Adapting the correction for CFAT application in time domain

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ADAPTING THE CORRECTION FOR CFAT APPLICATION IN TIME DOMAIN

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1. EXTENDED ABSTRACT

1.1 Context

This paper aims at presenting the potential interest of the CFAT (Corrected Force Analysis Technique) in the identification of a turbulent boundary layer excitation in time domain. This kind of excitation can be separated into two distinct components, the first being the convective part. This part encompasses the pressure field induced by the turbulences themselves. These turbulences are also generating acoustic waves, which contribute to the excitation as well.

The proper discrimination of the acoustic part from the convective one appears at first sight as problematic. Energetically speaking, the acoustic component is estimated to be between 30dB and 60 dB lower than its convective counterpart [1]. In order to effectively detect the acoustic part, a wavenumber approach should be privileged, as the two components of the turbulent boundary layer excitation have distinct wavenumber signatures : the convective part is located at high wavenumbers, whereas the acoustic part is located at low wavenumbers.

The existing methods that allow the separation of the two parts of a turbulent boundary layer excitation, can be classified into two categories. The first categories rely on the use of an array of pressure sensors and of wavenumber filtering [2]. Derivatives of this method have proven to be capable of identifying both components [3], although being highly sensor demanding and noise sensitive.

The second category regroups methods that analyse the vibration field of a structure, and take advantage of its natural wavenumber filtering to isolate the acoustic component [4]. These methods are generally easy to set up, but work on a specific wavenumber range. The CFAT can be filed in this category, and has already been used to identify the acoustic part of a turbulent boundary layer in the low wavenumber domain. [5].

1.2 Aim

An extension of the CFAT is proposed, which would allow load identification in time domain. This new approach would allow the study of transient signal, as well as spatially uncorrelated ones, such as turbulent boundary layer excitation. Moreover, knowing the time evolution of its acoustic component would provide data to sharpen models and simulations of turbulent boundary layers.

The approach is overall similar to the classical CFAT [6] : starting from the equation of motion of a structure, the partial derivatives with respect to space are discretized using a finite difference scheme. In order to take into account the bias introduced by this approximation, a correction is put into the equation. The time domain extension adds the need to design a filter to discretize the time dependent operator. For example, the equation of acceleration of flexural waves, refered as $\gamma(x,t)$ on a beam is considered :

$$f(x,t) = EI \iint \frac{\partial^4 \gamma}{\partial x^4} dt^2 + \rho S \gamma(x,t), \tag{1}$$

where E is the Young Modulus, S the beam section, ρ the density, and I the flexural moment of inertia. In this equation, the time operator is a double integration, that can be modelled using the trapezoidal rule. As the use of such filtering introduces another bias, the corrective coefficient must be adapted consequently. This new expression is :

$$\mu(\omega) = \frac{(k_N \Delta)^4 \tan^2\left(\frac{\omega T_s}{2}\right)}{\left(\omega T_s \left(1 - \cos(k_N \Delta)\right)^2\right)}, \tag{2}$$

T_s being the sampling period, and k_N the flexural wavenumber. The new correction being frequency dependant, it has to be translated into the time domain. This can be done numerically, using inverse fast-Fourier transform algorithm. Beforehand, as the CFAT method is only valid on a specific frequency range, a Tukey window is applied to the correction. Eventually, the correction, after being windowed and expressed in the time domain, is convolued. The resulting CFAT expression thus can be written :

$$f_{CFAT}(x,t) = EI\tilde{\mu}(t) * I^{2t}_{T_s}(\delta^{4x}_{\Delta}(\gamma(x,t)) + \rho S \gamma(x,t). \tag{3}$$

In this equation, the trapezoidal rule, represented as the term $I^{2t}_{T_s}$ is applied to the finite difference scheme, labeled δ^{4x}_{Δ}. The term $\tilde{\mu}(t)$ is the windowed correction in the time domain. The correction is only applied on the stiffness part of the equation, however the Tukey window must be applied on both terms. In order to assess the ability of the method to properly identify an excitation field, the ratio between the CFAT identified force (without using the Tukey

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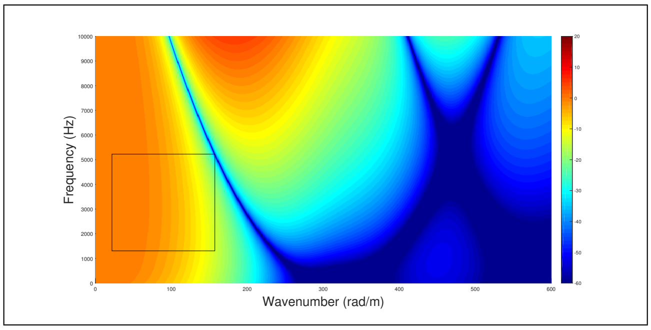

Figure 1. Ratio between the CFAT load identification and the actual load

window) and the real one is displayed in the dual domain (frequency wavenumber)in figure 1.

In this figure, the black rectangle encircles the area that is effectively identified. The Tukey window is used to keep only this part of the excitation. In this area, the ratio is equal to 0 dB for low wavenumbers, so the load is identified accurately. However, for higher wavenumber, the filtering effect of the finite difference scheme lessen the quality of the identification, diminishing the ratio down to -20 dB. Other filters can be used as alternative to address this issue, whose frequency response allows a more accurate identification [7].

1.3 Simulation results

An acceleration field fullfilling equation (2) is considered. The excitation field is given by the following equation :

$$f(x_0, t) = f_0 e^{j\frac{\omega_2+\omega_1}{2}t} sinc\left(\frac{\omega_2-\omega_1}{2}t\right), \qquad (4)$$

where f_0 is the amplitude of the excitation, and (x_0, t_0) a set of random coordinates in time and space. The frequency terms ω_1 and ω_2 are respectively the minimum and maximum frequency between which the method can be applied. The acceleration field resulting from this excitation is injected into equation (3). The excitation obtained using CFAT is then compared to the initial excitation at position $x = x_0$:

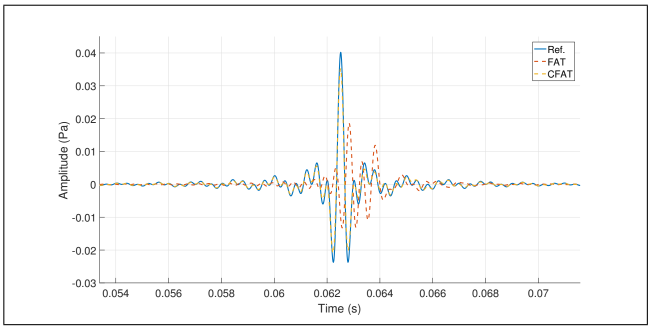

Figure 2. Comparison of the reference signal (blue plain line) with the FAT (red dashed line) and CFAT (yellow dotted line) identified effort

Figure 2 displays the excitation defined in (4), projected at position $x = x_0$, along with the FAT and CFAT identified excitation at the same position. In this situation, the CFAT allows a more accurate identification, and lessens the impact of secondary lobes. As the CFAT method tends to underestimate the amplitude of the identified excitation, this result was predictable.

1.4 Conclusion

The CFAT extension in time domain requires the introduction of a filter that can approximate the time operator of the equation of motion. This approximation led to the calculation of a corrective coefficient that could take its effect into consideration. As the method is frequency limited, a window should be applied in order to get rid of frequency information out of its range. For low wavenumbers the method allows an accurate identification of the excitation, although the finite difference scheme effect lowers the amplitude of the reconstructed excitation for higher wavenumbers. This effect could be lessened by using other spatial filter designs.

The method can be used effectively to identify an impulse excitation. The use of this method in order to identify a turbulent boundary layer excitation would require an extension of the method to plates.

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