}^{\mathbf{K}}_b} F(h^b)$ and $\max_{\mathbf{h}^w\in\mathcal{H}^{\mathbf{K}}_w} F(h^w)$).

Remark 3. Nevertheless, if the subset $\mathcal{X}^{sub}$ is too small, the problem can be unbounded since the primal problem over the set $\mathcal{X}^{sub}$ is infeasible. To deal with this problem at the beginning of the algorithm, we compute the optimal assignment of capacity and transportation for $\alpha = 1$ ($\mathbf{K}^{\alpha=1}, \mathbf{h}^{1,\alpha=1}$) and we add the constraint $W \geq F(h^{1,\alpha=1}, K^{\alpha=1})$ to obtain a capacity assignment.

Our algorithm (Algorithm 1) starts ($k = 0$) with LB equal to the solution of Problem (11), the capacity assignment $\boldsymbol{K}^0 = \boldsymbol{K}^{int}$ and empty set, $\mathcal{X}^{sub} := \emptyset$.

At each iteration k, a vector $\mathbf{x}^* \in \mathcal{X}$ is computed by solving $\max\{F(h^w)\ |\mathbf{h}^w \in \mathcal{H}^{\mathbf{K}^{k-1}}_w\}$. Then the set $\mathcal{X}^{sub}$ is updated $\mathcal{X}^{sub} = \mathcal{X}^{sub} \cup \{\mathbf{x}^*\}$. Moreover, the UB is computed by solving $\min\{F(h^b)\ |\mathbf{h}^b \in \mathcal{H}^{\mathbf{K}^{k-1}}_b\}$ and $\max\{F(h^w)\ |\mathbf{h}^w \in \mathcal{H}^{\mathbf{K}^{k-1}}_w\}$. Then a new capacity assignment with the new LB is computed by solving Model (12) for the set $\mathcal{X}^{sub}$. If a termination criterion is fulfilled (Step 2 of Algorithm 1), then the algorithm stops with the assignment of capacity, which is an approximation of an optimal affection.

Theorem 5.2. *For $\widehat{K}_{i,e} = a$, $a \in \{\mathbb{N} | K^{min}_i \leq a \leq K^{max}_i\}$, $\widehat{K}^{inf}_{i,e} > a-1$ and $\widehat{K}^{sup}_{i,e} < a-1$, Algorithm (1) gives an approximation of the optimal solution of Problem (6) in a finite number of steps for a given tolerance $\epsilon > 0$.*

Proof. For $\widehat{K}_{i,e} = a$, $a \in \{\mathbb{N} | K^{min}_i \leq a \leq K^{max}_i\}$, $\widehat{K}^{inf}_{i,e} > a-1$ and $\widehat{K}^{sup}_{i,e} < a-1$, Model (10) is equivalent to Model (12). Since, Model (10) has the same optimal value than the problem expressed in (6). Therefore, the optimal solution and the corresponding objective value given by Model (12) are equivalent to those given by the problem $\mathcal{P}$ (see expression 6). The proof of convergence of Algorithm (1) for Model (12) is similar to the one given in Guillaume, Kobylanski, and Zielinski (2012). Hence,

Algorithm 1: Finding an optimal capacity assignment.

h! **Input:** Parameters of Model (12), LB, a convergence tolerance parameter $\epsilon > 0$.
Output: A capacity assignment $\mathbf{K}^*$.
Step 0. $k := 1$, $\boldsymbol{K}^0 = \boldsymbol{K}^{int}$, $\mathcal{X}^{sub} := \emptyset$.
Step 1. Compute the new vector $\boldsymbol{x}^k$ and UB for capacity assignment $\boldsymbol{K}^{k-1}$: $\max\{F(h^w) \ |\mathbf{h}^w \in \mathcal{H}_w^{\mathbf{K}^{k-1}}\}$ and $\min\{F(h^b) \ |\mathbf{h}^b \in \mathcal{H}_b^{\mathbf{K}^{k-1}}\}$, $\mathcal{X}^{sub} = \mathcal{X}^{sub} \cup \{\mathbf{x^k}\}$
Step 2. $\Delta := UB - LB$. If $LB > 1$ then $\Delta := \Delta/LB$. If $\Delta \leq \epsilon$ then output $\boldsymbol{K}^k$, and STOP.
Step 3. Compute the new capacity assignment $\mathbf{K^k}$ for set $\mathcal{X}^{sub}$ and LB using Model (12).
Step 4. $k := k + 1$ and go to Step 1.

Table 1. Experimental results.

$\lvert\mathbb{C}\rvert$	6	8	10	12
$\lvert\Pi\rvert$	12	16	20	24
Time min(s)	0.63	28	48	93
Time avg(s)	3.2	171	434	1180
Time max(s)	22	562	2934	3450
Nb min it	2	3	3	2
Nb avg it	4	6	6	5
Nb max it	14	12	18	15

Algorithm (1) gives an approximate solution with tolerance error ϵ from the exact solution found by Problem (6). ■

In order to check the efficiency of Algorithm 1, we performed some computational tests on a set of artificial instances inspired from real data provided by the SDIS of Haute-Garonne Department. The generated data have the same order of magnitude as those recovered from the SDIS (number of Emergency Vehicles, number of Rescue Centers, Deployment Plans, average times needed for an EV from a given RC to reach the scene of accident).
For every $|\mathbb{C}| = 6, 8, 10, 12$ with $|\Pi| = 2.|\mathbb{C}|$, 30 instances of Problem (10) were generated. In every instance, $\forall \pi \in \Pi$ and $\forall (i,j) \in \mathbb{C}$, the following parameters are randomly generated: DP_π, $t_{i,\pi}$ the average times that respect the constraints due to DPs, $T_{i,j}^{max}$ the maximal transfer between RC_i and RC_j, and T_i^{RC} the maximal transfer for a given RC_i. The latter is randomly chosen from the set $\{0, 1, 2, \ldots, 5\}$. We assume that $\forall i \in \mathbb{C}$, $K_i^{max} - K_i^{min} = 5$, and both demand and initial capacity are randomly generated to satisfy the constraints $\sum_{i\in\mathbb{C}} K_i^{int} = \sum_{\pi\in\Pi} D_\pi$.

In order to solve the generated instances, we used GUROBI solver and a computer equipped with an Intel Core i7-4500U 1.8 GHz. Table 1 presents the minimal, average and maximal computation times in seconds, and the minimal, average and maximal number of iterations required to find approximations of optimal capacity assignment with the convergence tolerance parameter $\epsilon = 0.0001$. Thus, the evaluations of the computed capacity assignments are no more than 0.01% from optimality. As can be seen from the results obtained, Algorithm 1 enables quite large problems with up to 12 RCs and 24 DPs to be solved. The number of iterations seems independent of the size of the problem. However, the resolution time increases with the size of the problem due to the computation at each iteration of two MIPs.

6. Illustration with an example

In this section, we show how practitioners can use our model in a decision support approach. In fact, it is difficult to give an optimistic degree for practitioners. Hence, the optimistic degree is more often used to compute a set of possible assignments taking optimism into account. Here, we illustrate the method on an area of a department such as *Haute-Garonne* (France). We consider 7 RCs and 14 DPs. The initial, maximal and minimal capacities of RCs are:
K^{init} =(18, 17, 17, 9, 16, 20, 11)
K^{max} =(20, 20, 20, 10, 18, 20, 13)
K^{min} =(15, 15, 15, 5, 13, 15, 8)
The simultaneous requirements are: D_{π} =(7, 11, 13, 5, 10, 9, 8, 7, 6, 5, 5, 5, 10, 7), and the maximal transfers for RCs are: T^{RC} =(5, 4, 3, 5, 4, 5, 5). The maximal transferred quantities and the average times are given in Tables 2-3.

Table 2. The maximal transferred quantities.

T	RC_1	RC_2	RC_3	RC_4	RC_5	RC_6	RC_7
RC_1	0	4	4	2	3	3	3
RC_2	2	0	2	3	3	1	4
RC_3	2	1	0	2	1	3	1
RC_4	3	3	0	0	4	4	0
RC_4	1	4	3	4	0	2	2
RC_6	0	0	4	3	0	0	0
RC_7	3	1	1	4	4	3	0

Table 3. The average times.

t	RC_1	RC_2	RC_3	RC_4	RC_5	RC_6	RC_7
DP_1	19	15	5	13	3	6	8
DP_2	5	13	18	10	21	16	25
DP_3	14	19	12	8	5	1	6
DP_4	6	5	7	3	12	15	19
DP_5	6	17	5	12	8	9	22
DP_6	18	19	7	3	1	15	11
DP_7	24	20	8	15	12	4	1
DP_8	8	18	14	3	10	19	11
DP_9	5	14	17	15	8	10	1
DP_{10}	11	19	5	22	16	7	26
DP_{11}	12	7	23	18	14	4	8
DP_{12}	3	17	12	19	9	8	6
DP_{13}	6	5	20	18	11	14	13
DP_{14}	4	10	8	9	14	17	13

The proposed method allows the decision maker to estimate the performance of the redeployment for the best-case sequence and the worst-case sequence. To chose a redeployment plan the decision maker computes a set of optimal solutions which focus more or less on the worst/best-case sequences (using the parameter α). Table 4 summarizes different results depending on $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. The expected time for the best-case sequence is given in column *Best* and the expected time for the worst-case sequence in column *Worst*. First, we can remark that the expected times for the best sequence and the worst one are different. Moreover, we can not optimize both expected time at the same time since they are opposite (when one decreases the other increases). Nevertheless, in this example we only have three different optimal solutions according to α parameters. One for the optimistic DM ($\alpha \in [0.8, 1]$), one for the pessimistic DM ($\alpha \in [0, 0.3]$) and one other ($\alpha \in [0.4, 0.7]$). It can be seen that

(i) if we optimize $Best$, the best solution corresponds to the optimistic DM, (ii) if we optimize $Worst$, the best solution is given for a pessimistic DM, and (iii) RC_1, RC_4 and RC_6 are independent of the level of pessimism of the DM. On the other hand, RC_2 and RC_3 have variations of 3 EVs.

We can see that the solution, for $\alpha \in [0.4, 0.7]$, is a good compromise since it is just 1 second worse than the pessimistic ($\alpha = 1$) optimal solution and just 3 seconds worse than the optimistic ($\alpha = 0$) optimal solution. Moreover the proposed approach enables other aspects of the solution to be taken into account such as its stability (number of redeployed EVs). As can be seen from these results, the pessimistic solution seems to be the most stable.

Table 4. Intervals of the expected evaluations of the solutions.

			K_1	K_2	K_3	K_4	K_5	K_6	K_7
		Initial	18	17	17	9	16	20	11
α	*Best*	*Worst*	K_1	K_2	K_3	K_4	K_5	K_6	K_7
$[0.8, 1]$	3min25s	8min35s	19	15	18	9	16	19	12
$[0.4, 0.7]$	3min28s	8min22s	19	17	15	9	17	19	12
$[0, 0.3]$	3min31s	8min21s	19	18	15	9	17	19	11

This study is the first step towards a decision tool that takes into account the optimism of the decision maker (by using the parameter α) and human resource constraints. Our approach could be applied to define a redeployment plan by computing a set of optimal solutions which focus more or less on the worst/best-case sequences.

7. Conclusion

In this paper, we deal with the re-assignment problem of EVs to RCs based on the simultaneity of requirement of EVs, taking into account the uncertainty on the sequences of accident. The studied problem is a specific redeployment problem which can be situated between the Covering Facility Location Problem and the Real Time Redeployment Problem. Nevertheless, the focus is different as we introduce the notion of DPs and the simultaneity of requirements, which have not been studied in the literature. Based on the Hurwicz criterion, we propose a non-probabilistic two-stage approach to deal with the uncertainty which is due to the simultaneous requirement of EVs. For a given degree of optimism of the decision maker, the proposed iterative algorithm calculates the optimal mid-term redeployment of capacity. A study of the performance of the proposed algorithm and an illustration from the DM point of view are presented. It shows that the algorithm is applicable to a reasonable size of problem. In future work, we want to add constraints on the maximal time to reach an accident and improve the algorithm by investigating a colony generation approach to solve the master problem. Moreover, we want to investigate a multi-criteria optimization to take into account the cost of transferring capacity, even if the expected rescue time is the most important criterion. Another perspective is to couple the possibility of modifying the DP with the capacity redeployment, since the DP impacts the load of each RC. In other words, by modifying the DPs, there would be fewer changes in the initial capacities of the RCs.

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