Dynamics under location uncertainty

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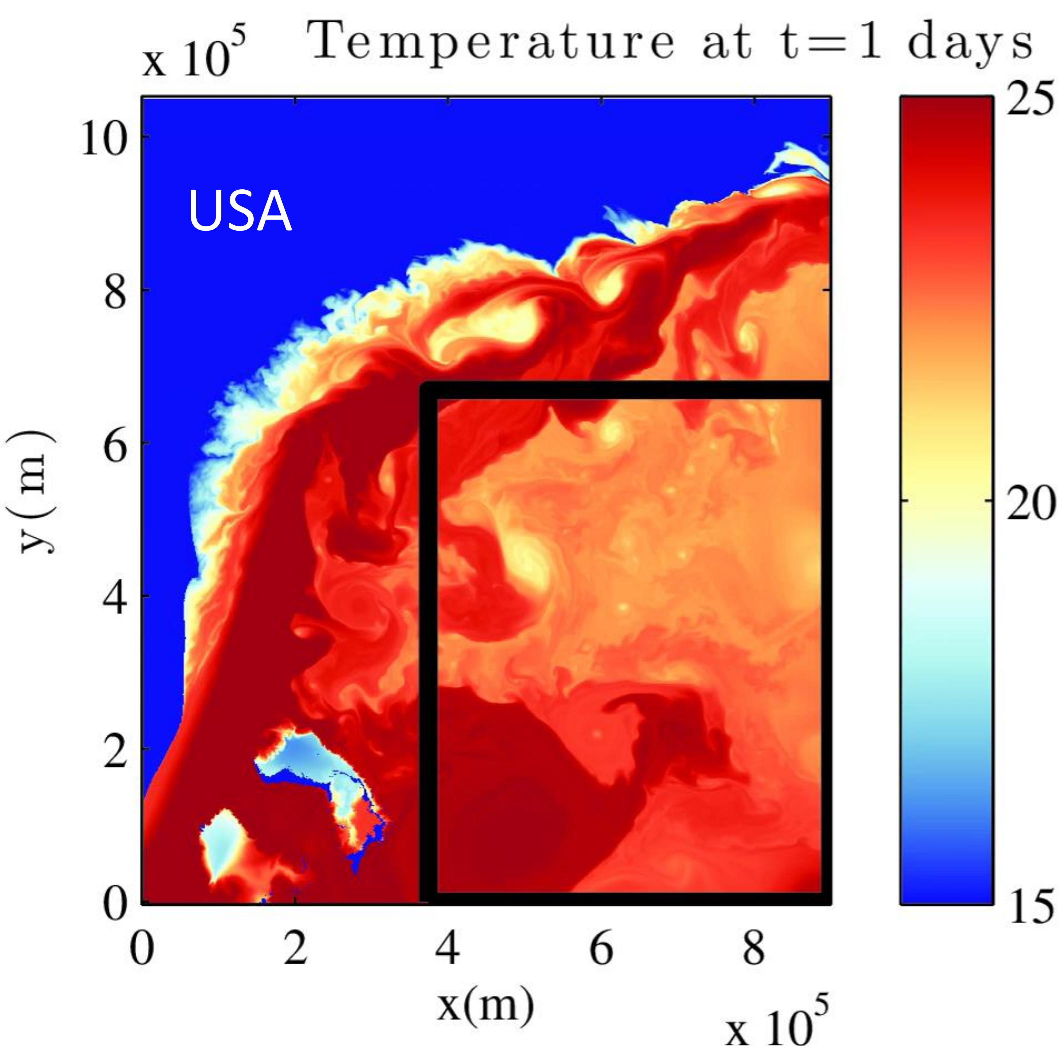
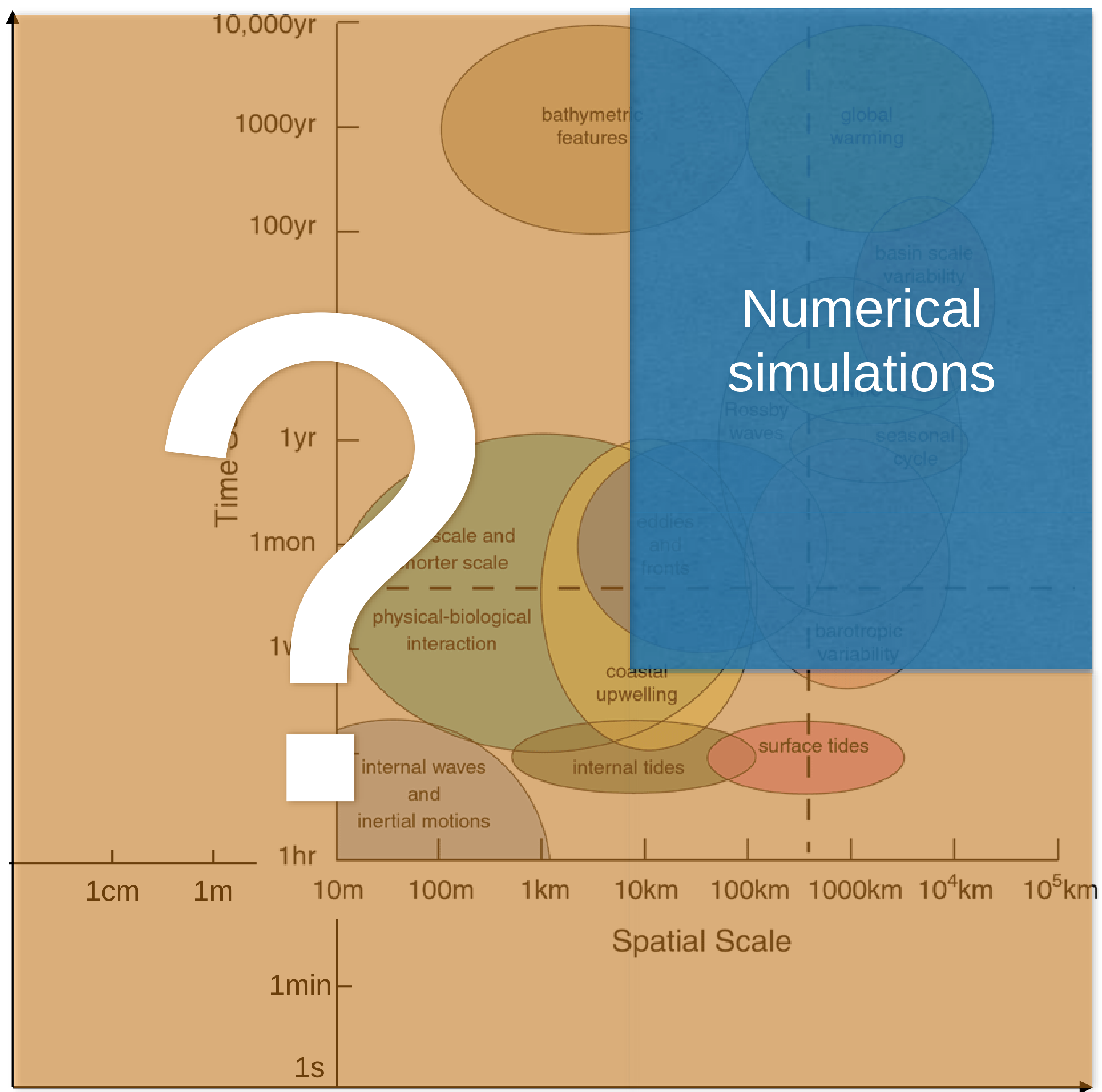
Motivations

Scales and resolutions in the ocean

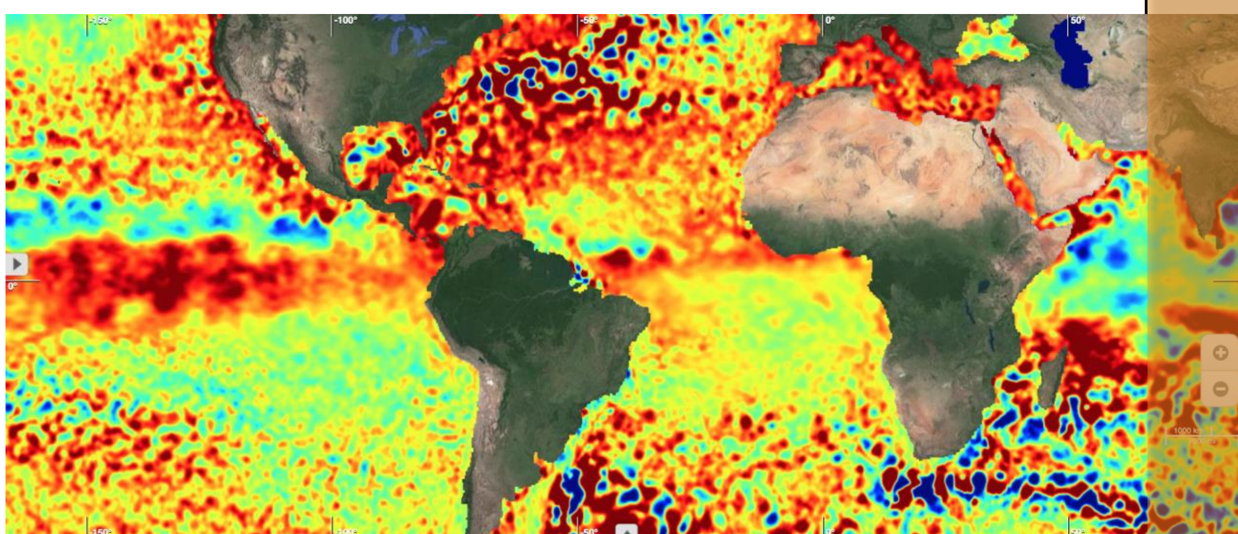


Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

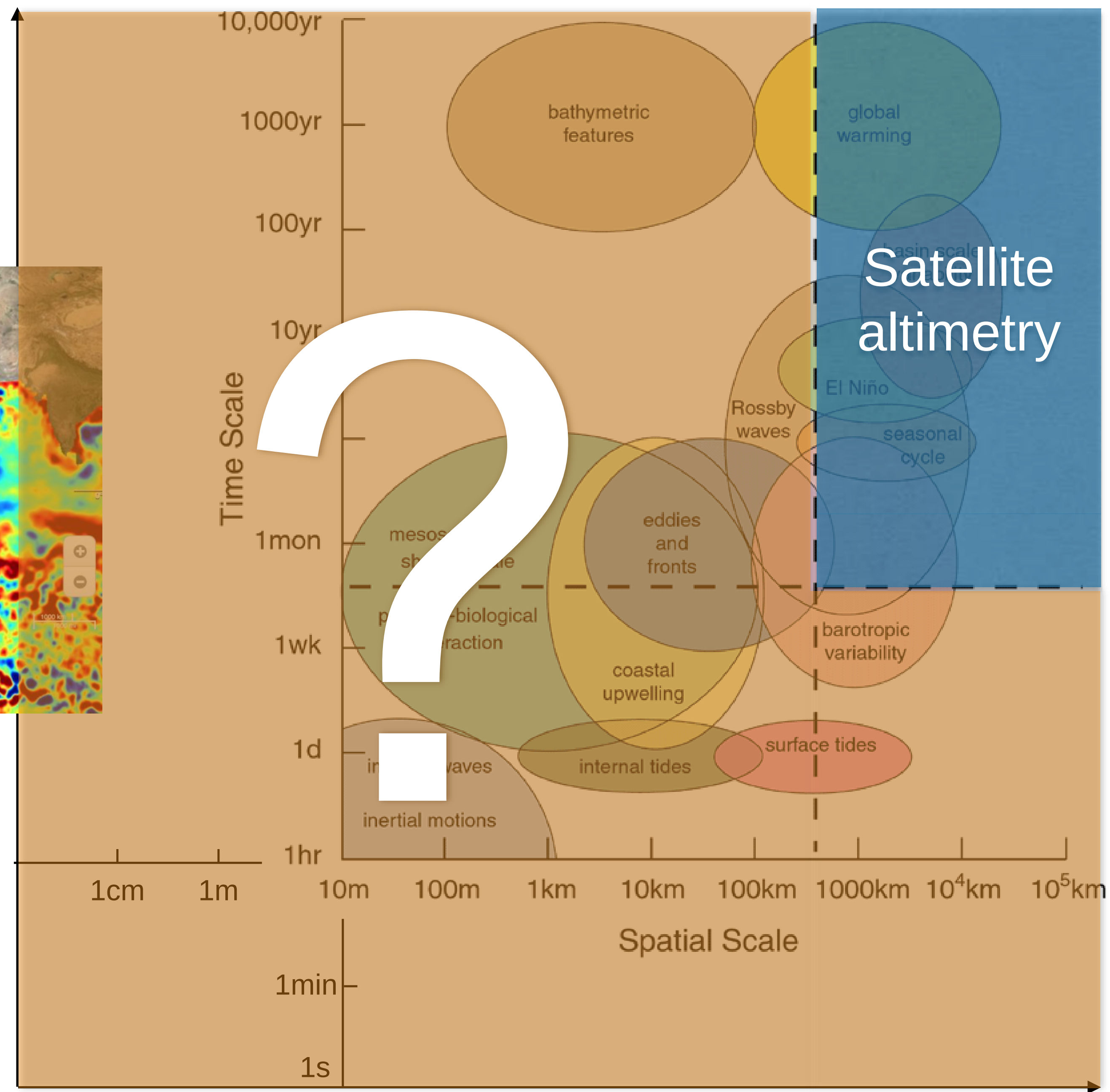


Source:
D. B. Chelton

Scales and resolutions in the ocean



SSH anomaly
(Source: Ocean Datalab)



Source:
D. B. Chelton

Motivations

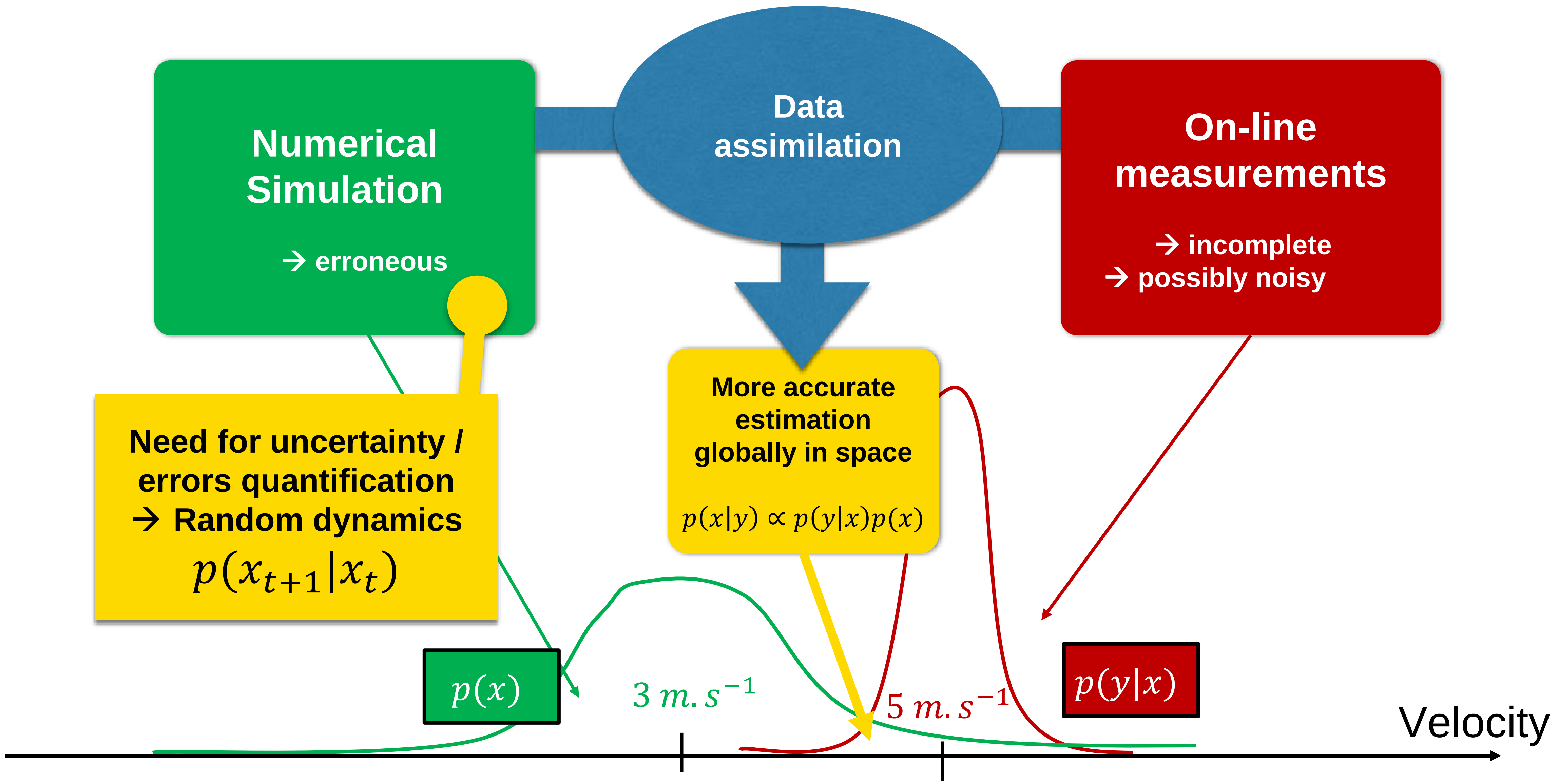
- More rigorously identified sudgrid dynamics effects (closure problem)

- Quantification of modeling errors (UQ)

  Ensemble forecasts and data assimilation

Data Assimilation =

Combining simulations and measurements



Part I

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Dynamics under location uncertainty (LU)

1. Stochastic transport

2. Unresolved velocity parametrization

3. Numerical example with oceanic currents

I.1.
Stochastic transport

Large scale velocity:

$$\bar{v} = w$$

Small scale velocity:

$$v' = \sigma \dot{B}$$

Variance tensor

$$a = \overline{v'(v')^T}\,\tau$$

Idea of LU :

Adding random velocity

Resolved large scales velocity

White-in-time small scales velocity

$$v = \bar{v} + v'$$

Dynamics:

e.g. advection of tracer Θ

Large scale velocity:

$$\bar{v} = w$$

Small scale velocity:

$$v' = \sigma \dot{B}$$

Variance tensor

$$a = \overline{v'(v')^T}\,\tau$$

$$\frac{D\Theta}{Dt} = 0$$

Conservative multiplicative random forcing

Stratonovich notations:

$$\partial_t \Theta + (w^* + \sigma \circ \dot{B}) \cdot \nabla \Theta = 0$$

Advection of tracer Θ

Large scale velocity:

$\bar{v} = w$

Small scale velocity:

$v' = \sigma \dot{B}$

Variance tensor

$a = \overline{v'(v')^T}\,\tau$

Ito-Wentzell formula (Kunita 1990)

Multiplicative random forcing

Balanced energy exchanges

Ito notations:

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Stratonovich drift : « Drift correction » in Ito notations

Large scale velocity:

$$\bar{v} = w$$

Small scale velocity:

$$v' = \sigma \dot{B}$$

Variance tensor

$$a = \overline{v'(v')^T}\,\tau$$

I.2. Unresolved velocity parametrization

$$\sigma = ?$$

Large scale velocity:

$$\bar{v} = w$$

Small scale velocity:

$$v' = \sigma \dot{B}$$

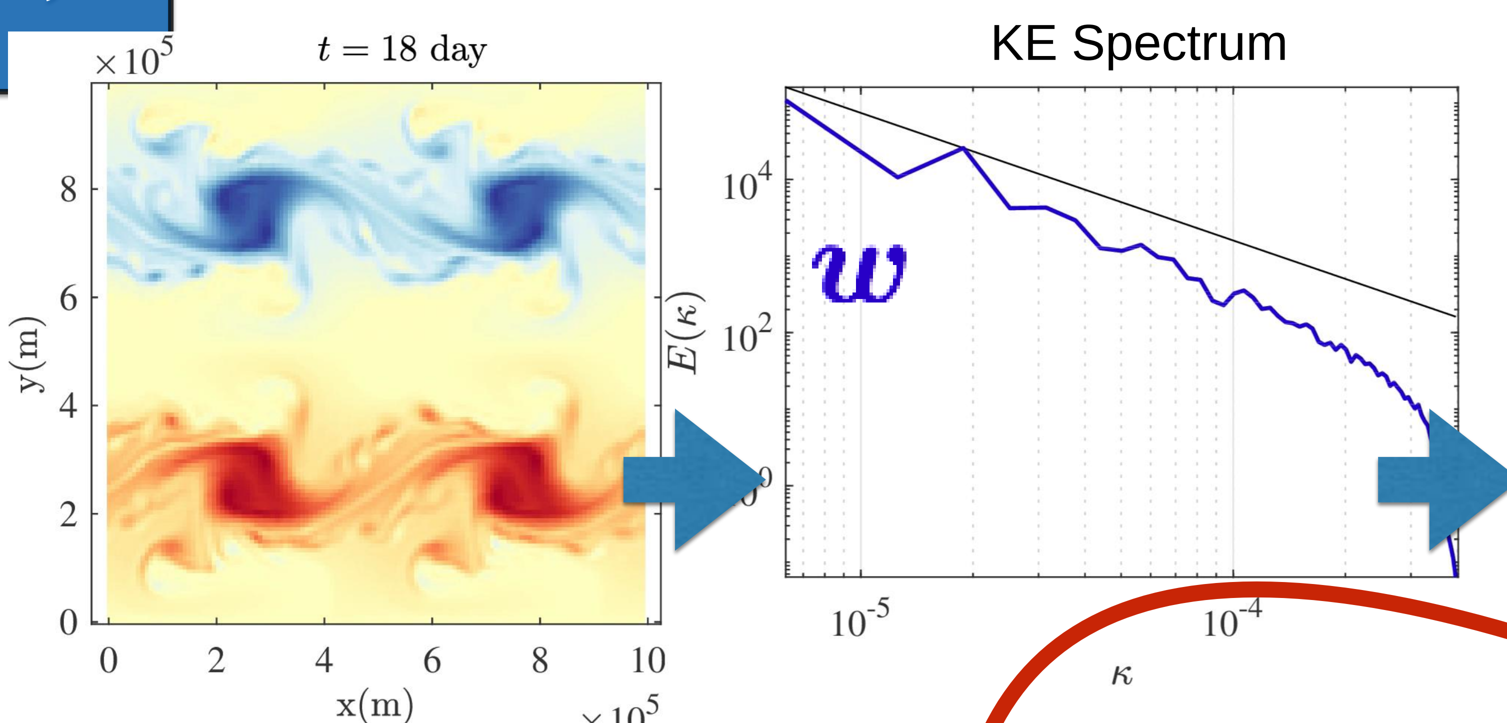
Variance tensor

$$a = \overline{v'(v')^T}\,\tau$$

Unresolved velocity parametrization

Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



$$v' = \sigma\,\dot{B} = \text{(filter)} * \text{(white noise)}$$

Reference:
Resseguier, Pan & Fox-Kemper 2020

I.3.
Numerical example with oceanic currents

SQG

$$\frac{Db}{Dt} = \underbrace{-\alpha_{HV}\Delta^4 b}_{\text{Hyper-viscosity}} \underbrace{- \frac{1}{\tau}(b - F)}_{\text{Forcing}}$$

$$u = \left(\text{cst}.\nabla^{\perp}\Delta^{-1/2}\right) b$$

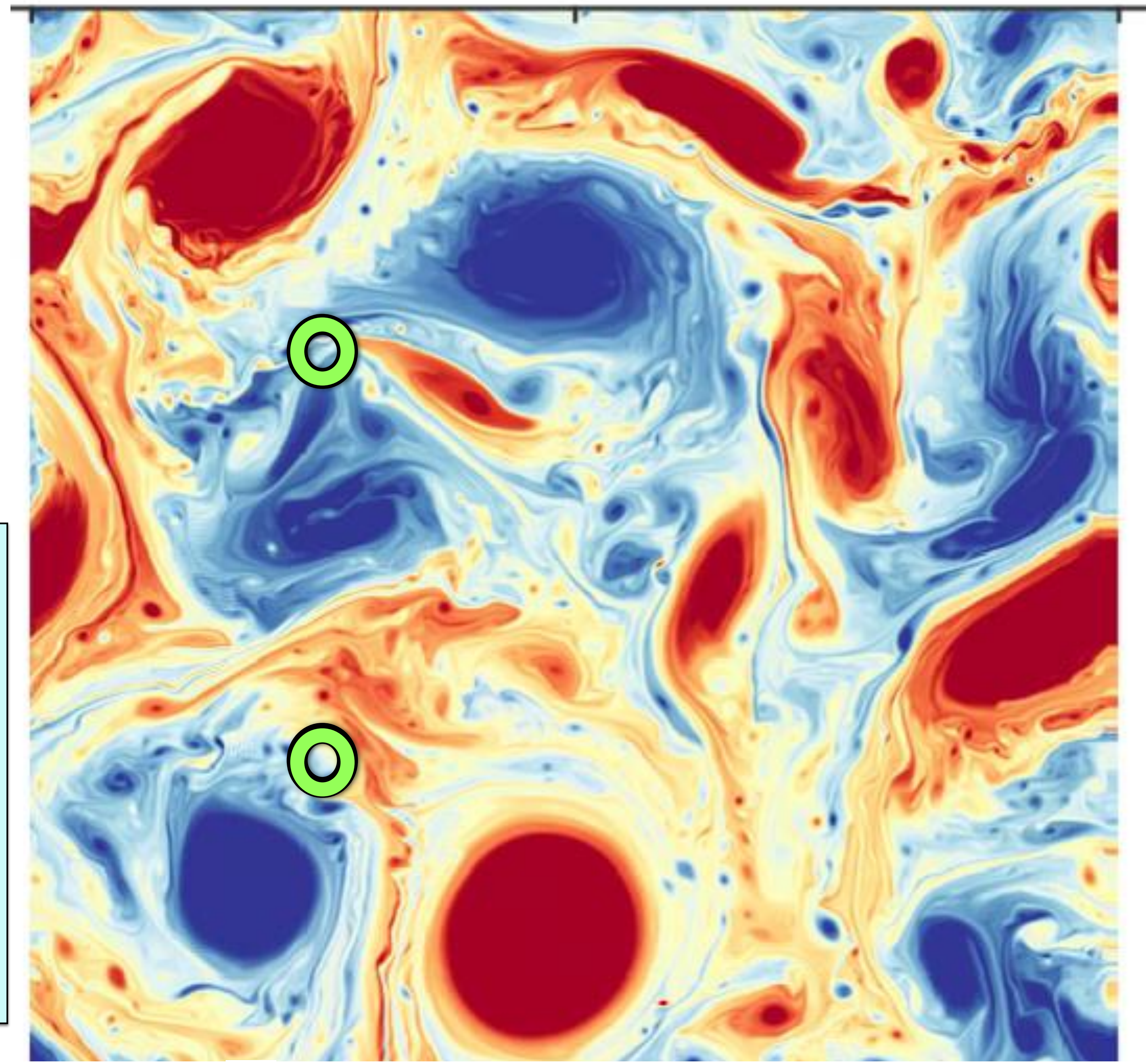
Reference flow:

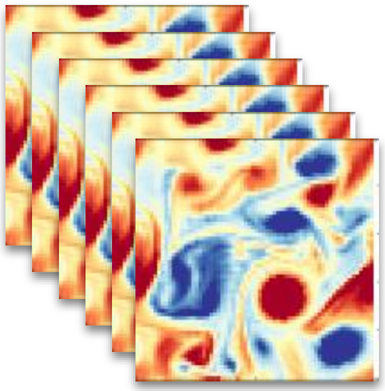
deterministic

SQG

512 x 512

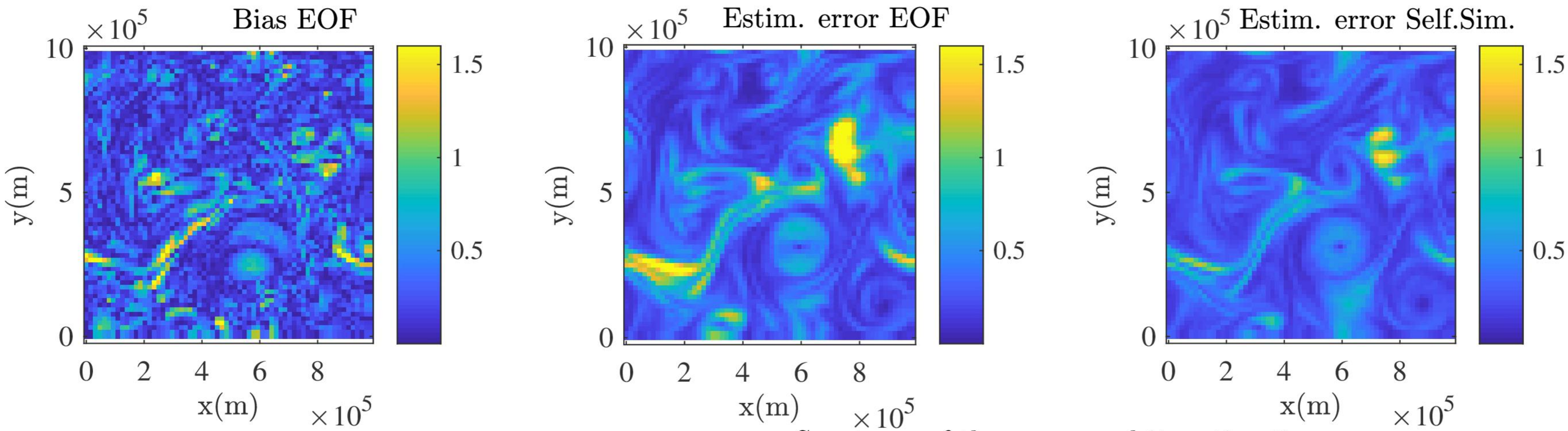
$t = 100$ day

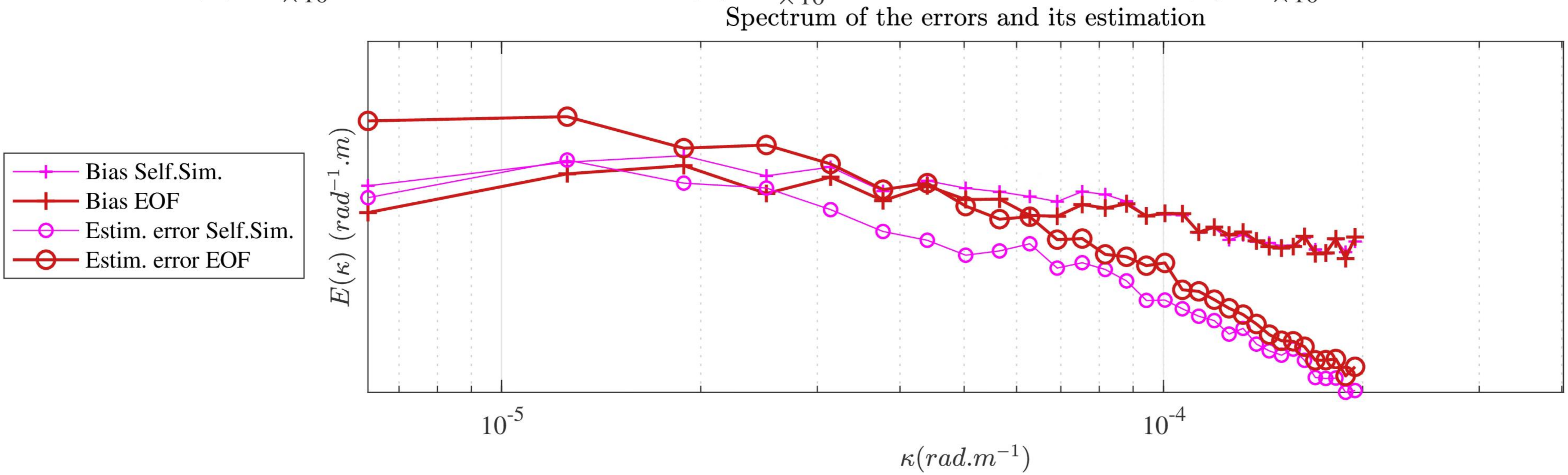


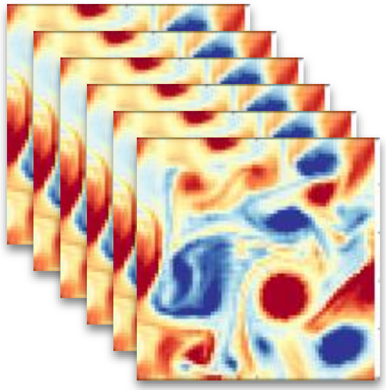


200 realizations at 64 x 64

Ensemble : uncertainty quantification





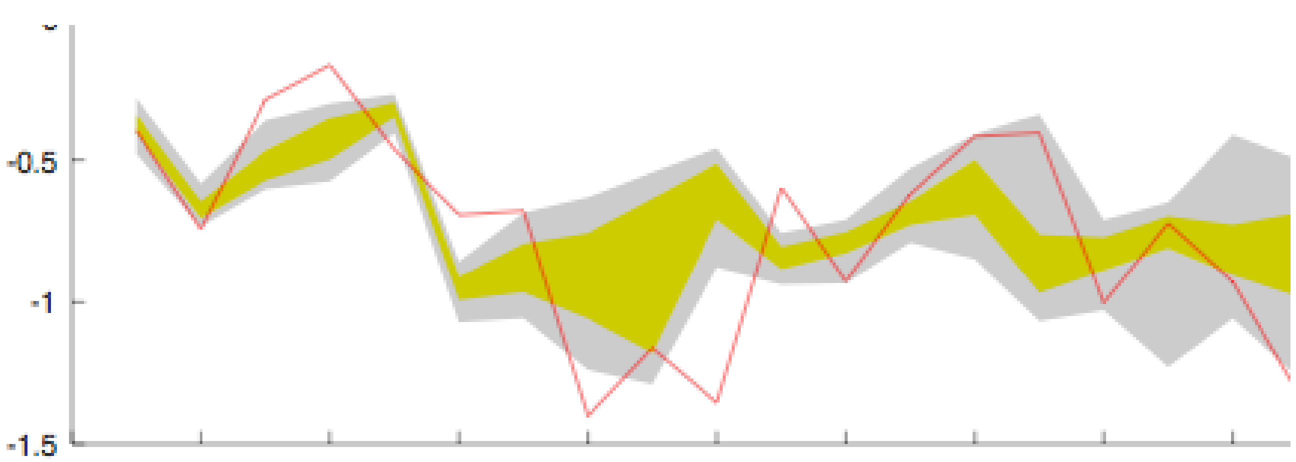


20 realizations
at 64 x 64

Ensemble : uncertainty quantification

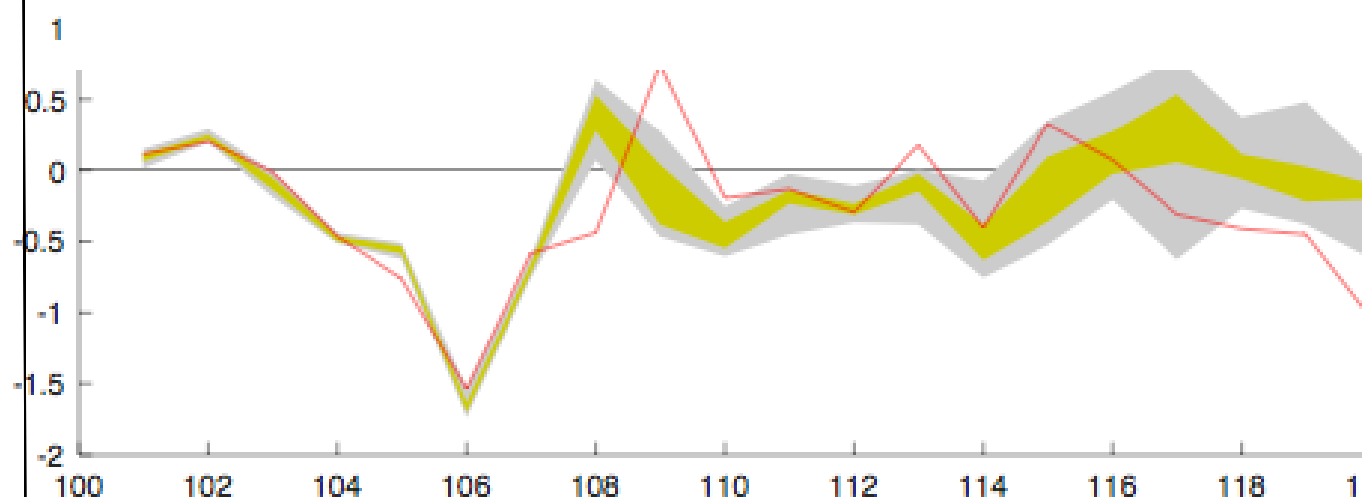
At point (250km,500km)

ADSD



At point (500km,500km)

ADSD



References :

Mikulevicius &
Rozovskii, 2004
Flandoli, 2011

LU

Memin, 2014
Resseguier et al. 2017 a, b, c
Cai et al. 2017
Chapron et al. 2018
Yang & Memin 2019

SALT

Holm, 2015
Holm and
Tyranowski, 2016
Arnaudon et al. 2017

Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al. 2018 a, b
Cotter and al. 2019

Cotter and al. 2017 Resseguier et al. 2020 a, b

Partial conclusion

Models under location uncertainty enables accurate uncertainty quantification to improve data assimilation

But also :

- blindly describe unresolved triades
- Stabilization / destabilization
- Model derivation
- Instabilities and bifurcations triggered

Part II

-

Wave-turbulence interaction

1. Swell-current interactions
2. Time decorrelation (LU)
3. Numeric simulations
4. Simple cases and analytic solutions for the time-uncorrelated model (LU)

II.1.
Swell - current interaction

Large scale group velocity:

$\bar{v}$ $+C_g^0$

Small scale group velocity:

v'

Wave:

$ae^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g||k||}$

Dispersion ratio

At the first order in steepness :

Wave phase transport

Frequency without currents

$$\frac{D\phi}{Dt} = \omega_0(k) = \sqrt{g||k||}$$

transport with a current velocity : $v = \bar{v} + v'$

Large scale group velocity:

$\bar{v}$ $+C_g^0$

Small scale group velocity:

v'

Wave:

$ae^{\frac{i}{\epsilon}\phi}$

Doppler frequenc

$\omega_0 =$

$\sqrt{g\|k\|}$

General case

Non-linear coupling

Fonction of k

Main contribution

$$\dot{X}_r = C_g = C_g^0 + \bar{v} + v'$$

Random ray

Change orientation of C_g^0

Stochastic refraction and contraction/dilatation

$$\dot{k} = -\nabla(\bar{v} + v')^T k$$

Fonction of x

Wave- vector

Conservation of action

$$\dot{A} = -\nabla \cdot u_g \; A \qquad A = \frac{a^2}{\omega_0}$$

Amplitude

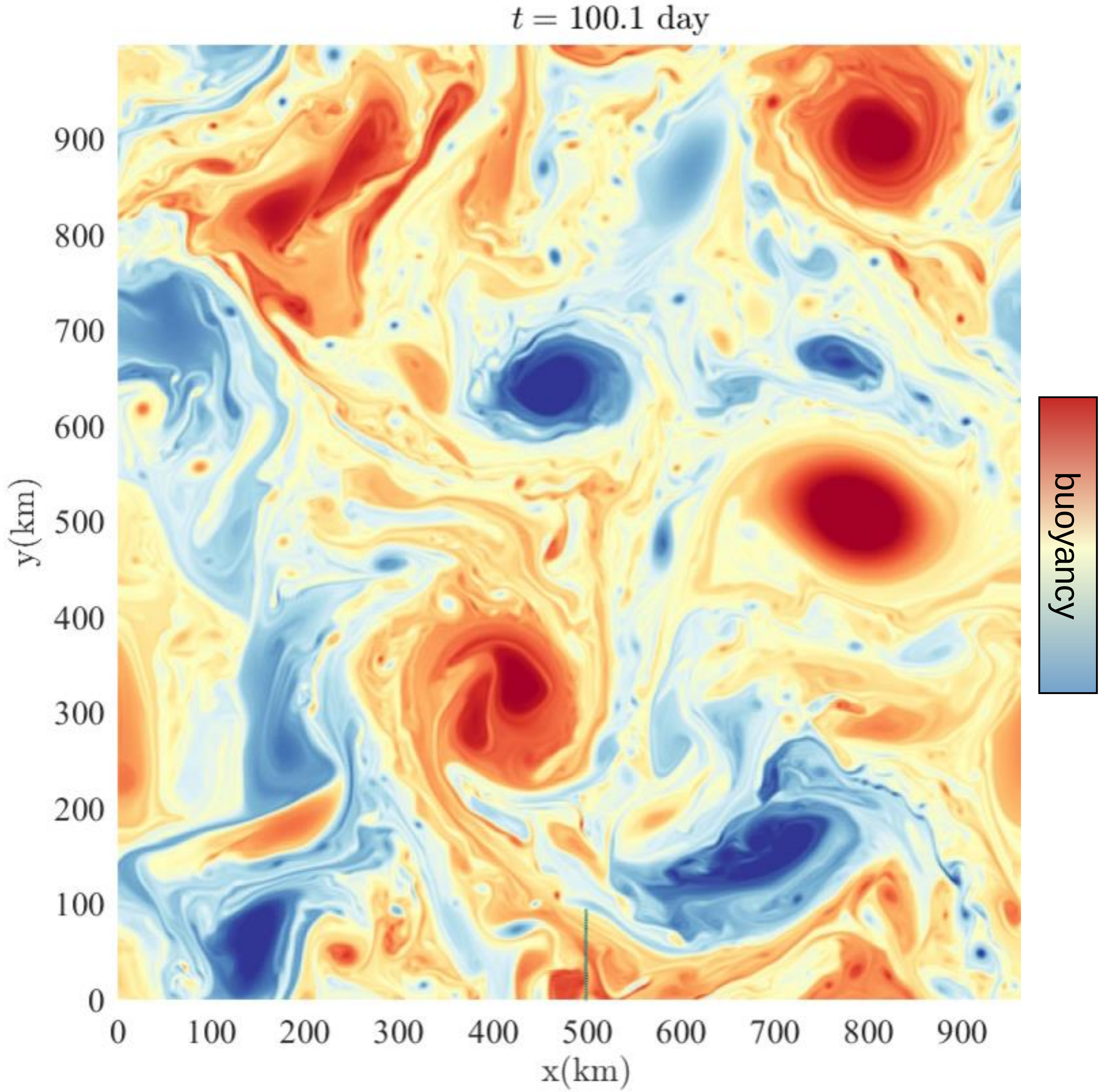
II.3.
Time decorrelation (LU)

Numerical example

like running in the middle of a crowd

$$v = \bar{v} + v'$$

(without modeling / simplification)



Large scale group velocity:

$\bar{v}$ $\boxed{+C_g^0}$

Small scale group velocity:

v'

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Time decorrelation assumption for v' (LU)

- Advantages :
  - Simpler / analytic formula
    ⇨ physical comprehension & lighter CPU for simulations
  - No precise knowledge needed about v' time dependency

- Validity :

$$\epsilon = \frac{(\textbf{Along-ray } v' \text{ correlation time})}{\left(\begin{array}{c}\text{characteristic time of}\\ \text{wave group properties } \textbf{evolution}\end{array}\right)} = \frac{\left(\frac{l_{v'}}{\|C_g^0\|}\right)}{\left(\frac{1}{\|\nabla v\|}\right)} \ll 1$$

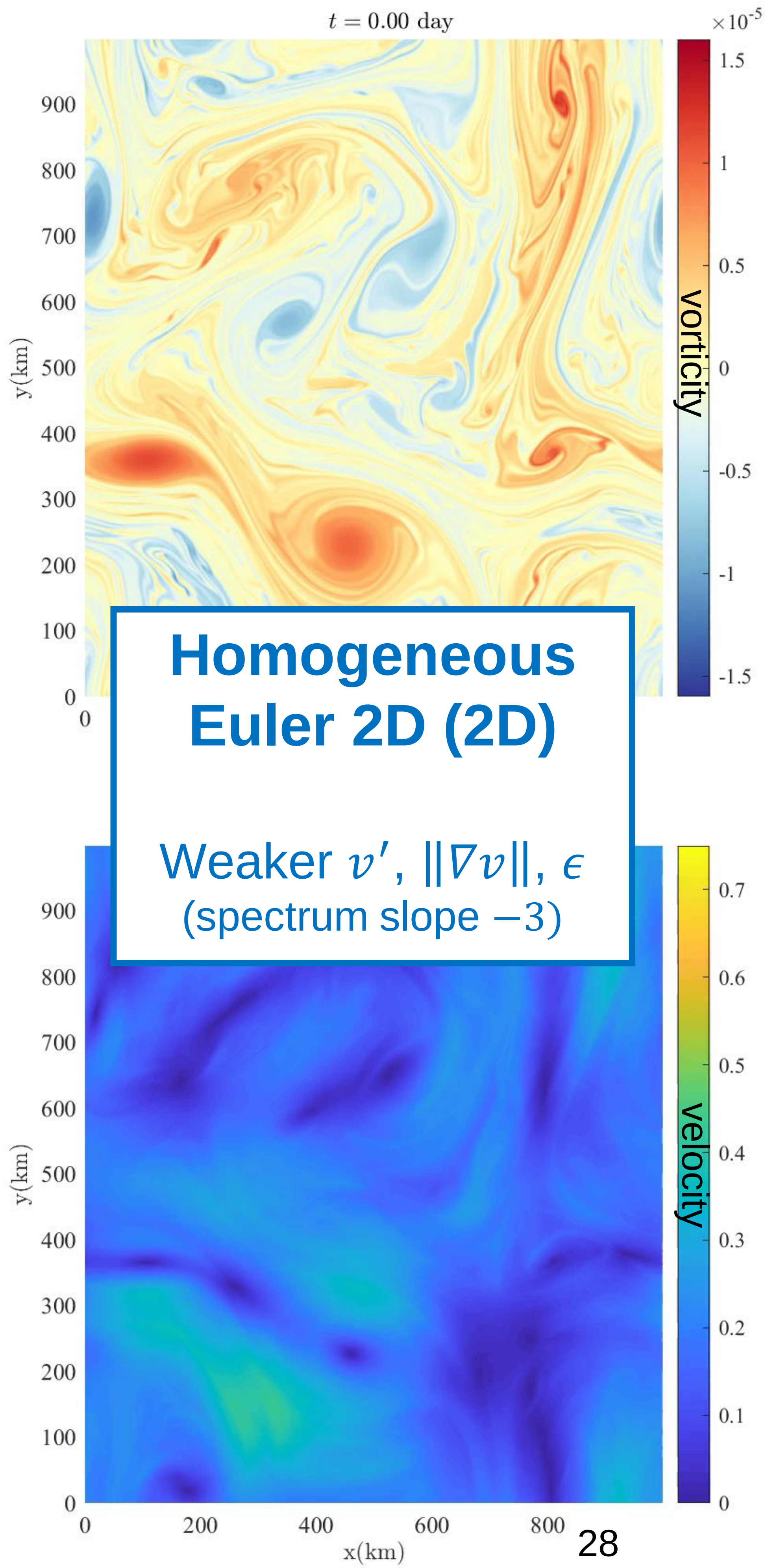
- Limitations :
  - Swells $\left(\|C_g^0\| \gg 1\right)$
  - Small-scale currents $(l_{v'} \ll 1)$
  - Moderate current gradients $(\|\nabla v\| \ll 1)$ (⇨ moderate $\|\nabla v'\|$ **!**)

II.3.
Numerical simulations

2 test flows

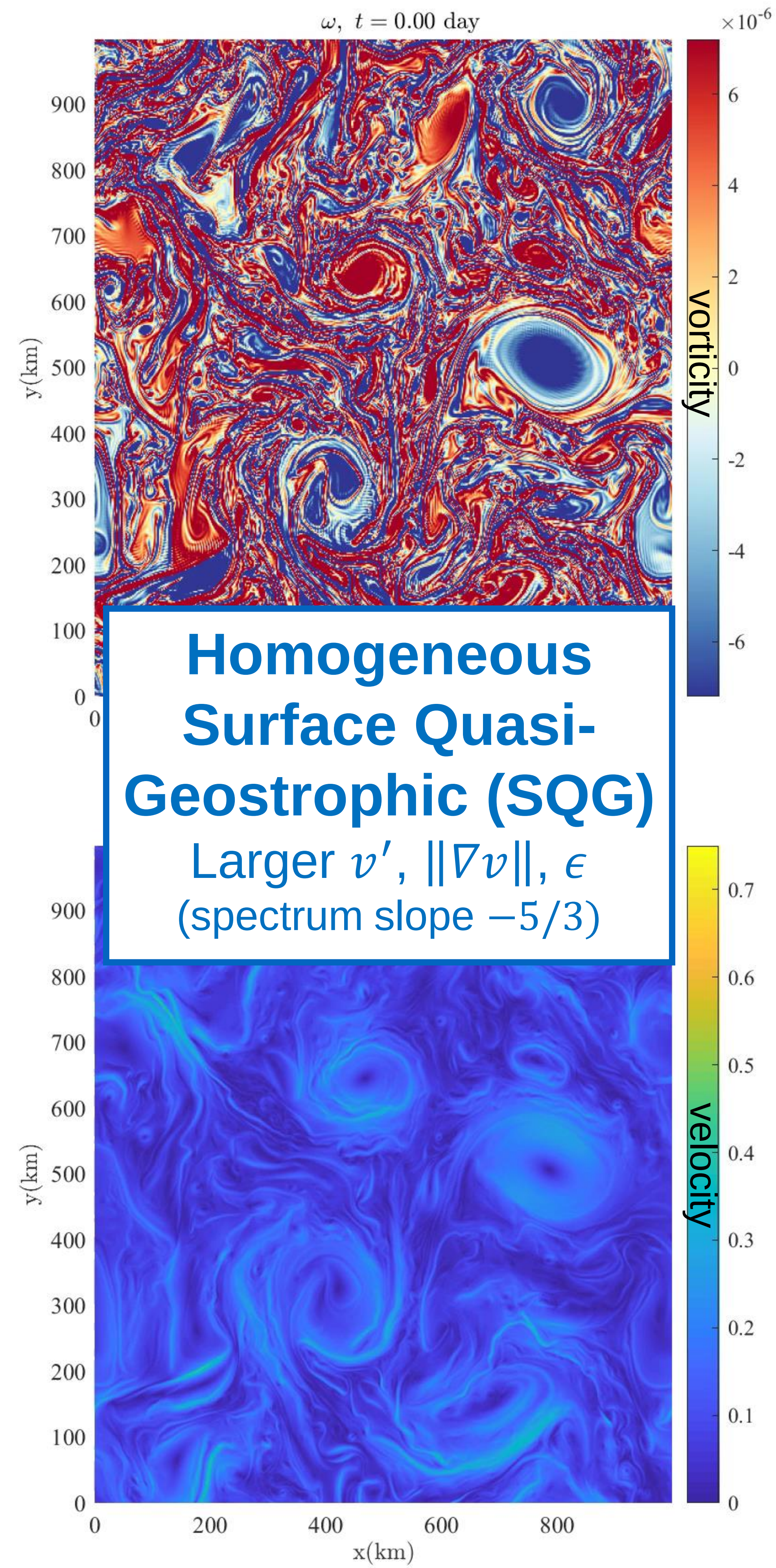
Euler
VS
SQG

($U \sim 0.1$ m.s^{-1},
$\lambda_{\text{wave}} = 300$ m)



Homogeneous Euler 2D (2D)

Weaker v', $\|\nabla v\|$, ϵ
(spectrum slope -3)

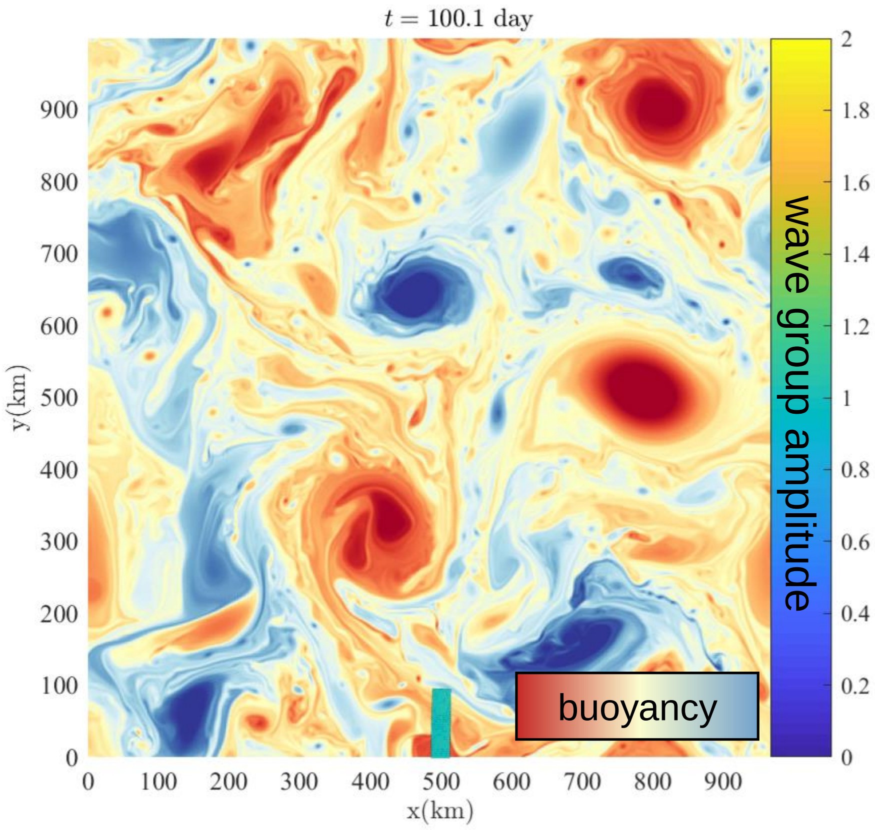


Homogeneous Surface Quasi-Geostrophic (SQG)

Larger v', $\|\nabla v\|$, ϵ
(spectrum slope $-5/3$)

Time-**uncorrelated** model for v'

$v = \bar{v} + v'$

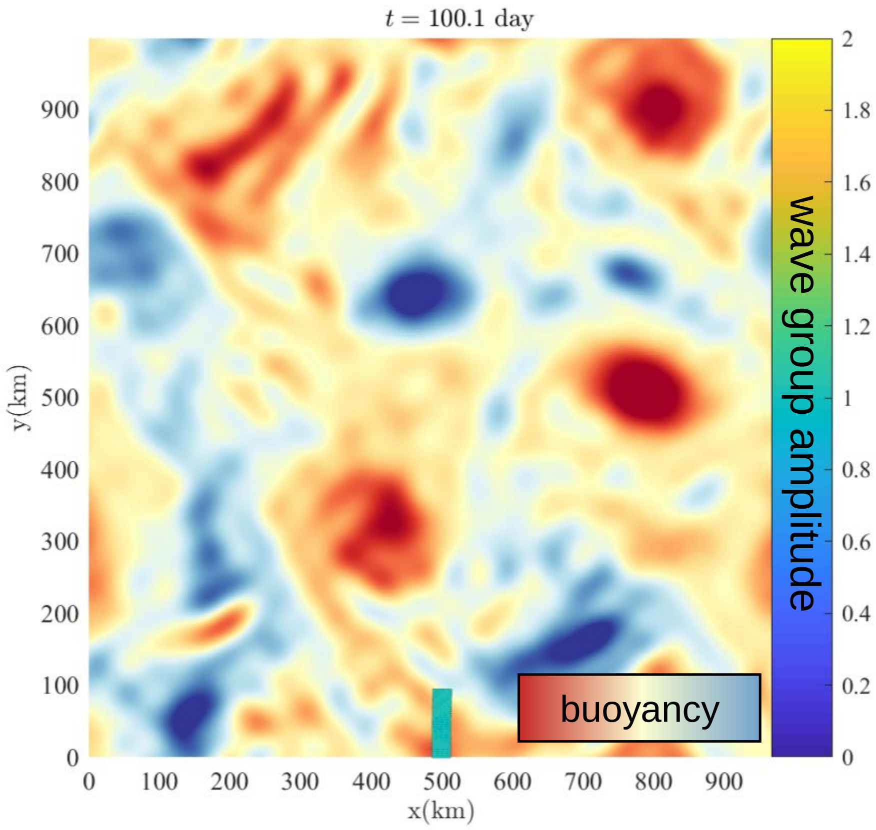


Deterministic reference:

wave groups in

512 x 512 **SQG** flow

$v = \bar{v}$

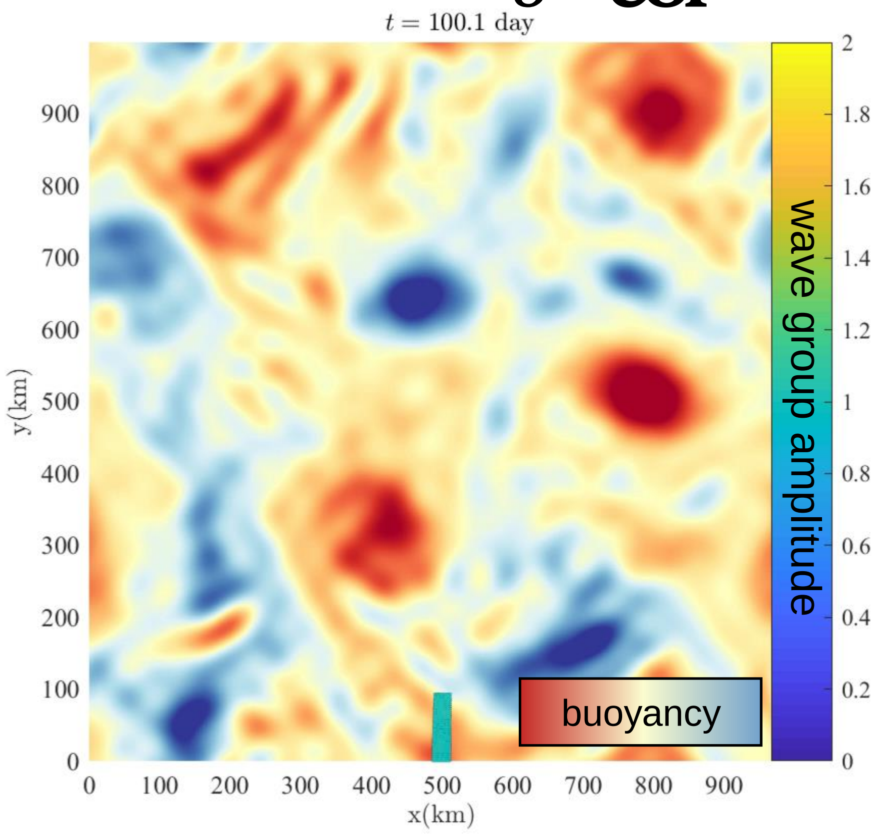


Deterministic benchmark:

wave groups in

smoothed flow $\bar{v}$ (**32 x 32**)

$v = \bar{v} + v'_{\delta-\mathrm{cor}}$



Our random model

wave groups in

smoothed flow $\bar{v}$ (**32 x 32**)

+ time-**uncorrelated** v'

Large scale group velocity:

$\bar{v}$ $+C_g^0$

Small scale group velocity:

v'

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

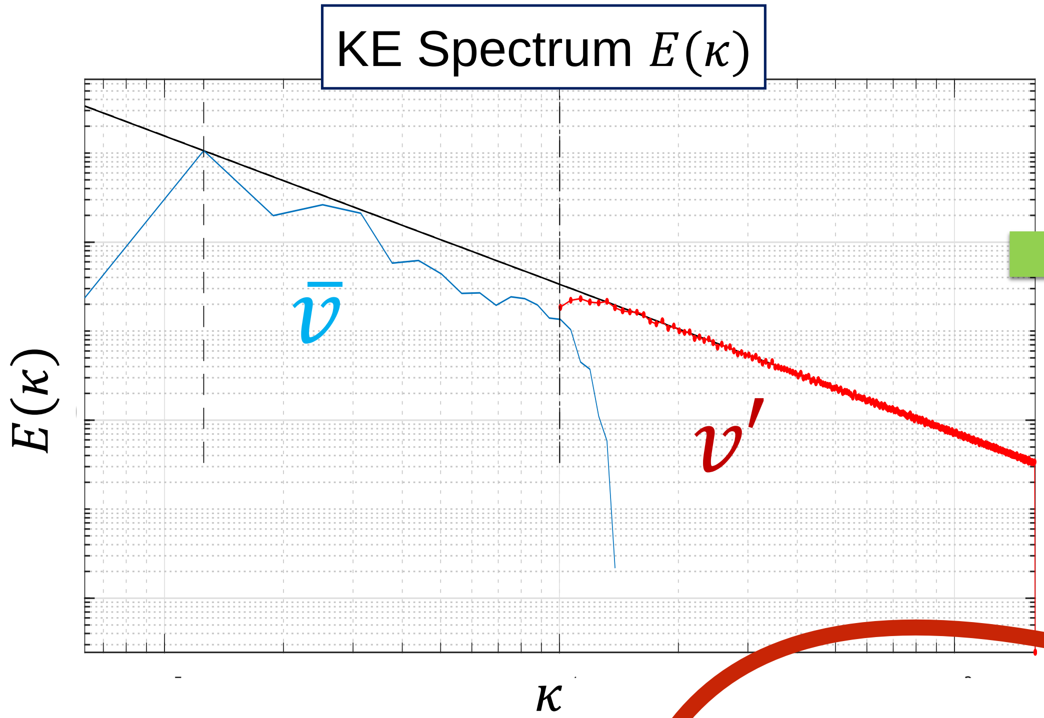
Time-**uncorrelated** model for v'

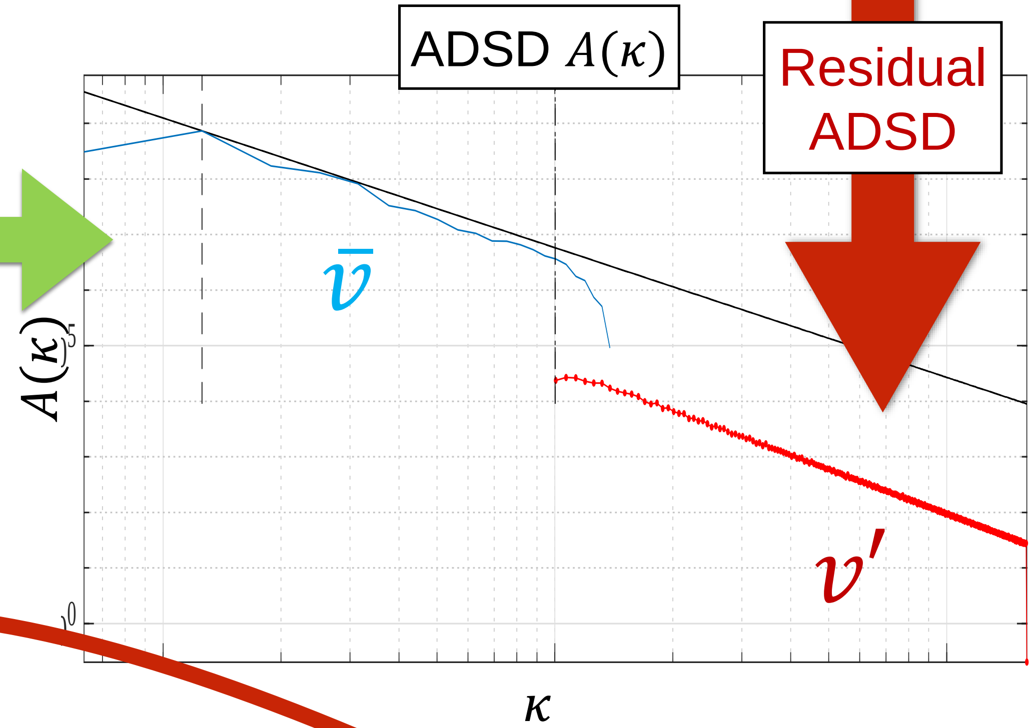
Absolute Diffusivity Spectral Density

$$A(\kappa) = E(\kappa)\,\tau(\kappa)$$

Correlation time along ray

$$\tau(\kappa) = \tau_{ray}(\kappa) = \frac{1/\kappa}{\|C_g^0\|}$$





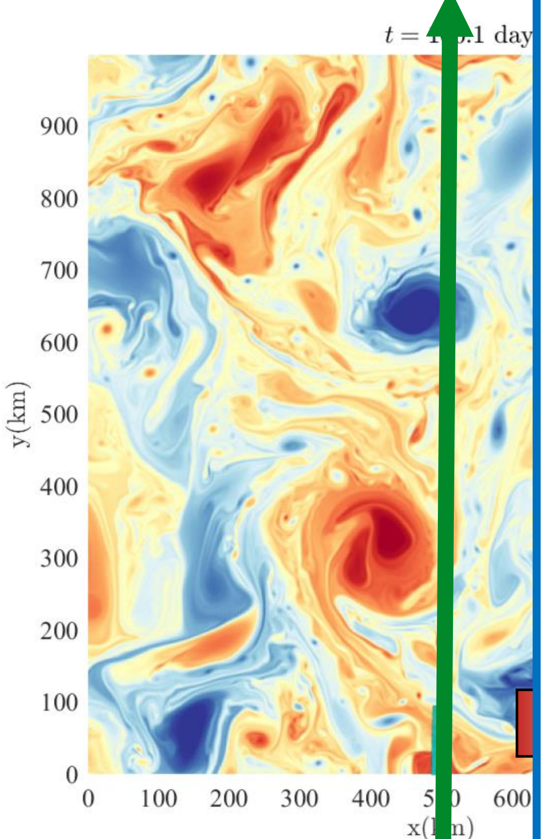
$$v' = \sigma\,\dot{B} = \text{(filter)} * \text{(white noise)}$$

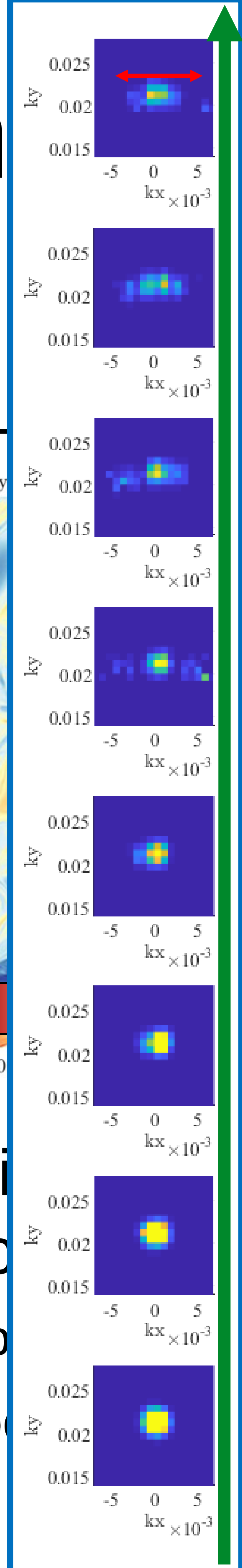
Reference:
Resseguier, Pan & Fox-Kemper 2020

Tim orrelated lel for v' :

s resolution c aints

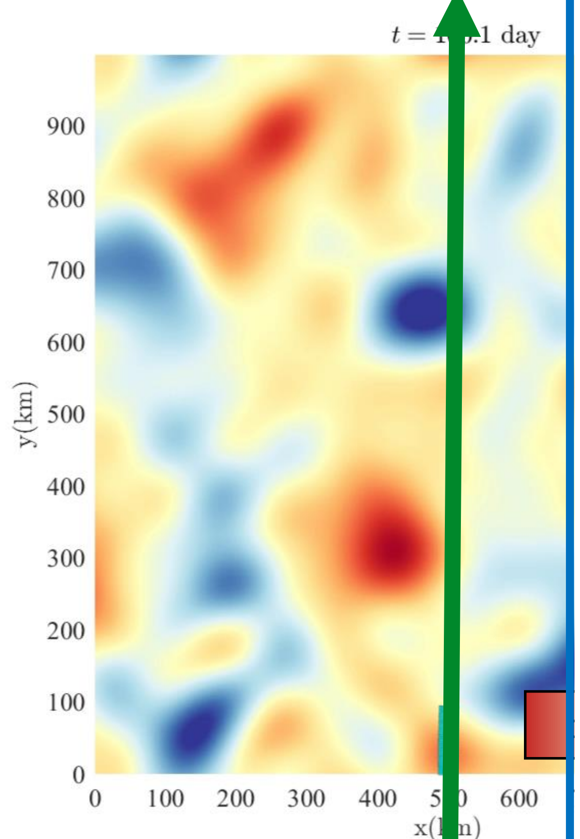
$v = \bar{v} +$

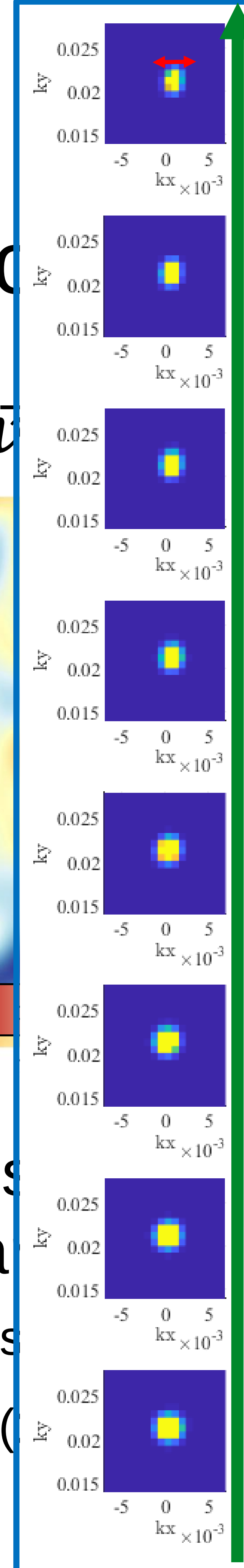




Determini
referenc
wave group
512 x 512 **SQ**

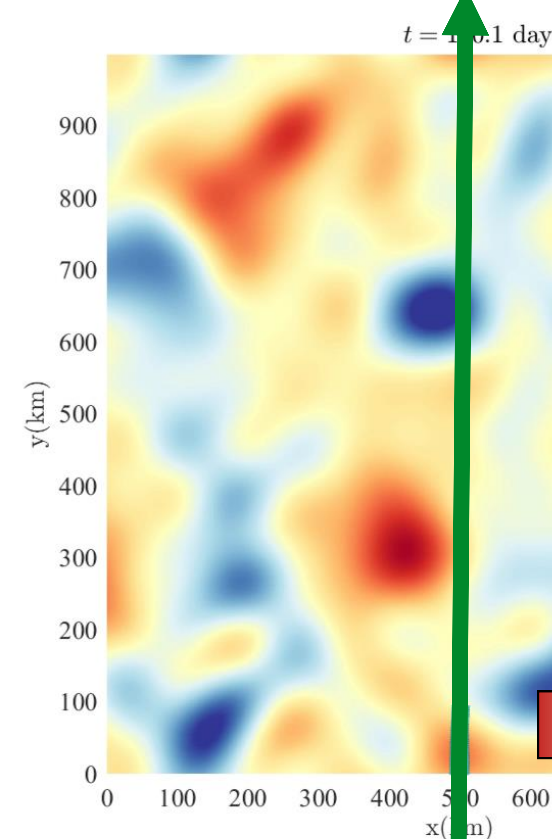
$v = \bar{v}$

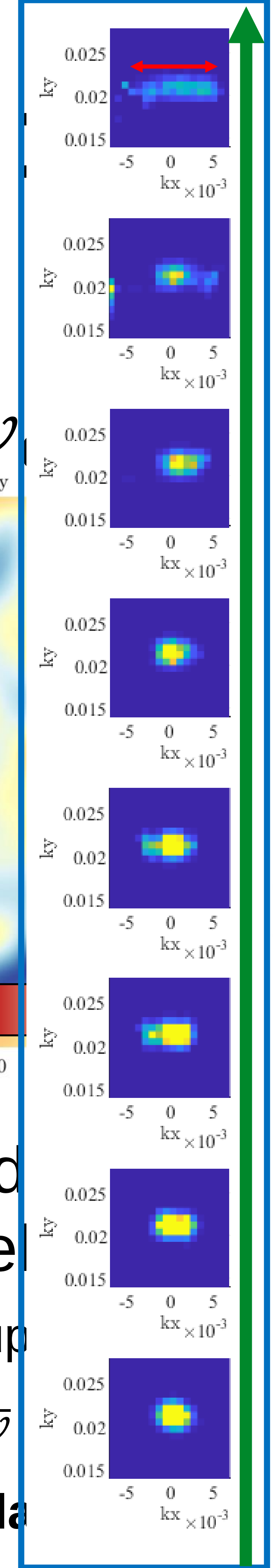




Determinis
benchma
wave groups
smoothed flow $\bar{v}$ (

$v = \bar{v} + v$





Our rand
model
wave group
smoothed flow $\bar{v}$
+ time-**correla**

Large scale group velocity:

$\bar{v}$ $\boxed{+C_g^0}$

Small scale group velocity:

v'

Wave:

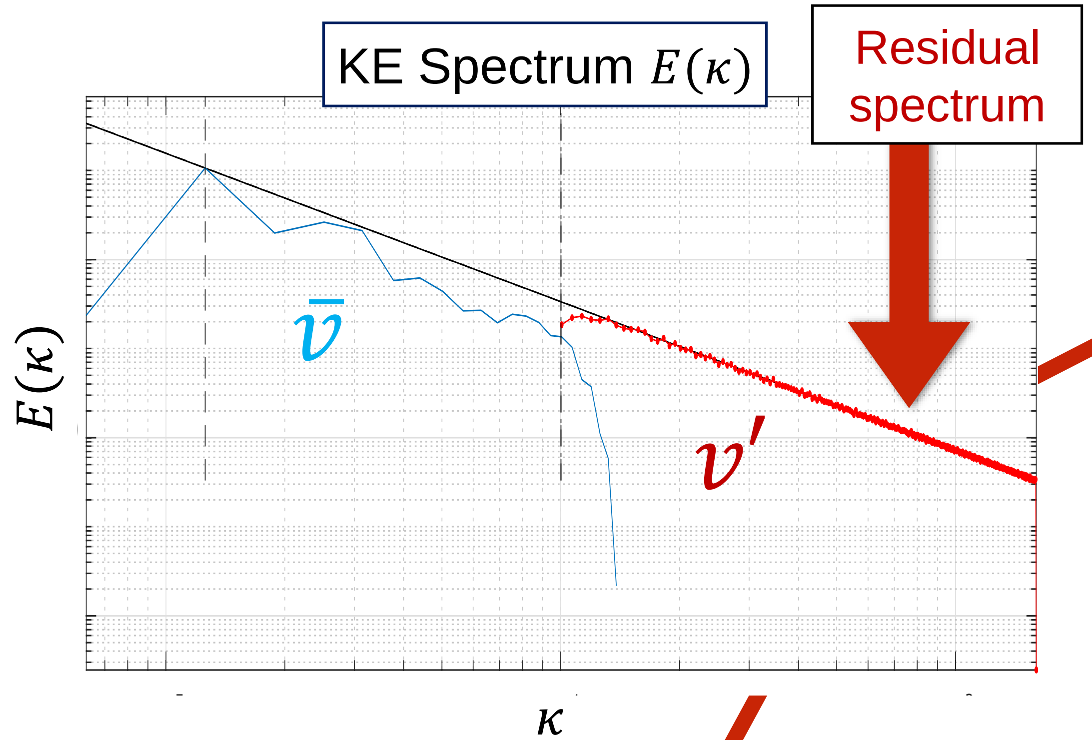
$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Time-**correlated** model for v':

less resolution constraints



Current correlation time

$$\tau(\kappa) = \tau_{cur}(\kappa) = \frac{1/\kappa}{v_k} = \frac{1/\kappa}{\sqrt{\kappa\, E(\kappa)}}$$

Multiscale noise $\dot{B}_c$:

spatial white noise

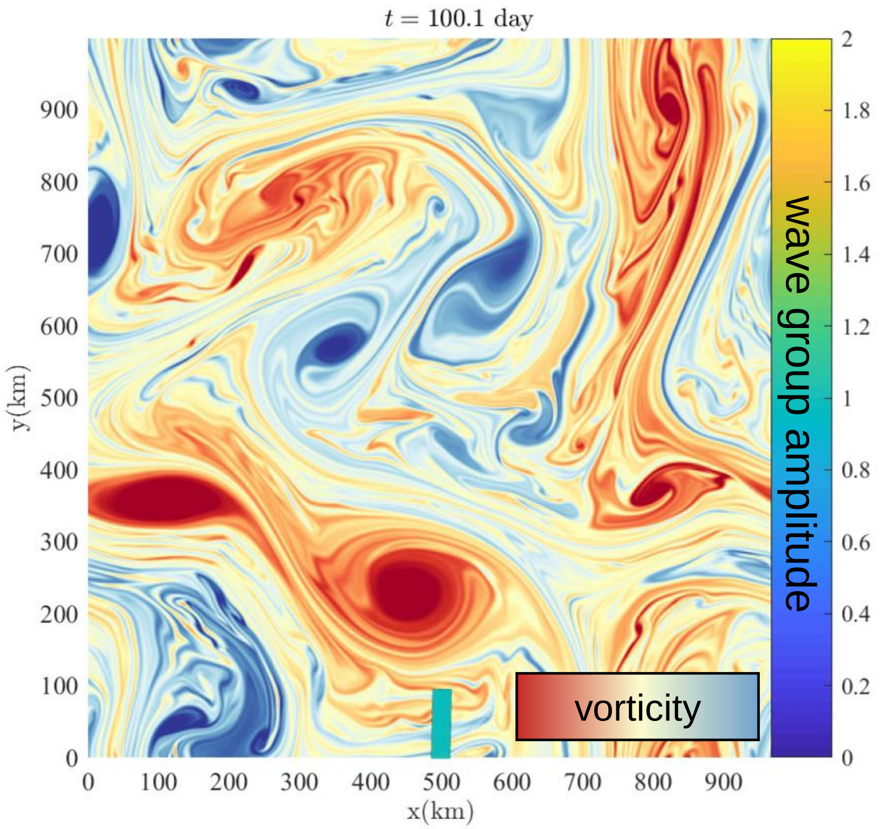
with time scale $\tau(k)$ at the scale k:

$$\frac{d}{dt}\widehat{\dot{B}_c} = -\frac{1}{\tau(k)}\widehat{\dot{B}_c} + \sqrt{\frac{2}{\tau(k)}}\,(\widehat{white\ noise})$$

$$v' = (\text{filter}) * (\text{time}-\text{correl. noise})$$

Time-**uncorrelated** model for v'

$$v = \bar{v} + v'$$

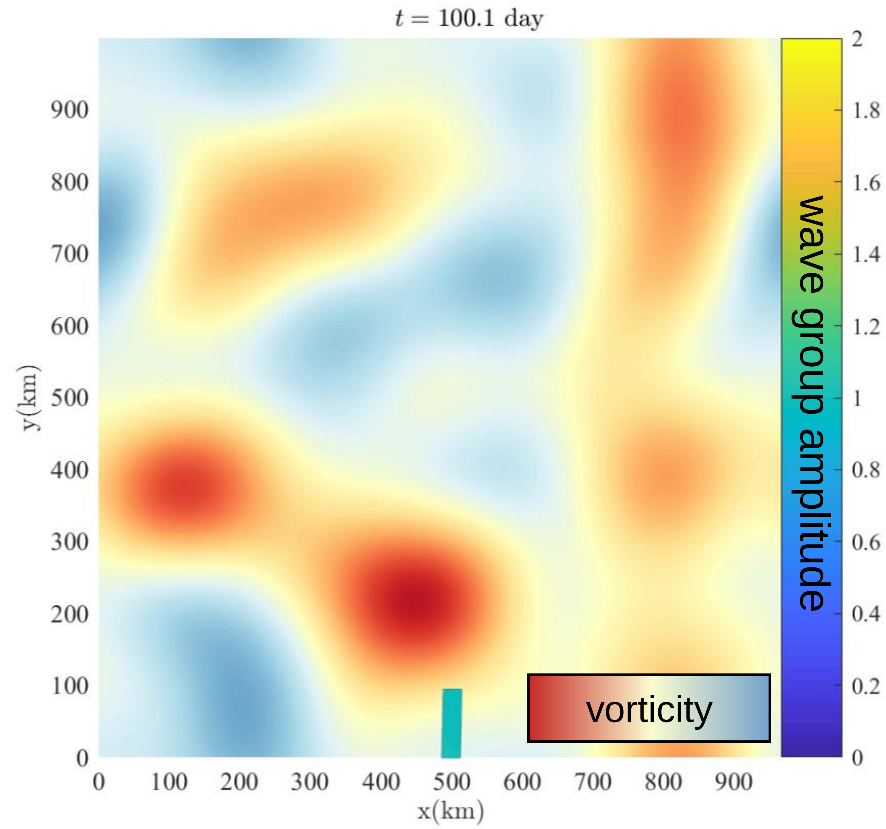


Deterministic reference:

wave groups in

512 x 512 **2D Euler** flow

$$v = \bar{v}$$

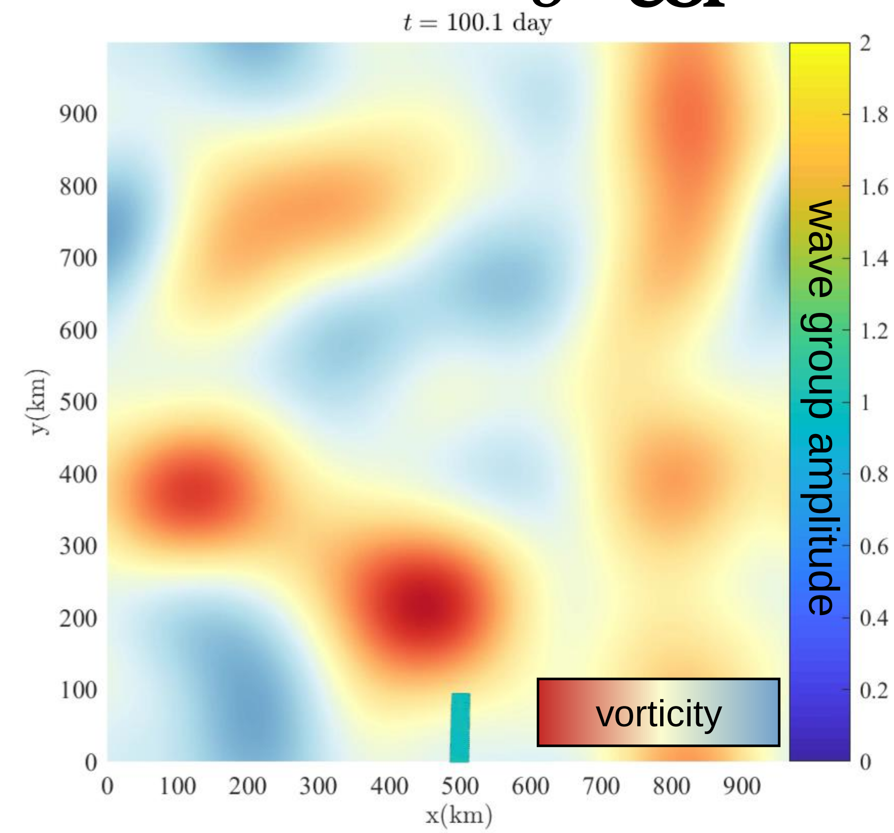


Deterministic benchmark:

wave groups in

smoothed flow $\bar{v}$ (**8 x 8**)

$$v = \bar{v} + v'_{\delta\mathrm{-cor}}$$



Our random model

wave groups in

smoothed flow $\bar{v}$ (**8 x 8**)

+ time-**uncorrelated** v'

II.4.
Simple cases and analytic solutions for the time-uncorrelated model

Large scale group velocity:

$\bar{v}$ $\boxed{+C_g^0}$

Wave:

$a e^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Simple linear case 1:

stationary deterministic, divergence-free and linear in x large-scale velocity

Stretching of phase

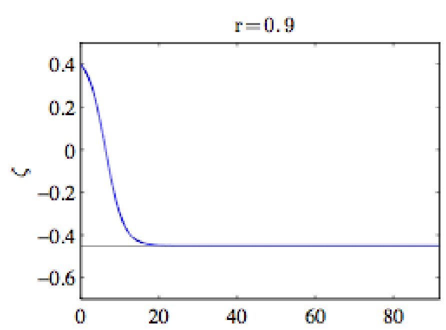
$$\dot{k} = -\nabla \bar{v}^T k$$

Ratio vorticity / strain rate

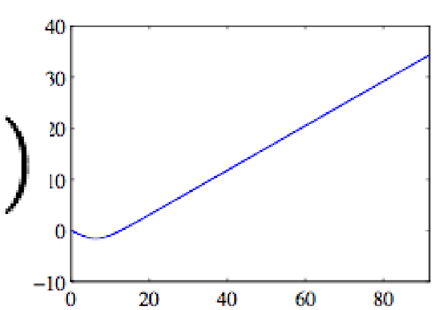
$$r = \frac{\omega}{\sigma_w}$$

$|r| < 1$

Wave shortening

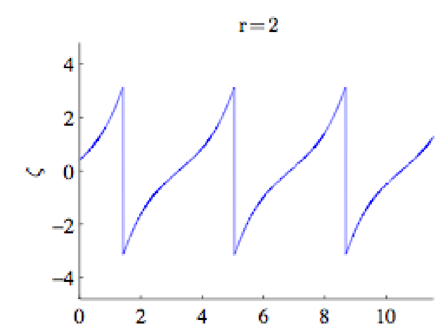


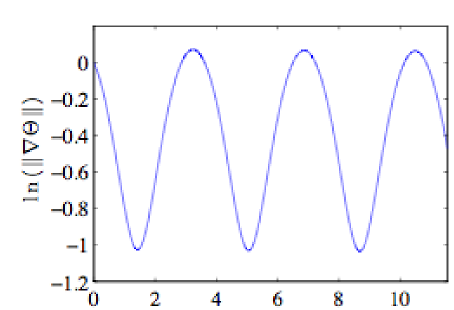
$ln\left(\|k\|\right)$



$|r| > 1$

Wave rotation





$ln\left(\|k\|\right)$

$$\frac{D\zeta}{D\tau} = r - \cos(\zeta)$$

$$\frac{D}{D\tau} \; ln\left(\|k\|\right) = -\sin(\zeta)$$

Large scale group velocity:

$+C_g^0$

Small scale group velocity:

v'

Wave:

$ae^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Simple linear case 2:

no large-scale current
+ homogeneous and divergence-free
small-scale velocity

$$\beta(x,t)e^{i(\omega_0 t + k\cdot\sigma B_t)}$$

Wave with **Brownian phase**

Stochastic stretching of phase

$$\dot{k} = -\nabla(v')^T k$$

Log-normal wave- number

Large scale group velocity:

$\bar{v}$ $+C_g^0$

Small scale group velocity:

v'

Wave:

$ae^{\frac{i}{\epsilon}\phi}$

Doppler frequency:

$\omega_0 = \sqrt{g\|k\|}$

Simple linear case 3

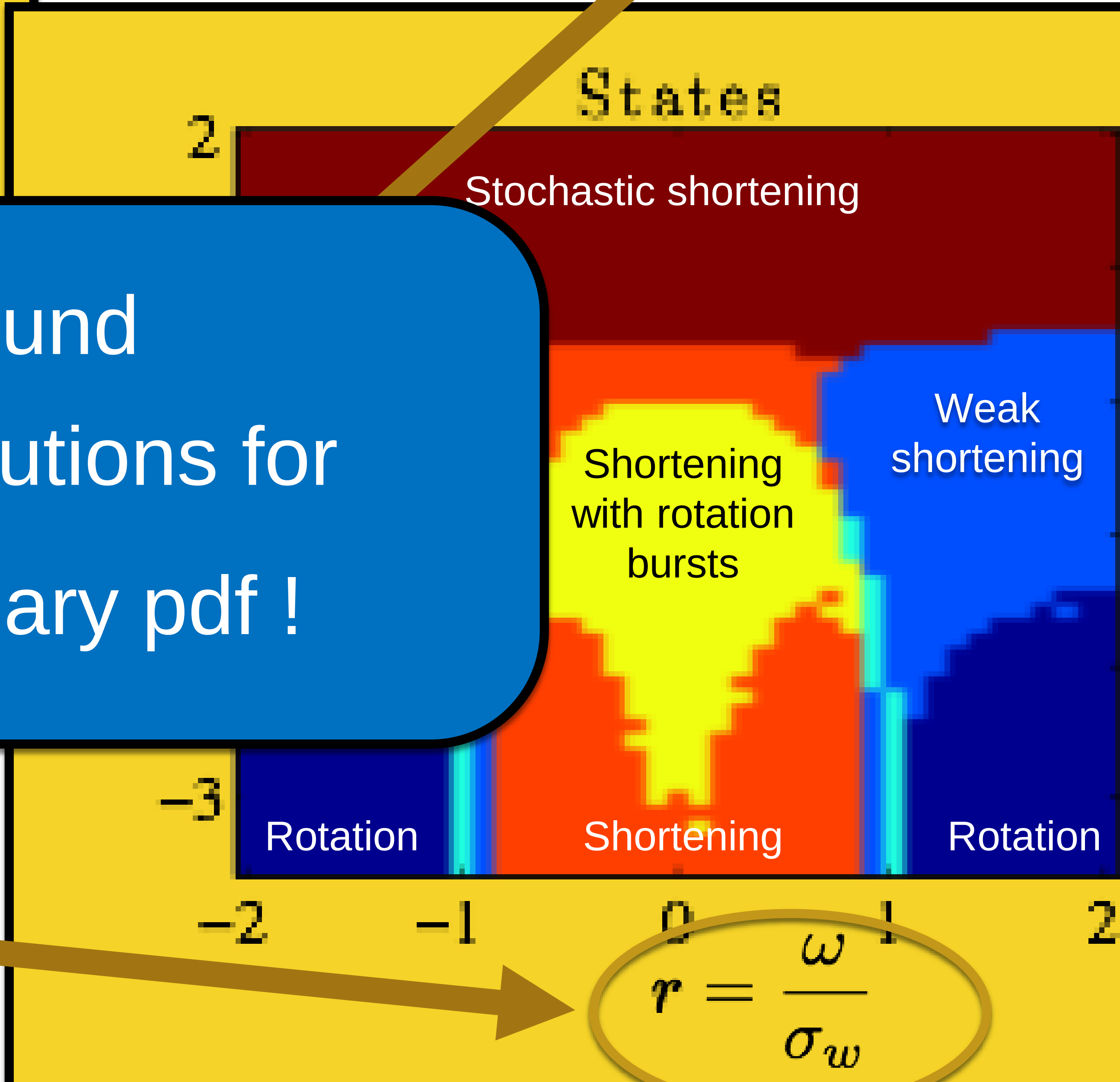
divergence-free and linear-in-x large-scale current
+ homogeneous and divergence-free
small-scale velocity

Stochastic stretching of phase

$$\dot{k} = -\nabla(\bar{v} + v')^T k$$

Depends on small scale's statistics

We found analytic solutions for the stationary pdf !



Ratio
vorticity / strain rate

Partial conclusion

- Theoretical comprehension + Numerical demonstration
  of our wave-current interaction models skills
  with different types of currents (local / non-local, homogeneous / heterogeneous currents)

- 3 stochastic currents models for small-scale current v'
  1. Time-uncorrelated :
     - ✓ Good skills, partial analytic solution, but resolution limitations for its application
  2. Multiscale time-correlated :
     - ✓ Always works
  3. Hybrid model

Part III
-
Reduced order model & data assimilation

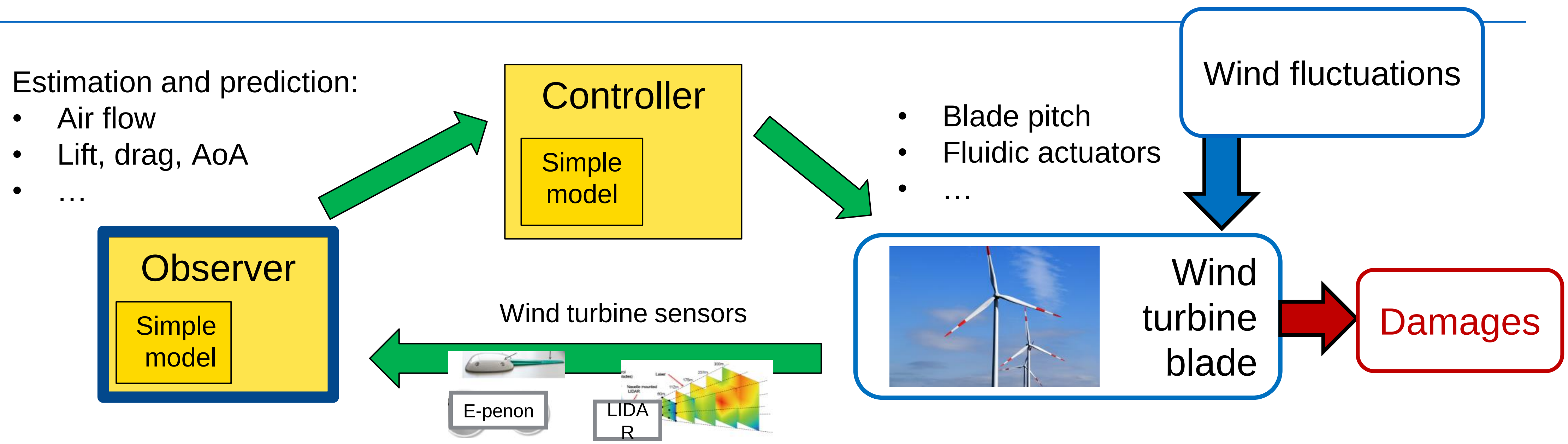
1. Application
2. Reduced order models
3. Data assimilation with reduced LU models

III.1.
Application

Application

Estimate and predict unsteady aerodynamism -- in real-time –
for better active control loops

- Lower wind turbine maintenance costs
- Longer wind turbine life cycle



Which simple model?   How to combine model & measurements?

III.2.
Reduced order model

Reduced order model (ROM)

Solution of an PDE with the form:

Time

Space

Parameters (if any)

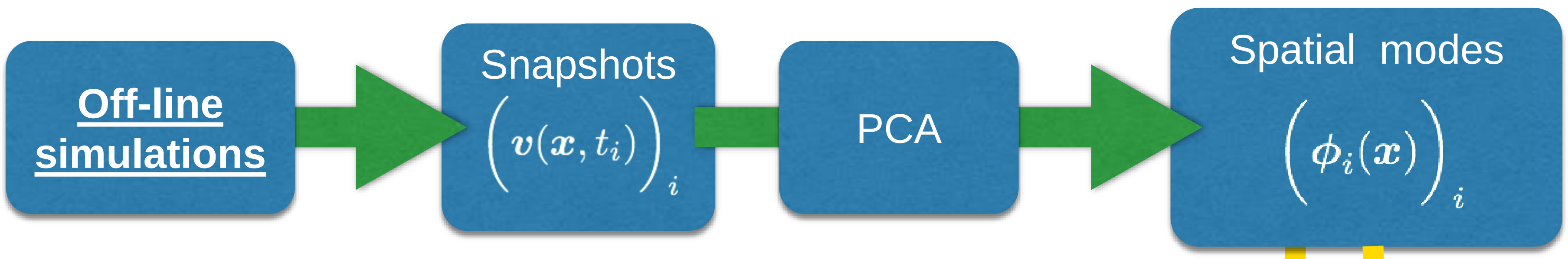
$$v(x,t,\alpha) \approx \sum_{i=o}^{n} \boxed{b_i(t)}\,\boxed{\phi_i(x)}\,\cancel{\boxed{\gamma_i(\alpha)}}$$

Order of magnitude examples in CFD

	Full space	Reduced space
Solution coordinates	$v_q(x_i,t))_{qi}$	$(b_i(t))_i$
Dimension	$M \times d \sim 10^7$	$n \sim 10 - 100$

POD-Galerkin

- <u>Principal Component Analysis (PCA)</u> on a *dataset* to reduce the dimensionality:



- <u>Approximation</u>:

Resolved modes

$$v(x,t) \approx \sum_{i=o}^{n} \boxed{b_i(t)}\, \phi_i(x)$$

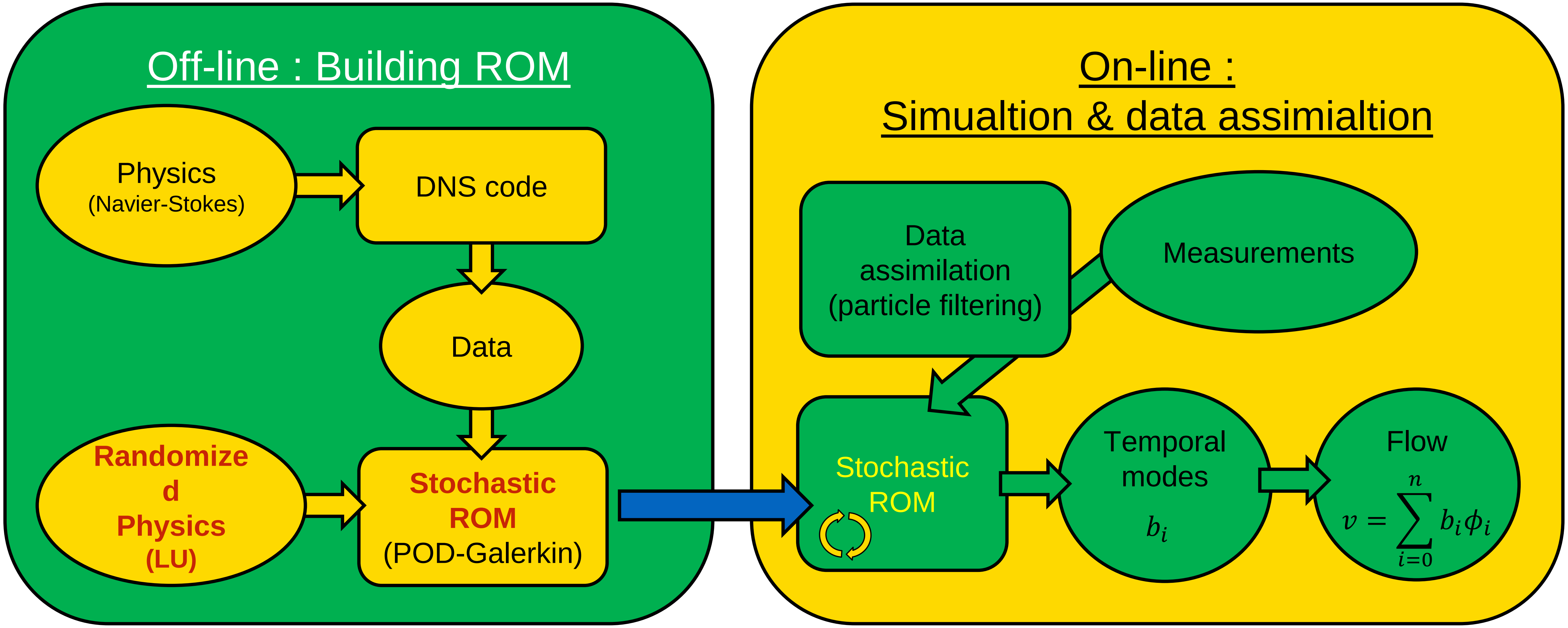
- <u>Projection of the "physics"</u> onto the spatial modes :

$\int_\Omega dx\, \phi_i(x) \cdot$ (*Physical equation* (e.g. Navier-Stokes))

→ **ROM for very fast simulation of temporal modes**

III.3.
Data assimilation with reduced LU models

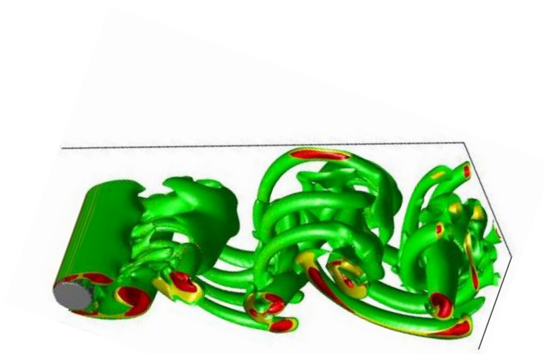
Big picture with reduced LU models



Numerical results : 3D Wake at Reynolds 300

Reduced order models with **8 degrees of freedom**

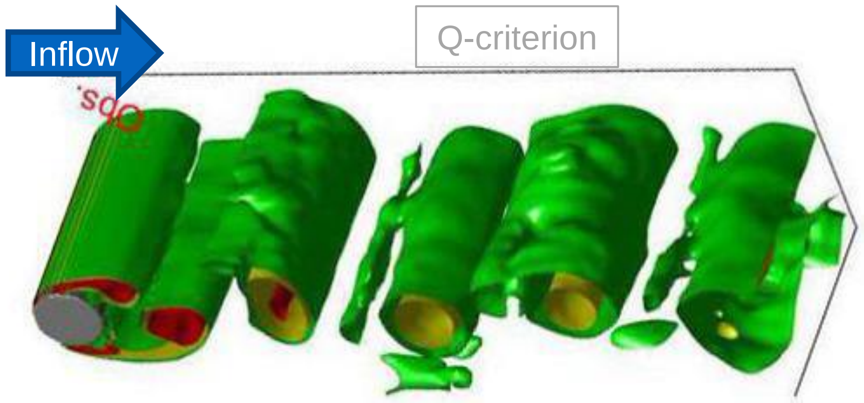
- only 1 PIV spatial point (local, blurred and noisy measure) is assimilated 10 times by vortex shedding cycle

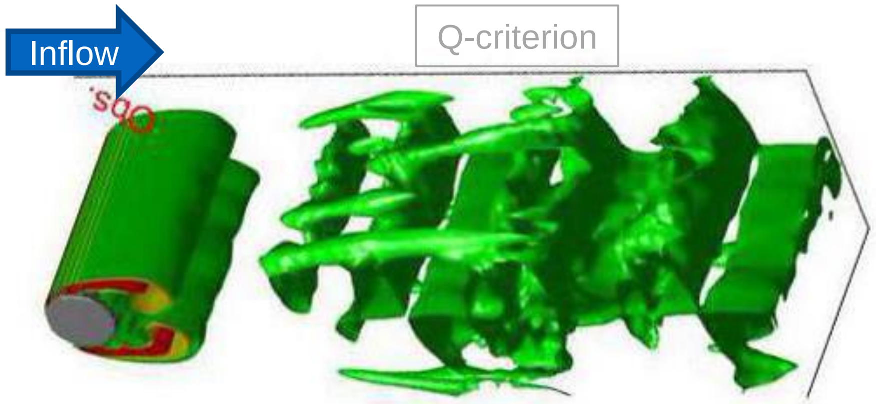


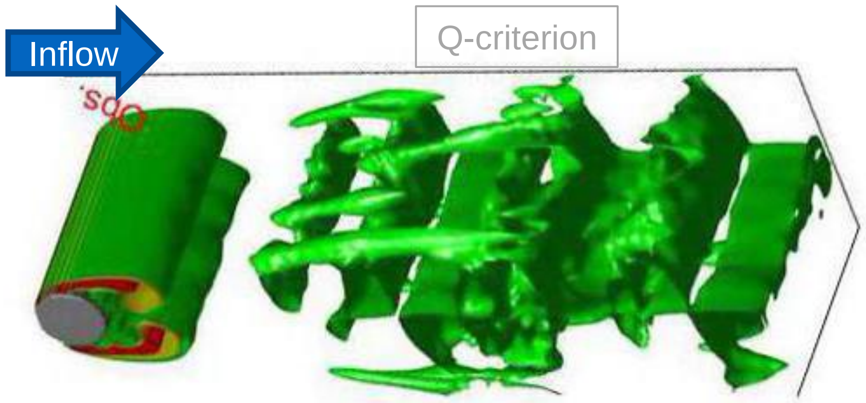
Reference :
PCA-projection of the "true" simulation (10^7-dof DNS)
(Optimal from 8-**dof** linear decomposition)

Our method :
POD-Galerkin with Navier-Stokes under location uncertainty (LUM)

State-of-art :
POD-Galerkin with Navier-Stokes + optimally tuned eddy viscosity & additive noise







Partial conclusion

- Reduced order model (ROM) : for very fast and robust CFD
  ($10^7 \rightarrow 6$ degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LU
- Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LU
- First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

Conclusion

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When subgrid information is missing,

models under location uncertainty (LU) and its time-correlated variants

are stochastic closures that

- Improves CFD and wave-current interaction simulations
- Quantify numerical model error for data assimilation purposes