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To cite this version:

Cyril Calmettes, Emmanuel Perrey-Debain, Emmanuel Lefrançois, Julien Caillet. Calculation of a Multi-Port Scattering Matrix for the Acoustic Power Flow Using Finite Element Method. Forum Acusticum, Dec 2020, Lyon, France. pp.383-385, 10.48465/fa.2020.0576 . hal-03221404

HAL Id: hal-03221404
https://hal.archives-ouvertes.fr/hal-03221404

Submitted on 20 May 2021

CALCULATION OF A MULTI-PORT SCATTERING MATRIX FOR THE ACOUSTIC POWER FLOW USING FINITE ELEMENT METHOD

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ABSTRACT

Duct acoustic network modelling is commonly carried out using the transfer matrix formalism which is limited to the low frequency range. The aim of this work is to extend it to higher frequencies by taking into account the multi-mode acoustic propagation. The first step is to compute, via finite element discretisation (FEM), the multi-port multi-modal scattering matrix of each element. The second step is to transform it into a scattering matrix for the acoustic power, relying on assumptions which are often used for the study of medium-to-high frequency broadband noise.

The method is applied to typical elements such as expansion chamber mufflers and air conditioning veins. In all cases, the power-flow model is compared to the FEM solution in terms of Transmission Losses. It is concluded that this simplified model is a reliable tool for the analysis of complex networks encountered in HVAC systems.

1. INTRODUCTION

In helicopters, cabin and cockpit heating is performed by taking hot air from engine compression stage. This heat is mixed to ambient or external air into a HVAC through an injector in order to reach the temperature setpoint. Acoustically, it results in a high frequency hot jet noise source propagating through the distribution system ducts before it radiates into the cabin and cockpit.

The matrix transfer method which is widely used in the industry is limited to the propagation of plane waves in the ducts and is therefore not suited in the present context. Though more stable numerically, the scattering matrix formalism for plane waves suffers from the same limitations but is better suited for network modelling as it allows an easy connection between sub-systems [1, 2]. In order to treat high frequency noise propagation in the system, the aim of this work is to develop a power flow scattering matrix formalism containing multi-modal information. This is achieved by first calculating the multi-modal pressure scattering matrix of a sub-system via the Finite Element Method (FEM).

2. THEORY

FEM allows to calculate duct modes shapes and associated cut-off frequencies at each port of the element, or subsystem. Following Kirby's work on matching methods [3], a scattering matrix for the acoustic pressure propagating in a multi-mode context is constructed. The latter links incoming mode coefficients $\mathsf{A}_j^+ = (A_{j,1}^+, A_{j,2}^+, \ldots)^t$ at port j with outgoing ones $\mathsf{A}_i^- = (A_{i,1}^-, A_{i,2}^-, \ldots)^t$ at port i as illustrated in Figure 1. Mathematically, this means that

$$\mathsf{A}_i^- = \sum_j \mathsf{S}_{i,j} \mathsf{A}_j^+, \tag{1}$$

where the block matrix $\mathsf{S}_{i,j}$ is the scattering matrix from port j to i and the summation is made over the total number of ports.

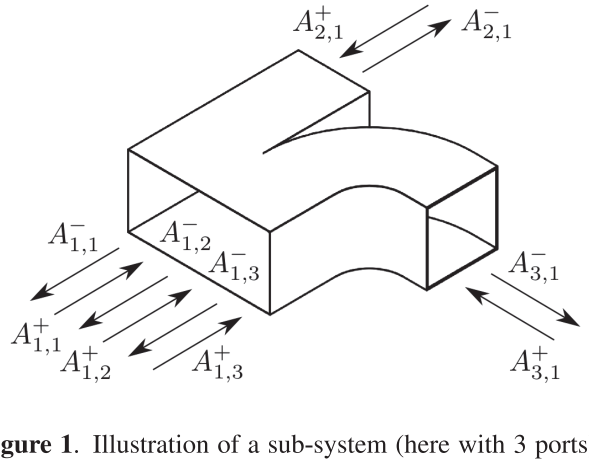

Figure 1. Illustration of a sub-system (here with 3 ports) showing incoming and outgoing acoustic pressure waves.

In order to transform it into a multi-port scattering matrix for the power flow, it is assumed that : (i) all propagating modes are uncorrelated and (ii) each mode carries the same quantity of acoustic energy. With this in mind, coefficients S_{ij}^W of the power scattering are defined as the power loss from port j to port i can be written as

$$S_{ij}^W = \frac{\langle W_i^- \rangle}{\langle W_j^+ \rangle} = \frac{(\mathsf{A}_j^+)^\dagger \mathrm{diag}(\mathsf{S}_{i,j}^\dagger \mathsf{K}_i \mathsf{S}_{i,j}) \mathsf{A}_j^+}{(\mathsf{A}_j^+)^\dagger \mathsf{K}_j \mathsf{A}_j^+} \tag{2}$$

Where K_i is the diagonal matrix containing the wave numbers for the propagating modes in port i. The calculation is carried out with assumption (ii) which means that at each port the quantity $k_n|A_n|^2$ is constant and we can take $A_n = (k_n)^{-1/2}$, here index n refers to the mode number.

3. RESULTS

The accuracy of the power method is assessed in the case of two chained 2-ports sub-systems where each sub-system consists of an expansion chamber with or without porous material. The main body length of the expansion chamber of cylindrical shape is set to L = 40 cm while inner and outer radius r_a and r_b are respectively set to 4 and 6 cm. The chamber is either filled with air (purely reactive) or with a rock mineral wool placed between r_a and r_b (dissipative case). Here the equivalent fluid model is employed and Biot parameters are taken from [4] : tortuosity α_∞ = 1, resistivity σ = 14066 N s m^{-4}, porosity Φ = 0·954, viscous and thermal characteristics Λ = 91·2 μm and Λ' = 182·4 μm. Air parameters are: air mass density ρ = 1·213 kg m^{-3}, speed of sound c = 342·2 m s^{-1}, dynamic viscosity μ = 1·84 $\times$ 10^{-5} kg m^{-1} s^{-1}, heat capacity ratio γ = 1·4 and Prandtl number Pr = 0·71. The same FEM mesh illustrated in Figure 2 is used in both configurations.

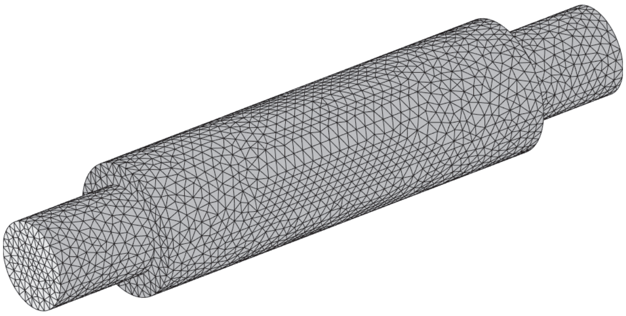

Figure 2. FEM mesh of the expansion chamber.

Figure 3 shows the Transmission Losses (TL) due to the association of two similar expansion chambers connected to each other. Results are averaged over the 1/3-octave frequency band as

$$\mathrm{TL} = -10\log\left(\frac{\int_{f_{min}}^{f_{max}}\langle W_t\rangle\,df}{\int_{f_{min}}^{f_{max}}\langle W_i\rangle\,df}\right) \qquad (3)$$

Results referred as ‘Pressure matrix’ correspond to the exact FEM calculation wheres ’Power matrix’ corresponds to the 2-ports simplified scattering model expresses in terms of acoustic power. It can be observed that discrepancies do not exceed 3 dB, which is acceptable from an engineering point of view. These results are encouraging as this purely reactive scenario is probably not ideal to assess the power method due to resonance effects which can not be properly predicted with the method.

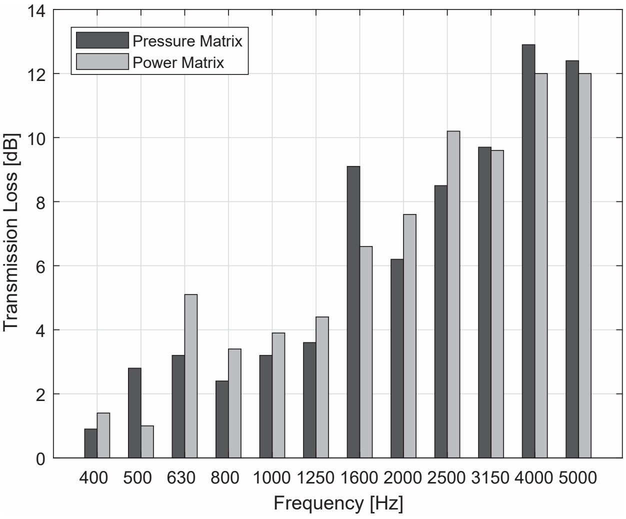

Figure 3. TL computed with the ’Pressure matrix’ (exact) and the ’Power matrix’ for two rigid expansion chambers.

Figure 4 shows the same comparison except that the second expansion chamber is now filled with a porous material.

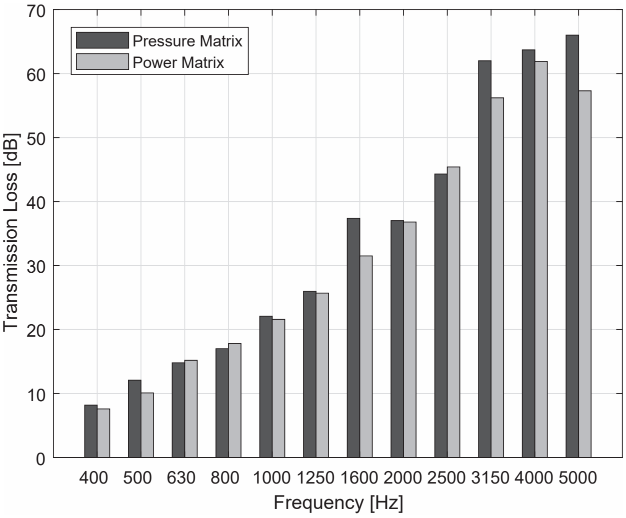

Figure 4. TL computed with the ’Pressure matrix’ (exact) and the ’Power matrix’ for a porous and a rigid expansion chamber.

Results are in good agreement and it is observed that the power method tends to under estimate the TL. Differences between the two methods tend to increase at high frequencies showing a maximum deviation of 8 dB attenuation at 5 kHz. Nevertheless, the method is reliable for engineering purposes with potential applications for the simulation of acoustic waves in complex realistic networks.

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