Bifractal nature of turbulent reaction waves at high Damköhler and Karlovitz numbers

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ABSTRACT

Governing physical mechanisms of the influence of Kolmogorov turbulence on a reaction wave (e.g., a premixed flame) are often discussed by adopting (combustion) regime diagrams. While two limiting regimes associated with (i) a high Damköhler number Da , but a low Karlovitz number Ka , or (ii) a low Da , but a high Ka drew significant amount of attention, the third limiting regime associated with (iii) $Da \gg 1$ and $Ka \gg 1$ has yet been beyond the mainstream discussions in the literature. The present work aims at filling this knowledge gap by adapting the contemporary understanding of the fundamentals of the regimes (i) and (ii) in order to describe the basic features of the influence of intense turbulence on a reaction wave in the regime (iii). More specifically, in that regime, the entire turbulence spectrum is divided in two subranges: small-scale and large-scale eddies whose influence on the reaction wave is modeled similarly to the regimes (ii) and (i), respectively. Accordingly, the surface of the reaction wave is hypothesized to be a bifractal with two different fractal dimensions of $D_f = 8/3$ and $7/3$ at small and large scales, respectively. The boundary between the two ranges is found by equating the local eddy turn-over time to the laminar-wave time scale. Finally, a simple scaling of $U_T \propto u'$ is obtained for the turbulent consumption velocity at $Da \gg 1$ and $Ka \gg 1$. Here, u' is the rms turbulent velocity.

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I. INTRODUCTION

The acceleration of expansion of reaction waves, caused by turbulence, is well documented in various research areas ranging from turbulent combustion,^{1–5} deflagration-to-detonation transition,⁶ or chemical reactions in aqueous solutions^{7,8} under terrestrial conditions to evolution of thermonuclear Ia supernovae^{9–12} in the universe. Due to the multiscale and highly nonlinear (e.g., in the case of combustion) nature of this phenomenon, substantial simplifications (e.g., single-step chemistry, characterization of fluid state with a single scalar, constant density, and adiabatic conditions) are commonly invoked to elucidate the physical mechanisms that control the influence of turbulence on the propagation of reaction waves. Such an approach is based on the seminal work by Damköhler,¹³ who hypothesized that acceleration of a premixed flame by turbulence is caused by two different physical mechanisms in two limiting regimes of the influence of turbulence on combustion, associated with a large or a small ratio of an integral turbulence length scale L to a

laminar flame thickness δ_L . More specifically, he has hypothesized as follows:

1. If $L \gg \delta_L$, the influence of turbulence on a premixed flame is reduced to an increase in the area of the flame surface wrinkled by turbulent eddies (“for coarse-body turbulence with a great mixing length compared to the thickness of the laminar combustion zone with roughened and ultimately scattered flame front, in which, as a result, the surface of the flame is enlarged and the flame speed referred to the flow cross section rises beyond the laminar value”; see p. 3 in the paper by Damköhler¹³).
2. If $L \ll \delta_L$, the influence of turbulence on a premixed flame is reduced to amplification of mixing (“for turbulence bodies smaller than the laminar flame thickness ... the flame speed increases by reason of the fact that all transport processes between the reaction zone and the preparatory zone in

the flame surface are amplified, owing to the turbulent mixing motion,” see p. 42 in the cited paper).

These hypotheses are known as Damköhler’s first and second hypotheses (DaI and DaII, in the following), respectively.

In the middle of the 1980s, these ideas were developed by Borghi,¹⁴ Williams,¹⁵ and Peters,¹⁶ who pioneered the use of combustion regime diagrams to discuss the governing physical mechanisms of the influence of turbulence on premixed flames by (i) exploring self-propagation of a dynamically passive, single-reaction wave in constant-density turbulence and (ii) considering a ratio of the rms turbulent velocity u' to the laminar flame speed S_L to be another important criterion for differentiating the governing physical mechanisms referred to. A combustion regime diagram is a 2D sector in a plane, e.g., $(u'/S_L, L/\delta_L)$, divided in several domains associated with different governing physical mechanisms. For instance, DaI and DaII are often assumed to hold in different regions. Boundaries of these two and other regions are commonly associated with curves of $Da = \text{const.}$ and $Ka = \text{const.}$. Here,

$$Da = \frac{\tau_T}{\tau_f} = \frac{L}{\delta_L} \frac{S_L}{u'} \quad (1)$$

is the Damköhler number and

$$\begin{aligned} Ka &= \frac{\tau_f}{\tau_K} = \left(\frac{\delta_L}{\eta_K} \right)^2 = \left(\frac{u_K}{S_L} \right)^2 = \frac{Re_t^{1/2}}{Da} \\ &= \left(\frac{u'}{S_L} \right)^2 Re_t^{-1/2} = \left(\frac{u'}{S_L} \right)^{3/2} \left(\frac{\delta_L}{L} \right)^{1/2} \end{aligned} \quad (2)$$

is the Karlovitz number; time scales $\tau_f = \delta_L/S_L$ and $\tau_T = L/u'$ characterize the laminar premixed flame and large turbulent eddies, respectively; the Kolmogorov microscales $u_K = \eta_K/\tau_K$, $\eta_K = (\nu^3/\bar{\epsilon})^{1/4}$, and $\tau_K = (\nu/\bar{\epsilon})^{1/2}$ characterize the smallest turbulent eddies; $\bar{\epsilon} = 2\nu S_{ij}S_{ij} \propto u'^3/L$ is the mean dissipation rate,^{17,18} with the summation convention applying to repeated indices; $S_{ij} = 0.5(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the rate-of-strain tensor; ν is the kinematic viscosity of the unburned mixture;

$$Re_t = \frac{u'L}{\nu} = \frac{u'L}{S_L\delta_L} \quad (3)$$

is the turbulent Reynolds number; and $\delta_L = \nu/S_L$ for simplicity, i.e., the Schmidt number $Sc = \nu/D = 1$, where D is the molecular diffusivity of the considered reactant. Note that any pair from a set of $\{u'/S_L, L/\delta_L, Da, Ka, Re_t\}$ can be used to draw coordinate axes or boundary curves in a combustion regime diagram.

Despite strong simplifications, the regime-diagram approach has clearly demonstrated the importance for understanding the fundamental mechanisms governing more complex phenomena, e.g., premixed turbulent burning. Accordingly, the diagrams are widely accepted as an appropriate framework for speculating what physical mechanisms govern the influence of turbulence on a reaction wave and under what conditions. For instance, such regime diagrams are discussed in almost every second archival publication on premixed turbulent combustion, even though important effects such as (i) preferential diffusion,^{2,5,19–21} (ii) variable density,^{22–24} and (iii) complex chemistry^{25–27} are well documented for flames but

disregarded in many combustion regime diagrams. While the following discussion addresses the influence of constant-density turbulence on a dynamically passive single-reaction wave, terms “flame,” “combustion,” and “burning” will be used to link the study with the aforementioned regime diagrams.

When discussing various regimes and governing physical mechanisms of premixed turbulent burning, changes of the regimes and/or mechanisms are commonly linked with an increase in u'/S_L and Ka in parallel with a decrease in Da . In other words, a pathway from conditions characterized by $Da \gg 1$ and $Ka \ll 1$ to conditions characterized by $Da \ll 1$ and $Ka \gg 1$ is addressed. Such a narrowing of considered range of parameters is associated with two facts. First, in a typical experimental study of turbulent premixed flames, a ratio of u'/S_L (or L/δ_L) is varied in a wide (small, respectively) range. Accordingly, Da is decreased and Ka is increased when u'/S_L is increased. Second, since $Da = Re_t^{1/2}/Ka$, both Da and Ka could be high only if Re_t is very high. For the above reasons, the regimes of (i) $Da \gg 1$, $Ka \ll 1$ and (ii) $Da \ll 1$, $Ka \gg 1$ have received plenty of attention, and current understanding of these two regimes is briefly summarized in Sec. II.

On the contrary, the regime of $Da \gg 1$, $Ka \gg 1$ has yet been beyond the scope of mainstream discussions in the combustion literature. The goal of the present communication and, in particular, Sec. III is to contribute to filling this knowledge gap by discussing basic features of turbulent reaction waves characterized by $Da \gg 1$ and $Ka \gg 1$. This subject may appear to be purely academic because turbulent Reynolds numbers required for both Da and Ka , which are much larger than unity, are unlikely to be reached in any laboratory burner or industrial combustion chamber [*a fortiori*, such Re_t have never been reached in a direct numerical simulation (DNS) of a turbulent flame]. Nevertheless, sufficiently high Re_t may be reached in thermonuclear Ia supernovae and, probably, locally in large-scale terrestrial fires.

II. TWO COMMON REGIMES

In this section, two widely discussed limiting regimes of premixed turbulent combustion are briefly addressed in order to lay foundations for exploring one more limiting regime in Sec. III. Following the common practice of construction of combustion regime diagrams,^{3,14–16} phenomena caused by preferential diffusion, thermal expansion, complex chemistry, and non-adiabatic effects are beyond the scope of the following discussion. The turbulent Reynolds number is considered to be much larger than unity. Note that the fourth limiting regime of $Da \ll 1$ and $Ka \ll 1$ is irrelevant to turbulent reacting flows because $Re_t = Da^2 Ka^2$ is much less than unity in such a case.

A. High Damköhler and low Karlovitz numbers

In this regime,

$$\frac{L}{\delta_L} = \sqrt{Re_t Da} \gg 1 \quad (4)$$

and

$$\frac{u'}{S_L} = \sqrt{\frac{Re_t}{Da}}. \quad (5)$$

While constraints of $Re_t \gg 1$ and $Da \gg 1$ are compatible with both $u'/S_L < 1$ and $u'/S_L > 1$, the turbulent Reynolds number is significantly larger than the Damköhler number in a typical turbulent premixed flame. Accordingly, we will consider S_L to be substantially less than u' , henceforth.

In this limiting regime, fundamentals of the influence of turbulence on the burning rate are well understood.^{3–5,14–16} The governing physical mechanism consists of wrinkling the thin ($\delta_L < \eta_K$, because $Ka < 1$) flame surface by turbulent eddies and turbulent burning velocity U_T is simply proportional to u' .^{13,28–31} This scaling is supported by recent Direct Numerical Simulation (DNS) data^{32,33} obtained by exploring the propagation of an infinitely thin front in turbulent medium and, therefore, associated with $Da \rightarrow \infty$ and $Ka \rightarrow 0$.

This scaling can also be obtained by considering premixed turbulent flames^{16,29,34} to be fractals,³⁵ with (i) the outer cut-off scale being proportional to L , (ii) the inner cut-off scale being equal to the Gibson scale,¹⁶

$$L_G = L \left(\frac{S_L}{u'} \right)^3 = L Ka^{-3/2} Re_t^{-3/4} = \eta_K Ka^{-3/2}, \quad (6)$$

and (iii) the fractal dimension D_f being equal to 7/3.²⁹ Under these assumptions,

$$U_T = S_L \frac{A_f}{A_0} = S_L \left(\frac{L}{L_G} \right)^{D_f-2} = u', \quad (7)$$

where A_f is the mean surface of the instantaneous wrinkled, thin, and inherently laminar flame and A_0 is the area of the mean surface of the turbulent flame. Such a simple model can be applied if $\eta_K < L_G < L$, i.e., if $Ka^{1/2} < 1$ and S_L is significantly smaller than u' , see Eq. (6).

The key point of this fractal model is a hypothesis that small-scale wrinkles on the flame surface are efficiently smoothed at scales smaller than L_G due to kinematic restoration,³¹ i.e., self-propagation of the local flame at speed S_L . Note that, according to the Kolmogorov scaling,¹⁷ the magnitude $\delta u_l \propto (\epsilon l)^{1/3}$ of the velocity difference in two points separated by distance l is on the order of S_L if $l = O(L_G)$.

B. Low Damköhler and high Karlovitz numbers

In earlier discussions of the combustion regime diagrams, this limiting case was commonly associated with the validity of DaII and was often interpreted in terms of appearance of instantaneous flames broadened by small-scale turbulent eddies. It is worth stressing, however, that such an interpretation does not follow from DaII.

Recent experimental and DNS data reviewed elsewhere^{36,37} indicate that reaction zones characterized by high rates of reactant consumption and heat release are thin even at high Ka and low Da reached in the reviewed studies. At the same time, mixing or preheat zones characterized by lower temperature and negligible major reaction rates can be substantially perturbed and broadened by turbulent eddies. Such a thin-reaction-zone regime of turbulent combustion was first highlighted by Peters.³¹ It has been attracting significant attention due to its relevance to burning of lean mixtures in modern gas turbine combustors and piston engines.

Recently, a phenomenological theory related directly to this combustion regime ($Da \ll 1$, $Ka \gg 1$, and $Re_t \gg 1$) was developed by considering the limiting case of an infinitely thin reaction zone (reaction sheet) attached to a thick mixing layer.^{36,38} Note that $Da \ll 1$ and $Ka \gg 1$ are finite in that study of infinitely thin reaction zones because the considered reaction wave contains not only the reaction sheet, but also a thick mixing layer where the scalar concentration is changed due to molecular diffusion (in the laminar flow) or/and turbulent transport (in a turbulent flow). The theory yields the following scaling for turbulent consumption velocity:

$$U_T \propto S_L Re_t^{1/2} \propto u' Da^{1/2} \quad (8)$$

in the constant-density and single-step-chemistry case. The scaling given by Eq. (8) was validated by a large amount of DNS data obtained from constant-density single-reaction waves propagating in intense turbulence under conditions of $Da < 1$, with the data also indicating thin reaction zones under the simulation conditions.³⁶ Moreover, at low Damköhler numbers, the same scaling was documented (i) in experiments on reaction-front propagation in aqueous solutions⁸ and flames³⁹ and (ii) in 2D DNSs of constant-density reaction waves,⁴⁰ 3D DNSs of thermonuclear deflagration,¹² and 3D DNSs of lean methane–air and hydrogen–air flames.⁴¹

Equation (8) was derived by hypothesizing that, at $Da \ll 1$, $Ka \gg 1$, and $Re_t \gg 1$, the reaction sheet affects the scalar field only within a very narrow layer adjacent to the sheet, whereas turbulent mixing controls variations in the scalar field outside the layer. Note that the hypothesis that, at a low Da , iso-scalar surfaces “may be approximated to behave locally at the small scales more or less as non-reactive scalars” was earlier put forward by Kerstein.⁴² Since the order-of-magnitude estimates performed by Sabelnikov *et al.*³⁶ indicated that the reaction-affected layer thickness was significantly less than η_K under the aforementioned conditions, the area A_f of the reaction sheet was hypothesized to be equal to the area of the iso-scalar surface of an inert scalar, associated with the outer boundary of the layer, with the latter area being controlled by the interaction of turbulent and molecular mixing. Finally, the following scaling:²

$$A_c \propto A_0 Re_t^{1/2} \quad (9)$$

for the area of an iso-scalar surface in the case of inert turbulent mixing was adopted to evaluate the area of the outer boundary of the reaction-affected layer and, hence, the area of the reaction sheet. Thus, within the framework of the summarized theory,^{36,38} both DaI and DaII hold simultaneously, i.e., an increase in U_T by u' is controlled by an increase in the reaction-sheet area due to wrinkles created by turbulent eddies, in line with DaI, but the magnitude of the area increase is controlled by turbulent mixing, in line with DaII.

It is worth stressing that the theory summarized above deals with the infinitely thin reaction sheet under adiabatic conditions. Accordingly, local combustion quenching cannot occur. In complex-chemistry non-adiabatic flames characterized by a finite thickness of a reaction zone, local combustion quenching and reignition could play an important role,⁴³ thus bounding the thin-reaction-zone regime with a very low $Da_{cr} \ll 1$ or/and a very high $Ka_{cr} \gg 1$. Accordingly, the regime discussed in this subsection should be called intermediately limiting regime. Here, the word

“intermediately” is skipped for brevity and because little is known about its boundary at very low Da and very high Ka . Recent experimental and DNS data reviewed elsewhere^{36,37} indicate that Ka_{cr} should be well above 100.

It is also worth noting that Eqs. (8) and (9) are consistent with a hypothesis^{44,45} that (i) the reaction sheet is fractal with $D_f = 8/3$ and, if $Sc \leq 1$, (ii) the inner cut-off scale is equal to the Obukhov–Corrsin scale $\eta_{OC} = (D^3/\bar{\epsilon})^{1/4} = Sc^{-3/4}\eta_K \propto Sc^{-3/4}LRe_t^{-3/4}$,^{46,47} which is much larger than the Gibson length scale at $Ka \gg 1$ if $Sc = O(1)$, see Eq. (6). Indeed, in such a case,

$$A_f \propto A_0 \left(\frac{L}{\eta_{OC}} \right)^{D_f-2} \propto A_0 Re_t^{1/2}, \quad (10)$$

in the considered case of $Sc = O(1)$. The value of $D_f = 8/3$ is supported by DNS data⁴⁵ obtained from highly turbulent premixed flames.

The use of η_{OC} in Eq. (10) is consistent with a hypothesis³¹ that smoothing of small-scale wrinkles on a flame surface in intense turbulence is controlled by molecular diffusion. This hypothesis is also supported by recent DNS data.^{48,49}

III. HIGH DAMKÖHLER AND KARLOVITZ NUMBERS

The above brief summary of fundamentals of two limiting regimes of premixed turbulent combustion implies that if premixed turbulent flames are fractals, then, the fractal dimension, smoothing mechanism, and inner cut-off scale are different at (i) $Da \gg 1$, $Ka \ll 1$; and (ii) $Da \ll 1$, $Ka \gg 1$. In particular, (i) $D_f = 7/3$ and (ii) $D_f = 8/3$, respectively. Such a scenario of an increase in D_f with decreasing Da and increasing Ka is supported by certain references and physical arguments discussed in detail elsewhere.^{44,45} Moreover, the smoothing mechanism changes from (i) kinematic restoration to (ii) molecular diffusion.³¹

Let us apply these ideas to the third limiting case of $Da \gg 1$ and $Ka \gg 1$, but $Ka < Ka_{cr}$. Following Refs. 36 and 38, the reaction zone is considered to be thin under such conditions. In this regime, wrinkles of the reaction zone can be smoothed by both the kinematic restoration and molecular mixing, with the change of the governing smoothing mechanism occurring at a crossover length scale l^* . The kinematic restoration dominates in the range of large scales $l^* < l < L$, whereas the molecular mixing prevails in the range of small scales $\eta_K < l < l^*$.

To find the crossover length scale l^* , let us (i) assume that it belongs to the inertial subrange, i.e., $\eta_K \ll l^* \ll L$, and (ii) introduce a Damköhler number Da_l ,

$$Da_l = \frac{\tau_l}{\tau_f} = \frac{l}{\tau_f \delta u_l}, \quad (11)$$

related to the considered length scale l . Here, $\delta u_l = (\bar{\epsilon}l)^{1/3}$ is the characteristic magnitude of velocity difference between two points separated by the distance l , $\tau_l = l/\delta u_l$, and Da_l increases from $Ka^{-1} \ll 1$ at $l \rightarrow \eta_K$ to $Da \gg 1$ at $l \rightarrow L$. Accordingly, the crossover length scale l^* may be associated with a constraint of $Da_{l^*} = 1$, i.e., $l^* = \sqrt{\bar{\epsilon}\tau_f^3}$ by equating the last ratio in Eq. (11) to unity. Thus,

$$l^* = \sqrt{\bar{\epsilon}\tau_f^3} = LDa^{-3/2} = \eta_K Ka^{3/2} = \delta_L Ka \quad (12)$$

and

$$\delta u_{l^*} = \sqrt{\bar{\epsilon}\tau_f} = S_L Ka = u' Da^{-1/2}. \quad (13)$$

In line with the assumption (i), $\eta_K \ll l^* \ll L$ because $Da \gg 1$ despite $Da_{l^*} = 1$. Moreover, $S_L \ll \delta u_{l^*} \ll u'$ in the considered case of $Ka \gg 1$ and $Da \gg 1$.

It is worth noting that the length and velocity scales defined by Eqs. (12) and (13) appeared in an unpublished manuscript by Prudnikov as early as in 1958.⁵⁰ He considered an impact of small-scale turbulence on flame propagation using a transport equation for the mean combustion progress variable $\bar{c}$ and neglecting the influence of the small-scale fluctuations in c on the mean reaction rate. Within the framework of such a simple model, he arrived at a conclusion that the small-scale turbulence broadened the flame locally, with the thickness and speed of the locally broadened flame being on the order of l^* and δu_{l^*} . Later, the same scales were obtained by Zimont,⁵¹ who also addressed locally thickened flames broadened by the small-scale turbulence. These scales were later used in other papers,^{31,42,52,53} which also addressed the locally thickened flames.

Let us separately explore the influence of small-scale ($l \ll l^*$) and large-scale ($l \gg l^*$) branches of the turbulence spectrum on a premixed flame.

A. Small-scale branch of turbulence spectrum

The time scale of small eddies characterized by $l \ll l^*$ is much less than the laminar flame time scale similar to the regime of $Ka \gg 1$ and $Da \ll 1$, discussed earlier. Moreover, $L_G \ll \eta_K \ll \delta_L$ if $Ka \gg 1$. Therefore, it is reasonable to assume that the governing physical mechanism of the influence of such small-scale eddies on a premixed flame is similar to the governing physical mechanism of the influence of turbulence on premixed combustion at $Ka \gg 1$ and $Da \ll 1$. Accordingly, in the range of scales of $l \ll l^*$, similarly to the physical scenario discussed in Sec. II B, (i) preheat (or mixing) zones are considered to be broadened by small-scale turbulent eddies, (ii) reaction zones are considered to be thin, and (iii) premixed turbulent flame is considered to be a fractal with dimension $D_f = 8/3$ and inner cut-off scale equal to $\eta_{OC} = O(\eta_K)$ if $Sc \leq 1$ and $Sc = O(1)$. Consequently, the contribution of eddies smaller than l^* to turbulent burning velocity can be estimated as follows:

$$u^* = S_L \left(\frac{l^*}{\eta_K} \right)^{D_f-2} = S_L Ka = u' Da^{-1/2} = (\bar{\epsilon}\tau_f)^{1/2}. \quad (14)$$

It is of interest that u^* does not depend on the turbulent Reynolds number, and this independence of u^* on Re_t is similar to independence of the inertial range of turbulence spectrum on Re_t .¹⁷

It is also worth noting that while Eq. (14) has been derived by assuming the fractal behavior of the flame surface at $l \ll l^*$, the same result could be obtained without clarifying physical mechanisms of the influence of the small-scale turbulence on a premixed flame. Starting from the Kolmogorov pioneering ideas, one may hypothesize that at $Da \gg 1$, $Ka \gg 1$, and $Re_t \gg 1$, the influence of turbulent eddies of a scale l on premixed combustion can be characterized (in statistical sense) using the length scale l , the mean dissipation rate $\bar{\epsilon}$, and the laminar flame time scale τ_f in a subrange of the inertial range ($\eta_K \ll l \ll L$) of the turbulence.⁵³ Subsequently, scaling for

the boundary l^* of this subrange and its contribution u^* to the turbulent burning velocity, i.e., Eqs. (12) and (14), respectively, result straightforwardly from dimensional reasoning. Accordingly, these estimates may hold even if other physical mechanisms of the influence of the small-scale turbulent eddies on premixed combustion, e.g., local extinction and reignition,⁴³ play an important role.

B. Large-scale branch of turbulence spectrum

The large-scale ($l \gg l^*$) eddies are characterized by a time scale larger than τ_f . Accordingly, their influence on a premixed flame may be associated with wrinkling of a reacting layer of thickness l^* that is broadened by the small-scale ($l \ll l^*$) turbulent eddies, propagates at speed u^* , and is characterized by the time scale τ_f . Moreover, substitution of S_L with u^* in Eq. (6) yields the counterpart L_G^* of the Gibson scale equal to the crossover scale l^* . Therefore, kinematic restoration efficiently smooths wrinkles of the reaction-layer surface whose length scale is smaller than l^* and the inner cut-off scale for large-scale wrinkles of the reacting layer may be associated with $L_G^* = l^*$. Such features are similar to the basic features of the influence of turbulence on combustion at $Da \gg 1$ and $Ka \ll 1$; see Sec. II A. Consequently, it is tempting to assume that, at $l > l^*$, the reacting-layer surface is a fractal with $D_f = 7/3$ and inner cut-off scale equal to l^* . Therefore, the turbulent burning velocity scales as follows:

$$U_T = u^* \left(\frac{L}{l^*} \right)^{D_f-2} = u'. \quad (15)$$

Thus, this simple scaling holds not only at $Da \gg 1$ and $Ka \ll 1$ but also at $Da \gg 1$ and $Ka \gg 1$, i.e., if the laminar flame time scale is much smaller than the time scale $\tau_T = L/u'$ of the largest turbulent eddies, the burning velocity is proportional to the rms turbulent velocity independently of the Kolmogorov time scale.

It is worth noting that the influence of the large-scale turbulent eddies on a premixed flame thickened by the small-scale turbulent eddies was earlier addressed by Zimont,⁵¹ Ronney and Yakhot,⁵² and Kerstein⁴² using different arguments. In particular, Kerstein⁴² has claimed that U_T should be proportional to u' because “from the perspective of large-scale processes, the thickened flame is functionally equivalent under these conditions to a laminar flame, with” l^* and u^* corresponding to δ_L and S_L , respectively, see p. 3 in the cited paper.

C. Flame as a bifractal

Unlike the work of Kerstein⁴² and other earlier studies, in the present paper, Eq. (15) is obtained by hypothesizing (for the first time to the best of the authors' knowledge) the bifractal nature of a premixed turbulent flame at $Da \gg 1$ and $Ka \gg 1$. In other words, the small-scale and large-scale wrinkles of a flame surface are characterized by two different fractal dimensions $D_{f,1} = 8/3$ and $D_{f,2} = 7/3$, respectively. It is also worth stressing that, in both (small-scale and large-scale) subranges of the turbulence spectrum, the major effect of turbulent eddies on a premixed flame is the wrinkling of the flame surface. However, the increase in the flame surface area by the small turbulent eddies is similar to an increase in an iso-scalar surface area by turbulent eddies in the case of inert mixing. On the contrary, an increase in the flame surface area by the large-scale eddies

depends not only on turbulence characteristics, but also on the flame speed. This difference is associated with different physical mechanisms of smoothing the flame surface at inner cut-off scales for the two fractals. At the smallest scales, the smoothing is controlled by molecular diffusion. At the boundary l^* between the two fractals, the smoothing is controlled by the kinematic restoration due to the propagation of a thickened flame at a speed equal to u^* . Thus, the total flame surface area and the turbulent consumption velocity scale are

$$\begin{aligned} A_f &= A_0 \left(\frac{l^*}{\eta_K} \right)^{D_{f,1}-2} \left(\frac{L}{l^*} \right)^{D_{f,2}-2} \\ &= A_0 \left(\frac{l^*}{\eta_K} \right)^{2/3} \left(\frac{L}{l^*} \right)^{1/3} \\ &= A_0 Ka^{1/2} Re_l^{-1/4} = A_0 Ka Da^{1/2}, \end{aligned} \quad (16)$$

$$U_T = S_L \frac{A_f}{A_0} = S_L Ka Da^{1/2} = u', \quad (17)$$

respectively.

Bifractals were already addressed in other research fields^{54–56} including fluid mechanics.^{57–60} For instance, Sreenivasan *et al.*⁶¹ discussed turbulent mixing in the case of a large Sc and argued that the fractal dimensions should be different for scales larger and smaller than the Kolmogorov length scale, see Fig. 2(a) in the cited paper. However, turbulent reacting waves (e.g., premixed flames) characterized by $Da \gg 1$ and $Ka \gg 1$ have never been considered to be bifractals to the best of the present authors' knowledge.

Finally, it is worth noting that, similarly to a typical asymptotic theory, transition between two fractals is considered to occur sharply at $l = l^*$. In reality, the transition may occur in a range of scales, with the fractal dimension decreasing from $D_{f,1} = 8/3$ at the small-scale boundary of the transition range to $D_{f,2} = 7/3$ at its large-scale boundary. If this transition range belongs to the Kolmogorov inertial range, then the local (in the wavenumber space) $D_{f,l}$ should depend on l , the mean dissipation rate $\bar{\epsilon}$, and laminar flame characteristics. For instance, if the time scale τ_f is assumed to be the primary characteristic of the laminar flame, in line with Eqs. (12)–(14), then $D_{f,l}$ (i) should depend on $\bar{\epsilon} \tau_f^3 / l^2$ for dimensional reasoning and (ii) should decrease with l , in line with the hypothesized transition from $D_{f,1} = 8/3$ to $D_{f,2} = 7/3$.

IV. CONCLUSIONS

Turbulent reaction waves characterized by asymptotically high Damköhler, Karlovitz, and Reynolds numbers, but a unity-order Schmidt number, are hypothesized to be bifractals, with the fractal dimension of the reaction-wave surface changing from 8/3 at small (when compared to $l^* = L Da^{-3/2} = \eta_K Ka^{3/2}$) scales to 7/3 at large scales. Under the aforementioned conditions, the surface is considered to be smoothed by the molecular mixing and the kinematic restoration at the small and large scales, respectively.

In the studied limiting case, a simple scaling of $U_T \propto u'$ is obtained for turbulent consumption velocity.

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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