



**HAL**  
open science

# Financial planning & optimal retirement timing for physically intensive occupations

Edouard Ribes

► **To cite this version:**

Edouard Ribes. Financial planning & optimal retirement timing for physically intensive occupations. 2022. hal-03219182v2

**HAL Id: hal-03219182**

**<https://hal.science/hal-03219182v2>**

Preprint submitted on 17 May 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Financial planning & optimal retirement timing for physically intensive occupations

Edouard Ribes, Mines Paristech

15/05/2022

## Abstract.

On average, O.E.C.D. statistics<sup>1</sup> shows that 4 out of 10 workers in developed geographies occupy a “blue-collar” type of position. Since those professions are physically demanding and come with a toll on one’s health (which in turns translates into additional healthcare expenses), the length of an individual’s active period must be carefully weighted. For blue collar workers seeking to accumulate capital, the decision to partake to the workforce results from a tradeoff. On one hand, labor generates capital (through savings) over the short run. On the other, it forces the individual to decumulate more capital over the long run as labor depreciates one’s health stock.

This article therefore offers a discrete financial model (discounted cashflow) to help make such decisions. Insights are generated by subjecting the model to a parametric sensitivity analysis to explore the various optimal retirement decisions available to blue collar workers. It notably shows that whilst most developed countries require individuals to work for about 40 years, early retirement can represent an economically viable solution. Early retirement should however not be considered as a “one-size-fits-all” kind of solution for “blue collar” roles as its benefits depend on a host of occupational factors. The core proposition developed in this article is indeed that retirement decisions should be heterogeneous. The primary reason is that different jobs yield different levels of physical intensity and therefore drive different retirement decisions. The second reason is that individuals journey across the labor market as learning and promotions opportunities arise. This potentially creates options for blue collar workers to move from physically taxing positions to non-taxing ones (e.g. white collar positions), which could delay their retirement decision.

This paper also put a specific emphasis on the impact financial literacy has retirements behaviors. For those with a strong predilection for present consumption and little interest in savings and investments, retirement is not an option. Since this kind of pattern is common amongst workers occupying blue collars roles, this article highlights that educational programs could be of use. Non forward looking behaviors indeed translates into an extra financial pressure which constrains those individuals to work until the end of their life under increasingly challenging health conditions.

**Keywords.** Health; Retirement; Financial planning

## Compliance with Ethical Standards:

**Funding.** The authors did not receive support from any organization for the submitted work.

**Conflict of interest.** The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers’ bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing

---

<sup>1</sup> See the O.E.C.D. WISE database: <https://www.oecd.org/employment/skills-for-employment-indicators.htm>

arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

**Informed consent.** None.

## Introduction

Research in the field of financial planning dates back as far as the 60s. Historically speaking, associated research has however been heavily geared towards company/firm level applications (e.g. optimal trading programs for banks, investments scheduling applications for industrial companies, workforce/manpower planning etc..) (see (Andriosopoulos, Doumpos, Pardalos, & Zopounidis, 2019) for a review and discussion).

This is something that is starting to change. The digitalization of the financial eco-system (i.e. the so called “fintech” movement) has been reducing financial intermediation costs and therefore has led to a democratization of financial services for retail investors (Gomber, Kauffman, Parker, & Weber, 2018). In the case of financial planning services, this means that services are no longer limited to firms but are being offered via platforms to individuals (independently of their wealth profile).

At this level, financial planning is about helping individuals achieve a balanced budget meeting their everyday needs. Those needs, according to the definition proposed by the O.E.C.D, include housing, food, clothing, leisure, and health. Among those elements, health has started to take in increasingly prominent place in developed countries. Associated expenditures have been on the rise, up from 3.8% of household budgets in the 60s to 8%+ in the late 90s (Nghiem & Connelly, 2017). Drivers causing this increase are well-known (income growth, the ageing of the population, technological progress, and the widespread availability of health insurance). But if the situation has started to stabilize in certain countries such as the E.U. ones (Hitiris & Nixon, 2001), the situation is not yet under control everywhere across the globe. One of the worst scenario (for developed countries) is for instance encountered in the US where about 25% of the population cannot, as of today, cope with health related bills (Dickman, Himmelstein, & Woolhandler, 2017) and where the situation keeps on worsening (according to the O.E.C.D statistics health expenses in the US represented 20%+ of households expenditures in 2018).

One of the acknowledged problems associated with raising healthcare costs has been that inequalities across households have been increasing. This is for instance reflected by the fact that nowadays life expectancy between the wealthiest constituents of the population and its lower quartile (income wise) varies by 10+ years (Chetty, et al., 2016). It is also made blatant by the fact that an important fringe of the population (8%+ in the US (Berchick, Hood, & Barnett, 2019), 2%+ in the EU (Toth, 2019)) still cannot afford/ does not have access to basic medical insurance. Additionally, primary health insurance scheme coverage can vary widely. As a result, the least wealthy part of O.E.C.D countries’ population has a hard time managing their budget and health bills.

Health expenses are however pervasive in the sense that they increase exponentially over time as one’s abilities start to naturally decline (Shang & Goldman, 2008). This is normally met by constituting precautionary savings whilst active (Cagetti, 2003). For the lower quartile (income wise) of O.E.C.D. populations, a first challenge lies with individual’s saving behavior (i.e. less wealthy individuals tend to live in the now – a well know behavioral trait (Love & Phelan, 2015)). An additional challenge arises for those involved in physically demanding occupations which are having an impact on their health. In this case, a potential conflict arises between (option A) working and saving whilst deprecating their health stock at a faster pace than what would naturally occur and (option B) stop working but having to face important healthcare related bills<sup>2</sup>. In short, a prominent question nowadays for a large fringe of the population is: when is it optimal to retire?

---

<sup>2</sup> Note that in this context, health issues refer to conditions that cannot be recovered from (e.g. chronic musculoskeletal conditions), but only controlled by “artificial” means (i.e. a therapy).

This paper proposes a first layer of answers to the question of optimal retirement timing. Answers are provided by setting up a discrete toy model representing the cashflow of a blue collar worker. The model is then incrementally built to encompass features representing the professional journey (e.g. decrease in ones' health stock) and financial behavior of an individual (e.g. time preferences). The underlying assumption of this exercise is that forward looking individuals seek to maximize the capital they accumulate through their life whilst having to meet increasing living expenses<sup>3</sup>. To meet this goal, they enter the workforce and transform their earnings into capital once the expenses associated to their living standards are met or decumulate acquired capital. In such setup, one would intuitively expect that individuals aiming at accumulating capital would work for as long as they can. But in the case of blue collar worker, an additional element weights in: working exerts a significant toll on one's health and if it creates capital, it also generates extra living expenses at a later stage of life and become the source of a predictable decumulation. A question of optimal retirement timing therefore naturally arises and is addressed by subjecting the model to a set of sensitivity analysis across the various parametric conditions a blue collar worker could encounter in today's complex and heterogenous societies.

In terms of audience, the model proposed throughout the following sections could be of use to financial services firms which produce retirement products (e.g. banks, insurance companies, pension funds) as well as financial products distributors (e.g. independent financial advisors). It could ,for instance, be embarked in the sales pipeline of distributors as a digital pre-sale tool to convince workers in physically demanding occupations to invest or buy specific retirement products in line with their optimal retirement plan. The model could also be used (in a more traditional and non-commercial fashion) by large corporations with an important share of blue collar workers (e.g. constructions or manufacturing ones) as their human resource departments design and shape specific and private retirement programs (complementary to public pensions) for their workforce. Note that this list is, of course, not exhaustive and other use cases may exist.

Structure wise, this paper will start by a literature review of the health economics models currently in place and use it to highlight the theoretical contribution of the article (section I). Section II will then gradually build the model and exemplify its properties. Section III will finally offer a discussion around the impact of location/market specific parameters, individuals' time preference as well as promotion opportunities on the proposed financial plan. This discussion ends with a highlight on potential future research directions. A short conclusion will then follow.

---

<sup>3</sup> This notably accounts for the fact that ageing requires higher level of health care.

## I – Literature review:

By tackling the topic of financial planning and retirement preparation for workers in physically intensive occupations, this article contributes to three strands of the economic literature. First, the financial planning concepts introduced in this article build upon the current discussions on individual savings throughout his/her lifecycle. Second, by offering an occupational nuance in the debates around the evolution of retirement policies, this paper appears to provide a new view on the required heterogeneity of the associated future frameworks. Finally, it also contributes to the current burgeoning health economics literature by mixing social concepts (notably the role of labor in personal health) to more classical biological ones.

### a. Individual lifecycle, savings and retirement preparation:

The “theoretical” financial principles structuring an individual lifecycle are well understood. Individuals accumulate capital, until the point where the decline in physical and cognitive abilities forces them to retire as they can no longer contribute efficiently to the society (Ando & Modigliani, 1963). In the absence of income stemming from labor, retirees then rely on pensions (primarily public ones which are complemented by private schemes). Public pensions only provide a portion of the income individuals would have whilst active. For instance, public pension replacement rates are in the 40% - 60% range across a large range of European countries. To smooth their consumption and welfare over their entire lifecycle, active workers thus save a portion of their income and decumulate the associated wealth when retiring (e.g. by converting it into an annuity (Yaari, 1965)).

However, individuals/households still face important execution challenges with the previously described pattern. As seen in the micro economic review of (Gomes, Hoyem, Hu, & Ravina, 2020), the primary challenge is one of adequacy: a large portion of individuals and households do not save enough to sustain retirement. On this front, the views documented in the literature have evolved over the past decades. Early studies indeed found (by combining a mixture of surveys and micro economic modeling) that about 60% to 70% of the population in mature countries did save enough for retirement (Engen, Gale, & Uccello, 2005) (Scholz, Seshadri, & Khitatrakun, 2006) (Hurd, Rohwedder, & Willis, 2012). However recent empirical discussions using panels of micro economic data (e.g. account balances extracts and survey) have highlighted that the situation may have worsened over the past two decades (Munnell, Webb, & Delorme, 2006) (Lusardi & Mitchell, 2011) (Purcell, 2012) (Gomes, Hoyem, Hu, & Ravina, 2020). According to (Gomes, Hoyem, Hu, & Ravina, 2020), a majority of workers (75%+) now does not have enough wealth to transition into retirement.

The reasons advanced for this shift pertain to both individual reasons (higher risk aversion, lack of financial literacy) as well as systemic changes (general decline in returns for financial products, rising healthcare costs). In terms of orders of magnitude, the average individual has now more than 50% chance to have little to no personal savings and therefore to see his/her consumption level fall by 20%+ compared to the ideal levels described by the literature (i.e. active consumption level minus 20% to account for changes in food expenditures – (Hurd & Rohwedder, 2003)).

This naturally calls for the development of tools and methods which could help individuals better assess their future financial needs, something that this paper will contribute to. The lifecycle model in section II indeed provides an optimal individual saving program which accounts for (rising) health care costs, while section III discusses the differences that may arise between forward looking and present oriented individuals.

## b. Ageing & retirement policies evolution:

If individuals can plan and act on their own to ensure a degree of financial health and stability throughout their lifecycle, they rely on basic social schemes to frame their actions. When it comes to retirement planning, the main societal component shaping individuals decision revolves around the design of public pensions. In most mature countries, those systems are yet ongoing changes (Fanti & Gori, 2012). The decline in birth rates, as well as the increased longevity of individuals, is indeed increasing the ratio of retired of individuals to active workers. The consequence is that public pension systems, which operate as a wealth transfer mechanism from active individuals to retired ones, becomes less and less sustainable.

The phenomenon is not new and past decades of research on this very topic have given birth to a large body of macroeconomic literature (Feldstein & Liebman, 2002). One of the dominating approach nowadays revolves around estimating the core parameters of public pensions with overlapping generation models (Diamond, 1965) which are tailored to specific questions (i.e. impact of migration policies, impact of subsidies to health care technologies etc...). At a generic level, public pensions are defined by three elements (Casamatta & Batté, 2016): the level of taxation affecting the revenue of active workers, the replacement rate given to retired individual and the minimal mandated retirement age in place locally. Initial views on public pensions systems (Blanchard, 1985) (Mierau & Turnovsky, 2014; Buiters, 1988) (Weil, 1989) were that the retirement age would have to be postponed and that replacement rates would have to shrink to account for current demographic changes.

However, recent long term equilibrium studies have highlighted that demographic shifts may not be a problem for the current existing economic eco system, but that it may induce some cyclical instability over the short run (Fanti & Gori, 2013) (Fanti, Gori, & Sodini, 2013). The core rationale was that local fertility changes are indeed compensated by migration patterns and that ageing (as highlighted by the forecasts provided by the UN<sup>4</sup>) will soon stabilize. The discussion has therefore recently shifted towards the options available to smooth the transition towards this expected end state. On that front, the debate is still open. Some views indeed highlight that increasing retirement age may be detrimental to economic growth over the next couple of years (Fanti, 2014). Other estimates that retirement rate should be increased as it reduces the natural welfare reduction that will occur in mature countries due to demographic shifts (Vogel, Ludwig, & Börsch-Supan, 2017).

If the macro-economic debate is far from being closed, several sub streams of discussions have started to emerge around differentiating retirement policies across socio-economic segments of the population. One of those streams pertains to a potential differentiation between white and blue collar workers (Pilipiec, Groot, & Pavlova, 2021). Polls have indeed already indicated that workers in demanding occupations could potential retire early (Vermeer, Mastrogiacomo, & Van Soest, 2016), whilst other empirical nuggets of research have shown that white collar workers can, on average, actually work longer (Belbase, Sanzenbacher, & Gillis, 2015) as the decline in cognitive abilities is not as sharp as what is experienced with respect to physical capabilities.

On that front, this paper contributes to the existing literature by providing a model where optimal retirement age can be assessed by occupational categories, once the said categories are quantified according to their degree of physical intensity. This notably shows that heterogeneity is expected amongst blue collar workers (when it comes to a potential early retirement decision). Besides, it further stresses that socio-economic traits (beyond the general macro considerations of growth, state budget balance etc...) may have a significant weight in the design of public pensions.

---

<sup>4</sup> <https://population.un.org/wpp/>

### c. Health economics modeling:

Modeling in the health economics space is rooted in the concepts developed by (Grossman, 1972). In this framework, an individual possesses a stock of health, which naturally depreciates over time. Individuals can then invest in their health stock (through healthcare expenditures for instance) to maximize their own utility over their entire lifecycle. This seminal model was then further generalized by (Muurinen & Le Grand, 1985) to notably study the impact of education on individual's health-related investments decisions.

Grossman's model has sparked several discussions over the past decades as the academic community has been challenging some of its underlying assumptions. A first wave of comments made by the community aimed at improving the way the model accounts for the human aging process. This included challenges around the underlying assumption around an individual's ability to repair/replenish one's health stock (Case & Deaton, 2005) (Galama T. , 2015) or its decontextualized view of an individual's life (family, social status etc... ) (Kohn & Patrick, 2008) (Sepehri, 2015). Additional comments also emerged around the model's complexity and its lack of tractability (i.e. the absence of closed forms solutions) (Strulik, 2015).

Despite some critics, the seminal nature of the concepts behind Grossman's model is well recognized and it gave birth to a large empirical literature (see (L'haridon, Messe, & Wolff, 2018) for a review). The best route to further our knowledge from a theoretical standpoint was then highlighted in the final address of (Muurinen & Le Grand, 1985) and remains unchanged as of today. Their recommendation was not to try to challenge Grossman's framework but rather to build simpler sub models out of it and use them to study very specific questions.

One of those modeling questions (which has recently gained traction amongst the academic community) pertains to the mechanisms linking health and an individual's retirement decisions. One of the reasons for this interest lies in the fact that if within developed countries governments have started to increase the legal retirement age<sup>5</sup> , an increasingly significant part of the population (25%+) is still retiring early (Beehr, 1986). Modeling efforts exploring the motivations for this unexpected individual behavior fall in two main categories.

On one hand, models have been used to assess the effect of individual income on health (including expenditures) and retirement age. (Dalgaard & Strulik, 2012) found that an increase in individual income was leading to higher health and that utility considerations would in turn lead individuals to seek an early retirement. This was also echoed in (Bloom & Canning, 2014) and (Kuhn, Wrzaczek, Prskawetz, & Feichtinger, 2015) but somewhat challenged in (Prettner & Canning, 2014), whose proposition was that early retirement was mainly driven by monetary incentives and institutional constraints rather than by income considerations. Another interesting view on the topic is the one of (Galama, Kapteyn, & Fonseca, 2013) whose model shows that workers with higher human capital / education invest more in health and, because they stay healthier, retire later than those with lower human capital whose health deteriorates faster.

On the other hand, specific models have also been used to assess the effects of specific biological facts on retirement decisions. For instance, (Lau, 2012) studied the effects of changing mortality rates across individuals' lifecycle<sup>6</sup> and has shown that mortality reductions at older ages delay

---

<sup>5</sup> Notably to account for the recent increase in life expectancy and the subsequent financial burden associated to public pension.

<sup>6</sup> Rapid science advances have led to a drastic reduction of mortality rates for active individuals and have improved our control over major health issues. The pace of the change is such that individuals can benefit from those progresses during their lifetime.

retirement unambiguously, but that mortality reductions at younger ages may lead to earlier retirement. More recently, other models (such as the one of (L'haridon, Messe, & Wolff, 2018)) have also started to explore the problem through another angle - how does activity reduction (because of retirement) benefit health.

The idea of this paper is to revisit the question of the optimal retirement age by blending biological and income considerations. The proposal is to review how the retirement decision is made in the case of individuals whose occupation naturally takes a toll on their health. This applies for instance to all “blue collar” workers who still represent as of today an important share of the active population of developed countries (for instance about 40% in the UK and in Germany according to the O.E.C.D WISE database). From what can be seen on the health economics literature, the problem appears unaddressed. Besides, it increments the work done by (Galama, Kapteyn, & Fonseca, 2013) by looking at low-income occupations and studying what happens when health is not a product of an investment but the motor for income generation. Additionally, it appears to address some of the recommended areas of future investigations (notably around contextualized retirement decisions) suggested by (Sepehri, 2015).

## II – Model & Results:

### a. A simple retirement model:

There are two periods in an individual's life. During the first period, the person is a productive member of the society and earns a net income  $I$  every year. This represents a person's contribution as a worker. During the second period, the canonical view (Skirbekk, 2004) is that a person's abilities decline to the point that no income can be generated through labor. Two income sources are then available. On one hand, the state provides a public pension to replace one's income at a rate<sup>7</sup>  $\theta$  (annuities scheme)<sup>8</sup>. On the other, individuals can withdraw some of the accumulated capital ( $C$ ) during their productive phase. This capital stems from saving a portion  $s \in [0; 1]$  of its income during the activity period and investing it in financial products (with an expected return of  $r$ ).

The lifecycle of an individual can then be described by two main parameters: the length of its active/ productive period ( $\tau_A$ ) and the length of its retirement phase ( $\tau_R$ ). Being active generates a total of  $I(1 - s) \cdot \tau_A$  and savings realized during that time yield a capital of  $C(\tau_A) = \sum_{t \leq \tau_A} I \cdot s \cdot (1 + r)^{t-1} = I \cdot s \cdot (1 + r)^{\tau_A} \cdot \frac{1 - (1 + r)^{-\tau_A}}{r}$ . During the retirement phase, the individual gets a yearly annuity of  $\theta \cdot I$ . The annuity is completed by withdrawal(s)  $w(t) \geq 0$ , whilst the acquired capital still yields dividends (i.e.  $C(t + 1) = (C(t) - w(t)) \cdot (1 + r)$ ). If the individual has no time preference, the earnings associated to his/her lifecycle  $\Pi$  can be summarized as:

$$\Pi = I(1 - s) \cdot \tau_A + \theta \cdot I \cdot \tau_R + \sum_{k \in [0; \tau_R]} w(\tau_A + k)$$

Given that an individual cannot withdraw more than the accumulated capital, the following set of recursive constraints appears:

$$\forall k < \tau_R; w(\tau_A + k) \leq C(\tau_A + k) = C(\tau_A) \cdot (1 + r)^k - \sum_{i < k} (1 + r)^i \cdot w(\tau_A + i)$$

<sup>7</sup> As a result, when retiring, an individual gets a pension worth  $\theta \cdot I$  until death.

<sup>8</sup> Note that it is not the aim of this paper to discuss potential public pension policies adjustments. As such, both replacement rates (and taxes) are assumed constant over time. This could however represent an area of additional research.

In this context, assume that both life cycle periods and individual income are given exogenously (i.e.  $\tau_A, \tau_R, I$ ). The underlying idea behind this assumption would be that one's productive abilities (incl. wages) are not something he/she can choose (i.e.  $\tau_A$  and  $I$ ), nor is his/her life expectancy (i.e.  $\tau_A + \tau_R$ ). If this is something that will be re-discussed later, this means, for now, that an individual's earning is potentially subject to an optimization in terms of a saving and withdrawal program. This then translates into the following constrained linear programming maximization problem:

$$\left\{ \begin{array}{l} \max_{\{s; w(\tau_A); \dots; w(\tau_A + \tau_R)\}} \Pi(s; w(\tau_A); \dots; w(\tau_A + \tau_R)) \\ \forall k \in \llbracket 0; \tau_R \rrbracket; 0 \leq w(\tau_A + k) \leq I \cdot s \cdot \frac{((1+r)^{\tau_A+1} - 1)}{r} \cdot (1+r)^k - \sum_{i < k} (1+r)^i \cdot w(\tau_A + i) \\ s \in [0; 1] \end{array} \right.$$

**Property 1.** *In the absence of time preference, an individual would seek to maximize his/her total earnings over his/her lifecycle (i.e.  $\Pi$ ) by investing all of his/her income during the activity period (i.e.  $s = 1$ ) and by only withdrawing money at the very last period of his/her life (i.e.  $\forall k < \tau_R; w(\tau_A + k) = 0$ ). Total earnings are then given by:*

$$\Pi = I \cdot \left( \theta \cdot \tau_R + \frac{((1+r)^{\tau_A} - 1) \cdot (1+r)^{\tau_R}}{r} \right)$$

**Proof.** If an individual has 1€ available in time  $t$ , he/she can consume it immediately or invest it and consume it after  $T$  periods yielding an output of  $(1+r)^T$ . In the absence of time preference or constraints, individuals will therefore save to maximize their wealth. The same behavior will conduct them to delay withdraw as much as possible.

The behavior described by property (1) revolves around accumulating wealth. Without any constraint on his/her budget, a wealth maximizing individual will simply invest and stock capital to finally pass it to the next generation. Note that this will be further discussed in the following sections, as individuals notably need to protect a portion of their income to ensure basic life expenditures (e.g. food, health etc...).

The behavior described by property (1) revolves around accumulating wealth. Without any constraint on budget, a wealth maximizing individual will simply invest and stock capital to finally pass it to the next generation. Note that this will be further discussed in the following sections, as individuals notably need to protect a portion of their income to ensure basic life expenditures (e.g. food, health etc...).

**Example.** Assume that an individual is active for about  $\tau_A = 40$  years with an income of  $I = 70k\$/year$  and lives another  $\tau_R = 20$  years when retired. Assume that the individual lives in a market where he/she can invest in a financial product yielding a return  $r = 3\%$  per year. Under the modeling assumptions described in this section, individuals will invest all their earnings while active, get their pensions and only make a withdrawal before dying. The total wealth ( $\Pi$ ) collected over their lifecycle is then linearly increasing with the replacement rate of the pension they get. It ranges from  $\Pi = 9.6M\%$  when the replacement rate is worth  $\theta = 5\%$  to  $\Pi = 10.4M\%$  for replacement rate at  $\theta = 60\%$ .

This behavior is optimal whatever the structure of the lifecycle (i.e. whatever the length of the active (resp. retirement) period [ $\tau_A$  (resp.  $\tau_R$ )]). Now, while individuals have little control over their life expectancy (i.e.  $L = \tau_A + \tau_R$ ), they can potentially choose whether to stay active or to

retire. In other more quantitative terms, an individual can, given the results highlighted in property 1, solve the following additional optimization program:

$$\max_{\tau_A} \Pi(\tau_A) = \max_{\tau_A} I \cdot \left( \theta \cdot (L - \tau_A) + \frac{((1+r)^{\tau_A} - 1) \cdot (1+r)^{L-\tau_A}}{r} \right)$$

**Lemma 1.** *In the absence of time preferences or budget constraints over their lifecycle and assuming the rate of return of financial products are small (i.e.  $r \ll 1$ ), individuals choose to retire as late as possible to maximize their wealth.*

**Proof.** The result displayed in lemma 1 can be obtained by leveraging the result highlighted in property (1) and solving  $\partial_{\tau_A} \Pi = I \cdot \left( -\theta + \ln(1+r) \cdot \frac{(1+r)^{L-\tau_A}}{r} \right) = 0$ . In this case, the optimal length of the activity period is given as  $\tau_A = \left( L - \frac{\ln\left(\frac{r\theta}{\ln(1+r)}\right)}{\ln(1+r)} \right)$ . Given that  $r \ll 1$ ;  $\theta \frac{r}{\ln(1+r)} = \theta(1+r + \dots)$  and  $\tau_A > L$  if  $\theta \cdot r < (1-\theta)$ .

#### b. Accounting for living standards:

The previous section has shown that individuals seeking to maximize their wealth save and capitalize as much as possible when they are not subject to any constraints (property 1). In that set up, they also tend to retire as late as possible (lemma 1). To make the model more realistic, living standard must yet be accounted for: individuals indeed need to earn at least  $\Phi$  to cater for their primary needs<sup>9</sup> (food/housing etc...). This adds a series of additional constraints to the problem described in the previous section:

$$\begin{cases} I \cdot (1-s) \geq \Phi \\ \forall t \in [\tau_A; \tau_A + \tau_R]; I \cdot \theta + w(t) \geq \Phi \end{cases}$$

When accounting for living standards, three potential cases appear. First, an individual can face difficulties providing for his/her primary needs while active (i.e.  $I \leq \Phi$ ). In this instance, the problem will also carry on during the retirement period as public pension only provide an annuity equals to a fraction of the wages earned while working (i.e.  $\theta \cdot I < I \leq \Phi$ ) and no savings are available. The discussion of this case is not in scope for this study.

The second possibility would be that an individual earns enough while active and that the public pension enables him/her to live comfortably (i.e.  $\Phi \leq \theta \cdot I < I$ ). In this case, the problem is similar to the one described in the previous section: the individual will accumulate capital up to  $\Pi = (I - \Phi) \cdot \left( \theta \cdot \tau_R + \frac{((1+r)^{\tau_A-1}) \cdot ((1+r)^{\tau_R})}{r} \right)$  and retire as late as possible if given the choice. Eventually the individual could even continue saving while retired as his/her needs are met.

But in a number of occasions, if individuals earn enough while active, public pensions are not sufficient to enable them to live up to their own standards (i.e.  $\theta \cdot I < \Phi < I$ ).

**Lemma 2.** *When public pensions are not sufficient to ensure a living standard of  $\Phi$ , for an individual to be able to withdraw just enough to meet his/her standards (i.e.  $w(t) = \Phi - \theta \cdot I$ ), the length of his/her activity period must be such that:*

<sup>9</sup> For now (i.e. throughout section III), living standards will be assumed constant. This will however be subject to a revision in the next section.

$$\tau_A > -\frac{\ln\left((1+r) \cdot \frac{((I-\Phi) + (\Phi - \theta \cdot I) \cdot (1+r)^{-L})}{(I \cdot (1-\theta \cdot (1+r)) + \Phi \cdot r)}\right)}{\ln(1+r)}$$

**Corollary 1.** Retired Individuals withdrawing  $w(t) = \Phi - \theta \cdot I$  to meet their livings standards will see their capital decrease unless the following condition is met:

$$\tau_A > \frac{\ln\left(1 + (\Phi - \theta \cdot I) \cdot \frac{(1+r)}{I - \Phi}\right)}{\ln(1+r)} - 1$$

**Proof.** When an individual withdraws  $w(t) = \Phi - \theta \cdot I$ , his/her capital follows  $C(t+1) = (C(t) - (\Phi - \theta \cdot I)) \cdot (1+r)$ . This translates into:

$$C(t) = (1+r)^{(t-\tau_A)} \cdot \left( C(\tau_A) - (\Phi - \theta \cdot I) \cdot \frac{(1+r)}{r} \right) + (\Phi - \theta \cdot I) \cdot \frac{(1+r)}{r}$$

Since  $C(\tau_A) = (I - \Phi) \cdot \frac{((1+r)^{\tau_A+1} - 1)}{r}$ , the condition highlighted in corollary 1 simply ensures that an individuals' capital is always positive and grows. Finally, meeting the condition  $C(L) \geq 0$  (with  $L = \tau_A + \tau_R$ ) yields the result highlighted in lemma 2.

**Property 2.** When public pensions are not sufficient to ensure a living standard of  $\Phi$ , individuals who maximize their wealth ( $\Pi$ ) and who have worked long enough (see lemma 2) save as much as possible while active (i.e.  $I - \Phi$ ) and then withdraw the minimum necessary to reach their standard (i.e.  $\forall k < \tau_R, w(\tau_A + k) = \Phi - I \cdot \theta$ ). Their overall wealth is then given by:

$$\Pi = \frac{(1+r)^{(\tau_R)}}{r} \cdot \left( (I - \Phi) \cdot ((1+r)^{\tau_A+1} - 1) - (\Phi - \theta \cdot I) \cdot (1+r) \right) + (\Phi - \theta \cdot I) \cdot \frac{(1+r)}{r}$$

**Proof.** Property 2 naturally stems from the considerations developed in property 1.

**Lemma 3.** Individuals who seek to maximize their wealth work for as long they can.

**Proof.** When individuals do not have enough savings to ensure a proper living standard during retirement, they keep on working. Those with sufficient savings have a wealth (see property 2) such that:

$$\partial_{\tau_A} \Pi = \frac{\ln(1+r)}{r} \cdot (1+r)^{(L-\tau_A)} \cdot \left( \underbrace{(I - \Phi)}_{>0} + \underbrace{(\Phi - \theta \cdot I)}_{>0} \cdot (1+r) \right) > 0$$

**Example.** Consider the example developed in the previous section<sup>10</sup> (i.e.  $L = 60$  years,  $I = 70k\$ \setminus year$ ,  $\theta = 30\%$ ,  $r = 3\%$ ). Lemma 2 can be used to understand how long individual need to work (i.e.  $\tau_A$ ) to have enough capital to sustain a given living standard (i.e.  $\Phi$ ) until their death (i.e. where does the capital availability boundary stand?). Conversely, corollary 1 can be used to assess how long does an individual need to work to keep capitalizing whilst retired (i.e. where does the capital increase boundary stands?). Results presented in figure 1 show that for an individual to cover

<sup>10</sup> Note that in this context, the early stage of an individual's life (i.e. the first 20 years) are excluded of the analysis. This period is normally dedicated to education and does not enable individuals to be productive members of the society.

living standard expenses of  $\Phi = 80\%$ .<sup>1</sup> he/she must at least work for a period of  $\tau_A > 30$  years. Besides, he/she will only generate some excess of capital while retired if they have worked long enough for the capital to be sufficient to generate meaningful dividends (i.e.  $\tau_A > 40$  years).

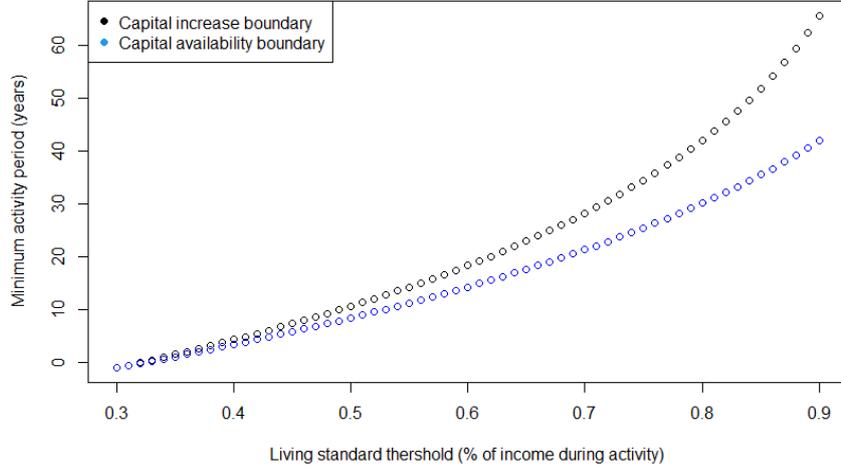


Figure 1 - Duration of the activity period to achieve certain capital related objectives under living standards constraints.

In summary, this section has shown that the financial plan of a person who must meet a set living standard is not so different from the one of unconstrained individuals. Over their lifecycle, constrained persons will indeed seek to work for as long as possible. The only difference revolves around whether they will consume or keep on increasing their capital, which has been shown to be dependent on the length of the activity period.

### c. Impact of health on retirement decisions:

Recent research has shown that the amount required to ensure a decent living (i.e.  $\Phi$ ) is actually variable during an individual lifecycle. A key component driving individual expenditures is indeed related to out-of-pocket expenses as individuals age and their health deteriorates (i.e.  $\partial_t \Phi(t) > 0$ ). This decline is due to two main factors. On one hand the natural ageing process comes with increasing health shocks. On the other, labor further erodes one's health capital<sup>11</sup>. Therefore, when planning for their financial well-being, individuals are potentially faced with a challenge: if being active increases their wealth as they can invest a portion of their income and yield dividends, it may also lead to future financial hurdles. This section will thus build upon the previous ones by embedding notions of variable health expenditures in the proposed lifecycle model. The resulting model will then be analyzed to understand how individuals can optimally plan their finances and organize their lifecycle.

Over each period of time  $t$  (e.g. a year), individuals face some health challenges which are taking them a portion  $p(t) \in [0; 1]$  of their available time (s.t.  $\partial_t p \geq 0$ ). Every moment spent in this state comes at a cost<sup>12</sup>  $\phi$ . Assuming that their overall living expenditures are mainly evolving according to his health status, their living standard can be modeled as:  $\Phi(t) = \phi \cdot p(t)$ .

<sup>11</sup> For instance, consider a worker exerting a physical activity such as a stonemason, an electrician or a plumber.

<sup>12</sup> In the paper, health issues are not assumed to impact individuals' income during their working period (i.e.  $I$  remains unchanged). The idea here is twofold. First, health issues are relatively rare whilst working, because deferred to a later part of life. Second, public health insurances provide coverage for the couple of days where

To understand how health considerations can impact an individual's decisions, additional modeling is required. Assume that the portion of time where an individual is healthy decrease at a rate  $v(t)$  over each period (i.e.  $p(t+1) = p(t) \cdot (1 - v(t)) + v$ ) and that the beginning of one's life is exempt of health issues (i.e.  $p(0) = 0$ ). Now, a simple way of looking at the rate at which health declines is to assume that an inactive individual experiences a natural deterioration at a rate  $v(t) = v_N \in [0; 1]$ , whilst working comes with some extra burden and a rate of health decay worth  $v(t) = v_A + v_N \leq v_N$ . In this context (and leveraging the notations introduced in the previous sections), health issues obey the following rules:

$$\begin{cases} \forall t < \tau_A, p(t) = 1 - (v_A + v_N)^t \\ \forall t \in [\tau_A; L]; p(t) = (1 - (v_A + v_N)^{\tau_A}) \cdot v_N^{t - \tau_A} \end{cases} \quad (1)$$

**Lemma 4.** *When faced with variable health expenditures, an individual who saves and invests as much as he/she can while active (with an expected rate of return of  $r$ ) therefore accumulates a capital worth:*

$$C(\tau_A) = \frac{(1+r)^{\tau_A+1} - 1}{r} \cdot (I - \phi) + \phi \frac{1 - ((1+r) \cdot (v_A + v_N))^{\tau_A+1}}{1 - (1+r) \cdot (v_A + v_N)} \quad (2)$$

**Proof.** At every time step  $k$ , active individuals can save up to  $I - \phi \cdot p(k)$ . The capital they accumulate over  $\tau_A$  active periods is therefore given by:  $C(\tau_A) = \sum (1+r)^k \cdot (I - \phi \cdot p(k))$ . Leveraging equation (1) then yields the proposed result.

**Corollary 2.** *If individuals earn enough while active to cover for the expenditures they would face in the worst possible health conditions (i.e.  $I > \phi$ ), the capital they accumulate while active keeps on increasing (i.e.  $C(\tau_A + 1) > C(\tau_A)$ ). Otherwise (i.e.  $I \leq \phi$ ), it is not interesting for an individual to work for more the following numbers of periods:*

$$I \leq \phi \rightarrow \tau_A < \frac{\ln\left(1 - \frac{I}{\phi}\right)}{\ln(v_A + v_N)}$$

**Lemma 5.** *Once retired, individuals, who withdraw the bare minimum necessary to meet their living expenditures (i.e.  $\phi \cdot p(t) - \theta \cdot I$ ), see their capital follow ( $\forall k > 0$ ):*

$$\begin{aligned} C(\tau_A + k) = & (1+r)^k \cdot C(\tau_A) - (\phi - \theta \cdot I) \cdot (1+r) \cdot \frac{((1+r)^k - 1)}{r} \\ & + \phi \cdot (v_A + v_N)^{\tau_A} \cdot v_N^k \cdot \left(\frac{1+r}{v_N}\right) \cdot \frac{1 - \left(\frac{1+r}{v_N}\right)^k}{1 - \left(\frac{1+r}{v_N}\right)} \end{aligned} \quad (3)$$

**Proof.** Post retirement (i.e.  $t > \tau_A$ ), individual's capital evolution is driven by:  $C(t+1) = (C(t) - (\phi \cdot p(t) - \theta \cdot I)) \cdot (1+r)$ . Leveraging equation (1) then leads to ( $\forall k > 0$ ):

$$C(\tau_A + k) = (1+r)^k \cdot C(\tau_A) - \sum_{l=1}^k (1+r)^l \cdot ((\phi - \theta \cdot I)) + \sum_{l=1}^k (1+r)^l \cdot (\phi \cdot (v_A + v_N)^{\tau_A} \cdot v_N^{k-l})$$

Further developments then yield the proposed result.

---

individuals are on sick leave. A natural extension (although not in scope of this paper) could be to consider the impact of an additional private health insurance on the retirement decision.

Note that lemma (5) implies that individuals whose pension is high enough to cover the worse possible health expenditures (i.e.  $\phi < \theta \cdot I$ ) always experience an increase in capital.

**Property 3.** *In a context where health deteriorates over a lifecycle of length  $L$ , individuals (active over  $\tau_A$  periods) maximize their wealth by investing as much as possible while working and then only withdrawing the bare minimum during their retirement. Their resulting wealth is given by:*

$$\begin{aligned} \Pi(\tau_A) = & \left( \frac{(1+r)^{L+1} - (1+r)^{L-\tau_A}}{r} \cdot (I - \phi) + \phi \frac{(1+r)^{L-\tau_A} - (1+r)^{L+1} (v_A + v_N)^{\tau_A+1}}{1 - (1+r) \cdot (v_A + v_N)} \right) \\ & - (\phi - \theta \cdot I) \cdot (1+r) \cdot \frac{((1+r)^{L-\tau_A} - 1)}{r} \\ & + \phi \cdot (v_A + v_N)^{\tau_A} \cdot \left( \frac{1+r}{v_N} \right) \cdot \frac{v_N^{L-\tau_A} - (1+r)^{L-\tau_A}}{1 - \left( \frac{1+r}{v_N} \right)} \quad (4) \end{aligned}$$

Equation (4) can then be used to assess the length of an individual's activity period which maximizes their wealth. If this problem does not lead to a closed formula, it can easily be solved through simulations (see the following example).

**Example.** Assume that an individual lives in a socio-economic context similar to the one depicted in the previous sections<sup>13</sup> (i.e.  $r = 3\%$ ,  $L = 60$  years,  $I = 70k\$/year$ ) and that health care expenses can rise up to  $\phi = 90k\$/year$ . A simple grid search can then be used to assess what is the optimal activity period for a given health decay profile  $(v_A, v_N)$ . Results (displayed in figure 2) show that 3 scenarios can arise. When the natural decay rate of individuals' health is low (i.e.  $v_N > 0.97$ ), they will not work at all if labor decreases their health in an important manner (i.e.  $v_A < -0.05$  in the example). A second possibility consists in having individuals work until the end of their life. This occurs when health conditions are poor (i.e.  $v_N < 0.97$  &  $v_A < -0.05$ ) or excellent (i.e.  $v_N > 0.99$  &  $v_A > -0.01$ ). An intermediary case then arises where workers optimize their activity period. Note that the higher the healthcare expenses, the more workers are interested in optimizing the duration of their activity period (see figures 3 & 4).

An interesting behavior of the model lies within its sensitivity to the rate at which working conditions deprecate individual's health stock (namely  $v_A$ ). The numerical example detailed above indeed shows that the harsher workers' environment (i.e. the lower  $v_A$ ), the more individuals exhibit extreme behaviors. As seen in the following figures (2,3,4), poor working conditions (i.e. low  $v_A$ ) are linked to individuals either working during their entire life or to them forfeiting the job opportunity. Moreover, workers compensation has a counterintuitive effect on individual behavior. The model indeed shows that with the same degree of penibility (i.e. same  $v_A$ ) and a set level of natural health decay (i.e.  $v_N$ ), increasing workers' compensation will reduce individuals' interest in the job. This can be explained as higher levels of pay lead to the generation of excess savings (once basic living standards are met) that can be accumulated and capitalized on to subsidize natural health issues related to old age without having to work.

<sup>13</sup> The underlying assumption here is that everyone has access to financial products. Given that equity investments are nowadays fully democratized by online brokerage platforms (e.g. Degiro, Robinhood etc...), access is a question of financial literacy. Some aspects of this problem will be further discussed in section V.

Evolution of the optimum activity period with health decay conditions

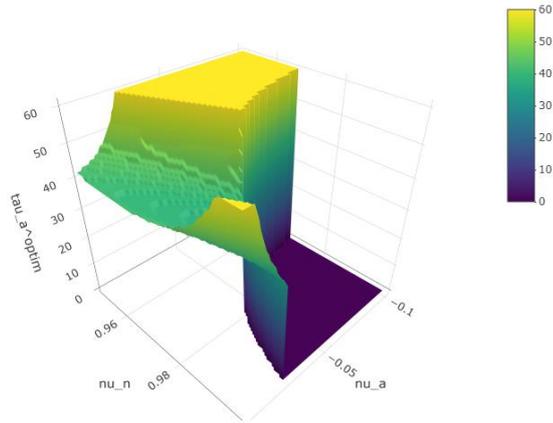


Figure 2 - Optimum activity period length ( $I=90k\$/year$ )

Evolution of the optimum activity period with health decay conditions

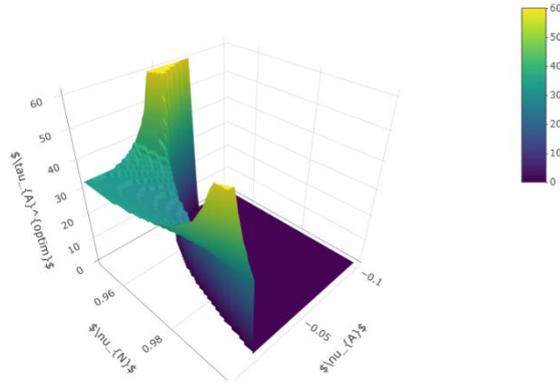


Figure 3 - Optimum activity period length ( $I=120k\$/year$ )

Evolution of the optimum activity period with health decay conditions

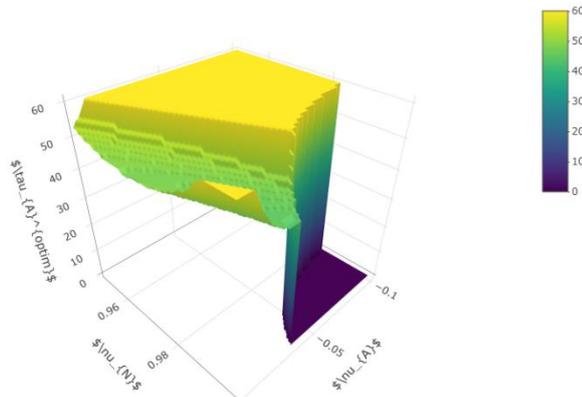


Figure 4 - Optimum activity period length ( $I=70k\$/year$ )

### III – Discussions:

#### a. Sensitivity of individual's retirement behavior on the local context:

Given an occupation (and the associated health stock impact), the retirement timing selected by an individual can vary based on two exogeneous (and location based) components. On one hand, retirement will be impacted by the local price of healthcare, reflected by the ratio  $\frac{\phi}{I}$ . On the other, the retirement decision will change depending in the level of replacement rate (i.e.  $\theta$ ) offered by local public pension schemes. For instance, average replacement rates in the U.K. are around 10% versus 60% to 80% in southern European countries such as France, Italy or Spain.

A sensitivity analysis was therefore performed with the proposed model to understand the impact of location/ social context (i.e. a pair of parameters  $(\frac{\phi}{I}; \theta)$ ) on individuals' behavior. Results displayed on figure 5 show that the retirement decision is more sensitive to the price of healthcare than to the replacement rates offered by public pensions. As expected, the more expensive the healthcare system compared to their job-dependent income, the longer individuals are incentivized to work. Besides, the sensitivity analysis also shows that if public pensions are too high and if there is no eligibility criteria for public pensions (in terms of numbers of years/ months spent being active), individuals have no incentive to work.

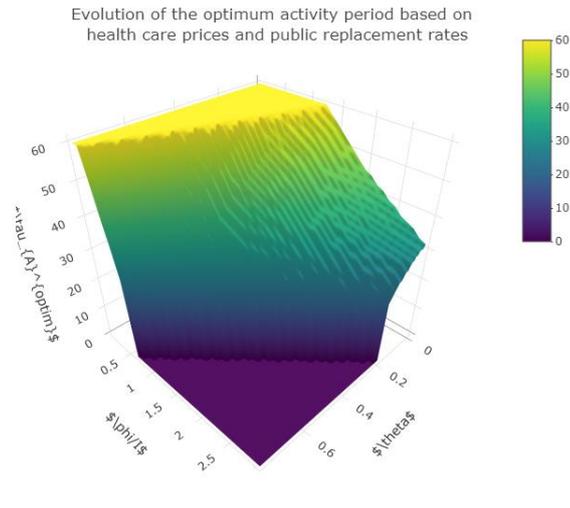


Figure 5 -Optimum activity period length ( $v_A = -0.01, v_N = 0.999$ )

A natural sub-area of discussion then arises as most states/countries offer public pensions which size depends on the amount of time spent contributing to the society (i.e.  $\theta(\tau_A)$ ). Looking at the rules framing public pensions schemes across developed countries, a few commonalities appear. Usually, replacement rates evolve between a maximum  $\theta_{Max}$  and a minimum  $\theta_{Min}$  and if the maximum replacement rate can be claimed after a period  $\tau_S$  of activity, early retirement leads to a loss of  $\gamma$  points per year.

$$\theta(\tau_A) = \min(\theta_{Max}; \max(\theta_{Min}; \theta_{Max} + \gamma \cdot (\tau_A - \tau_S)))$$

Such a functional form can easily be embedded equation (4) to assess how optimal retirement timing evolves depending on healthcare prices. Assuming for instance that  $\theta_{Max}=60\%$ ,  $\theta_{Min} = 10\%$ ,  $\tau_S = 40$  and  $\gamma = -1.25\%$ , the proposed model can then be used to decide when to optimally retire depending on healthcare prices. Results displayed in figure 6 show that as long as maximum healthcare prices are below annual working wages (i.e.  $\phi < I$ ), individuals will keep on working until they die and that individuals do not work if healthcare prices are five times higher than working

wages (i.e. deprecating one health stock is too expensive). Interestingly, when  $\frac{\phi}{I} \in [1; 5]$ , individuals retire early. This could explain why individuals in physically demanding and low-income occupations could be driven towards early (if not very early) retirement.

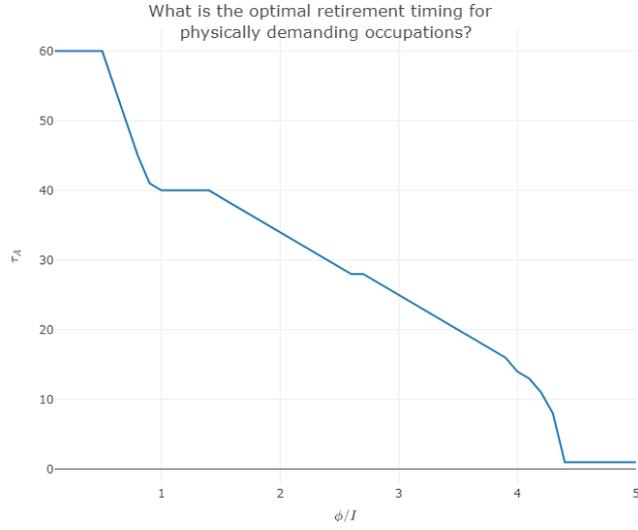


Figure 6 - Optimum activity period length based on healthcare prices ( $v_A = -0.01, v_N = 0.999$ )

#### b. Sensitivity of the proposed financial plan to individual's time preferences:

Looking back at the insights derived from the literature, it also appears that individuals with physically demanding jobs not only have a low income but also present a lack of financial knowledge/literacy (Love & Phelan, 2015). As a result, their time preferences are often biased towards the present and the decisions they make depart from canonical frameworks (from which the model developed in this paper is derived). An option to assess how such a behavior may impact the optimal financial and retirement plan proposed in the previous section consists in introducing a utility  $U$  in the equation describing individual's wealth ( $\Pi$ ). Note that, in this context,  $U(x, w)$  represents the perceived utility an individual has of wealth  $w$  in  $x$  units of time. The outcome of this adjustment is that wealth decisions become a recursive program depending in the current point in time:

$$\begin{aligned} \Pi(s, w, t, \tau_A) = & \sum_{t \leq k < \tau_A} U\left(k - t, \left(I(1 - s(t)) - \phi \cdot p(t)\right)\right) \\ & + \sum_{\tau_A \leq k < L} U\left(k - t, \left(\theta \cdot I - \phi \cdot p(t) + w(t)\right)\right) + U(L - t, C(L)) \end{aligned}$$

Two cases can then potentially occur. In the first instance, if an individual perceives that saving yields a better outcome than immediate consumption (i.e.  $U(t + 1, 1 + r) > U(t, 1)$ ), the plan highlighted throughout sections II to IV in terms of investments and withdrawal still holds. Consequently, individuals can also make an "early" retirement decision based on the physical hardship entailed by their occupation.

However, if individuals' preferences are skewed towards the present (i.e.  $U(t + 1, 1 + r) \leq U(t, 1)$ ), they will postpone their investments. Given the recurring nature of those decisions, this means that, at every point in time, they will keep delaying the start of potential provisions (i.e.  $s(t) = 0$ ) and won't have any capital to withdraw from (i.e.  $C(t) = 0$ ). Their overall wealth will therefore follow:

$$\Pi(t, \tau_A) = \sum_{t \leq k < \tau_A} U(k - t, (I - \phi \cdot p(t))) + \sum_{\tau_A \leq k < L} U(k - t, (\theta \cdot I - \phi \cdot p(t)))$$

This type of behavior will also potentially impact the definition of an optimal retirement timing. When reviewing their options at a point in time  $t$ , individuals will indeed delay their retirement decision by one period as long as it appears in their interest to do so (i.e.  $\Pi(t, t+1) > \Pi(t, t)$ ). This means that they will delay retirement if:

$$\begin{aligned} & U\left(0, \left(I - \phi \cdot \left((1 - (v_A + v_N)^t\right)\right)\right)\right) - U\left(t, \left(I \cdot \theta - \phi \cdot \left((1 - (v_A + v_N)^{t-1} \cdot v_N\right)\right)\right)\right) \\ & + \sum_{t < k < L} U\left(k, \left(\theta \cdot I - \phi \cdot \left((1 - (v_A + v_N)^t \cdot v_N^{k-t}\right)\right)\right)\right) \\ & - \sum_{t < k < L} U\left(k, \left(\theta \cdot I - \phi \cdot \left((1 - (v_A + v_N)^{t-1} \cdot v_N^{t-k+1}\right)\right)\right)\right) > 0 \end{aligned}$$

Now to progress further and qualify a potential delay in the retirement decision, it is necessary to assume a functional form for individual's utility. Let us take two examples. In the first instance (scenario A), assume that individuals have hyperbolic time preferences (i.e.  $U(k, w) = \frac{w}{1 + \epsilon k}$ ). In this scenario, they will only invest if  $r > \epsilon$ . Otherwise (i.e. when  $r \leq \epsilon$ ), they will not have any capital available and when considering retirement, they will keep working if the following condition is met:

$$(1 - \theta) + \frac{\phi}{I} \cdot (v_A + v_N)^{t-1} \cdot (v_A) \cdot \left( \sum_{t \leq k < L} \frac{v_N^{k-t}}{1 + \epsilon \cdot (k - t)} \right) > 0$$

Similarly, let us consider a second scenario (scenario B) where individuals discount future preferences exponentially (i.e. assume  $U(k, w) = w \cdot \delta^k$ ,  $\delta \in [0; 1]$ ). In this case, they will delay their investments if  $\delta > \frac{1}{(1+r)}$  and when that is the case also delay their retirement if the next condition is satisfied:

$$\Pi(t, t+1) > \Pi(t, t) \leftrightarrow (1 - \theta) + \frac{\phi}{I} \cdot (v_A + v_N)^{t-1} \cdot v_A \cdot \sum_{0 \leq k < L-t} \delta^k \cdot v_N^k > 0$$

**Example.** The functional forms highlighted in this sub-section can be used to compare the decision made by individuals on retirement timing based on their time preferences. Assume that individuals' health stock is naturally decreasing at a rate  $v_N = 0.99$ , that their occupation yields a health depreciation rate of  $v_A = -0.01$ . Also assume that replacement rates offered by the public pensions are worth  $\theta = 30\%$  and that financial investments yield a return of  $r = 3\%$ .

When individuals have hyperbolic time preference (scenario A), it can be seen from simulations that individuals either choose to work until the end of their life if healthcare expenditures are small compared their income or avoid work. Besides the more individuals prefer the present (i.e. the higher  $\epsilon$ ), the higher their tolerance towards an expensive healthcare system (see figure 7). The same pattern holds for individual with exponential time preferences (scenario B) (see figure 8).

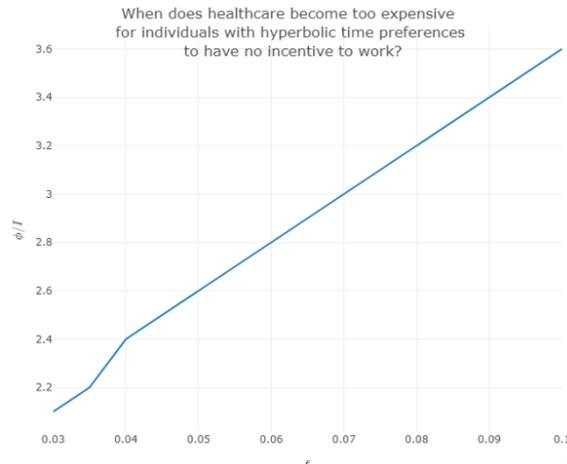


Figure 7 - Sensitivity of retirement decisions to time preferences and healthcare prices (scenario A)

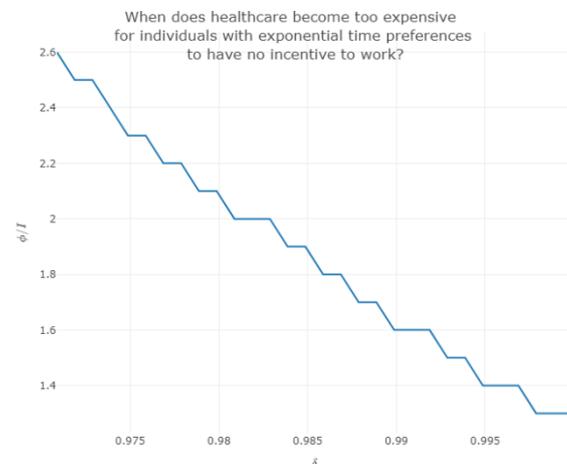


Figure 8 - Sensitivity of retirement decisions to time preferences and healthcare prices (scenario B)

### c. Sensitivity of the retirement plan to individual's learning curve:

The mechanisms described in section (II) implicitly assume that individuals have a set productivity and therefore income over their entire professional life. It is however known that workers learn on the job and that the associated productivity gains are transferred back through compensation adjustments (Lazear E. , 1981) (Anzanello & Fogliatto, 2011). Learning can mainly occur alongside two dimensions. On one hand, workers can improve their technical capabilities on the job. For example, in the case of blue collar workers and in a manufacturing context, this can result in more units of a good being produced per unit of time. This is a well-known phenomenon that gave rise (amongst other thing) to piece wise compensation schemes (Lazear E. , 2000). On the other hand, blue collar workers can develop non-technical related capabilities and progress, through a promotion mechanism, towards a white collar type of situation. This generally occurs with a probability  $\Psi$ , which is labor market dependent, after a learning period of  $\tau_p$ . Now, those two adjustments to a blue collar worker career could potentially affect individual retirement behavior. This sub-section will therefore review the impact of such learning opportunities to the model proposed earlier on.

The acquisition of additional technical skills resulting in a productivity boost can be easily integrated in the model. It is indeed sufficient to consider that individuals earning  $I(t)$  is time dependent and increasing with experience ( $\forall(t, t') \in [1; L]^2; t' \geq t; I(t') \geq I(t)$ ). Under those

circumstances, forward looking individuals still exhibit a savings behavior similar to the one detailed in lemma (4). Individuals save as much possible whilst active and accumulate during their  $\tau_A$  years of activity a capital of:

$$C(\tau_A) = \sum_{t=0}^{\tau_A} (1+r)^{\tau_A-t} \cdot (I(t) - \phi \cdot (1 - (v_A + v_N)^t))$$

As in the previous section, a stopping time arises where forward looking individuals retire and their decumulation behavior then follows lemma (5) and corollary (2). On that front, considerations of learning yield an impact which is similar to what was observed in the simulations of section (II) with various levels of  $I$ : the sharper the learning curve (i.e. the more productive they become with time and get rewarded/compensated for it), the shorter the length of stay of individuals in physically demanding occupations.

Now, if the acquisition of technical capabilities on a set physically demanding occupation may accelerate individual retirement decision, a second parameter has to be integrated in the discussion. Blue collar worker can indeed move up in a white collar type of position (with an income  $I_p > I$ ) through the acquisition of non-technical skills with probability  $\Psi \in [0; 1]$ . Let us call:

- $\tau_A^*$  be the maximum amount of time an individual will spend in a blue collar occupation given an occupational and healthcare system set up ( $v_A; v_N$ )
- $\tau_p < \tau_A^*$  the time after which the blue collar worker has acquired enough non-technical related skills throughout his/her professional journey to occupy a white collar type of position.

Assume that in white collar position, individuals do not deprecate their health stock. Two options then arise for blue collar workers:

- Option 1: individuals end up being promoted to a white collar position before reaching a point where the depreciation of their health stock in exchange for compensation is not interesting anymore (i.e.  $\tau_A^*$ ). This occurs with a chance  $\Psi \cdot \sum_{k=1}^{\tau_A^* - \tau_p} (1 - \Psi)^k = (1 - \Psi) \cdot (1 - (1 - \Psi)^{\tau_A^* - \tau_p})$ . Once promoted, they virtually never retire as they can accumulate more capital without any additional costs. As a result, the capital accumulated by individuals at the end of their life is, on average, given by:

$$C(\tau_p) = \left( \sum_{t=0}^{\tau_p-1} (1+r)^{\tau_p-1-t} \cdot (I - \phi(1 - (v_A + v_N)^t)) \right) \cdot (1+r)^{L-\tau_p+1} + \Psi \cdot \sum_{k=1}^{\tau_A^* - \tau_p} \left( (1 - \Psi)^k \left( \sum_{t=\tau_p+k}^L (1+r)^{\tau_A-t} \cdot (I_p - \phi(1 - (v_A + v_N)^{\tau_p+k} (v_N)^{t-\tau_p-k})) \right) + \sum_{t=\tau_p}^{\tau_p+k} (1+r)^{\tau_A-t} \cdot (I - \phi(1 - (v_A + v_N)^t)) \right) \right)$$

- Option 2: Individuals do not get the chance to move up and retire after  $\tau_A^*$  years of activity. This occurs with a probability  $\left(1 - \Psi \cdot \sum_{k=1}^{\tau_A^* - \tau_p} (1 - \Psi)^k\right)$  and yields a capital similar to what was described earlier in lemma (4) & (5).

In summary, learning has an impact on individual retirement behavior. If the acquisition of technical skills can be accounted for to estimate properly the optimality of retirement timing under the constraint of health depreciation, it does not change drastically blue collars behavior. However, the acquisition of non-technical competencies creates an opportunity for individuals to shave off the question of the toll on one's health and generates a potential heterogeneity in retirement decisions. One on hand, workers who get the chance to be promoted virtually work until the end of their life in the absence of any other constraints, whilst non promoted workers retire as soon as the depreciation of their health stock becomes too costly.

#### d. Limitations and future research directions:

There are three main limitations in this article, which could benefit from further explorations. First, the educational journey of an individual prior to him/her joining the workforce is not discussed. This could be the subject of two further developments. On one hand, it would be interesting to integrate the cost of education in the model and the financial constraints (i.e. a debt) it creates at the onset of an individual's professional life. This extra burden could likely delay blue collar retirement decision. On the other, the time required for the society to shape a worker (i.e. the speed of the educational system) potentially correlates with a depreciation in one's health stock, which reduces its propensity to work and therefore increases its likelihood of retiring early.

Second, the model assumes that when retired, individuals decumulate wealth on top of perceiving public pensions. Though this has been and still remains the generic pattern observed in today's western societies, alternative livelihood options for retired workers in their post-retirement years could be further considered to nuance the initial description. Introducing extra revenue streams at the end of an individual lifecycle would likely accelerate individuals' retirement decision, but additional considerations would be required to assess what kind of positions could be suitable and what kind of system would have to put in place to manage such transitions.

Finally, the model assumes that compensation for a blue-collar occupation (i.e. the parameter  $I$ ) is given as an exogeneous variable. But as highlighted by the model, high level of compensations in physically demanding occupations lead to rapid retirement decisions. The model could therefore suggest that a portion of workers in their prime age would potentially sit back/retire if compensation is too strong, which could destabilize the pension system. The model could therefore be revisited with a more macro-economic lense to introduce a systemic feedback loop between retirement decisions, public pension fundings and wages (see (Fanti, 2015) for an example). The intuition would indeed be that balancing the pension system would limit wages and incentivize a longer working life.

The considerations embarked in this paper also highlight future avenues of research, notably at a macroeconomic level. First, it would be beneficial to empirically test the level of correlation between blue collar workers retirement decisions (namely  $\tau_A$ ) with the performance of healthcare systems (i.e.  $v_N, \phi$ ) and with occupational contexts (i.e.  $v_A$ ). This could potentially help stress test the model and help assess whether different behaviors may exist across geographies & economies (to notably answer the question of whether or not differences arise between developed and emerging countries).

Second, it would be interesting to explore the policy implications of the model, notably with respect to retirement design policies as well as health care system policies. Embedding macro-economic

considerations of capital and public pension systems could indeed remove model dependencies towards exogenous variables (e.g. wages) and make it to evolve towards a predictive tool rather a financial planning one. When it comes to healthcare policies, the model could be revised and build upon to help inform discussions around the nature of the health care sector and the way it takes care of the retired workers. Discussions could, for instance, be centered around the share of the private sector in the overall societal effort associated the care of our elders and around its regulation.

Finally, transgenerational considerations could be embedded in a revision of the model. The current set up indeed assumes that workers operate as single individuals and not as households. Integrating the care of children and financial constraints on their education (as well as the associated overall societal benefits) could prove interesting and a more accurate/optimal retirement planning for blue collar workers.

## Conclusions:

Developed countries have an important share of workers who have physically demanding occupations. Since their activity exerts a toll on their health stock and since this depreciation comes at a price, they face a natural financial optimization problem in terms of the length of their activity period.

This paper thus offers a model to help them better assess the trade-offs between having relatively high earnings while active with an impact on their health and retiring. This naturally translates into a saving program and a recommendation around their optimal retirement timing. The model has four notable traits.

First, it highlights that the more expensive the healthcare system, the earlier individuals should retire when they occupy a position which is physically taxing. For instance, under assumptions mimicking the public pension policies present in several developed countries, simulations shows that when healthcare expenses represent more than twice the annual earnings of a working person, some individuals in physically intensive occupations should retire up to 10 years earlier than what is recommended by most public mandated pension scheme.

Second, the model highlights that retirement policies should be differentiated across blue collar positions since they come with different working conditions. For instance, the model shows that between two occupations  $i$  &  $ii$ , when the working conditions in job  $ii$  yields a health stock depreciation which is twice as important as in job  $i$  (i.e.  $\frac{v_A^i}{2} > v_A^{ii}$ ) workers in occupation  $ii$  should retire 5 to 10 years earlier than their counterpart in job  $i$ .

Third, the model shows that when learning opportunities exist for blue collar worker to move out of a physically intensive occupation to a non-taxing one (e.g. a white collar one), retirement behaviors become heterogenous across the blue collar population. Workers who are able to switch jobs before a certain stopping time carry on working while the ones who don't extract themselves from their occupation retire. When provided with the right learning and promotion opportunities, physically taxing occupations could therefore be associated to a form of social ladder.

Finally, this paper also illustrates that individuals' behavior is yet subject change to change based on the time preferences of individuals. If they have a strong preference for immediate consumption, the model states that they will either work across their entire life or have no incentive to work at all. Behaviors in these cases are set by a combination of healthcare prices and time preferences.

## References

- Ahmed, J., Barber, B., & Odean, T. (2018). Made poorer by choice: Worker outcomes in social security vs. private retirement accounts. *Journal of Banking & Finance*, 311-322.
- Ando, A., & Modigliani, F. (1963). The "life cycle" hypothesis of saving: Aggregate implications and tests. . *The American economic review*, 55-84.
- Andriosopoulos, D., Doumpos, M., Pardalos, P., & Zopounidis, C. (2019). Computational approaches and data analytics in financial services: A literature review. . *Journal of the Operational Research Society*, 1581-1599.
- Anzanello, M., & Fogliatto, F. (2011). Learning curve models and applications: Literature review and research directions. . *International Journal of Industrial Ergonomics*, 573-583.
- Badarinza, C., Campbell, J., & Ramadorai, T. (2016). International comparative household finance. . *Annual Review of Economics*, 111-144.
- Beehr, T. (1986). The process of retirement: A review and recommendations for future investigation. . *Personnel psychology*, 31-55.
- Belbase, A., Sanzenbacher, G., & Gillis, C. (2015). Does age-related decline in ability correspond with retirement age?. *Center for Retirement Research at Boston College Working Paper*.
- Berchick, E., Hood, E., & Barnett, J. (2019). Health insurance coverage in the United States: 2018 . *Washington, DC: US Department of Commerce*, 2.
- Blanchard, O. (1985). Debt, deficits, and finite horizons. . *Journal of political economy*, 223-247.
- Bloom, D., & Canning, D. M. (2014). Optimal retirement with increasing longevity. . *The Scandinavian journal of economics*, 838-858.
- Buiter, W. (1988). Death, birth, productivity growth and debt neutrality. . *The Economic Journal*, 279-293.
- Cagetti, M. (2003). Wealth accumulation over the life cycle and precautionary savings. . *Journal of Business & Economic Statistics*, 339-353.
- Casamatta, G., & Batté, L. (2016). Handbook of the economics of population aging. In *The political economy of population aging* (pp. 381-444).
- Case, A., & Deaton, A. (2005). Broken down by work and sex: How our health declines. In *Analyses in the Economics of Aging*, 185-212.
- Chetty, R., Stepner, M., Abraham, S., Lin, S., Scuderi, B., Turner, N., . . . Cutler, D. (2016). The association between income and life expectancy in the United States, 2001-2014. *Jama*, 1750-1766.
- Dalgaard, C., & Strulik, H. (2012). The Genesis of the Golden Age-Accounting for the Rise in Health and Leisure. *Univ. of Copenhagen Dept. of Economics Discussion Paper*.
- Diamond, P. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 1126-50.
- Dickman, S., Himmelstein, D., & Woolhandler, S. (2017). Inequality and the health-care system in the USA. *The Lancet*, 1431-1441.

- Engen, E., Gale, W., & Uccello, C. (2005). Effects of stock market fluctuations on the adequacy of retirement wealth accumulation. *Review of Income and Wealth*, 397-418.
- Fanti, L. (2014). Raising the mandatory retirement age and its effect on long-run income and pay-as-you-go (PAYG) pensions. *Metroeconomica*, 619-645.
- Fanti, L. (2015). Growth, PAYG pension systems crisis and mandatory age of retirement. *Economics Bulletin*, 1160-1167.
- Fanti, L., & Gori, L. (2012). Fertility and PAYG pensions in the overlapping generations model. *Journal of Population Economics*, 955-961.
- Fanti, L., & Gori, L. (2013). Fertility-related pensions and cyclical instability. *Journal of Population Economics*, 1209-1232.
- Fanti, L., Gori, L., & Sodini, M. (2013). Complex dynamics in an OLG model of neoclassical growth with endogenous retirement age and public pensions. *Nonlinear Analysis: Real World Applications*, 829-841.
- Feldstein, M., & Liebman, J. (2002). *Handbook of public economics. Social security*. 2245-2324.
- Friend, I., & Blume, M. (1975). The demand for risky assets. *The American Economic Review*, 900-922.
- Galama, T. (2015). A contribution to health-capital theory. *CESR-Schaeffer Working Paper*.
- Galama, T., Kapteyn, A., & Fonseca, R. M. (2013). A health production model with endogenous retirement. *Health economics*, 883-902.
- Gomber, P., Kauffman, R., Parker, C., & Weber, B. (2018). On the fintech revolution: Interpreting the forces of innovation, disruption, and transformation in financial services. *Journal of Management Information Systems*, 220-265.
- Gomes, F., Hoyem, K., Hu, W., & Ravina, E. (2020). Retirement savings adequacy in US defined contribution plans. *Available at SSRN 3294422*.
- Grossman, M. (1972). On the concept of health capital and the demand for health. *Journal of Political Economy*, 223-55.
- Guiso, L., Haliassos, M., & Jappelli, T. e. (2002). Household portfolios. *MIT press*.
- Haliassos, M., & Bertaut, C. (1995). Why do so few hold stocks?. *the economic Journal*, 1110-1129.
- Hitiris, T., & Nixon, J. (2001). Convergence of health care expenditure in the EU countries. *Applied Economics Letters*, 223-228.
- Hurd, M., & Rohwedder, S. (2003). The retirement-consumption puzzle: Anticipated and actual declines in spending at retirement. *NBER*.
- Hurd, M., Rohwedder, S., & Willis, R. (2012). *Economic Preparation for Retirement*. University of Chicago Press.
- Kohn, J., & Patrick, R. (2008). Health and wealth: a dynamic demand for medical care. *HEA 2007 6th World Congress*.

- Kuhn, M., Wrzaczek, S., Prskawetz, A., & Feichtinger, G. (2015). Optimal choice of health and retirement in a life-cycle model. . *Journal of Economic Theory*, 186-212.
- L'haridon, O., Messe, P., & Wolff, F. (2018). Quels effets de la retraite sur la santé? *Revue française d'économie*, 103-154.
- Lau, S. S.-R. (2012). Mortality transition and differential incentives for early retirement. . *Journal of Economic Theory*, 261-283.
- Lazear, E. (1979). Why is there mandatory retirement? *Journal of political economy*, 1261-1284.
- Lazear, E. (1981). Agency, earnings profiles, productivity, and hours restrictions. . *The American Economic Review*, 606-620.
- Lazear, E. (2000). Performance pay and productivity. . *American Economic Review*, 1346-1361.
- Love, D., & Phelan, G. (2015). Hyperbolic discounting and life-cycle portfolio choice. *Journal of Pension Economics and Finance*.
- Lusardi, A., & Mitchell, O. (2011). Financial literacy and planning: Implications for retirement wellbeing. *National Bureau of Economic Research*.
- Mankiw, N. Z. (1991). The consumption of stockholders and nonstockholders. *Journal of financial Economics*, 97-112.
- Mierau, J., & Turnovsky, S. (2014). Demography, growth, and inequality. . *Economic Theory*, 29-68.
- Munnell, A., Webb, A., & Delorme, L. (2006). A new national retirement risk index. *Issue in brief*.
- Muurinen, J., & Le Grand, J. (1985). The economic analysis of inequalities in health. . *Social science & medicine*, 1029-1035.
- Nghiem, S., & Connelly, L. (2017). Convergence and determinants of health expenditures in OECD countries. . *Health economics review*.
- Pilipiec, P., Groot, W., & Pavlova, M. (2021). The effect of an increase of the retirement age on the health, well-being, and labor force participation of older workers: a systematic literature review. . *Journal of Population Ageing*, 271-315.
- Prettner, K., & Canning, D. (2014). Increasing life expectancy and optimal retirement in general equilibrium. *Economic Theory*, 191-217.
- Purcell, P. (2012). Income replacement ratios in the health and retirement study. *Soc. Sec. Bull*.
- Rocha, R., Vittas, D., & Rudolph, H. (2010). The payout phase of pension systems: a comparison of five countries. . *World Bank Policy Research Working Paper*.
- Scholz, J., Seshadri, A., & Khitatrakun, S. (2006). Are Americans saving "optimally" for retirement?. . *Journal of political economy*, 607-643.
- Sepehri, A. (2015). A critique of Grossman's canonical model of health capital. . *International Journal of Health Services*, 762-778.
- Shang, B., & Goldman, D. (2008). Does age or life expectancy better predict health care expenditures? *Health Economics*, 487-501.

- Skirbekk, V. (2004). Age and individual productivity: A literature survey. . *Vienna yearbook of population research*, 133-153.
- Strulik, H. (2015). A closed-form solution for the health capital model. . *Journal of Demographic Economics*, 301-316.
- Toth, F. (2019). Prevalence and generosity of health insurance coverage: a comparison of EU member states. *Journal of Comparative Policy Analysis: Research and Practice*, 518-534.
- Vermeer, N., Mastrogiacomo, M., & Van Soest, A. (2016). Demanding occupations and the retirement age. *Labour Economics*, 159-170.
- Vogel, E., Ludwig, A., & Börsch-Supan, A. (2017). Aging and pension reform: extending the retirement age and human capital formation. . *Journal of Pension Economics & Finance*, 81-107.
- Weil, P. (1989). Overlapping families of infinitely-lived agents. . *Journal of public economics*, 183-198.
- Yaari, M. (1965). The Review of Economic Studies. *Uncertain lifetime, life insurance, and the theory of the consumer*. , 137-150.