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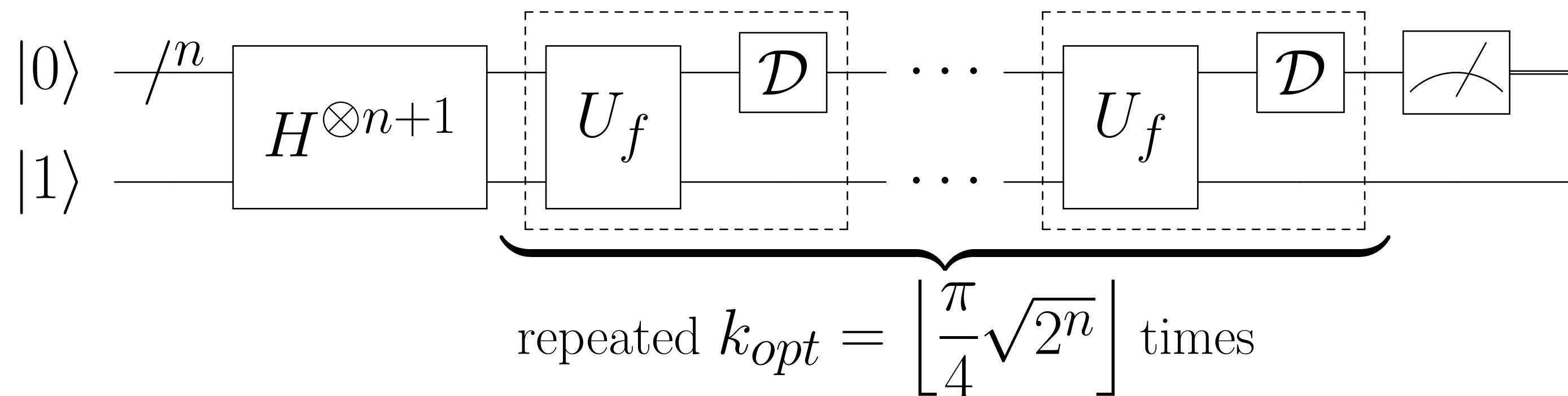


# NON-LOCALITY AND ENTANGLEMENT DETECTION WITH MERMIN POLYNOMIALS FOR GROVER'S ALGORITHM AND QUANTUM FOURIER TRANSFORM

Henri de Boutray, Hamza Jaffali, Frédéric Holweck, Alain Giorgetti & Pierre-Alain Masson

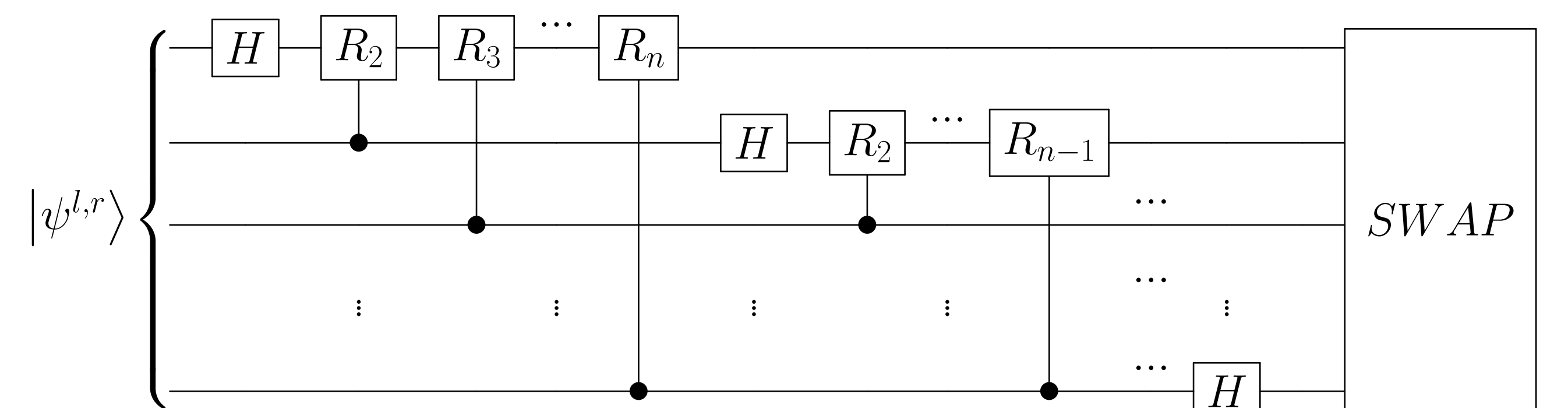
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## Grover's quantum algorithm



- Search of an item  $|\mathbf{x}_0\rangle$  in a list, based on an oracle
- Complexity:  $\mathcal{O}(\sqrt{2^n})$  (against  $\mathcal{O}(2^n)$  for classical search algorithms)

## Quantum Fourier Transform



Generally performed on periodic states defined by:  
 $|\varphi^{l,r}\rangle = \frac{1}{\sqrt{A}} \sum_{i=0}^{A-1} |l + ir\rangle$

- Quantum analogue of the discrete Fourier transform
- Complexity:  $\mathcal{O}(n^2)$  (against  $\mathcal{O}(n^{2n})$ )

## Mermin's polynomials

$$\begin{cases} M_1 = a_1 \\ \forall n \geq 2, M_n = \frac{1}{2} M_{n-1} \otimes (a_n + a'_n) + \frac{1}{2} M'_{n-1} \otimes (a_n - a'_n) \end{cases}$$

$(a_i)$  and  $(a'_i)$  are one-qubit observables with eigenvalues in  $\{-1, 1\}$

$\langle \varphi | M_n | \varphi \rangle > 1$  implies that  $|\varphi\rangle$  is non-local.

Example:

With  $(a_1, a_2, a'_1, a'_2) = (Z, X, (Z + X)/\sqrt{2}, (Z - X)/\sqrt{2})$ ,  $M_2$  corresponds to the operator used for Bell inequalities.

## Grover's algorithm evaluation

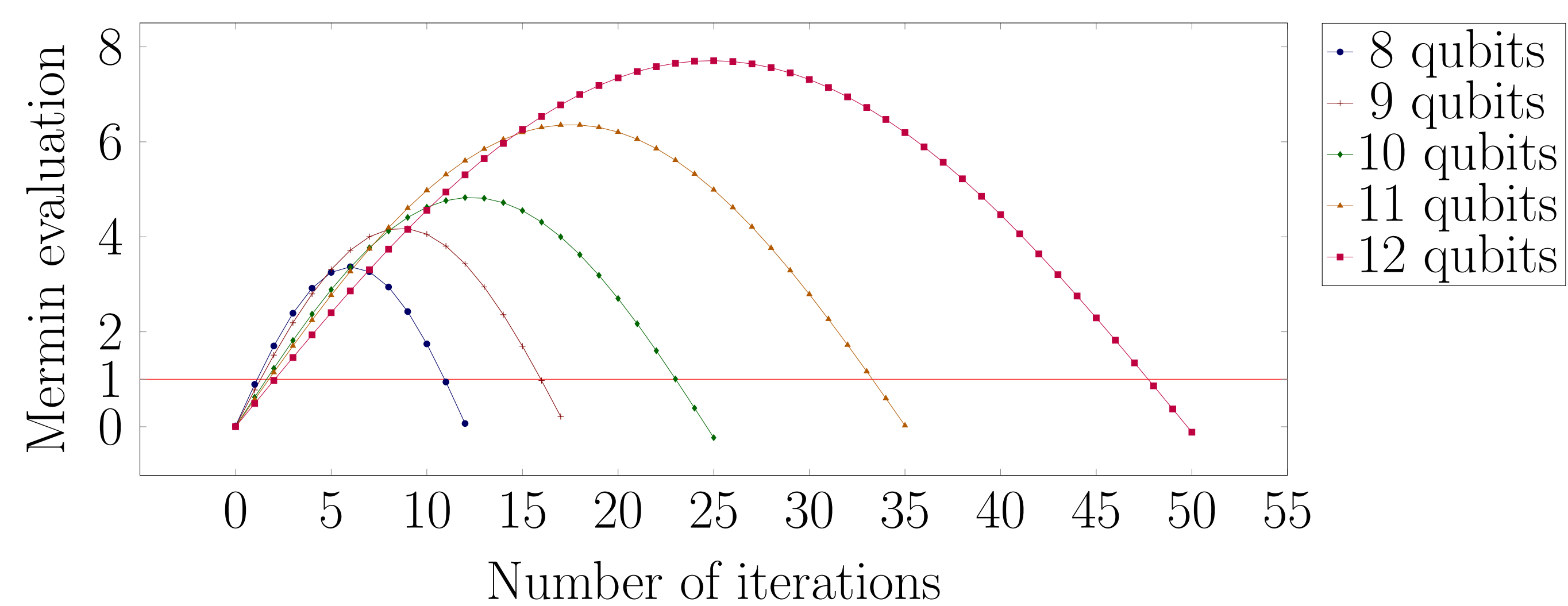
### Proposition ([JH19]):

- The states in Grover's algorithm are  $|\varphi_k\rangle = \alpha_k |\mathbf{x}_0\rangle + \beta_k |+\rangle^{\otimes n}$ , with  $(\alpha_0, \beta_0) = (0, 1)$  and  $(\alpha_{k_{opt}}, \beta_{k_{opt}}) \approx (1, 0)$ .
- For  $k$  close to  $k_{opt}/2$ ,  $|\varphi_k\rangle$  comes close to a state  $|\varphi_{ent}\rangle = (|\mathbf{x}_0\rangle + |+\rangle^{\otimes n})/K$  maximizing  $\langle \varphi | M_n | \varphi \rangle$ .

Evaluation method: find  $M_n$  such that  $\langle \varphi_{ent} | M_n | \varphi_{ent} \rangle$  is maximal.

Computing  $\langle \varphi_k | M_n | \varphi_k \rangle$  for every  $k$  with this  $M_n$  **positively** answers the following question:

"Is Grover's algorithm using entanglement to achieve quantum speedup?"

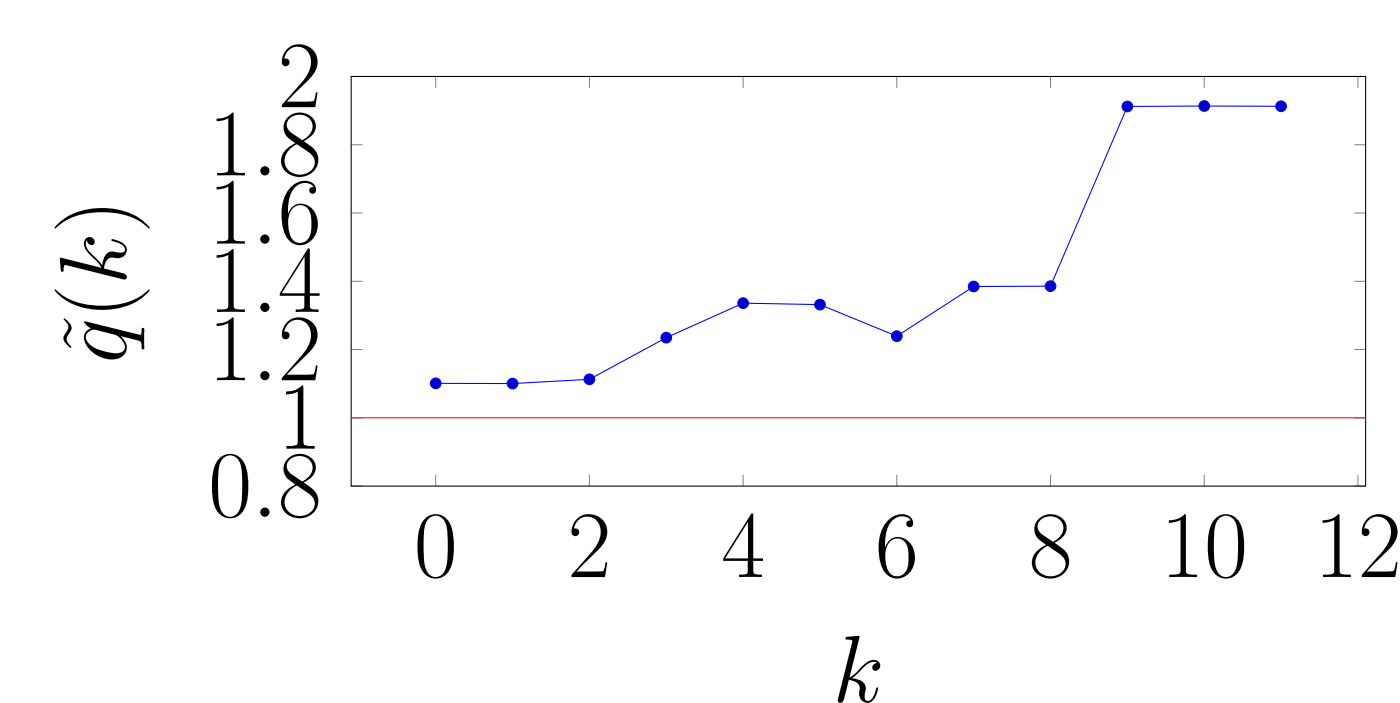


## QFT evaluation

$q : |\varphi\rangle \mapsto \max_{M_n} \langle \varphi | M_n | \varphi \rangle$  is a measure of entanglement.

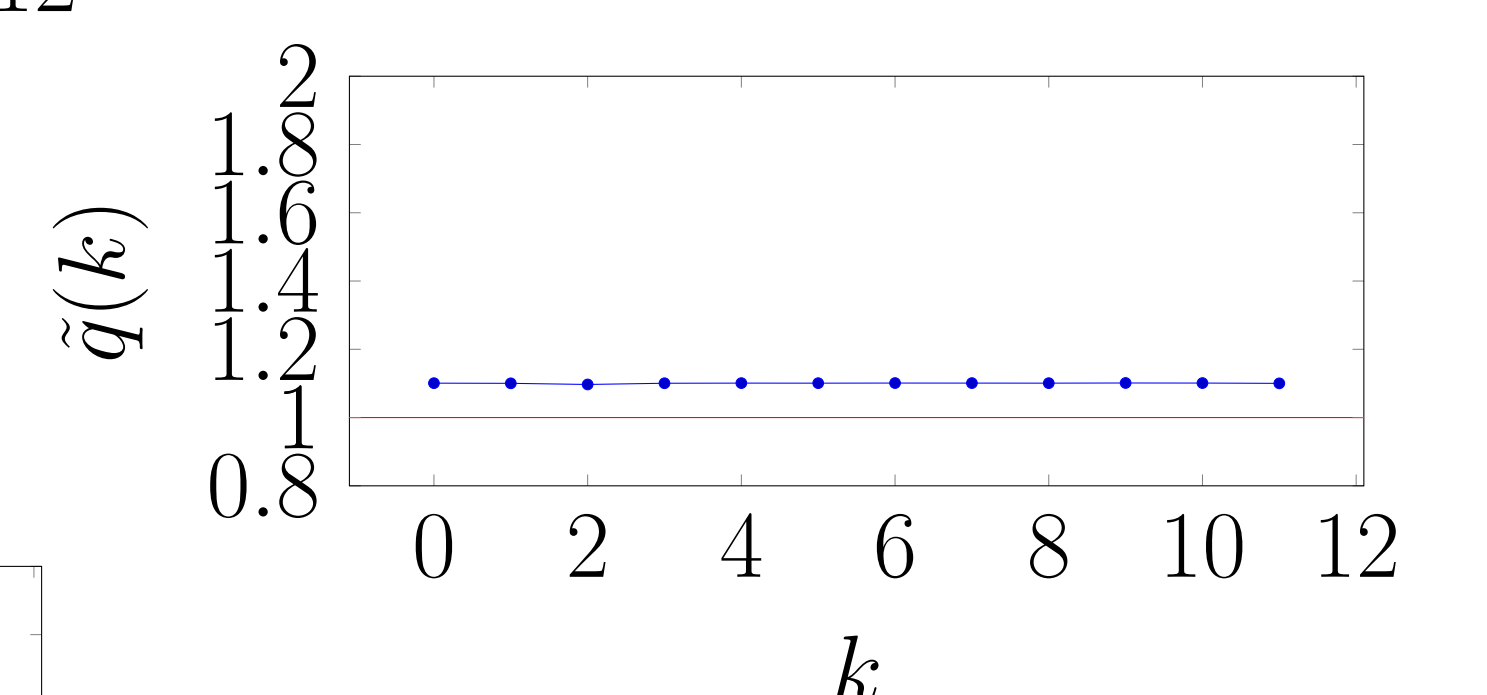
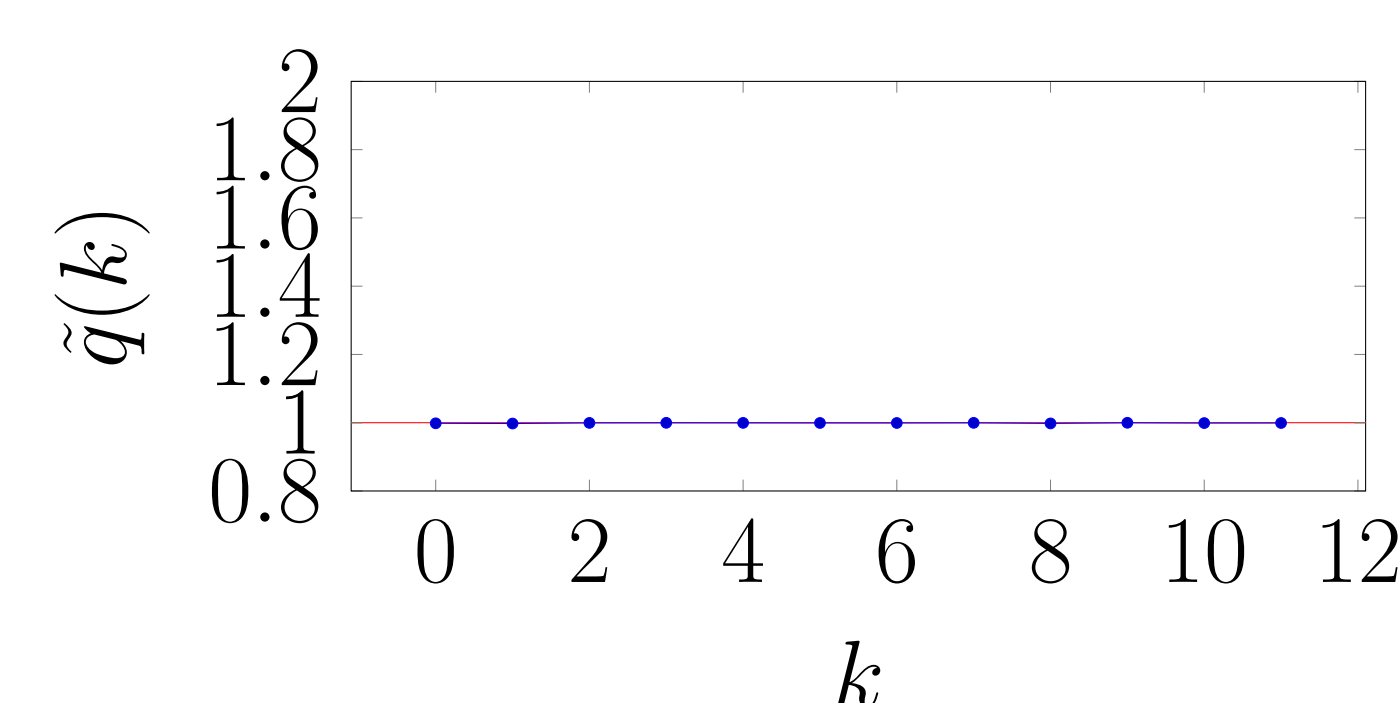
Evaluation method: for each state  $|\varphi_k\rangle$ , find  $\tilde{M}_n$  such that  $\langle \varphi_k | \tilde{M}_n | \varphi_k \rangle$  is maximal.

The corresponding experimental approximation  $\tilde{q}$  of  $q$  allows us to distinguish between three types of QFT runs in our experiments with  $n = 4$ :



Entangled states and variable measure (here for  $(l, r) = (9, 1)$ ).

Entangled states and constant measure (here for  $(l, r) = (2, 2)$ ).



Separable states (here for  $(l, r) = (8, 5)$ ).

One can also check some key points such as the fact that entanglement evaluation doesn't change during LOCC operations ( $H$  gates in this case).

## References

[dJH<sup>+</sup>20] H. de Boutray, H. Jaffali, F. Holweck, A. Giorgetti, and P.-A. Masson. Non-locality and Entanglement Detection with Mermin polynomials for Grover's algorithm and Quantum Fourier Transform. *Submitted in September 2020*, previous version at arXiv:2001.05192.

[JH19] H. Jaffali and F. Holweck. Quantum Entanglement involved in Grover's and Shor's algorithms: The four-qubit case. *Quantum Information Processing*, 18(5):133, May 2019.

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