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DISPERSIVE GRID-FREE ALGORITHM APPLIED ON REAL DATA FOR MODAL ESTIMATION IN OCEAN ACOUSTICS

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ABSTRACT

In underwater acoustic shallow-water environments act as dispersive waveguides. With a low-frequency source emitting in such environment, the signal can be described as the sum of few modal components, each associated to its own wavenumber. A precise estimation of these wavenumbers is essential to characterize the nature of the environment. In the continuation of previous works, we propose a new-grid free algorithm applied to time-shifted data allowing a super-resolution of the (f-k) diagram. Our method is validated on both simulated and real data.

1. INTRODUCTION

In normal mode theory, the low-frequency sound field in shallow water is composed of several normal modes at each frequency. The solutions of the modal equation with its boundary conditions are a series of modes [1]. Each mode is characterized by its own wavenumber. For a horizontal linear array (HLA), the received signal at frequency f can be written within the matrix form

$$\mathbf{y}_f = \mathbf{D}\mathbf{a}_f + \mathbf{n}_f, \tag{1}$$

where $\mathbf{y}_f$ is the received signal at frequency f, the elements in $\mathbf{a}_f$ are proportional to the modal amplitudes, $\mathbf{n}_f$ is an additive noise and $\mathbf{D} \in \mathcal{C}^{M\times N}$ is a dictionary made up of Fourier atoms, that constitutes a N-points discretization of the wavenumber subspace. A least square estimation of $\mathbf{a}_f$ is given by applying an inverse Fourier transform to $\mathbf{y}_f$. Repeating the process for each frequency of the source and stacking the obtained spectra results in a frequency-wavenumber (f-k) diagram representative of the waveguide dispersion [2, 3, 4]. While being simple, this approach is also highly sensitive to the antenna configuration and requires a large quantity of sensors to reach a satisfying resolution of spatial wavenumbers. Alternatively, since only few modes are expected at each frequency, sparse representations (SR) comes as a natural and intuitive model for the search of these few wavenumbers over a large dictionary of discretized wavenumbers. SR algorithms have been successfully applied to the modal estimation problem [4, 5]. However, these approaches are almost always looking for the wavenumbers within a discretized domain–*i.e.,* within ad discretized dictionary $\mathbf{D}$–that induces mismatch in the estimation. In [6] the authors propose a grid-free version of the SR optimization problem that is solved using the Convex optimization toolbox CVX and thus results in a high complexity and time consuming algorithm.

Some recent contributions [4, 5] proposed to take into account a propagation relation of the estimated modes between two successive frequencies. This was showned to provide a performance improvement compared to the naive approach.

We propose to include a mode propagation relation into an computationally light grid-free algorithm based on the well known greedy SR procedure that is Orthogonal Matching Pursuit (OMP) [7]. The proposed approach is also considering a shifted version of the signal that writes for each frequency f as $\mathbf{y}_f^{shift} = e^{(\frac{2i\pi f}{c})}\mathbf{y}_f$, where c is the celerity in the medium. This shift, proposed in [2, 3] is a method that allows a better separability of the wavenumbers and prevents aliasing in the wavenumber domain.

2. PROPOSED APPROACH

In modal theory, the horizontal wavenumbers k_{rm} associated to the propagating modes are linked to their vertical counter-parts k_{zm} by the dispersion relation, that is, for a given frequency f,

$$\left(\frac{2\pi f}{c}\right)^2 = k_{rm}(f)^2 + k_{zm}(f)^2. \tag{2}$$

Discretizing the frequency axis (with $f = \nu\Delta_f$, $\nu \in \{0, ..., F\}$, Δ_f the frequency spacing) and denoting $\tilde{k}_{rm}[\nu] \triangleq \tilde{k}_{rm}(\nu\Delta_f)$, the shifted wavenumbers attached to two successive indices are linked as [4] :

$$\left(\tilde{k}_{rm}[\nu+1]+\alpha(\nu+1)\right)^2 = \left(\tilde{k}_{rm}[\nu]+\frac{2\pi\Delta_f}{c}\nu\right)^2+(2\nu+1)\left(\frac{2\pi\Delta_f}{c}\right)^2+\epsilon[\nu]. \quad (3)$$

where $\epsilon[\nu]$ stands for the variation of the vertical components. In shallow-water environments, the vertical wavenumbers k_{zm} weakly depend on the frequency, the quantity ϵ is thus smaller than the other terms of the equation and can be neglected. The dispersive relation has been exploited in [4, 5, 8].

Our proposed approach mainly builds on a continuous version of OMP (COMP) that basically consists in i) selecting "the best" wavenumber from the grid to be part of the sparse representation and ii) initiating a gradient descent from this wavenumber to relax the grid constrained solution. The contribution of the obtained wavenumber is then removed from the residual signal and these steps are repeated up to a defined stopping criteria, *e.g.,* up to a maximum number of selection. In the continuation of [5], we propose to integrate the information provided by the dispersion relation (3) into the COMP SR process. Instead of performing COMP for each frequency independently, we propagate the SR obtained at frequency f to frequency $f+\Delta_f$. Concretely, if no wavenumbers have been activated in the SR of $\tilde{\mathbf{y}}_f$ then SR of $\tilde{\mathbf{y}}_{f+\Delta_f}$ is performed using standard COMP. On the contrary, if some wavenumbers have been activated, then the SR process starts with a propagation stage that consists in initiated gradient descents to each wavenumber satisfying (3). Depending on their relevance, the obtained wavenumbers are conserved in the SR support or ignored. Standard COMP in then applied to the remaining wavenumbers of the grid. The process is repeated for the whole frequencies so as to build the (f-k) diagram.

3. EXPERIMENT

We propose to compare the performance of our approach to those obtained with OMP, COMP and propagated SoBaP algorithm [5]. Tests are realised on a physically-realistic dataset simulated with a Pekeris waveguide [1].

Fig.1 shows the evolution of the Jaccard's distance [9] reached with four algorithms with respect to the number of sensor on the simulated dataset. Jaccard's distance is a criteria designed to quantitatively assess the detection performance of continuous algorithms. One can see that our approach, that combines both an off-grid selection of the wavenumbers and the propagation relation outperforms all other considered approaches no matter the number of sensors. The method has also been tested on experimental data collected in the North Sea and shows promising result.

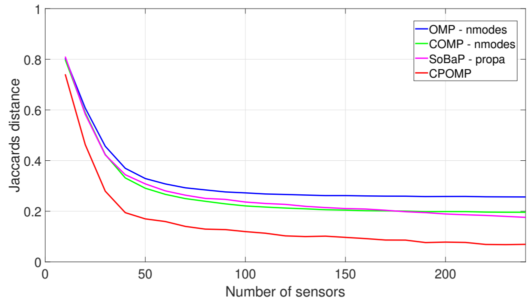

Figure 1. Evolution of the Jaccard's distance with respect to the number of sensors.

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