Mathematical model of retractions: Facts, analysis and recommendations

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Abstract

The high rate of new retraction from different publishers nowadays is alarming. By reading reasons or retractions notes, one will conclude that there are fair and unfair retractions. To protect the integrity of research, the practice of fair retraction should be performed and authors should be responsible for their wrong doings. On the other hand, attention should be devoted to unfair retractions, especially retraction notes indicating that the authors did not agree with the retraction. The aim of this paper is to provide a discussion by presenting first the statistical analysis of retraction data from ten different publishers ranging between the year 2000 and 2020. Secondly, to provide a forecast up to 2050 and see which of the publishers will have more or less retractions. The aim of such prediction is a wakeup call to authors, reviewers, editors and publishers to be more mindful of what they are doing. Most importantly, publishers must put all mechanisms in place to avoid unfair retractions. A list of possible causes of high rates of unfair and fair retractions have been presented to help different actors to take actions. A mathematical model with a system of seven ordinary differential equations depicting a possible scenario of retraction dynamic is constructed in this work. Different analyses were performed for deterministic and stochastic versions. Finally a discussion and recommendations were made to restore the dignity of those authors that have been victims of unfair retractions.

Keywords and phrases: Retractions, statistical analysis, unfair retractions, mathematical model of retractions, simulations and recommendations.

1 Introduction

Generally speaking, research or scientific papers are some pieces of academic writing in which the authors provide analysis, interpretations and even arguments based on comprehensive self-determining research. Scientists, to communicate their latest findings, write a scientific paper with the aim to communicate it to the rest of the world through a journal. However for this paper to be published in a journal, they are procedures to be followed. The paper is prepared by one or more than one authors, after identifying a suitable journal, the paper is submitted. The journal assigns the paper to an editor who is an expert in the field to check whether or not the paper is suitable for publication. If the editor finds the paper interesting, he then assigns it to two or more independent reviewers who are also experts in the field. The reviewers provide their report, and give their scientific opinion whether the paper will be accepted with (minor or major revision) or rejected. Finally, the editor will communicate the report to the corresponding author, if the editor suggest rejection, then the process stops, however, if revisions are required, the author will then perform the revisions and resubmit for further evaluations and finally if reviewers and editor are satisfied then the paper is accepted and send under production for publication. There are also few steps that have to be followed before the paper can appear online. However, it had been noticed that published papers imperfections, to solve this problem, a new concept was suggested, the retraction. For those who are not aware of this process, a retraction is in simple terms an action to remove from a journal a published paper. One of the earlier retractions can be traced back as early as 1756, the concerned paper in philosophical transactions of the royal society [4-13]. Since then, the number of retracted papers has increased each year, especially in the last 5 years the number of retracted papers has increased exponentially. Some authors have represented some analysis and even given suggestions to enhance the process of retraction, nevertheless, the number keeps on increasing. The question that has been asked by several academic structures is to know why more retraction? However, before giving some reasons that have been raised, we will like to note that, there are two types of retraction, fairly retractions where the paper contains serious errors that cannot be corrected, plagiarism, and many other unethical reasons. However, there are also unfair retractions where the paper is retracted due to unjustifiable reasons. In this paper, a statistical analysis of collected data from a retraction database will be performed [3]. A mathematical model depicting a dynamic about retraction will be proposed and studied using some mathematical analysis. A prediction will be done with the aim to warn researchers, publishers and editors to take serious steps to help low down the curve of retraction.

2 Statistical analysis of the retracted papers by some publishers

An effort made by a database has led to a collection of some retracted papers from different publishers and their respective journals. We have considered retracted research papers (Case report papers, conference abstract and other retractions are not considered here) from 2000 to 2020, the collected data were taken for 11 publishers including Elsevier, Springer, Wiley and son, Taylor & Francis, Hindawi, Springer-Nature, MPDI, AAAS, BMJ, De Gruyter. An approximate total number of retracted papers from each of the mentioned publishers is listed in table below. The table below shows that Elsevier, Springer and Wiley are the leading publishers in terms of yearly retraction, while BMJ is the publisher with less retractions. In this section, we perform some statistical analysis of collected for each publisher, our analysis will then consist of correlation of retraction between publishers, a chart representing yearly percentages of retractions for each publisher, an accumulative graph of retraction for each publisher from 2000 to 2020, prediction up to 2050 for each publisher and finally fitting using moving average. The predictions show that, only the year 2050, Elsevier would retract a maximum number of 800 papers, an average number of 600 or a minimum number of 400. With the same prediction, Springer would retract a maximum number of 650 papers, an average number of 550 papers or a minimum number of 450 papers. Wiley would retract a maximum number of 320, average of 230 or a minimum number of 90. But Springer Nature would in 2050 accumulate a maximum number of 70 papers, an average number of 55 or a minimum number of 40 papers. Nature would retract a maximum number of 35 papers, or an average of 25 or a minimum of 12. Hindawi which is also one of the mega publishers would retract 130 as maximum, 100 as average or 50 minimum. For MDPI, a maximum number of 80 papers would be retracted, or an average of 50 or a minimum of 9. For AAAS publishers, a maximum number of 6 papers would be retracted or even zero retraction, a similar prediction for the publisher BMJ 10 maximum or 1 as minimum. Finally, Taylor and Francis, would have a maximum number of 230 retractions, or a minimum number of 45.

Table 1 represents total numbers of the retractions made by 11 publishers from 2000 to 2020.

Publishers	**Total Retracted Papers**
Elsevier	2620
Springer	1549
Wiley	1018
Taylor&Francis	694
Nature	143
Hindawi	257
SpringerNature	218
MDPI	101
AAAS	84
De Gruyter	84
BMJ	49

Table 1. Total number of retracted papers by some publishers.

Table 2 presents a mutual relationship of the retraction data between 11 publishers from 2000 to 2020.

CORRELATION	Elsevier	Springer	Wiley	Springer Nature	Nature	Hindawi	MDPI	AAAS	Taylor&Francis	BMJ	De Gruyter
Elsevier	1										
Springer	0.916189	1									
Wiley	0.92465	0.898737	1								
Springer Nature	0.804486	0.791744	0.831442	1							
Nature	0.102922	0.135932	0.12175	-0.20397	1						
Hindawi	0.862343	0.863608	0.778476	0.838809	0.112377	1					
MDPI	0.784554	0799556	0.79006	0.938212	-0.26488	0.783097	1				
AAAS	-0.04911	-0.04999	-0.16482	0.070045	-0.06953	0.106321	0.042915	1			
Taylor&Francis	0.849466	0.868137	0.877066	0.830623	0.032196	0.819098	0.879921	-0.07893	1		
BMJ	0.337975	0.21122	0.262944	-0.18382	0.551592	0.114166	-0.18033	-0.3155	0.075131	1	
De Gruyter	0.563502	0.580592	0.499189	0.342301	0.401043	0.602726	0.294675	0.027687	0.388934	0.243715	1

Table 2. Correlation about publishers.

Accumulative numbers of retractions made by different publishers are represented in Figure 1. The graphs show that Elsevier, Springer and Wiley are the three leading publishers in terms of yearly retraction. Yearly numbers of retractions made by Elsevier, Springer, Wiley, Taylor & Francis, Hindawi, SpringerNature, Nature, MDPI, BMJ and AAAS are presented from Figure 2 to 12. MDPI and Hindawi although being mega publishers have less yearly retractions compared to the top three. At this point one would think that this is due to them being open access publishers, a statement that cannot be justified at this state as a proper investigation needs to be done to see if open access journals retract less papers.

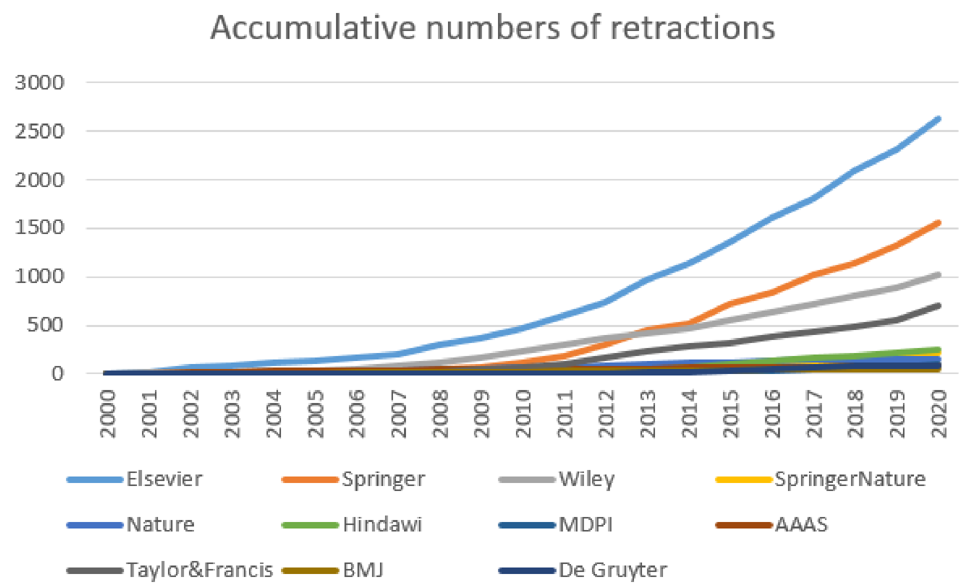

Figure 1. Accumulative data for retracted papers for the considered publishers.

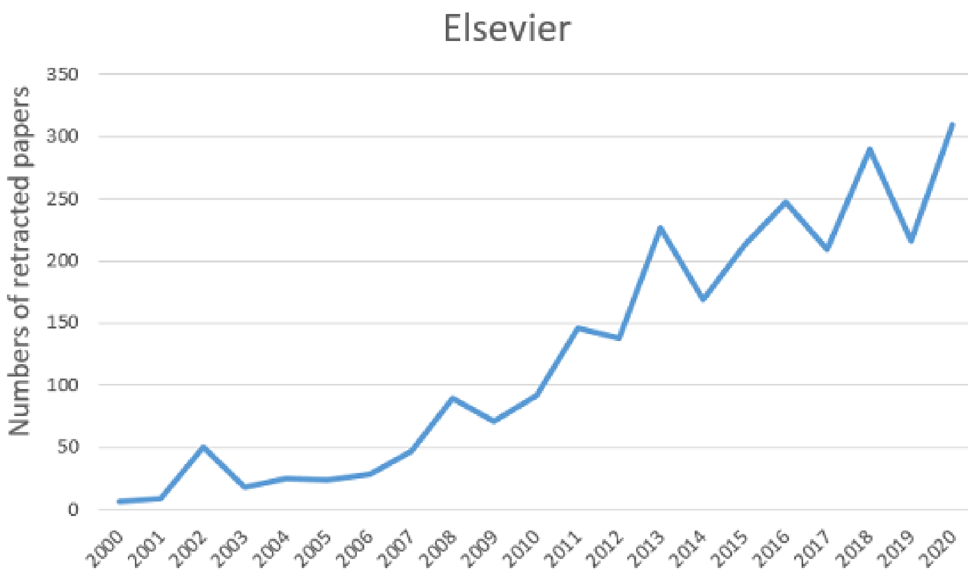

Figure 2. Numbers of retracted papers for Elsevier.

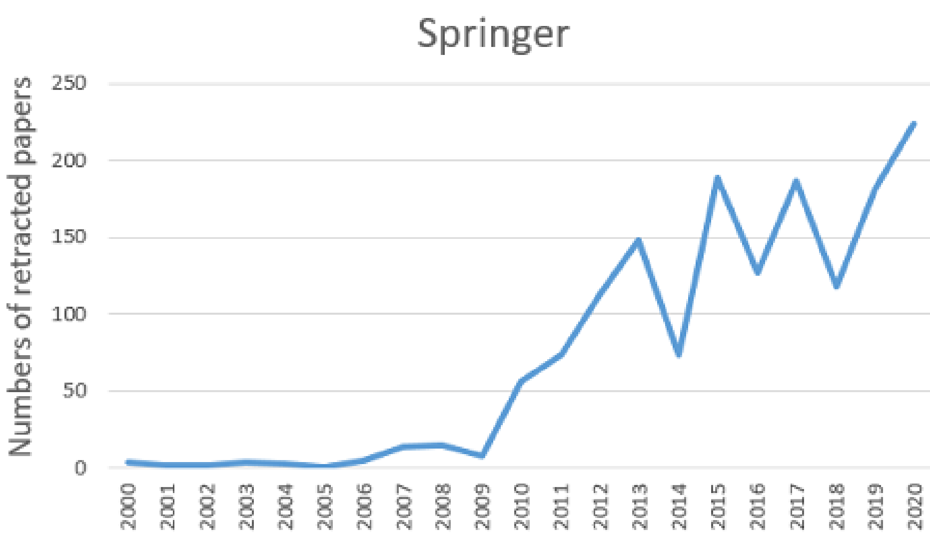

Figure 3.Numbers of retracted papers for Springer.

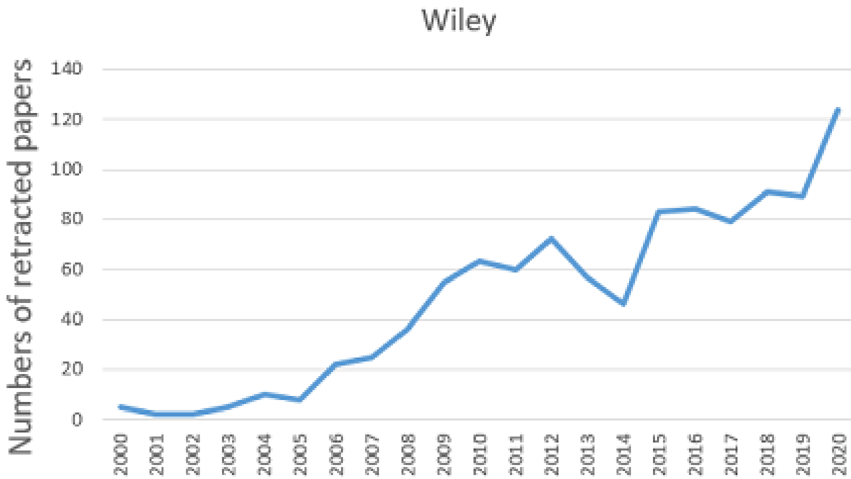

Figure 4.Numbers of retracted papers for Wiley.

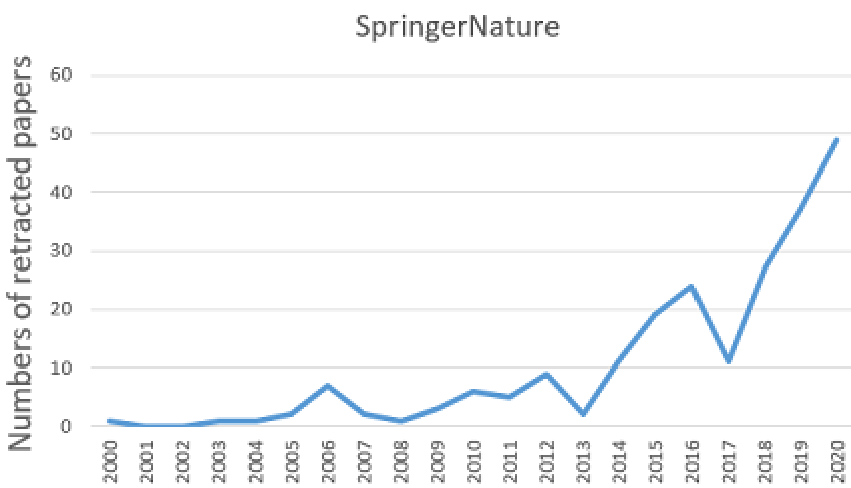

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Figure 5. Numbers of retracted papers for SpringerNature.

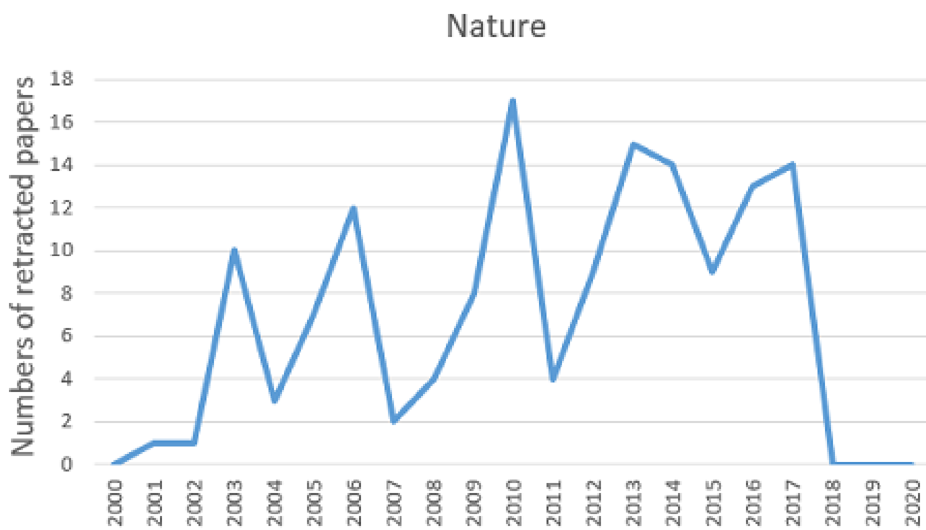

Figure 6. Numbers of retracted papers for Nature.

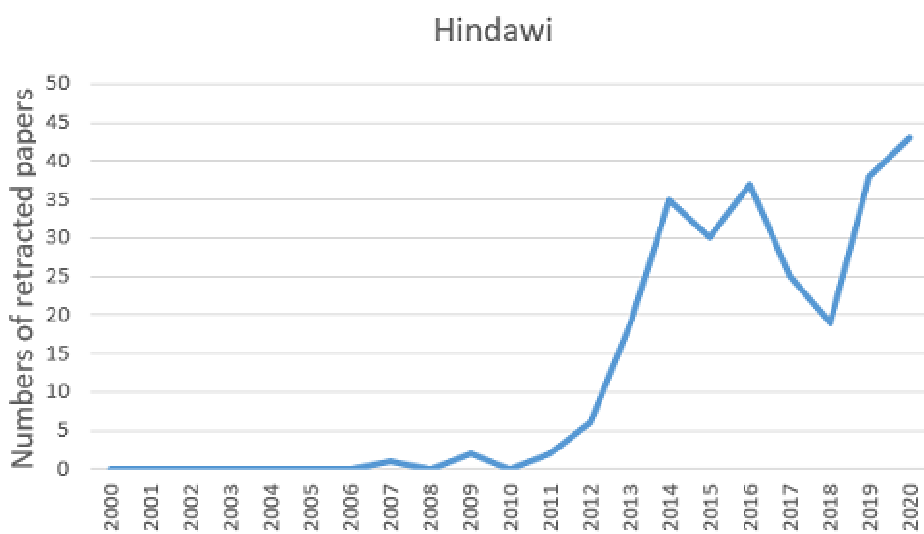

Figure 7. Numbers of retracted papers for Hindawi.

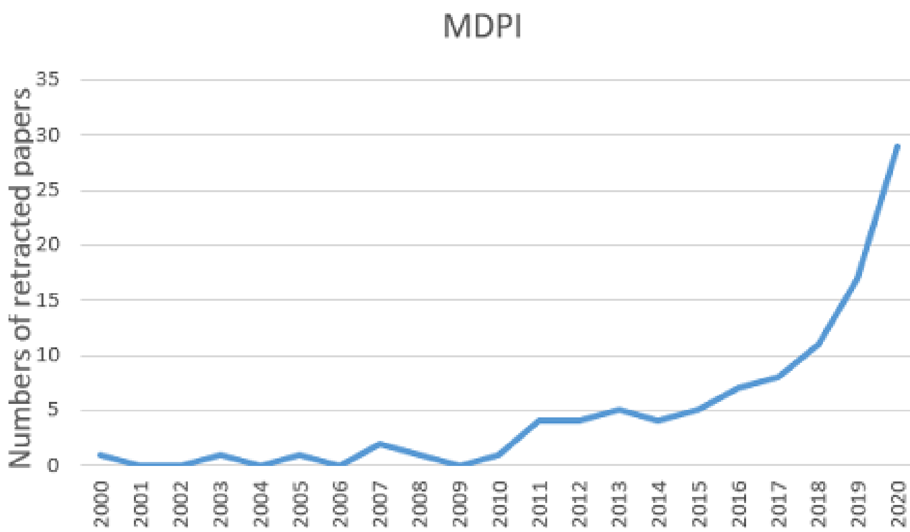

Figure 8. Numbers of retracted papers for MDPI.

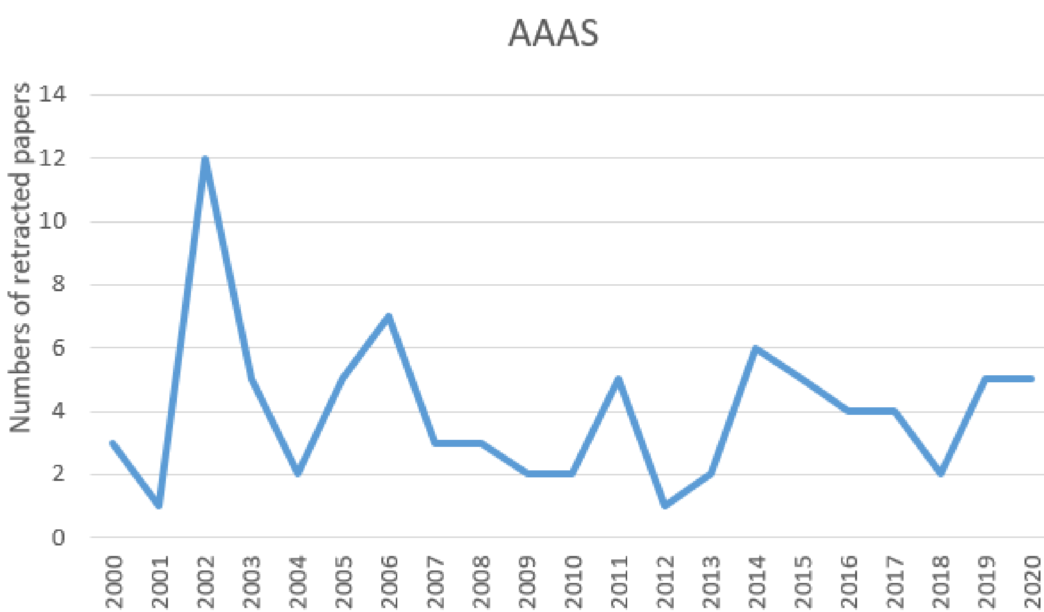

Figure 9. Numbers of retracted papers for AAAS.

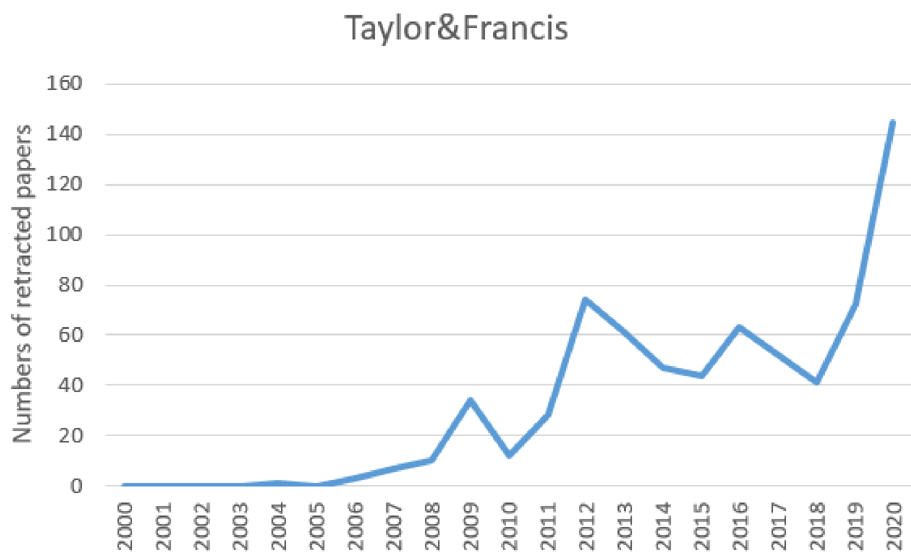

Figure 10. Numbers of retracted papers for Taylor&Francis.

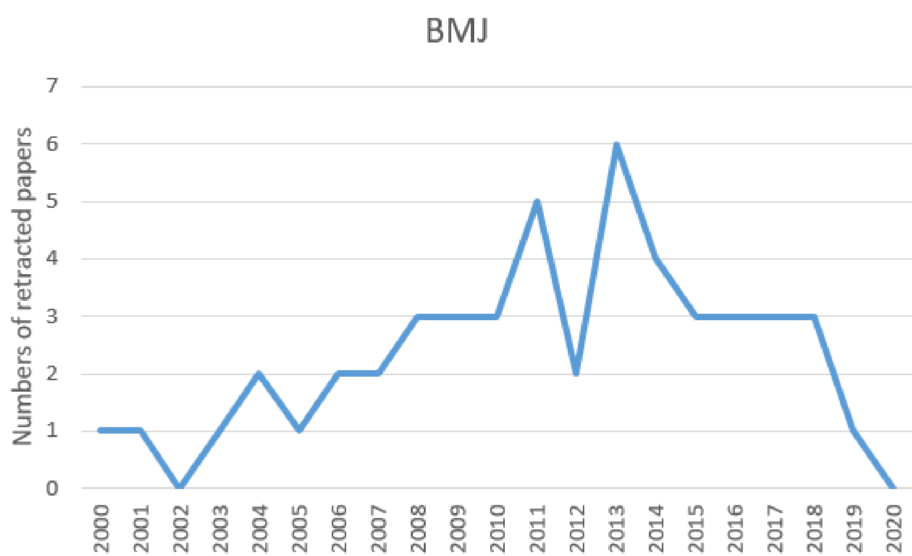

Figure 11. Numbers of retracted papers for BMJ.

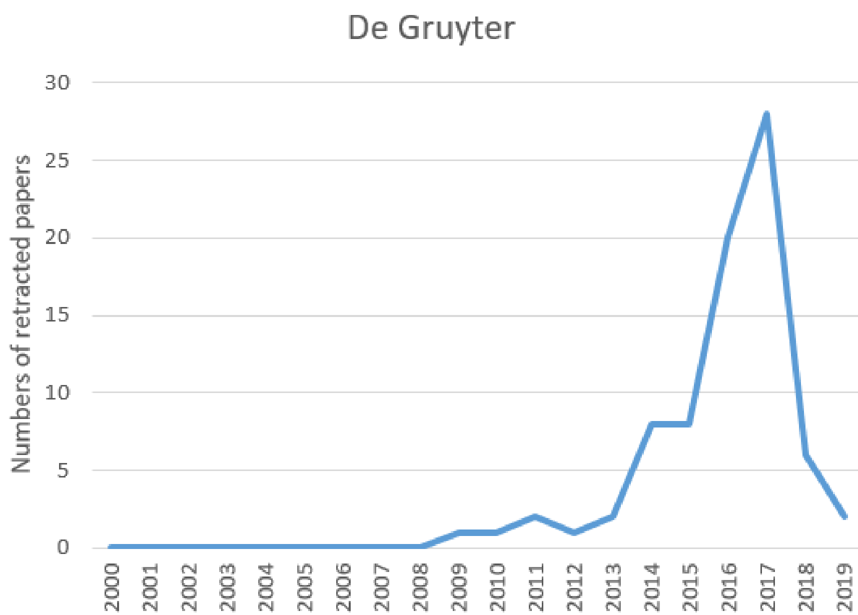

Figure 12. Numbers of retracted papers for De Gruyter.

We present in Figure 13 to 23 the charts depicting yearly percentages of retraction of the ten considered publishers.

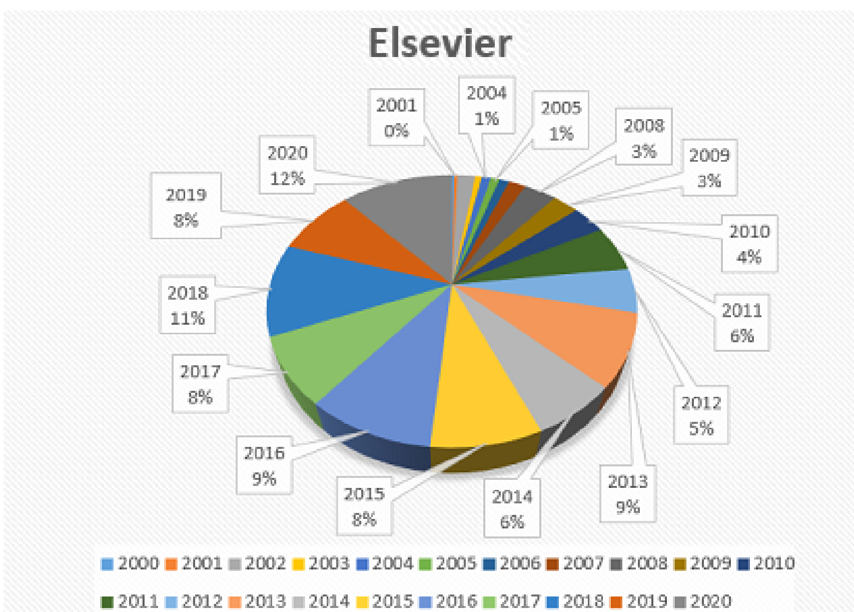

Figure 13. Numbers of retracted papers for Elsevier.

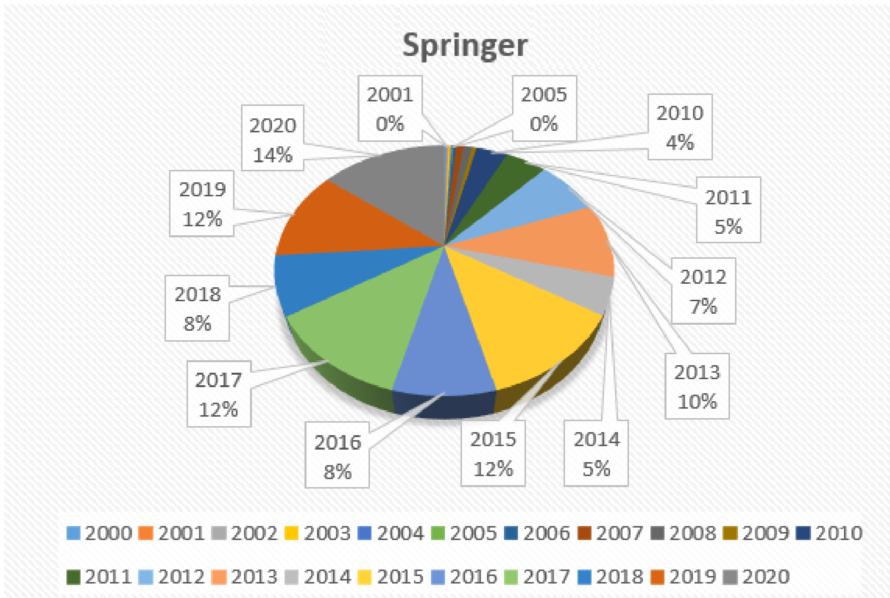

Figure 14. Numbers of retracted papers for Springer.

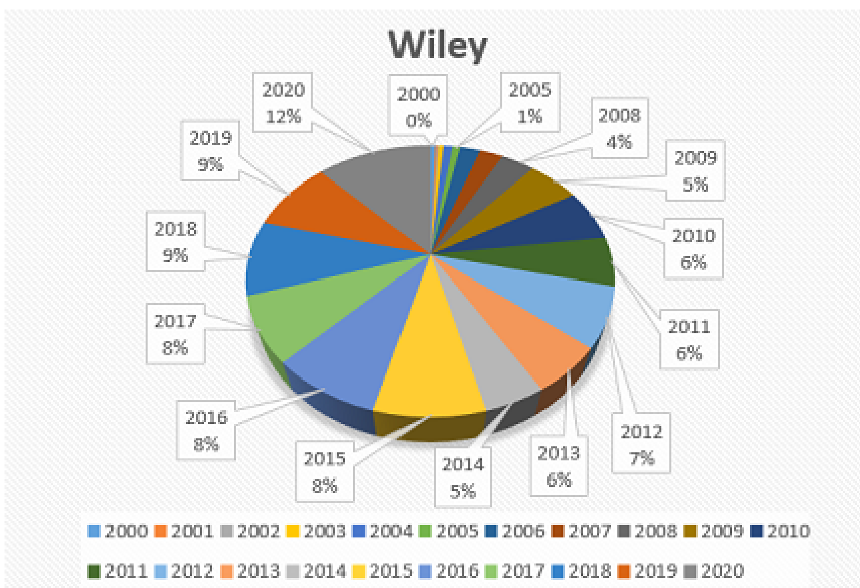

Figure 15. Numbers of retracted papers for Wiley.

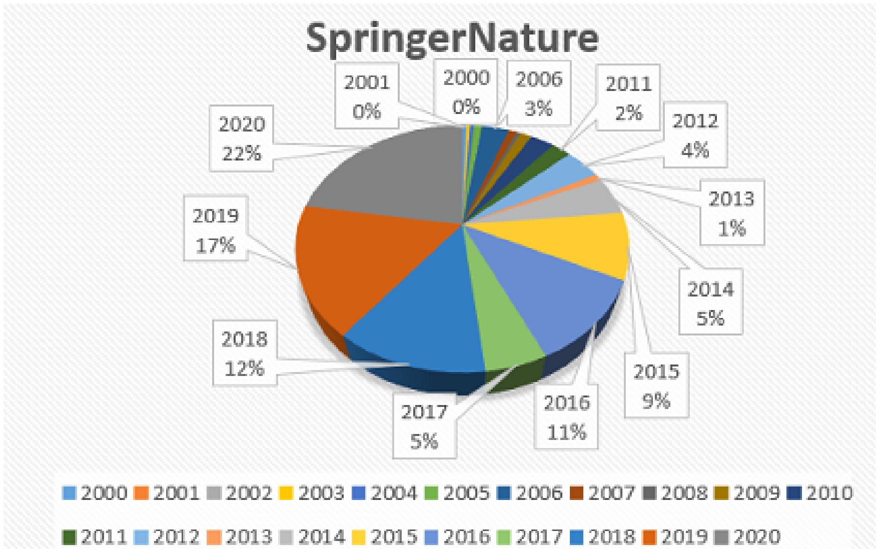

Figure 16. Numbers of retracted papers for SpringerNature.

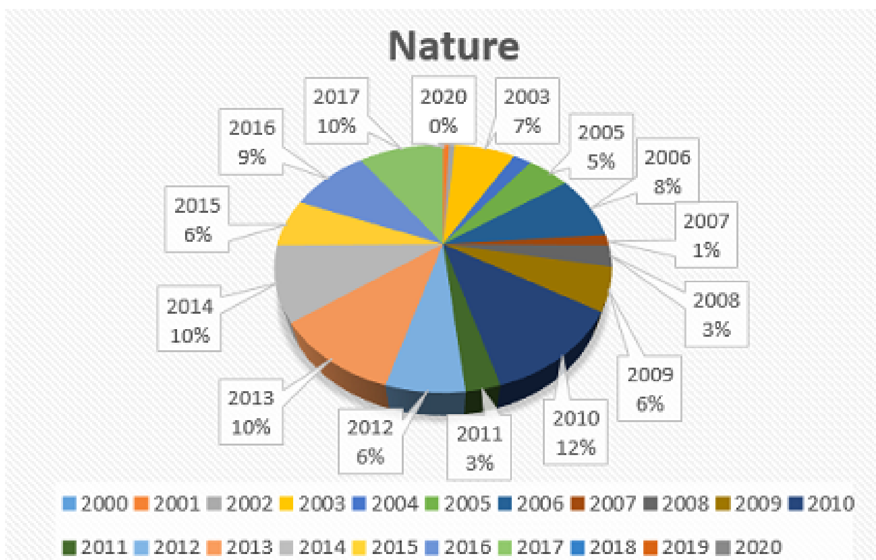

Figure 17. Numbers of retracted papers for Nature.

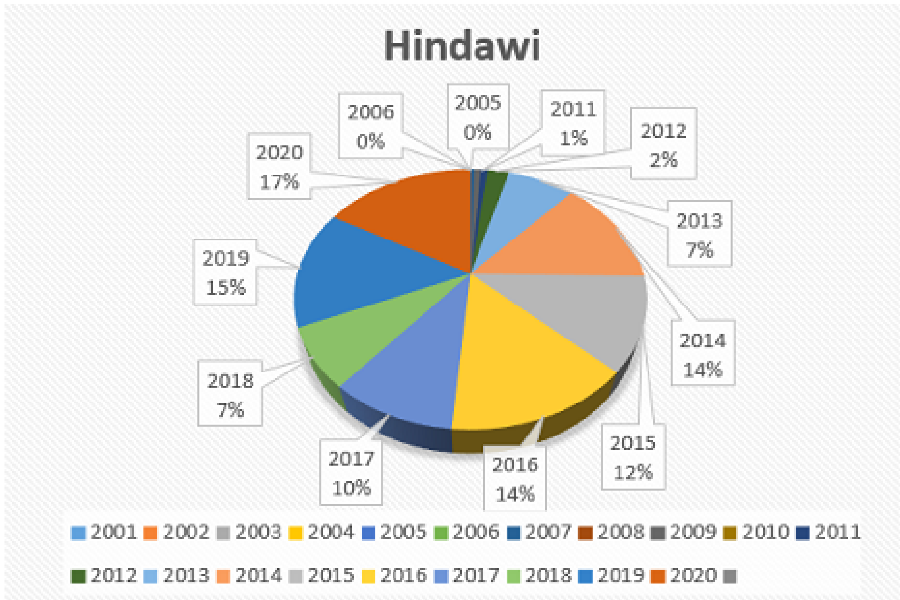

Figure 18. Numbers of retracted papers for Hindawi.

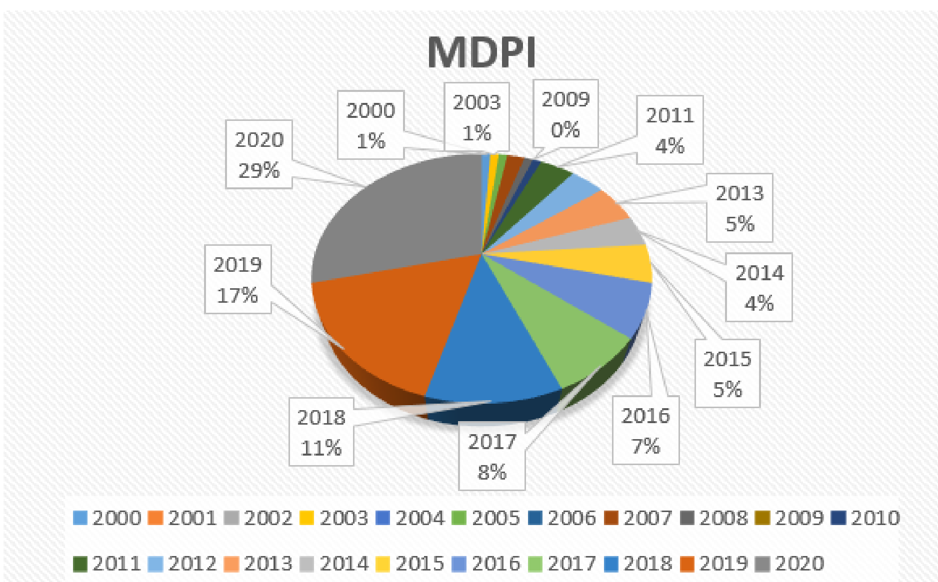

Figure 19. Numbers of retracted papers for MDPI.

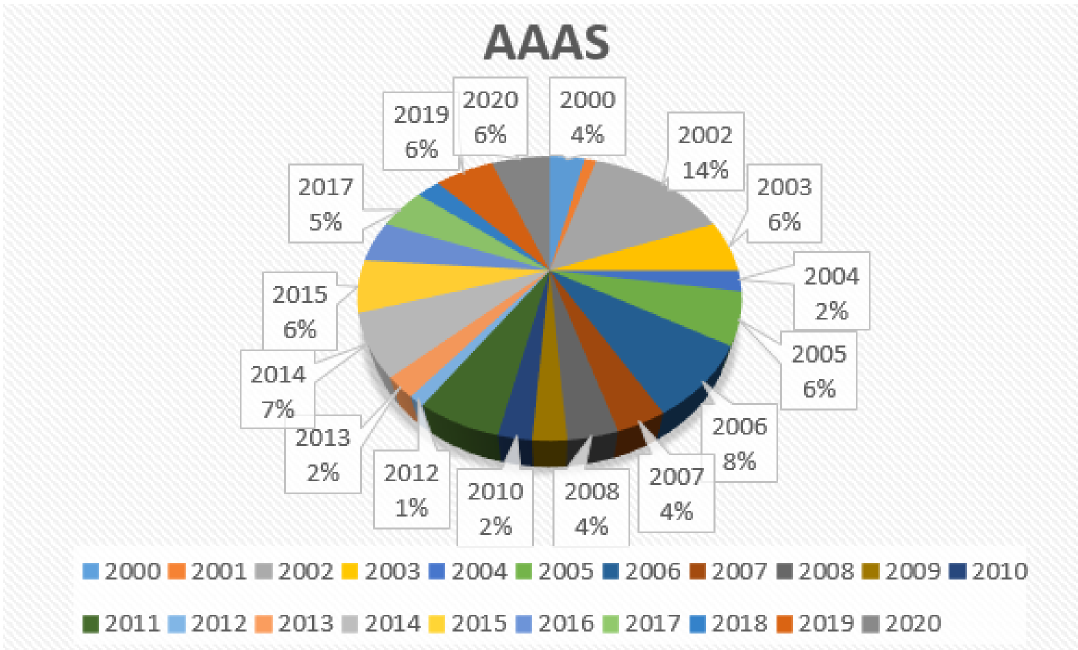

Figure 20. Numbers of retracted papers for AAAS.

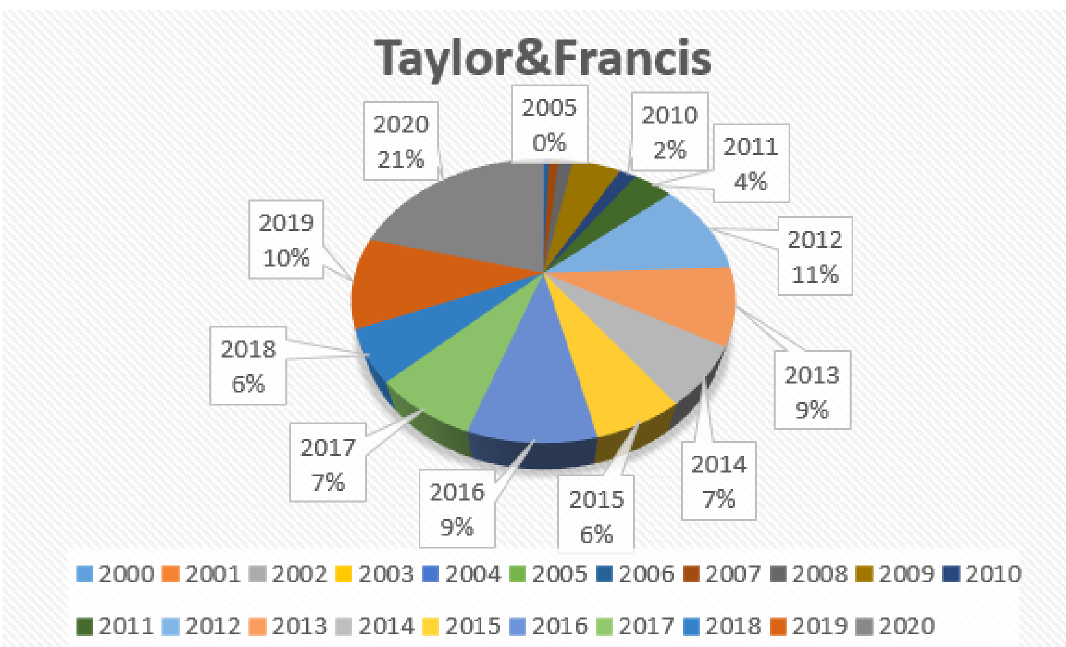

Figure 21. Numbers of retracted papers for Taylor&Francis.

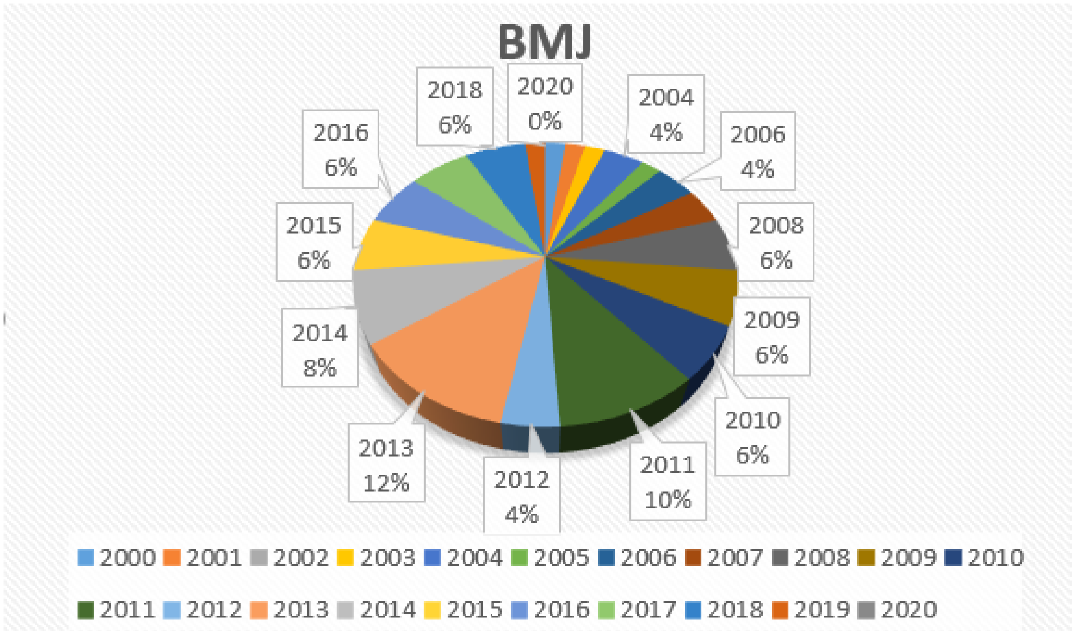

Figure 22. Numbers of retracted papers for BMJ.

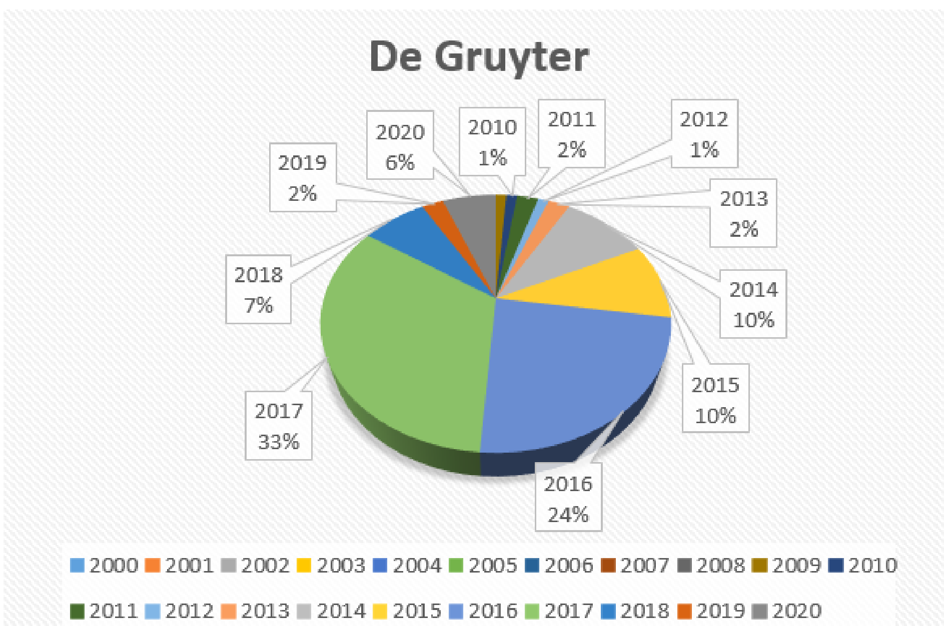

Figure 23. Numbers of retracted papers for De Gruyter.

Figures 24 to 34 represent predictions obtained from 95 percent of the forecast sheet for the ten chosen publishers from 2020 to 2050.

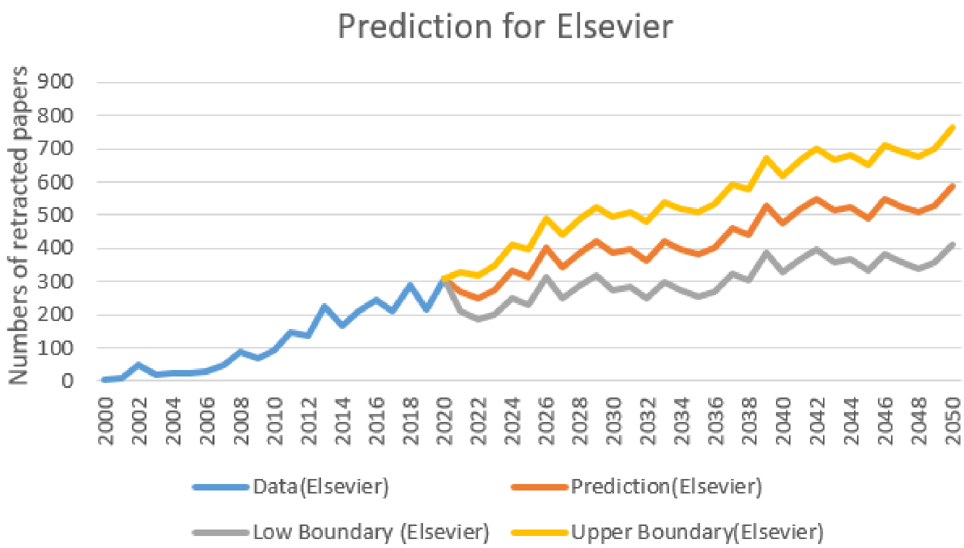

Figure 24. Prediction for numbers of retracted papers in Elsevier.

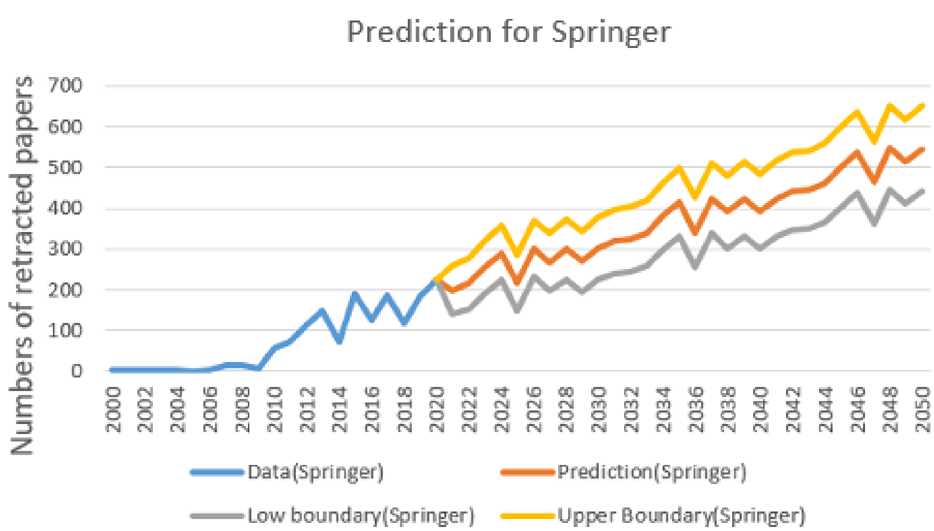

Figure 25. Prediction for numbers of retracted papers in Springer.

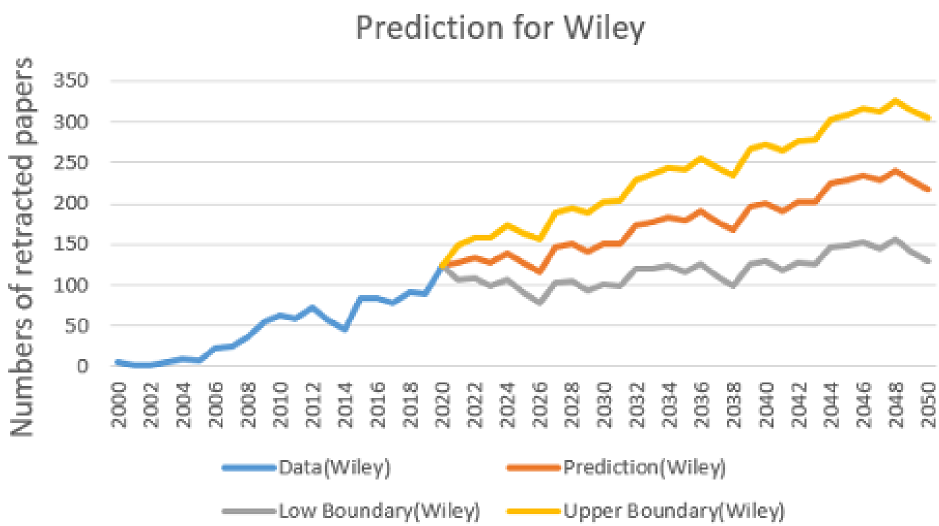

Figure 26. Prediction for numbers of retracted papers in Wiley.

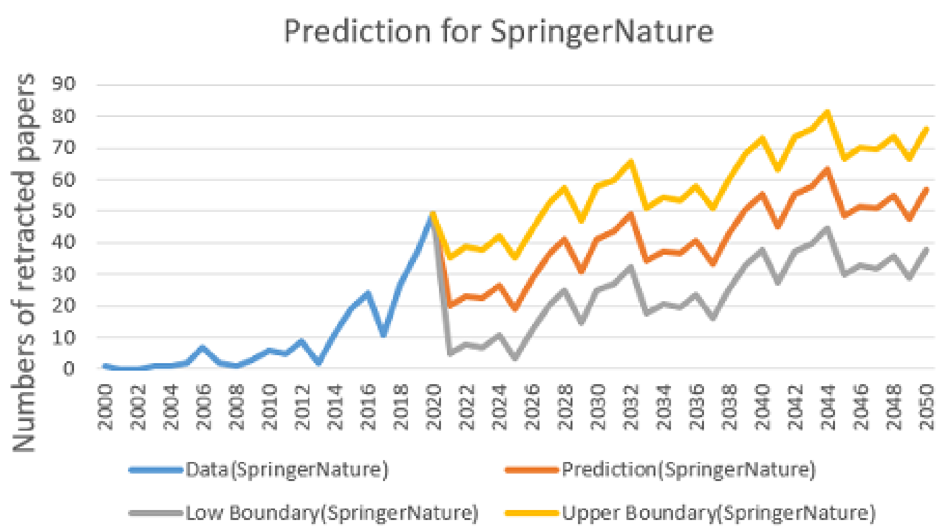

Figure 27. Prediction for numbers of retracted papers in SpringerNature.

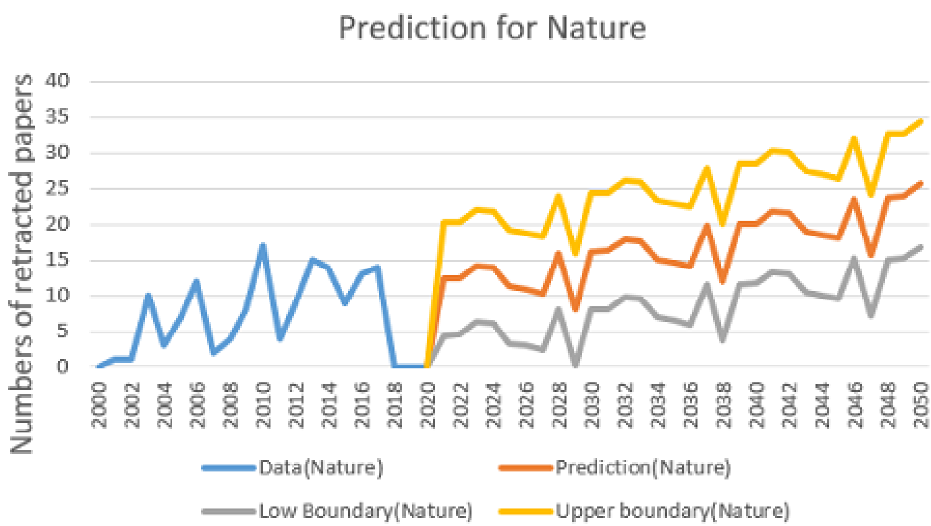

Figure 28. Prediction for numbers of retracted papers in Nature.

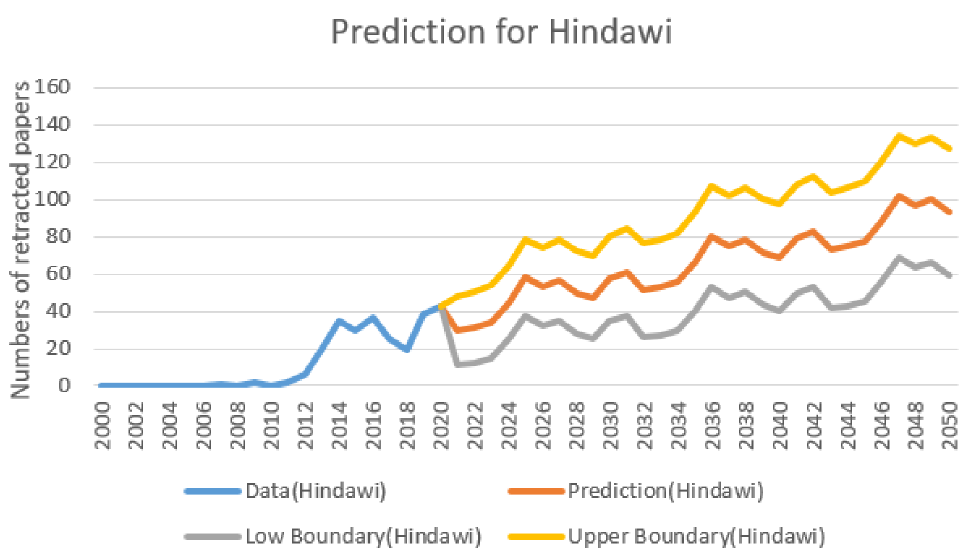

Figure 29. Prediction for numbers of retracted papers in Hindawi.

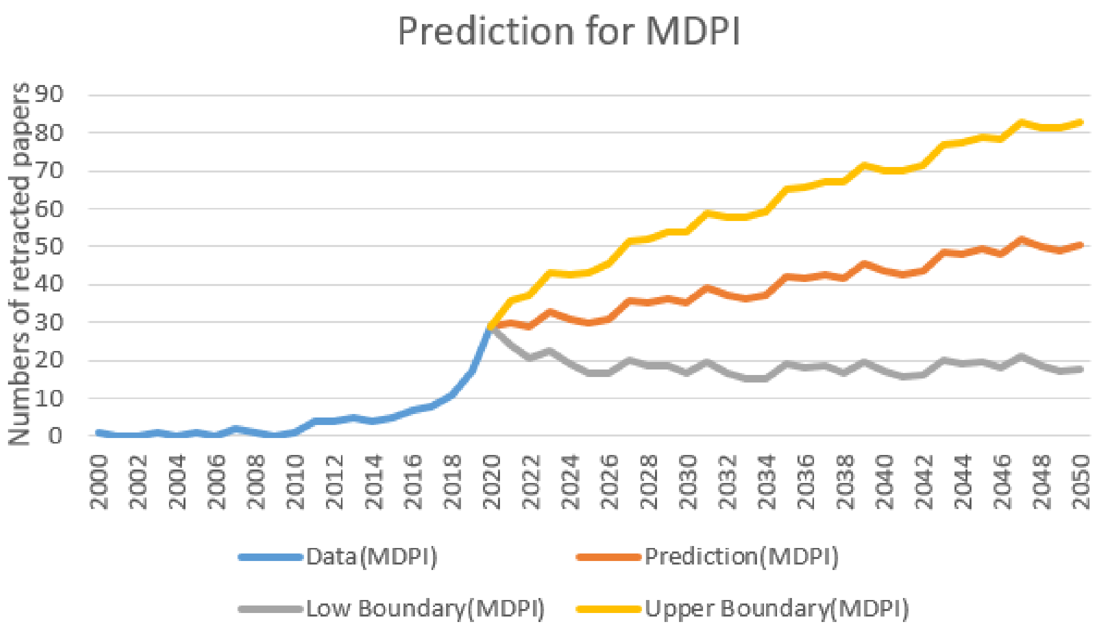

Figure 30. Prediction for numbers of retracted papers in MDPI.

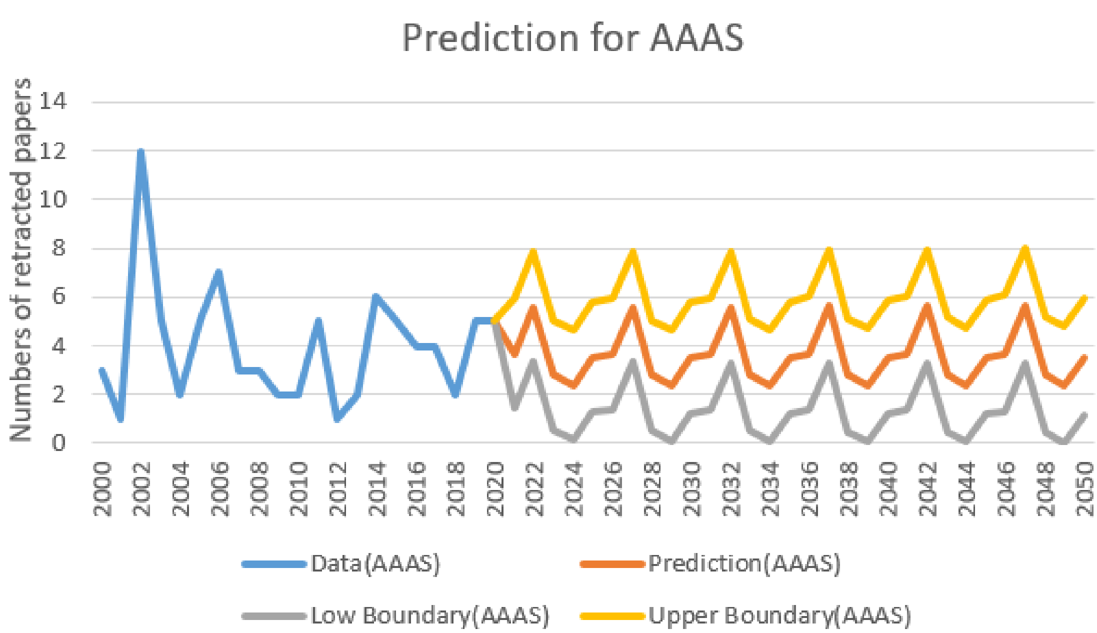

Figure 31. Prediction for numbers of retracted papers in AAAS.

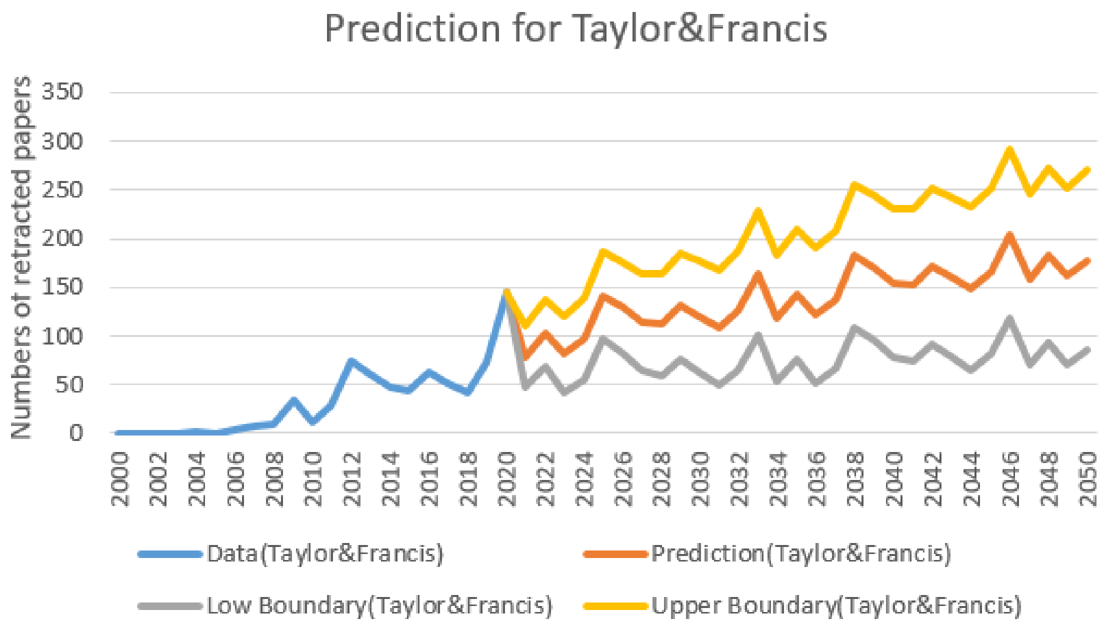

Figure 32. Prediction for numbers of retracted papers in Taylor&Francis.

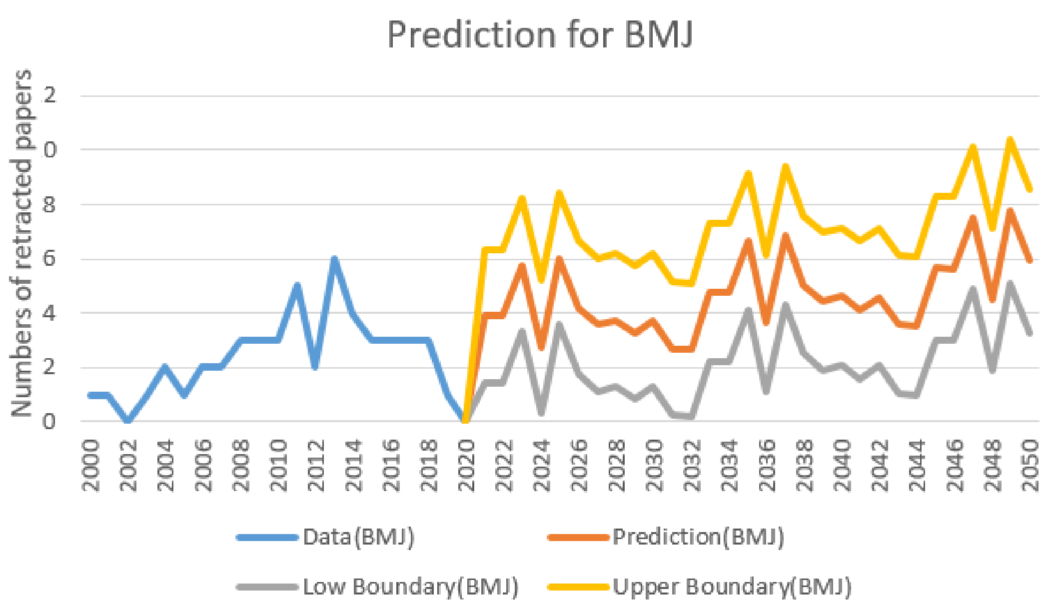

Figure 33. Prediction for numbers of retracted papers in BMJ.

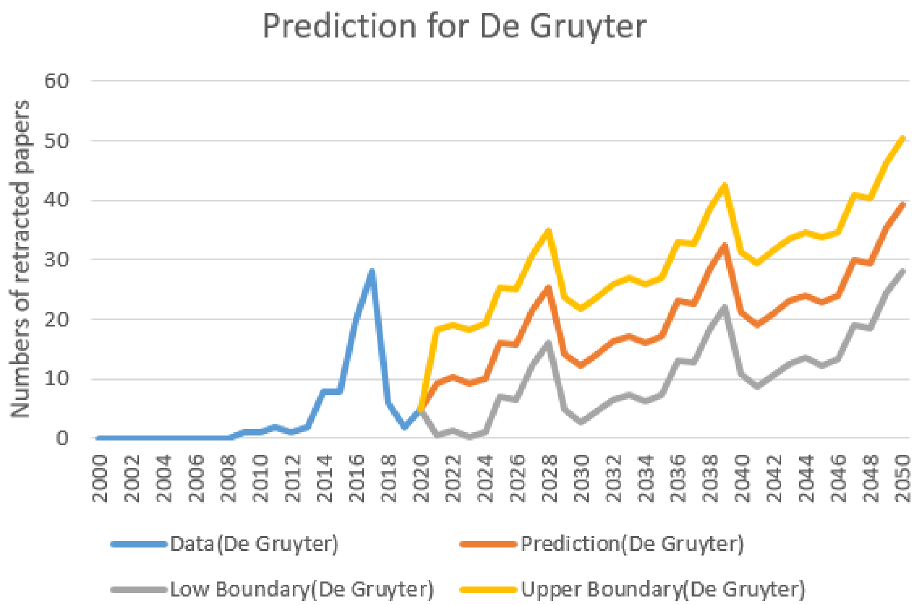

Figure 34. Prediction for numbers of retracted papers in De Gruyter.

In Figure 35 to 45, we attempt to fit collected data from each publisher using a statistical method called moving average.

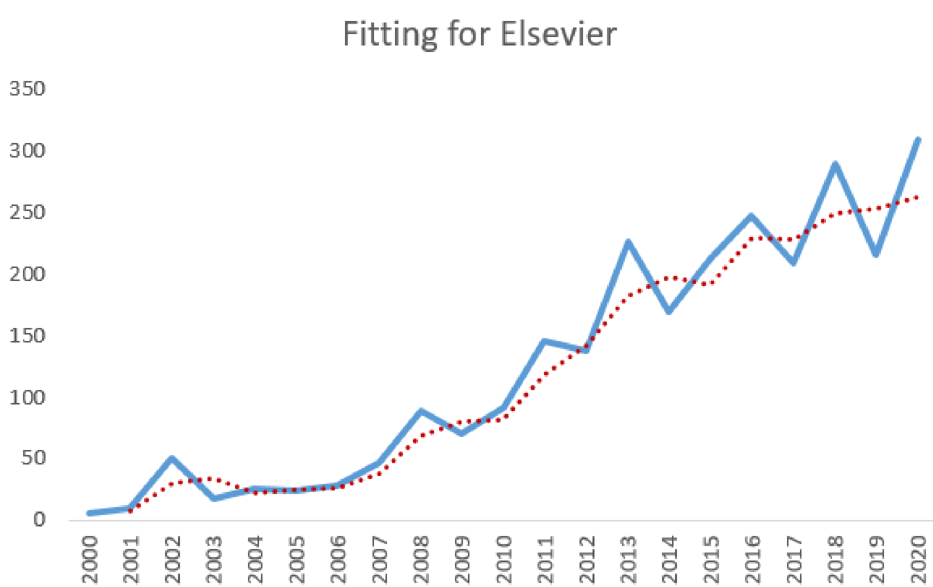

Figure 35. Fitting for retracted papers in Elsevier.

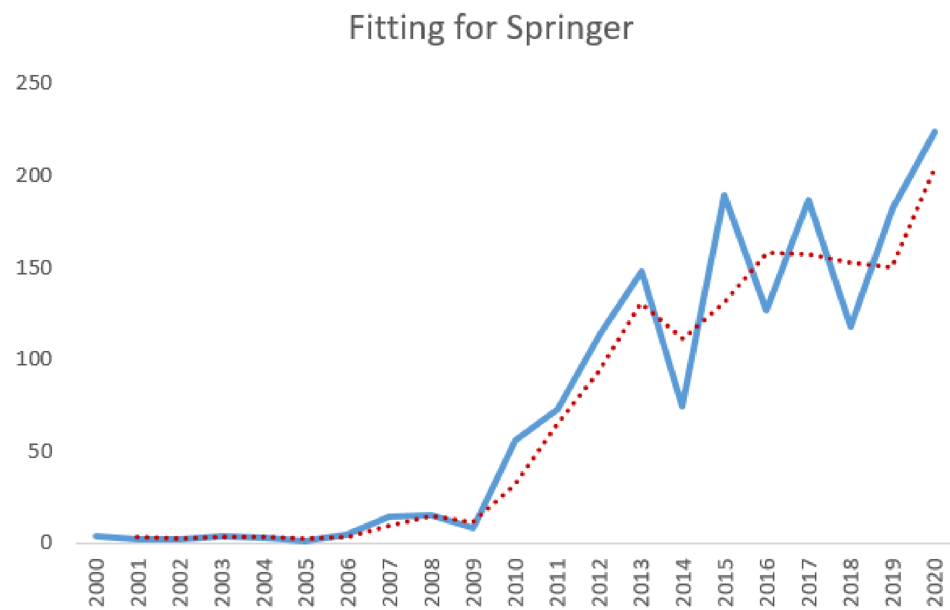

Figure 36. Fitting for retracted papers in Springer.

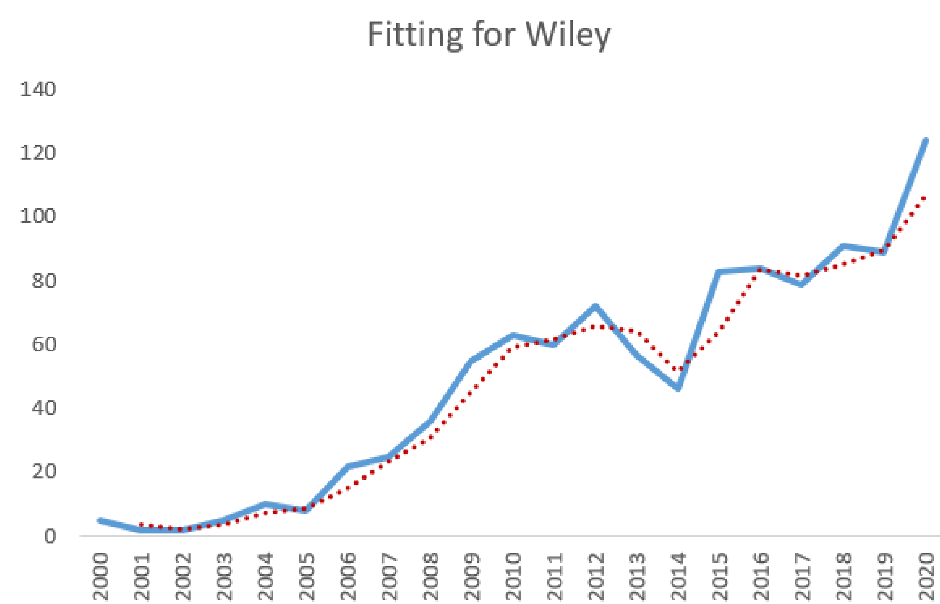

Figure 37. Fitting for retracted papers in Wiley.

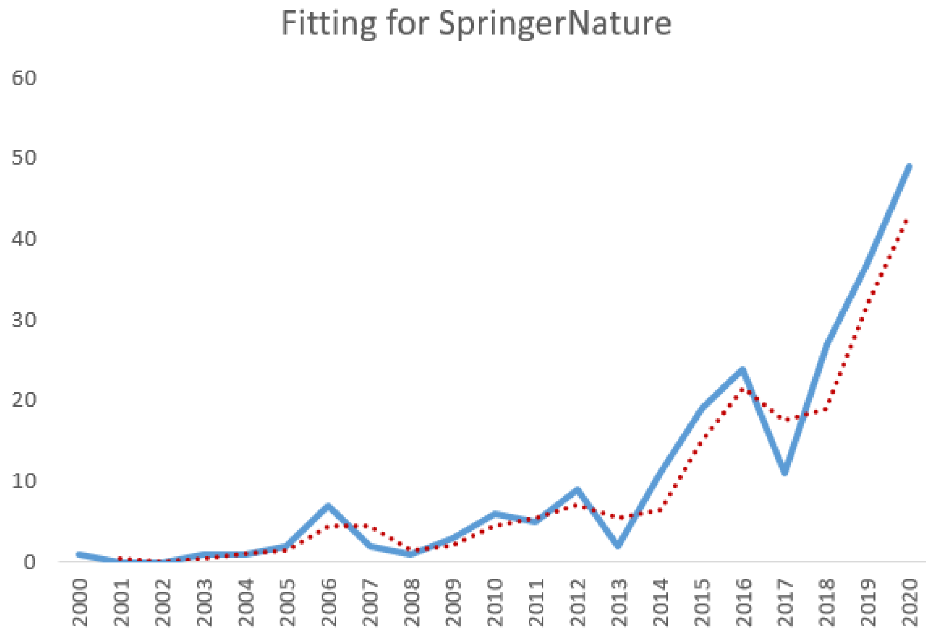

Figure 38. Fitting for retracted papers in SpringerNature.

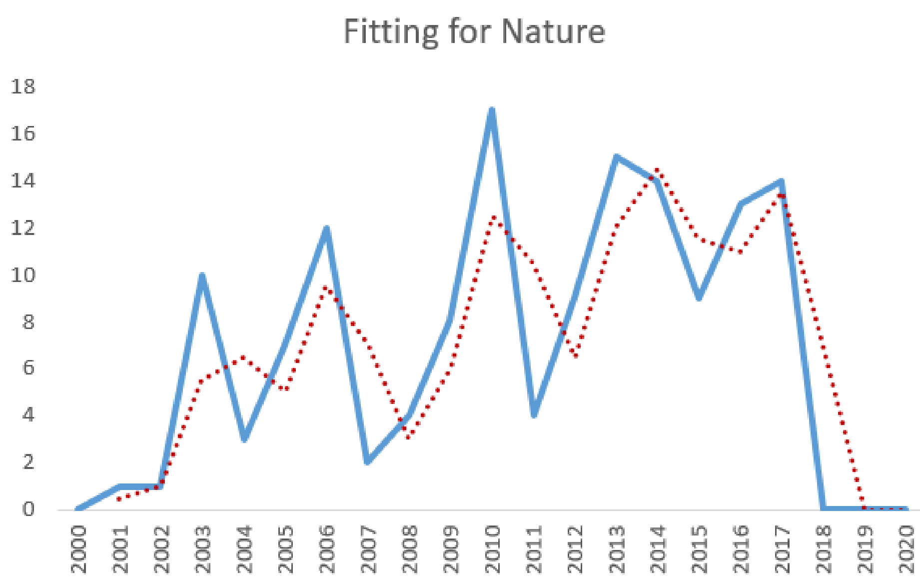

Figure 39. Fitting for retracted papers in Nature.

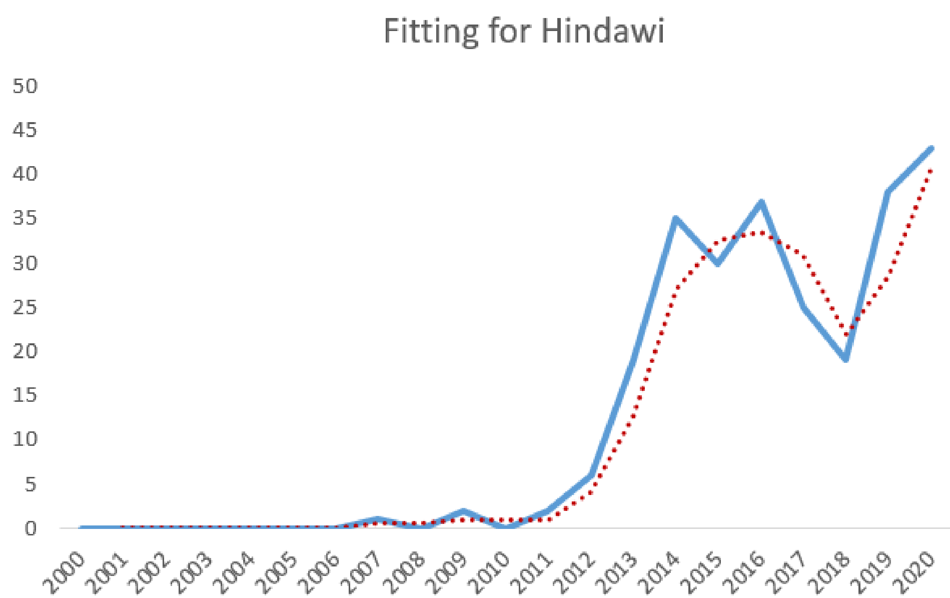

Figure 40. Fitting for retracted papers in Hindawi.

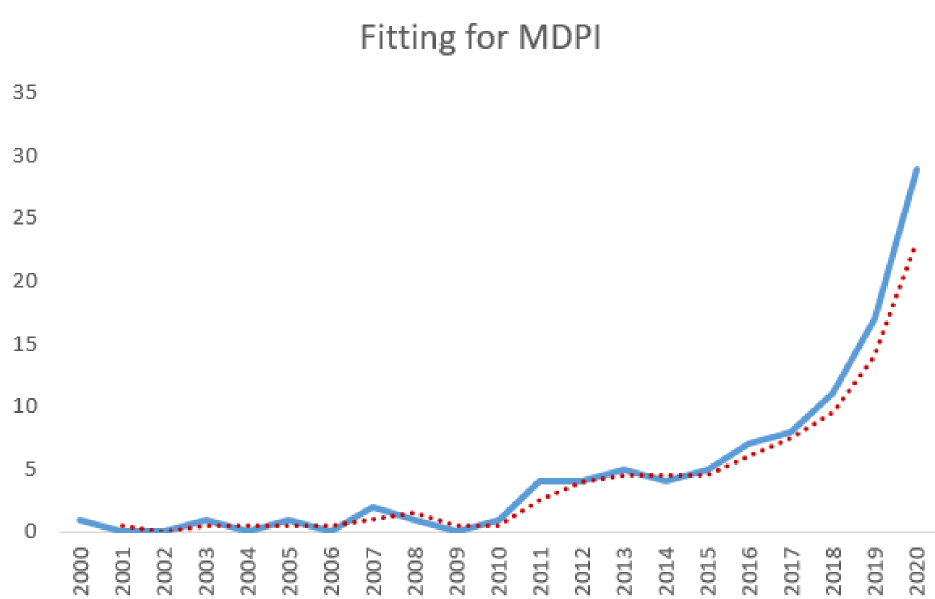

Figure 41. Fitting for retracted papers in MDPI.

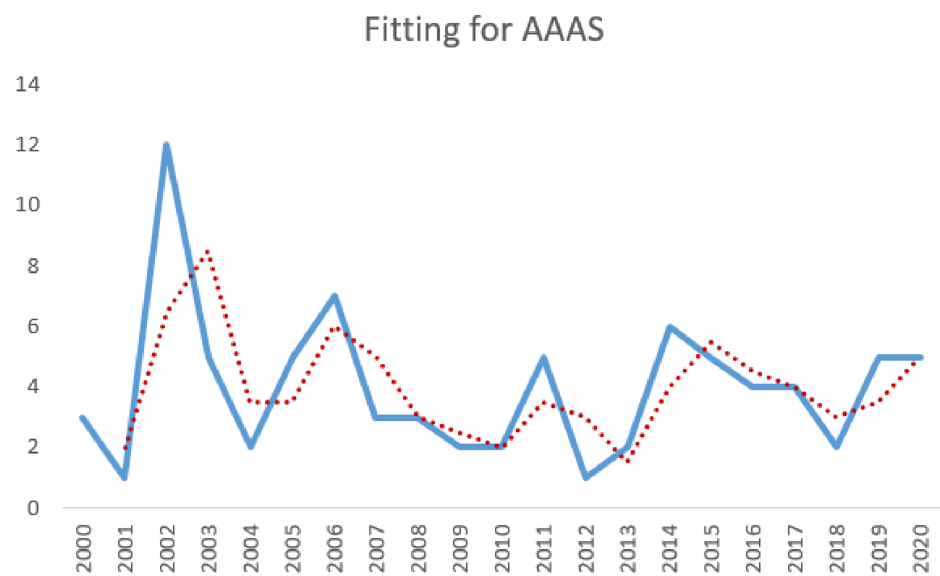

Figure 42. Fitting for retracted papers in AAAS.

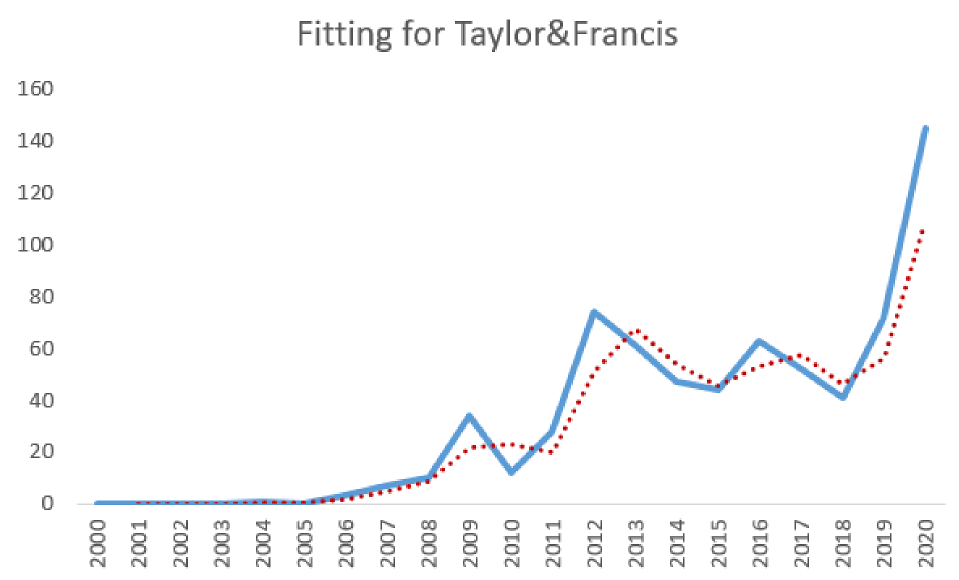

Figure 43. Fitting for retracted papers in Taylor&Francis.

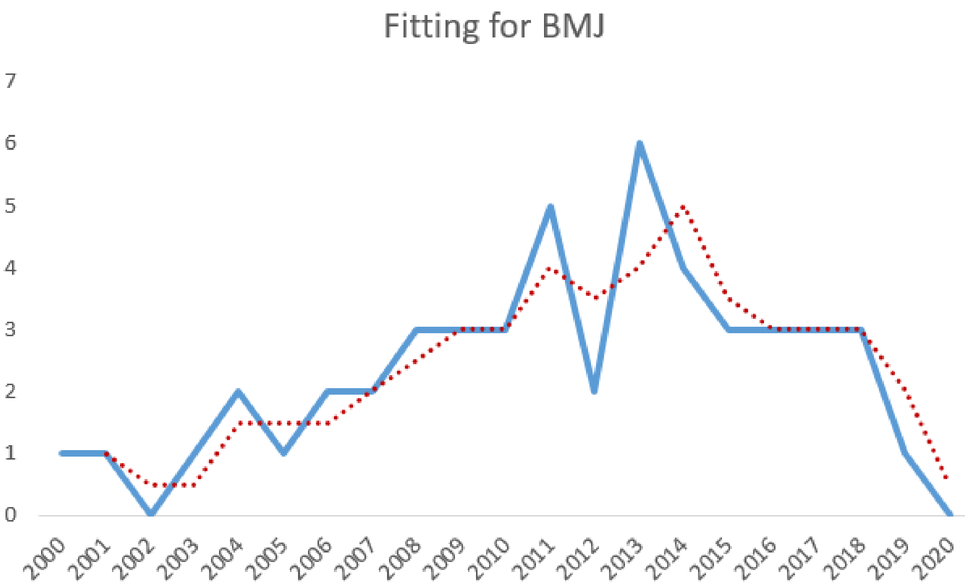

Figure 44. Fitting for retracted papers in BMJ.

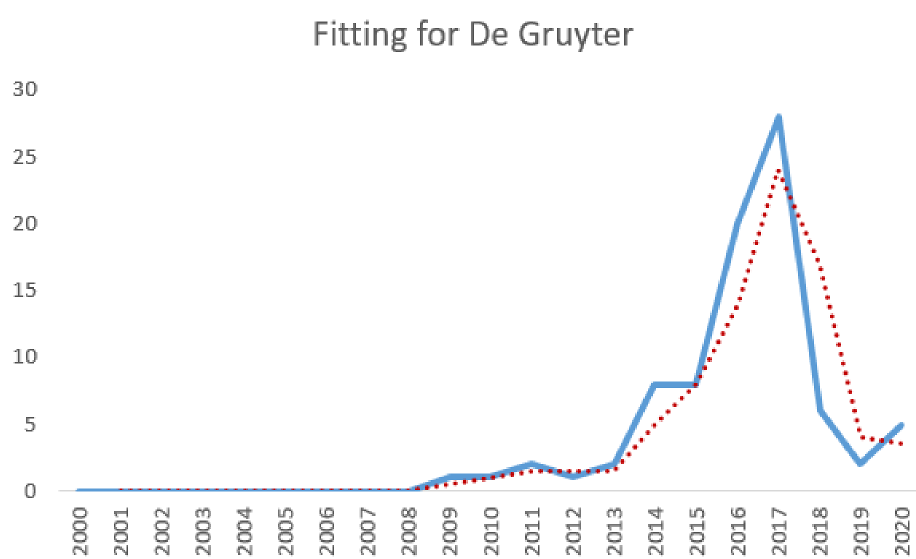

Figure 45. Fitting for retracted papers in De Gruyter.

With the analysis presented above, the next question one would like to answer is the following: Why is there a high rate of retraction? Some authors have reacted to this question and have attempted to supply a list of items that could be the main driving reasons for high numbers of retractions nowadays. Such a list will be presented below.

1. Authors are always under pressure to be productive with no financial support.
2. Reviewers are not doing their job properly only they are interested in getting citations.
3. Editors are loaded with high number of submissions therefore have no time to access the content of the papers properly.
4. Editors are discriminative.
5. The appointment of the editorial board is impartial as it only covers some specific continent.
6. Publishers have no experience, as they take impartial decisions.
7. Envy among peers, which lead some readers to target some authors.
8. Readers have connections with editorial boards or publishers.
9. Authors have 24 hours to check and reply for his galley-proofs which with no doubt put the author under pressure.
10. A sponsored company has to report retractions.
11. Pub peer allows anonymous individuals to post comments on published papers, even when those individuals are not experts in the field. Such comments are sometimes used to retract papers.
12. Some authors manipulate their results.
13. Plagiarism.

Beside the high number of retracted papers, the second question that has been raised by many authors is to know why only the author is the victim or responsible for the retraction? Except for manipulation of results, especially experimental research, the authors should not only be responsible for a retraction for the following reasons.

A submitted paper by an author does follow some steps before the paper can be accepted and published. Indeed the aim of submission is for the paper to be evaluated by some supposedly experts in the field.

a) After a submission, the editor-in-chief of the journal identifies a suitable editor who is expert in the field. The editor have to read the content of the paper and suggest rejection or the peer review process.

b) In case of peer reviewer process, two or more than three experts in the fields are approached by the handling editors, who will read the paper, evaluate the content of the paper and finally write reports that will also be evaluated by the handling editor.

c) If the reports suggest revisions, the handling editor will forward the comments to the corresponding author, who will be asked to perform those revisions and resubmit the paper for further evaluations.

d) After the resubmission the handling editor can accept the paper or send it again for second round for peer review if the paper is finally accepted by reviewers, the editor can accept it or still as for some revisions.

e) Finally if the paper is accepted by the handling editor, the paper is sent to production for publication.

3 A mathematical model of retraction with classical differentiation

Several reasons have been listed that could be considered as main causes of high rate of yearly retractions from different publishers, indeed this could vary from one journal to another accordingly to who is the editor in chief and who is the publisher. In big publishers like Elsevier there are more factors that could lead to their high numbers of yearly retraction, this will be a discussion for another research. In this section, we present a mathematical model depicting a possible scenario of the dynamic of retraction. To achieve this, we consider some classes including: $S(t)$ which is the class of published paper susceptible to be retracted, $R(t)$ is the class of retracted papers, $R_T(t)$ is the class of fairly retracted papers, $R_F(t)$ is the class of unfairly retracted papers, $B_E(t)$ is the class of unfair editors, who contribute to retracting unfairly papers published by some authors, $G_E(t)$ is the class of fair editors, they should be appointed more in different editorial board to insure fairness in terms of acceptance, rejections, and retraction. $D(t)$ is the class of papers that are reported by a retraction database. A possible system of ordinary differential equations depicting retraction scenario is given below as:

$$\begin{aligned}
\dot{S}(t) &= \Lambda - \beta S(G_E + \tau B_E) + \kappa_5 B_E \\
\dot{R}(t) &= \beta S(G_E + \tau B_E) - (1-\psi_1)\varphi_1 R - (1-\psi_2)\varphi_2 R \\
\dot{R_T}(t) &= \psi_1 \varphi_1 R - \kappa_1 R_T \\
\dot{R_F}(t) &= \psi_2 \varphi_2 R - \kappa_2 R_F \\
\dot{G_E}(t) &= (1-\psi_1)\varphi_1 R - \kappa_3 G_E \\
\dot{B_E}(t) &= (1-\psi_2)\varphi_2 R - (\kappa_4 + \kappa_5) B_E \\
\dot{D}(t) &= \kappa_1 R_T + \kappa_2 R_F + \kappa_3 G_E + \kappa_4 B_E
\end{aligned} \tag{1}$$

with the initial conditions

$$S(0) = S^0, R(0) = R^0, R_T(0) = R_F^0, G_E(0) = G_E^0, B_E(0) = B_E^0, D(0) = D^0. \tag{2}$$

In Figure 46, a diagram that takes into account retraction scenarios is provided to better understand such processes.

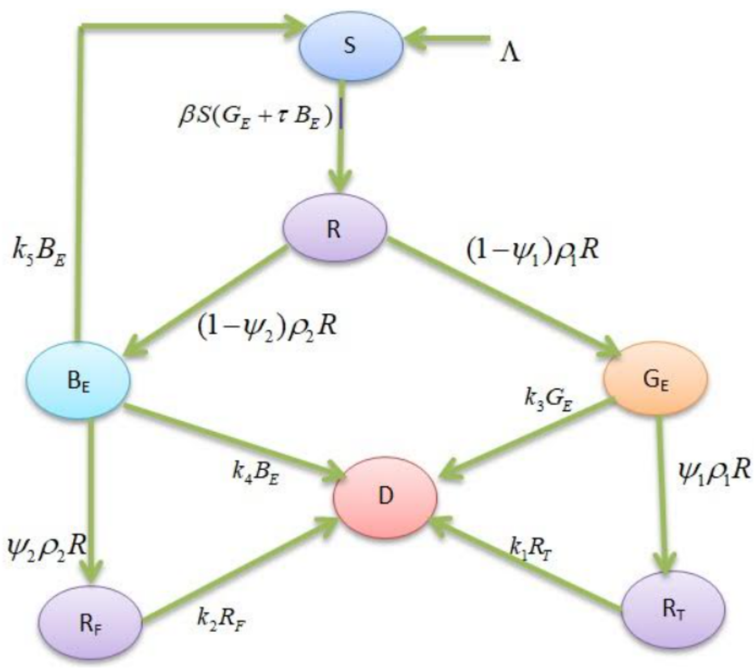

Figure 46. Flow chart for the retraction model.

Before proceeding, we insure that all solutions are positive.

$$\begin{aligned}
\dot{S}(t) &= \Lambda - \beta S\left(G_E + \tau B_E\right) + \kappa_5 B_E, \quad \forall t \geq 0 \\
&\geq -\beta S\left(G_E + \tau B_E\right), \quad \forall t \geq 0 \\
&\geq -\beta S\left(\left|G_E\right| + \tau\left|B_E\right|\right), \quad \forall t \geq 0, \\
&\geq -\beta S\left(\sup_{t\in D_{G_E}}\left|G_E\right| + \tau \sup_{t\in D_{B_E}}\left|B_E\right|\right), \quad \forall t \geq 0 \\
&\geq -\beta S\left(\left\|G_E\right\|_\infty + \tau\left\|B_E\right\|_\infty\right), \quad \forall t \geq 0 \\
&\geq -\beta S M, \quad \forall t \geq 0.
\end{aligned} \tag{3}$$

Since $G_E(t)$ represents the class of good editors and $B_E(t)$ represents editors with bad judgements, then

$$\left\|B_E\right\|_\infty < M_1 < \infty, \left\|G_E\right\|_\infty < M_2 < \infty. \tag{4}$$

Thus

$$\dot{S}(t) \geq -\beta M S(t), \quad \forall t \geq 0 \tag{5}$$

and

$$S(t) \geq S^0 \exp\left[-\beta M t\right], \quad \forall t \geq 0. \tag{6}$$

Using same routine, we should that $\forall t \geq 0$

$$\begin{array}{rcl} R\left(t\right) & \geq & R^{0}\exp\left[-\left(\left(1-\psi_{1}\right)\varphi_{1}+\left(1-\psi_{2}\right)\varphi_{2}\right)t\right] \\ R_{T}\left(t\right) & \geq & R_{T}^{0}\exp\left[-\kappa_{1}t\right] \\ R_{F}\left(t\right) & \geq & R_{F}^{0}\exp\left[-\kappa_{2}t\right] \\ G_{E}\left(t\right) & \geq & G_{E}^{0}\exp\left[-\kappa_{3}t\right] \\ B_{E}\left(t\right) & \geq & B_{E}^{0}\exp\left[-\left(\kappa_{4}+\kappa_{5}\right)t\right] \\ D\left(t\right) & \geq & 0 \end{array} \tag{7}$$

as the sum of positive functions.

We next show that under some conditions the retraction model has a unique system of solutions. We define the following norm

$$\left\|g\right\|_{\infty}=\inf_{t\in D_{g}}\left|g\left(t\right)\right|. \tag{8}$$

We reformulate the system of equations as

$$\begin{array}{rcl} \dot{S}\left(t\right) & = & F_{1}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right) \\ \dot{R}\left(t\right) & = & F_{2}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right) \\ \dot{R}_{T}\left(t\right) & = & F_{3}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right) \\ \dot{R}_{F}\left(t\right) & = & F_{4}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right) \\ \dot{G}_{E}\left(t\right) & = & F_{5}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right) \\ \dot{B}_{E}\left(t\right) & = & F_{6}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right) \\ \dot{D}\left(t\right) & = & F_{7}\left(t,S,R,R_{T},R_{F},G_{E},B_{E},D\right). \end{array} \tag{9}$$

The following properties need to be verified

(i) $\forall i\in\{1,2,3,4,5,6,7\}$

$$\left|F_{i}\left(x_{i},t\right)\right|^{2}\leq k_{i}\left(1+\left|x_{i}\right|^{2}\right). \tag{10}$$

(ii) $\forall i\in\{1,2,3,4,5,6,7\}$

$$\left|F_{i}\left(x_{i}^{1},t\right)-F_{i}\left(x_{i}^{2},t\right)\right|^{2}\leq\overline{k}_{i}\left|x_{i}^{1}-x_{i}^{2}\right|^{2}. \tag{11}$$

We start with the class $F_1(S,t)$

$$\begin{aligned} |F_1(S,t)|^2 &= |\Lambda - \beta S(G_E + \tau B_E) + \kappa_5 B_E|^2 \\ &\leq 3\Lambda^2 + 3\beta^2 |S|^2 |G_E + \tau B_E|^2 + 3\kappa_5^2 |B_E|^2 \\ &\leq 3\Lambda^2 + 6\beta^2 |S|^2 |G_E|^2 + 6\beta^2 |S|^2 \tau^2 |B_E|^2 + 3\kappa_5^2 |B_E|^2 \\ &\leq 3\Lambda^2 + 6\beta^2 |S|^2 \sup_{t\in D_{G_E}} \left|G_E^2\right|^2 + 6\beta^2 |S|^2 \tau^2 \sup_{t\in D_{B_E}} \left|B_E^2\right| + 3\kappa_5^2 \sup_{t\in D_{B_E}} \left|B_E^2\right| \\ &\leq \left(3\Lambda^2 + 3\kappa_5^2 \left\|B_E^2\right\|_\infty\right)\left(1 + \frac{6\beta^2\left(\left\|G_E^2\right\|_\infty + \tau^2\left\|B_E^2\right\|_\infty\right)}{3\Lambda^2 + 3\kappa_5^2\left\|B_E^2\right\|_\infty}|S|^2\right) \end{aligned} \tag{12}$$

under the condition that $\frac{6\beta^2\left(\left\|G_E^2\right\|_\infty + \tau^2\left\|B_E^2\right\|_\infty\right)}{3\Lambda^2 + 3\kappa_5^2\left\|B_E^2\right\|_\infty} < 1$, then

$$|F_1(S,t)|^2 \leq k_1\left(1 + |S|^2\right). \tag{13}$$

Also

$$\begin{aligned} \left|F_1\left(S^1,t\right) - F_1\left(S^2,t\right)\right|^2 &= \left|-\beta(G_E + \tau B_E)\left(S^1 - S^2\right)\right|^2 \\ &\leq \beta^2 |G_E + \tau B_E|^2 \left|S^1 - S^2\right|^2 \\ &\leq 2\beta^2\left|G_E^2\right| + 2\beta^2\tau^2\left|B_E^2\right|\left|S^1 - S^2\right|^2 \\ &\leq 2\beta^2\left(\sup_{t\in D_{G_E}}\left|G_E^2\right| + \tau^2\sup_{t\in D_{B_E}}\left|B_E^2\right|\right)\left|S^1 - S^2\right|^2 \\ &\leq 2\beta^2 K\left|S^1 - S^2\right|^2 \\ &\leq \bar{k}_1\left|S^1 - S^2\right|^2. \end{aligned} \tag{14}$$

Using same routine, we can have the following

$$\begin{aligned} |F_2(R,t)|^2 &= |\beta S(G_E + \tau B_E) - ((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2)R|^2 \\ &\leq 4\beta^2\left\|S^2\right\|_\infty\left(\left\|G_E^2\right\|_\infty + \tau^2\left\|B_E^2\right\|_\infty\right) + 2\left((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2\right)^2|R|^2 \\ &\leq 4\beta^2\left\|S^2\right\|_\infty K + 2\left((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2\right)^2|R|^2 \\ &\leq 4\beta^2\left\|S^2\right\|_\infty K\left(1 + \frac{2\left((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2\right)^2}{4\beta^2\left\|S^2\right\|_\infty K}|R|^2\right) \end{aligned} \tag{15}$$

under the condition that $\frac{2((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2)^2}{4\beta^2\|S^2\|_\infty K} < 1$, then

$$|F_2(R,t)|^2 \leq k_2\left(1 + |R|^2\right). \tag{16}$$

Also

$$\begin{aligned} \left|F_2\left(R^1,t\right) - F_2\left(R^2,t\right)\right|^2 &= \left|-\left((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2\right)\left(R^1 - R^2\right)\right|^2 \\ &\leq 2\left((1-\psi_1)^2\varphi_1^2 + (1-\psi_2)^2\varphi_2^2\right)\left|R^1 - R^2\right|^2 \\ &\leq \bar{k}_2\left|R^1 - R^2\right|^2. \end{aligned} \tag{17}$$

For the function F_3,

$$
\begin{aligned}
\left|F_3\left(R_T,t\right)\right|^2 &= \left|\psi_1\varphi_1 R-\kappa_1 R_T\right|^2 \\
&\leq 2\psi_1^2\varphi_1^2\left|R^2\right|+2\kappa_1^2\left|R_T\right|^2 \\
&\leq 2\psi_1^2\varphi_1^2\sup_{t\in D_R}\left|R^2\right|+2\kappa_1^2\sup_{t\in D_{R_T}}\left|R_T\right|^2 \\
&\leq 2\psi_1^2\varphi_1^2\left\|R^2\right\|_\infty\left(1+\frac{2\kappa_1^2}{2\psi_1^2\varphi_1^2\left\|R^2\right\|_\infty}\left|R_T\right|^2\right)
\end{aligned} \tag{18}
$$

under the condition that $\frac{2\kappa_1^2}{2\psi_1^2\varphi_1^2\|R^2\|_\infty}<1$, then

$$
\left|F_3\left(R_T,t\right)\right|^2\leq k_2\left(1+\left|R_T\right|^2\right). \tag{19}
$$

Also

$$
\begin{aligned}
\left|F_3\left(R_T^1,t\right)-F_3\left(R_T^2,t\right)\right|^2 &= \kappa_1^2\left|\left(R_T^1-R_T^2\right)\right|^2 \\
&\leq \frac{3}{2}\kappa_1^2\left|\left(R_T^1-R_T^2\right)\right|^2 \\
&\leq \overline{k}_3\left|\left(R_T^1-R_T^2\right)\right|^2.
\end{aligned} \tag{20}
$$

For the function F_4,

$$
\begin{aligned}
\left|F_4\left(R_F,t\right)\right|^2 &= \left|\psi_2\varphi_2 R-\kappa_2 R_F\right|^2 \\
&\leq 2\psi_2^2\varphi_2^2\left|R^2\right|+2\kappa_2^2\left|R_F\right|^2 \\
&\leq 2\psi_2^2\varphi_2^2\sup_{t\in D_R}\left|R^2\right|+2\kappa_2^2\sup_{t\in D_{R_F}}\left|R_F\right|^2 \\
&\leq 2\psi_2^2\varphi_2^2\left\|R^2\right\|_\infty\left(1+\frac{2\kappa_2^2}{2\psi_2^2\varphi_2^2\left\|R^2\right\|_\infty}\left|R_F\right|^2\right)
\end{aligned} \tag{21}
$$

under the condition that $\frac{2\kappa_2^2}{2\psi_2^2\varphi_2^2\|R^2\|_\infty}<1$, then

$$
\left|F_4\left(R_F,t\right)\right|^2\leq k_4\left(1+\left|R_F\right|^2\right). \tag{22}
$$

Also

$$
\begin{aligned}
\left|F_4\left(R_F^1,t\right)-F_4\left(R_F^2,t\right)\right|^2 &= \kappa_2^2\left|R_F^1-R_F^2\right|^2 \\
&\leq \frac{3}{2}\kappa_2^2\left|R_F^1-R_F^2\right|^2 \\
&\leq \overline{k}_3\left|R_F^1-R_F^2\right|^2.
\end{aligned} \tag{23}
$$

For the function F_5,

$$\begin{aligned} \left|F_5\left(G_E,t\right)\right|^2 &= \left|\left(1-\psi_1\right)\varphi_1 R-\kappa_3 G_E\right|^2 \\ &\leq 2\left(1-\psi_1\right)^2\varphi_1^2\left|R^2\right|+2\kappa_3^2\left|G_E\right|^2 \\ &\leq 2\left(1-\psi_1\right)^2\varphi_1^2\sup_{t\in D_R}\left|R^2\right|+2\kappa_3^2\sup_{t\in D_{R_T}}\left|G_E\right|^2 \\ &\leq 2\left(1-\psi_1\right)^2\varphi_1^2\left\|R^2\right\|_\infty\left(1+\frac{2\kappa_3^2}{2\left(1-\psi_1\right)^2\varphi_1^2\left\|R^2\right\|_\infty}\left|G_E\right|^2\right) \end{aligned} \tag{24}$$

under the condition that $\frac{2\kappa_3^2}{2(1-\psi_1)^2\varphi_1^2\|R^2\|_\infty}<1$, then

$$\left|F_5\left(G_E,t\right)\right|^2\leq k_5\left(1+\left|G_E\right|^2\right). \tag{25}$$

Also

$$\begin{aligned} \left|F_5\left(G_E^1,t\right)-F_5\left(G_E^2,t\right)\right|^2 &\leq \frac{3}{2}\kappa_3^2\left|G_E^1-G_E^2\right|^2 \\ &\leq \bar{k}_5\left|G_E^1-G_E^2\right|^2. \end{aligned} \tag{26}$$

For the function F_6,

$$\begin{aligned} \left|F_6\left(B_E,t\right)\right|^2 &= \left|\left(1-\psi_2\right)\varphi_2 R-\left(\kappa_4+\kappa_5\right)B_E\right|^2 \\ &\leq 2\left(1-\psi_2\right)^2\varphi_2^2\left|R^2\right|+4\left(\kappa_4^2+\kappa_5^2\right)\left|B_E\right|^2 \\ &\leq 2\left(1-\psi_2\right)^2\varphi_2^2\sup_{t\in D_R}\left|R^2\right|+4\left(\kappa_4^2+\kappa_5^2\right)\left|B_E\right|^2 \\ &\leq 2\left(1-\psi_2\right)^2\varphi_2^2\left\|R^2\right\|_\infty\left(1+\frac{4\left(\kappa_4^2+\kappa_5^2\right)}{2\left(1-\psi_2\right)^2\varphi_2^2\left\|R^2\right\|_\infty}\left|B_E\right|^2\right) \end{aligned} \tag{27}$$

under the condition that $\frac{4\left(\kappa_4^2+\kappa_5^2\right)}{2(1-\psi_2)^2\varphi_2^2\|R^2\|_\infty}<1$, then

$$\left|F_6\left(B_E,t\right)\right|^2\leq k_6\left(1+\left|B_E\right|^2\right). \tag{28}$$

Also

$$\begin{aligned} \left|F_6\left(B_E^1,t\right)-F_6\left(B_E^2,t\right)\right|^2 &\leq 2\left(\kappa_4^2+\kappa_5^2\right)\left|B_E^1-B_E^2\right|^2 \\ &\leq \bar{k}_6\left|G_E^1-G_E^2\right|^2. \end{aligned} \tag{29}$$

For the function F_7,

$$\begin{aligned} \left|F_7\left(D,t\right)\right|^2 &= \left|\kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E\right|^2 \\ &\leq \left|\kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E\right|^2\left(1+\varepsilon\left|D\right|^2\right) \\ &\leq 4\left(\kappa_1^2\sup_{t\in D_{R_T}}\left|R_T^2\right|+\kappa_2\sup_{t\in D_{R_F}}\left|R_F^2\right|+\kappa_3\sup_{t\in D_{G_E}}\left|G_E^2\right|+\kappa_4\sup_{t\in D_{B_E}}\left|B_E^2\right|\right)\left(1+\varepsilon\left|D\right|^2\right) \\ &\leq 4\left(\kappa_1^2\left\|R_T^2\right\|_\infty+\kappa_2\left\|R_F^2\right\|_\infty+\kappa_3\left\|G_E^2\right\|_\infty+\kappa_4\left\|B_E^2\right\|_\infty\right)\left(1+\varepsilon\left|D\right|^2\right) \end{aligned} \tag{30}$$

under the condition that $\varepsilon < 1$, then

$$\left|F_7\left(D,t\right)\right|^2 \leq k_6\left(1+\left|D\right|^2\right).$$

Also

$$\left|F_7\left(D^1,t\right)-F_7\left(D^2,t\right)\right|^2 \leq \overline{k}_7\left|D^1-D^2\right|^2.$$

Finally if

$$\max\left\{\begin{array}{c} \frac{6\beta^2\left(\left\|G_E^2\right\|_\infty+\tau^2\left\|B_E^2\right\|_\infty\right)}{3\Lambda^2+3\kappa_5^2\left\|B_E^2\right\|_\infty}, \frac{2((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)^2}{4\beta^2\|S^2\|_\infty K}, \frac{2\kappa_1^2}{2\psi_1^2\varphi_1^2\|R^2\|_\infty} \\ , \frac{2\kappa_2^2}{2\psi_2^2\varphi_2^2\|R^2\|_\infty}, \frac{2\kappa_3^2}{2(1-\psi_1)^2\varphi_1^2\|R^2\|_\infty}, \frac{4\left(\kappa_4^2+\kappa_5^2\right)}{2(1-\psi_2)^2\varphi_2^2\|R^2\|_\infty}, \varepsilon \end{array}\right\} < 1, \tag{31}$$

the model of the retraction has a unique system of solutions. In the absence of fake retractions and bad editors, the equilibrium points will be

$$\begin{aligned} \Lambda-\beta S^* G_E^* &= 0 \\ \beta S^* G_E^*-\left(1-\psi_1\right)\varphi_1 R^*-\left(1-\psi_2\right)\varphi_2 R^* &= 0 \\ \psi_1\varphi_1 R^*-\kappa_1 R_T^* &= 0 \\ \left(1-\psi_1\right)\varphi_1 R^*-\kappa_3 G_E^* &= 0. \end{aligned} \tag{32}$$

After solving above, we obtain

$$\begin{aligned} S^* &= \frac{\Lambda}{\beta G_E^*} = \frac{\kappa_3}{\beta\left(1-\psi_1\right)\varphi_1}\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right) \\ R^* &= \frac{\Lambda}{\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2} \\ R_T^* &= \frac{\psi_1\varphi_1}{\kappa_1}\frac{\Lambda}{\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2} \\ G_E^* &= \frac{\left(1-\psi_1\right)\varphi_1}{\kappa_3}\frac{\Lambda}{\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2}. \end{aligned} \tag{33}$$

However in the presence of false retraction and bad editors, the equilibrium points would be

$$\begin{aligned} \Lambda-\beta S^*\left(G_E^*+\tau B_E^*\right)+\kappa_5 B_E^* &= 0 \\ \beta S^*\left(G_E^*+\tau B_E^*\right)-\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R^* &= 0 \\ \psi_1\varphi_1 R^*-\kappa_1 R_T^* &= 0 \\ \psi_2\varphi_2 R^*-\kappa_2 R_F^* &= 0 \\ \left(1-\psi_1\right)\varphi_1 R^*-\kappa_3 G_E^* &= 0 \\ \left(1-\psi_2\right)\varphi_2 R^*-\left(\kappa_4+\kappa_5\right)B_E^* &= 0. \end{aligned} \tag{34}$$

By solving above system, we obtain

$$\begin{aligned}
R^* &= \frac{\Lambda(\kappa_4+\kappa_5)}{\kappa_5(1-\psi_1)\varphi_1+\kappa_4((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)}, \\
R_T^* &= \frac{\psi_1\varphi_1}{\kappa_1}\frac{\Lambda(\kappa_4+\kappa_5)}{\kappa_5(1-\psi_1)\varphi_1+\kappa_4((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)}, \\
R_F^* &= \frac{\psi_2\varphi_2}{\kappa_2}\frac{\Lambda(\kappa_4+\kappa_5)}{\kappa_5(1-\psi_1)\varphi_1+\kappa_4((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)}, \\
G_E^* &= \frac{(1-\psi_1)\varphi_1}{\kappa_3}\frac{\Lambda(\kappa_4+\kappa_5)}{\kappa_5(1-\psi_1)\varphi_1+\kappa_4((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)}, \\
B_E^* &= \frac{(1-\psi_2)\varphi_2}{(\kappa_4+\kappa_5)}\frac{\Lambda(\kappa_4+\kappa_5)}{\kappa_5(1-\psi_1)\varphi_1+\kappa_4((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)} \\
&= \frac{\Lambda(1-\psi_2)\varphi_2}{\kappa_5(1-\psi_1)\varphi_1+\kappa_4((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)}.
\end{aligned} \tag{35}$$

Finally

$$S^* = \frac{((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)R^*}{\beta(G_E^*+\tau B_E^*)}. \tag{36}$$

Before proceeding with the stability analysis of equilibrium points, we will first present the conditions under which the classes of $B_E(t)$ and $R_F(t)$ are declining. Using elementary calculus, we know the function $B_E(t)$ will decrease if

$$\begin{aligned}
\dot{B}_E(t) &< 0 \Rightarrow (1-\psi_2)\varphi_2 R-(\kappa_4+\kappa_5)B_E<0 \\
\frac{(1-\psi_2)\varphi_2}{(\kappa_4+\kappa_5)} &< \frac{B_E}{R}.
\end{aligned} \tag{37}$$

$R_F(t)$ will decline if and only if

$$\begin{aligned}
\dot{R}_F(t) &< 0 \Rightarrow \psi_2\varphi_2 R-\kappa_2 R_F<0 \\
\frac{\psi_2\varphi_2}{\kappa_2} &< \frac{R_F}{R}<1.
\end{aligned} \tag{38}$$

To investigate if classes will have concavity,we study the sign of their respective second derivatives. Therefore

$$\begin{aligned}
\frac{d\dot{B}_E}{dt} &= (1-\psi_2)\varphi_2\dot{R}-(\kappa_4+\kappa_5)\dot{B}_E \\
&= (1-\psi_2)\varphi_2[\beta S(G_E+\tau B_E)-((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2)R] \\
&\quad -(\kappa_4+\kappa_5)((1-\psi_2)\varphi_2 R-(\kappa_4+\kappa_5)B_E).
\end{aligned} \tag{39}$$

Then

$$\frac{d\dot{B}_E}{dt}<0 \tag{40}$$

if

$$\begin{aligned}&(1-\psi_2)\varphi_2\left[\beta S\left(G_E+\tau B_E\right)-\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)R\right]\\&-(\kappa_4+\kappa_5)\left((1-\psi_2)\varphi_2 R-(\kappa_4+\kappa_5)B_E\right)<0.\end{aligned} \tag{41}$$

A fortiori

$$\begin{aligned}&(1-\psi_2)\varphi_2\left[\beta\tau B_E-\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)R\right]\\&-(\kappa_4+\kappa_5)\left((1-\psi_2)\varphi_2 R-(\kappa_4+\kappa_5)B_E\right)<0\end{aligned} \tag{42}$$

and

$$\begin{aligned}&(1-\psi_2)\varphi_2\beta\tau B_E-(1-\psi_2)\varphi_2\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)R\\&-(\kappa_4+\kappa_5)(1-\psi_2)\varphi_2 R-(\kappa_4+\kappa_5)^2 B_E<0\end{aligned} \tag{43}$$

or

$$\left((1-\psi_2)\varphi_2\beta\tau+(\kappa_4+\kappa_5)^2\right)B_E-\left[\begin{array}{c}(1-\psi_2)\varphi_2\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)\\+(\kappa_4+\kappa_5)(1-\psi_2)\varphi_2\end{array}\right]R<0. \tag{44}$$

Therefore

$$\frac{(1-\psi_2)\varphi_2\beta\tau+(\kappa_4+\kappa_5)^2}{(1-\psi_2)\varphi_2\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)+(\kappa_4+\kappa_5)(1-\psi_2)\varphi_2}<\frac{R}{.B_E}. \tag{45}$$

Similarly

$$\begin{aligned}\frac{d\dot{R_F}}{dt}&=\psi_2\varphi_2\dot{R}-\kappa_2\dot{R_F}\\&=\psi_2\varphi_2\left(\begin{array}{c}\beta S\left(G_E+\tau B_E\right)\\-\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)R\end{array}\right)-\kappa_2\left(\psi_2\varphi_2 R-\kappa_2 R_F\right).\end{aligned} \tag{46}$$

Thus $\frac{d\dot{R_F}}{dt}<0$ if

$$\psi_2\varphi_2\left(\beta\tau B_E-\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2\right)R\right)-\kappa_2\psi_2\varphi_2 R+\kappa_2^2 R_F<0. \tag{47}$$

Since $S>R_F$ a fortiori $S\left(G_E+\tau B_E\right)>R_F$

$$\left(\psi_2\varphi_2\beta+\kappa_2^2\right)R_F-\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2+\kappa_2\psi_2\varphi_2\right)R<0. \tag{48}$$

Therefore $\frac{d\dot{R_F}}{dt}<0$ if and only if

$$\frac{R_F}{R}<\frac{(1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2+\kappa_2\psi_2\varphi_2}{\psi_2\varphi_2\beta+\kappa_2^2}. \tag{49}$$

We now investigate the sign of the Lyapunov energy associated to this system.

$$\begin{aligned} L\left(S,R,R_T,R_F,B_E,G_E,D\right) &= \left(S-S^*-S^*\log\frac{S^*}{S}\right)+\left(R-R^*-R^*\log\frac{R^*}{R}\right) \\ &+\left(R_T-R_T^*-R_T^*\log\frac{R_T^*}{R_T}\right)+\left(R_F-R_F^*-R_F^*\log\frac{R_F^*}{R_F}\right) \\ &+\left(B_E-B_E^*-B_E^*\log\frac{B_E^*}{B_E}\right)+\left(G_E-G_E^*-G_E^*\log\frac{G_E^*}{G_E}\right). \end{aligned} \tag{50}$$

By taking its derivative, we have

$$\begin{aligned} \frac{dL\left(t\right)}{dt} &= \left(1-\frac{S^*}{S}\right)\dot{S}+\left(1-\frac{R^*}{R}\right)\dot{R}+\left(1-\frac{R_T^*}{R_T}\right)\dot{R}_T \\ &+\left(1-\frac{R_F^*}{R_F}\right)\dot{R}_F+\left(1-\frac{B_E^*}{B_E}\right)\dot{B}_E+\left(1-\frac{G_E^*}{G_E}\right)\dot{G}_E. \end{aligned} \tag{51}$$

Putting all together, we obtain

$$\begin{aligned} \frac{dL\left(t\right)}{dt} &= \left(1-\frac{S^*}{S}\right)\left(\Lambda-\beta S\left(G_E+\tau B_E\right)+\kappa_5 B_E\right) \\ &+\left(1-\frac{R^*}{R}\right)\left(\beta S\left(G_E+\tau B_E\right)-\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R\right) \\ &+\left(1-\frac{R_T^*}{R_T}\right)\left(\psi_1\varphi_1 R-\kappa_1 R_T\right)+\left(1-\frac{R_F^*}{R_F}\right)\left(\psi_2\varphi_2 R-\kappa_2 R_F\right) \\ &+\left(1-\frac{B_E^*}{B_E}\right)\left(\left(1-\psi_2\right)\varphi_2 R-\left(\kappa_4+\kappa_5\right)B_E\right) \\ &+\left(1-\frac{G_E^*}{G_E}\right)\left(\left(1-\psi_1\right)\varphi_1 R-\kappa_3 G_E\right) \end{aligned} \tag{52}$$

and

$$\begin{aligned} \frac{dL\left(t\right)}{dt} &= \left(\frac{S-S^*}{S}\right)\left(\Lambda-\beta\left(S-S^*\right)\left(\left(G_E-G_E^*\right)+\tau\left(B_E-B_E^*\right)\right)+\kappa_5\left(B_E-B_E^*\right)\right) \\ &+\left(\frac{R-R^*}{R}\right)\left(\beta\left(S-S^*\right)\left(\left(G_E-G_E^*\right)+\tau\left(B_E-B_E^*\right)\right)-\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)\left(R-R^*\right)\right) \\ &+\left(\frac{R_T-R_T^*}{R_T}\right)\left(\psi_1\varphi_1\left(R-R^*\right)-\kappa_1\left(R_T-R_T^*\right)\right)+\left(\frac{R_F-R_F^*}{R_F}\right)\left(\psi_2\varphi_2\left(R-R^*\right)-\kappa_2\left(R_F-R_F^*\right)\right) \\ &+\left(\frac{B_E-B_E^*}{B_E}\right)\left(\left(1-\psi_2\right)\varphi_2\left(R-R^*\right)-\left(\kappa_4+\kappa_5\right)\left(B_E-B_E^*\right)\right) \\ &+\left(\frac{G_E-G_E^*}{G_E}\right)\left(\left(1-\psi_1\right)\varphi_1\left(R-R^*\right)-\kappa_3\left(G_E-G_E^*\right)\right). \end{aligned} \tag{53}$$

Arranging above, one can find

$$\begin{aligned}
\frac{dL(t)}{dt} &= -\frac{(S-S^*)^2}{S}\left(\beta G_E+\tau B_E^*\right)+\frac{(S-S^*)^2}{S}\left(\beta G_E^*+\tau B_E^*\right)+\Lambda+\kappa_5 B_E-\kappa_5 B_E^*-\frac{S^*}{S}\Lambda \qquad (54)\\
&+\kappa_5\frac{S^*}{S}B_E^*-\frac{(R-R^*)^2}{R}\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)+\beta S G_E-\beta S G_E^*-\beta S^* G_E\\
&+\beta S^* G_E^*+\beta\tau S B_E-\beta\tau S B_E^*-\beta\tau S^* B_E+\beta\tau S^* B_E^*-\frac{R^*}{R}\beta S G_E+\frac{R^*}{R}\beta S G_E^*-\kappa_5\frac{S^*}{S}B_E\\
&+\frac{R^*}{R}\beta S^* G_E-\frac{R^*}{R}\beta S^* G_E^*-\frac{R^*}{R}\beta\tau S B_E+\frac{R^*}{R}\beta\tau S B_E^*+\frac{R^*}{R}\beta\tau S^* B_E-\frac{R^*}{R}\beta\tau S^* B_E^*\\
&-\frac{(R_T-R_T^*)^2}{R_T}\kappa_1+\psi_1\varphi_1 R-\psi_1\varphi_1 R^*-\frac{R_T^*}{R_T}\psi_1\varphi_1 R+\frac{R_T^*}{R_T}\psi_1\varphi_1 R^*-\frac{(R_T-R_T^*)^2}{R_T}\kappa_2\\
&+\psi_2\varphi_2 R-\psi_2\varphi_2 R^*-\frac{R_F^*}{R_F}\psi_2\varphi_2 R+\frac{R_F^*}{R_F}\psi_2\varphi_2 R^*-\frac{(B_E-B_E^*)^2}{B_E}\left(\kappa_4+\kappa_5\right)+\left(1-\psi_2\right)\varphi_2 R\\
&-\left(1-\psi_2\right)\varphi_2 R^*-\frac{B_E^*}{B_E}\left(1-\psi_2\right)\varphi_2 R+\frac{B_E^*}{B_E}\left(1-\psi_2\right)\varphi_2 R^*-\frac{(G_E-G_E^*)^2}{G_E}\kappa_3\\
&+\left(1-\psi_1\right)\varphi_1 R-\left(1-\psi_1\right)\varphi_1 R^*-\frac{G_E^*}{G_E}\left(1-\psi_1\right)\varphi_1 R+\frac{G_E^*}{G_E}\left(1-\psi_1\right)\varphi_1 R^*.
\end{aligned}$$

Here, we write

$$\begin{aligned}
L_1 &= \frac{(S-S^*)^2}{S}\left(\beta G_E^*+\tau B_E^*\right)+\Lambda+\kappa_5 B_E+\kappa_5\frac{S^*}{S}B_E^*+\beta S G_E \qquad (55)\\
&+\beta S^* G_E^*+\beta\tau S B_E+\beta\tau S^* B_E^*+\frac{R^*}{R}\beta S G_E^*\\
&+\frac{R^*}{R}\beta S^* G_E+\frac{R^*}{R}\beta\tau S B_E^*+\frac{R^*}{R}\beta\tau S^* B_E\\
&+\psi_1\varphi_1 R+\frac{R_T^*}{R_T}\psi_1\varphi_1 R^*+\frac{B_E^*}{B_E}\left(1-\psi_2\right)\varphi_2 R^*\\
&+\psi_2\varphi_2 R+\frac{R_F^*}{R_F}\psi_2\varphi_2 R^*+\left(1-\psi_2\right)\varphi_2 R\\
&+\left(1-\psi_1\right)\varphi_1 R+\frac{G_E^*}{G_E}\left(1-\psi_1\right)\varphi_1 R^*
\end{aligned}$$

and

$$
\begin{aligned}
L_2 &= \frac{(S-S^*)^2}{S}\left(\beta G_E+\tau B_E^*\right)+\kappa_5 B_E^*+\frac{S^*}{S}\Lambda+\beta S G_E^*+\beta S^* G_E \\
&+\frac{(R-R^*)^2}{R}\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)+\beta\tau S B_E^*+\beta\tau S^* B_E \\
&+\frac{R^*}{R}\beta S G_E+\kappa_5\frac{S^*}{S}B_E+\frac{R^*}{R}\beta S^* G_E^*+\frac{R^*}{R}\beta\tau S B_E+\frac{R^*}{R}\beta\tau S^* B_E^* \\
&+\frac{(R_T-R_T^*)^2}{R_T}\kappa_1+\psi_1\varphi_1 R^*+\frac{R_T^*}{R_T}\psi_1\varphi_1 R+\frac{(R_T-R_T^*)^2}{R_T}\kappa_2 \\
&+\psi_2\varphi_2 R^*+\frac{R_F^*}{R_F}\psi_2\varphi_2 R+\frac{(B_E-B_E^*)^2}{B_E}\left(\kappa_4+\kappa_5\right)+\left(1-\psi_2\right)\varphi_2 R^* \\
&+\frac{B_E^*}{B_E}\left(1-\psi_2\right)\varphi_2 R+\frac{(G_E-G_E^*)^2}{G_E}\kappa_3+\left(1-\psi_1\right)\varphi_1 R^*+\frac{G_E^*}{G_E}\left(1-\psi_1\right)\varphi_1 R.
\end{aligned}
\tag{56}
$$

If $L_1 < L_2$, then the Lyapunov energy will be negative. If $L_2 < L_1$, then the energy will be positive. If $L_1 = L_2$, the situation right now will be unchanged.

4 Numerical solution for retraction model

In this section, we will present Atangana-Seda scheme to solve the suggested mathematical model with classical case. We recall our problem

$$
\begin{aligned}
\frac{d}{dt}S(t) &= \Lambda-\beta S\left(G_E+\tau B_E\right)+\kappa_5 B_E \\
\frac{d}{dt}R(t) &= \beta S\left(G_E+\tau B_E\right)-\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R \\
\frac{d}{dt}R_T(t) &= \psi_1\varphi_1 R-\kappa_1 R_T \\
\frac{d}{dt}R_F(t) &= \psi_2\varphi_2 R-\kappa_2 R_F \\
\frac{d}{dt}G_E(t) &= \left(1-\psi_1\right)\varphi_1 R-\kappa_3 G_E \\
\frac{d}{dt}B_E(t) &= \left(1-\psi_2\right)\varphi_2 R-\left(\kappa_4+\kappa_5\right)B_E \\
\frac{d}{dt}D(t) &= \kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E.
\end{aligned}
\tag{57}
$$

For simplicity, we write above equation as follows;

$$
\begin{aligned}
\frac{d}{dt}S(t) &= S^*(t,S,R,R_T,R_F,G_E,B_E,D) \\
\frac{d}{dt}R(t) &= R^*(t,S,R,R_T,R_F,G_E,B_E,D) \\
\frac{d}{dt}R_T(t) &= R_T^*(t,S,R,R_T,R_F,G_E,B_E,D) \\
\frac{d}{dt}R_F(t) &= R_F^*(t,S,R,R_T,R_F,G_E,B_E,D) \\
\frac{d}{dt}G_E(t) &= G_E^*(t,S,R,R_T,R_F,G_E,B_E,D) \\
\frac{d}{dt}B_E(t) &= B_E^*(t,S,R,R_T,R_F,G_E,B_E,D) \\
\frac{d}{dt}D(t) &= D^*(t,S,R,R_T,R_F,G_E,B_E,D).
\end{aligned}
\tag{58}
$$

After integrating above and putting Newton polynomial into these equations, we can solve our model as follows

$$
S^{n+1} = S^n + \left\{ \begin{array}{c} \frac{23}{12}S^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}S^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}S^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\} \tag{59}
$$

$$
R^{n+1} = R^n + \left\{ \begin{array}{c} \frac{23}{12}R^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}R^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}R^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\}
$$

$$
R_T^{n+1} = R_T^n + \left\{ \begin{array}{c} \frac{23}{12}R_T^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}R_T^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}R_T^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\}
$$

$$
R_F^{n+1} = R_F^n + \left\{ \begin{array}{c} \frac{23}{12}R_F^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}R_F^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}R_F^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\}
$$

$$
G_E^{n+1} = G_E^n + \left\{ \begin{array}{c} \frac{23}{12}G_E^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}G_E^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}G_E^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\}
$$

$$
B_E^{n+1} = B_E^n + \left\{ \begin{array}{c} \frac{23}{12}B_E^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}B_E^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}B_E^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\}
$$

$$
D^{n+1} = D^n + \left\{ \begin{array}{c} \frac{23}{12}D^*(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n)\,\Delta t \\ -\frac{4}{3}D^*\left(t_{n-1},S^{n-1},R^{n-1},R_T^{n-1},R_F^{n-1},G_E^{n-1},B_E^{n-1},D^{n-1}\right)\Delta t \\ +\frac{5}{12}D^*\left(t_{n-2},S^{n-2},R^{n-2},R_T^{n-2},R_F^{n-2},G_E^{n-2},B_E^{n-2},D^{n-2}\right)\Delta t \end{array} \right\}.
$$

5 A stochastic model of retraction with classical differentiation

In this section, we add stochastic component to our model as follows

$$
\begin{aligned}
dS(t) &= (\Lambda - \beta S(G_E + \tau B_E) + \kappa_5 B_E)\, dt + \sigma_1 S(t)\, dB_1(t) \\
dR(t) &= (\beta S(G_E + \tau B_E) - ((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2) R)\, dt + \sigma_2 R(t)\, dB_2(t) \\
dR_T(t) &= (\psi_1 \varphi_1 R - \kappa_1 R_T)\, dt + \sigma_3 R_T(t)\, dB_3(t) \\
dR_F(t) &= (\psi_2 \varphi_2 R - \kappa_2 R_F)\, dt + \sigma_4 R_F(t)\, dB_4(t) \\
dG_E(t) &= ((1-\psi_1)\varphi_1 R - \kappa_3 G_E)\, dt + \sigma_5 G_E(t)\, dB_5(t) \\
dB_E(t) &= ((1-\psi_2)\varphi_2 R - (\kappa_4 + \kappa_5) B_E)\, dt + \sigma_6 B_E(t)\, dB_6(t) \\
dD(t) &= (\kappa_1 R_T + \kappa_2 R_F + \kappa_3 G_E + \kappa_4 B_E)\, dt + \sigma_7 D(t)\, dB_7(t).
\end{aligned} \tag{60}
$$

where $B_i(t)$ and $\sigma_i, i = 1,2,3,4,5,6,7$ represents Brownian motion and density of randomness, respectively.

5.1 Extinction of retraction and B_E class

We present a discussion underpinning a possible extinction of retraction and B_E class. Let us consider the following formula [14-16]

$$\langle \gamma(t) \rangle = \frac{1}{t} \int_0^t \gamma(\tau)\, d\tau. \tag{61}$$

At the threshold R_0 for the retraction model is defined as

$$R_0 = \frac{\beta}{\left((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2 + \frac{\sigma_2^2}{2}\right)}. \tag{62}$$

Theorem. Under the condition that $R_0 > \frac{\sigma_1^2 \sigma_2^2 \sigma_3^2 \sigma_4^2 \sigma_5^2 \sigma_6^2 \sigma_7^2}{2}$ and $(S, R, R_F, R_T, G_E, B_E, D)$ represent the system solution of the retraction model, with initial condition $(S(0), R(0), R_F(0), R_T(0), G_E(0), B_E(0), D(0)) \in \mathbb{R}_+^7$. If $R_0 < 1$, then

$$\lim_{t\to\infty} \frac{\langle \log R(t) \rangle}{t} < 0, \lim_{t\to\infty} \frac{\langle \log R_F(t) \rangle}{t} < 0, \lim_{t\to\infty} \frac{\langle \log R_T(t) \rangle}{t} < 0 \text{ and } \lim_{t\to\infty} \frac{\langle \log B_E(t) \rangle}{t} < 0. \tag{63}$$

That is $R(t) \to 0$ exponentially which implies the retraction will cease with unit probability. Also

$$\begin{aligned}
\lim_{t\to\infty} \frac{1}{t}\int_0^t S(\tau)\,d\tau &= 0, \qquad (64)\\
\lim_{t\to\infty} \frac{1}{t}\int_0^t R(\tau)\,d\tau &= 0,\\
\lim_{t\to\infty} \frac{1}{t}\int_0^t R_F(\tau)\,d\tau &= 0,\\
\lim_{t\to\infty} \frac{1}{t}\int_0^t R_T(\tau)\,d\tau &= 0,\\
\lim_{t\to\infty} \frac{1}{t}\int_0^t G_E(\tau)\,d\tau &= 0,\\
\lim_{t\to\infty} \frac{1}{t}\int_0^t B_E(\tau)\,d\tau &= 0,\\
\lim_{t\to\infty} \frac{1}{t}\int_0^t D(\tau)\,d\tau &= 0.
\end{aligned}$$

Proof. To achieve our proof, we first convert the system to integral equation to obtain

$$\begin{aligned}
\frac{S(t)-S(0)}{t} &= \left(\Lambda - \beta\left\langle S\left(G_E+\tau B_E\right)\right\rangle + \kappa_5\left\langle B_E\right\rangle\right) + \frac{\sigma_1}{t}\int_0^t S(\tau)\,dB_1(\tau) \qquad (65)\\
\frac{R(t)-R(0)}{t} &= \left(\beta\left\langle S\left(G_E+\tau B_E\right)\right\rangle - \left(\left(1-\psi_1\right)\varphi_1 + \left(1-\psi_2\right)\varphi_2\right)\left\langle R\right\rangle\right) + \frac{\sigma_2}{t}\int_0^t R(\tau)\,dB_2(\tau)\\
\frac{R_T(t)-R_T(0)}{t} &= \left(\psi_1\varphi_1\left\langle R\right\rangle - \kappa_1\left\langle R_T\right\rangle\right) + \frac{\sigma_3}{t}\int_0^t R_T(\tau)\,dB_3(\tau)\\
\frac{R_F(t)-R_F(0)}{t} &= \left(\psi_2\varphi_2\left\langle R\right\rangle - \kappa_2\left\langle R_F\right\rangle\right) + \frac{\sigma_4}{t}\int_0^t R_F(\tau)\,dB_4(\tau)\\
\frac{G_E(t)-G_E(0)}{t} &= \left(\left(1-\psi_1\right)\varphi_1\left\langle R\right\rangle - \kappa_3\left\langle G_E\right\rangle\right) + \frac{\sigma_5}{t}\int_0^t G_E(\tau)\,dB_5(\tau)\\
\frac{B_E(t)-B_E(0)}{t} &= \left(\left(1-\psi_2\right)\varphi_2\left\langle R\right\rangle - \left(\kappa_4+\kappa_5\right)\left\langle B_E\right\rangle\right) + \frac{\sigma_6}{t}\int_0^t B_E(\tau)\,dB_6(\tau)\,.
\end{aligned}$$

Nevertheless, applying Ito formula on $R(t)$ class yields

$$d\log R(t) \le \beta - \left\{\left(1-\psi_1\right)\varphi_1 + \left(1-\psi_2\right)\varphi_2 + \frac{\sigma_2^2}{2}\right\} + \sigma_2 dB_2(\tau)\,. \qquad (66)$$

Integrating and dividing by t yields

$$\begin{aligned}\frac{\log R(t)-\log R(0)}{t} &= \beta-\left\{(1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2+\frac{\sigma_2^2}{2}\right\}+\frac{\sigma_2}{t}\int_0^t dB_2(\tau) \qquad (67)\\ &\leq \left\{(1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2+\frac{\sigma_2^2}{2}\right\}\left\{\frac{\beta}{(1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2+\frac{\sigma_2^2}{2}}-1\right\}\\ &\quad +\frac{\sigma_2}{t}\int_0^t dB_2(\tau)\end{aligned}$$

Noting that

$$M(t)=\frac{\sigma_2}{t}\int_0^t dB_2(\tau) \qquad (68)$$

A function that has been known to be local continuous martingale and $M(0)=0$. But by $\lim_{t\to\infty} M(t)$, we have

$$\lim_{t\to\infty}\sup\frac{M(t)}{t}=0. \qquad (69)$$

But $R_0<1$, thus

$$\lim_{t\to\infty}\sup\frac{\log R(t)}{t}\leq\left((1-\psi_1)\varphi_1+(1-\psi_2)\varphi_2+\frac{\sigma_2^2}{2}\right)(R_0-1)\leq 0. \qquad (70)$$

The above leads to

$$\lim_{t\to\infty}\langle R(t)\rangle=0. \qquad (71)$$

Then

$$\frac{R_T(t)-R_T(0)}{t}=(\psi_1\varphi_1\langle R\rangle-\kappa_1\langle R_T\rangle)+\frac{\sigma_3}{t}\int_0^t dB_3(\tau) \qquad (72)$$

and

$$\begin{aligned}\lim_{t\to\infty}\frac{R_T(t)-R_T(0)}{t} &= \lim_{t\to\infty}\psi_1\varphi_1\langle R\rangle-\lim_{t\to\infty}\kappa_1\langle R_T\rangle+\lim_{t\to\infty}\frac{\sigma_3}{t}\int_0^t dB_3(\tau) \qquad (73)\\ 0 &= \lim_{t\to\infty}\psi_1\varphi_1\langle R\rangle-\lim_{t\to\infty}\kappa_1\langle R_T\rangle.\end{aligned}$$

This implies

$$\lim_{t\to\infty}\langle R_T(t)\rangle=0. \qquad (74)$$

By using the same routine, we obtain

$$\lim_{t\to\infty}\langle R_F(t)\rangle=0, \qquad (75)$$

$$\lim_{t\to\infty}\langle G_E(t)\rangle=0, \qquad (76)$$

$$\lim_{t\to\infty}\langle B_E(t)\rangle=0, \qquad (77)$$

$$\lim_{t\to\infty}\langle S(t)\rangle=0 \qquad (78)$$

which completes the proof.

5.2 Existence of unique global positive solution

In this section, we present the existence of a unique positive solution of the suggested model.

Theorem. For the set of initial conditions $S^*(0) = (S(0), R(0), R_T(0), R_F(0), G_E(0), B_E(0), D(0)) \in \mathbb{R}^7_+$, there exists a nonnegative solution $S^*(t) = (S(t), R(t), R_T(t), R_F(t), G_E(t), B_E(t), D(t))$ of the stochastic model on $t \geq 0$ and the problem solution will maintain in $\mathbb{R}^7_+$ with unit probability.

Proof. As the coefficient of the equation are locally continuous in Lipschitz sense for the given initial size of population $(S(0), R(0), R_T(0), R_F(0), G_E(0), B_E(0), D(0)) \in \mathbb{R}^7_+$, so there must exist a unique solution (i.e. local solution) $(S(t), R(t), R_T(t), R_F(t), G_E(t), B_E(t), D(t))$ on $t \in [0, \kappa_e)$, where κ_e denote the explosion time. In order to show that actually the solution is global, one has to prove that in fact a.s. $\kappa_e = \infty$. Let us consider a positive real number l_0 and large enough so that all of the initial values of the states lie within $\left\{\frac{1}{l_0}, l_0\right\}$. Further, let us define the stopping time

$$\kappa_l = \left\{ \begin{array}{c} t \in [0, \kappa_e) : \frac{1}{l} \geq \min\{S(t), R(t), R_T(t), R_F(t), G_E(t), B_E(t), D(t)\} \\ \text{or } \max\{S(t), R(t), R_T(t), R_F(t), G_E(t), B_E(t), D(t)\} \geq l \end{array} \right\} \tag{79}$$

for each nonnegative integer l greater than or equal to l_0.

We assumed here that $\inf \phi = \infty$ whenever ϕ denotes the empty set. By looking into the definition of stopping time, one can say that κ_l is monotonically increasing $l \to \infty$. Set $\lim_{l \to \infty} \kappa_l = \kappa_\infty$ with $\kappa_e \geq \kappa_\infty$ a.s.

If for all nonnegative values of t, we show that $\kappa_\infty = \infty$ a.s. then we can say that $\kappa_e = \infty$ and a.s. $(S(t), R(t), R_T(t), R_F(t), G_E(t), B_E(t), D(t)) \in \mathbb{R}^7_+$. Thus, we have to prove that $\kappa_e = \infty$ a.s. If the conclusion is assumed to be false, then there must exist two constants $0 < T$ and $\epsilon \in (0, 1)$ such that

$$P\{T \geq \kappa_\infty\} > \epsilon. \tag{80}$$

Next, we will define a function $H : \mathbb{R}^7_+ \to \mathbb{R}_+$ from the C^2 space, such that

$$\begin{aligned} H(S, R, R_T, R_F, G_E, B_E, D) &= S + R + R_T + R_F + G_E + B_E + D - 7 \\ &\quad - (\log S + \log R + \log R_T + \log R_F + \log G_E + \log B_E + \log D). \end{aligned} \tag{81}$$

By using the fact that $\forall y > 0, y - 1 - \log y \geq 0$, one can notice that $H \geq 0$. Further assume that

$l_0 < l$ and $0 < T$ and by applying the Ito formula on above, we obtain

$$\begin{aligned}
dH\left(S,R,R_T,R_F,G_E,B_E,D\right) &= \left(1-\frac{1}{S}\right)dS+\sigma_1\left(S-1\right)dB_1\left(t\right) &(82)\\
&+\left(1-\frac{1}{R}\right)dR+\sigma_2\left(R-1\right)dB_2\left(t\right)\\
&+\left(1-\frac{1}{R_T}\right)dR_T+\sigma_3\left(R_T-1\right)dB_3\left(t\right)\\
&+\left(1-\frac{1}{R_F}\right)dR_F+\sigma_4\left(R_F-1\right)dB_4\left(t\right)\\
&+\left(1-\frac{1}{G_E}\right)dG_E+\sigma_5\left(G_E-1\right)dB_5\left(t\right)\\
&+\left(1-\frac{1}{B_E}\right)dB_E+\sigma_6\left(B_E-1\right)dB_6\left(t\right)\\
&+\left(1-\frac{1}{D}\right)dD+\sigma_7\left(D-1\right)dB_7\left(t\right) &(83)\\
&= LH\left(S,R,R_T,R_F,G_E,B_E,D\right)dt+\sigma_1\left(S-1\right)dB_1\left(t\right)\\
&\quad \sigma_2\left(R-1\right)dB_2\left(t\right)+\sigma_3\left(R_T-1\right)dB_3\left(t\right)\\
&\quad +\sigma_4\left(R_F-1\right)dB_4\left(t\right)+\sigma_5\left(G_E-1\right)dB_5\left(t\right)\\
&\quad +\sigma_6\left(B_E-1\right)dB_6\left(t\right)+\sigma_7\left(D-1\right)dB_7\left(t\right).
\end{aligned}$$

In above, $H:\mathbb{R}_+^7\to\mathbb{R}_+$ may be defined through the relation written below

$$\begin{aligned}
dH\left(S,R,R_T,R_F,G_E,B_E,D\right) &= \left(1-\frac{1}{S}\right)\left(\Lambda-\beta S\left(G_E+\tau B_E\right)+\kappa_5 B_E\right) &(84)\\
&+\left(1-\frac{1}{R}\right)\left(\begin{array}{c}\beta S\left(G_E+\tau B_E\right)\\ -\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R\end{array}\right)\\
&+\left(1-\frac{1}{R_T}\right)\left(\psi_1\varphi_1 R-\kappa_1 R_T\right)\\
&+\left(1-\frac{1}{R_F}\right)\left(\psi_2\varphi_2 R-\kappa_2 R_F\right)\\
&+\left(1-\frac{1}{G_E}\right)\left(\left(1-\psi_1\right)\varphi_1 R-\kappa_3 G_E\right)\\
&+\left(1-\frac{1}{B_E}\right)\left(\left(1-\psi_2\right)\varphi_2 R-\left(\kappa_4+\kappa_5\right)B_E\right)\\
&+\left(1-\frac{1}{D}\right)\left(\kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E\right) &(85)\\
&+\frac{\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+\sigma_5^2+\sigma_6^2+\sigma_7^2}{2}
\end{aligned}$$

and

$$
\begin{aligned}
LH\left(S,R,R_T,R_F,G_E,B_E,D\right) = {} & \Lambda+\beta\left(G_E+\tau B_E\right)+\kappa_5 B_E+\kappa_1 \\
& +\beta S\left(G_E+\tau B_E\right)+\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right) \\
& +\psi_2\varphi_2 R+\kappa_2+\left(1-\psi_1\right)\varphi_1 R+\kappa_3+\left(1-\psi_2\right)\varphi_2 R \\
& +\psi_1\varphi_1 R+\kappa_4+\kappa_5+\kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E \\
& +\left\{\begin{array}{c}
\frac{\Lambda}{S}+\beta S\left(G_E+\tau B_E\right)+\frac{\kappa_5 B_E}{S}+\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R \\
+\frac{\beta S\left(G_E+\tau B_E\right)}{R}+\psi_1\varphi_1\frac{R}{R_T}+\kappa_1 R_T+\psi_2\varphi_2\frac{R}{R_F}+\kappa_2 R_F \\
\left(1-\psi_1\right)\varphi_1\frac{R}{G_E}+\kappa_3 G_E+\left(\left(1-\psi_2\right)\varphi_2\frac{R}{B_E}+\left(\kappa_4+\kappa_5\right)B_E\right) \\
+\frac{1}{D}\left(\kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E\right)
\end{array}\right\} \\
& +\frac{\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+\sigma_5^2+\sigma_6^2+\sigma_7^2}{2} \\
\leq {} & \Lambda++\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)+\kappa_4+\kappa_5+\kappa_1+\kappa_2+\kappa_3 \\
& +\frac{\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+\sigma_5^2+\sigma_6^2+\sigma_7^2}{2}=K.
\end{aligned}
\tag{86}
$$

Here the formulation of K shows that it is positive and independent the state variables as well as independent variable. Therefore

$$
\begin{aligned}
dH\left(S,R,R_T,R_F,G_E,B_E,D\right) \leq {} & Kdt+\sigma_1\left(S-1\right)dB_1\left(t\right) \\
& +\sigma_2\left(R-1\right)dB_2\left(t\right)+\sigma_3\left(R_T-1\right)dB_3\left(t\right) \\
& +\sigma_4\left(R_F-1\right)dB_4\left(t\right)+\sigma_5\left(G_E-1\right)dB_5\left(t\right) \\
& +\sigma_6\left(B_E-1\right)dB_6\left(t\right)++\sigma_7\left(D-1\right)dB_7\left(t\right).
\end{aligned}
\tag{87}
$$

Integrating both side of above equation from 0 to $\kappa_l\wedge T$, we have

$$
\begin{aligned}
& E\left[H\left(S\left(\kappa_l\wedge T\right),R\left(\kappa_l\wedge T\right),R_T\left(\kappa_l\wedge T\right),R_F\left(\kappa_l\wedge T\right),G_E\left(\kappa_l\wedge T\right),B_E\left(\kappa_l\wedge T\right),D\left(\kappa_l\wedge T\right)\right)\right] \\
\leq {} & H\left(\left(S\left(0\right),R\left(0\right),R_T\left(0\right),R_F\left(0\right),G_E\left(0\right),B_E\left(0\right),D\left(0\right)\right)\right)+E\left[\int_0^{\kappa_l\wedge T}K\right] \\
\leq {} & H\left(\left(S\left(0\right),R\left(0\right),R_T\left(0\right),R_F\left(0\right),G_E\left(0\right),B_E\left(0\right),D\left(0\right)\right)\right)+TK.
\end{aligned}
\tag{88}
$$

Setting $\Omega_l=\{T\geq\kappa_l\}$ for $l_1\leq l$ and thus $P\left(\Omega_l\right)\geq\epsilon$. Note that for each w in Ω_l, there must exist at least one $F_c\left(\kappa_l,w\right),I\left(\kappa_l,w\right),I_P\left(\kappa_l,w\right),I_N\left(\kappa_l,w\right),R\left(\kappa_l,w\right),D\left(\kappa_l,w\right)$ which equals $\frac{1}{l}$ or l. Hence $\left(S\left(\kappa_l\right),R\left(\kappa_l\right),R_T\left(\kappa_l\right),R_F\left(\kappa_l\right),G_E\left(\kappa_l\right),B_E\left(\kappa_l\right),D\left(\kappa_l\right)\right)$ is not less than $l-\log l-1$ or $\log l-1+\frac{1}{l}$. As a result,

$$
\left(\log l-1+\frac{1}{l}\right)\wedge E\left(l-\log l-1\right)\leq H\left(S\left(\kappa_l\right),R\left(\kappa_l\right),R_T\left(\kappa_l\right),R_F\left(\kappa_l\right),G_E\left(\kappa_l\right),B_E\left(\kappa_l\right),D\left(\kappa_l\right)\right).
\tag{89}
$$

From above, we can write

$$
\begin{aligned}
& H\left(S\left(0\right),R\left(0\right),R_T\left(0\right),R_F\left(0\right),G_E\left(0\right),B_E\left(0\right),D\left(0\right)\right)+TK \\
\geq {} & E\left[1_{\Omega_w}H\left(S\left(\kappa_l\right),R\left(\kappa_l\right),R_T\left(\kappa_l\right),R_F\left(\kappa_l\right),G_E\left(\kappa_l\right),B_E\left(\kappa_l\right),D\left(\kappa_l\right)\right)\right] \\
\geq {} & \epsilon\left[\left(l-\log l-1\right)\wedge\left(\log l-1+\frac{1}{l}\right)\right].
\end{aligned}
\tag{90}
$$

Here the notation 1_{Ω_w} represents the indicator function of Ω. By letting $l \to \infty$ will lead to the contradiction $\infty > H\left(S\left(0\right), R\left(0\right), R_T\left(0\right), R_F\left(0\right), G_E\left(0\right), B_E\left(0\right), D\left(0\right)\right) + TK = \infty$, which implies that $\kappa_\infty = \infty$ a.s. and the completes the proof.

6 Optimal control for retraction model

In this section, we present optimality conditions for retraction model by using the Pontryagin's maximum principle [19]. It is reasonable to first talk about the goals before adding the control functions to the proposed model. Although we do not want any researcher's article to be retracted, but if there is truly a scientific mistake identified, retraction should be inevitable. What should be considered is the real reason behind the retracted article. In short, if there will be a retraction of any article, the decision should be made fairly. Although we do not want to see any article being retracted, we at least don't want to see wrongly retracted articles and malicious editors. Well-intentioned editors and at least fairly withdrawn articles will increase the trust in publishers and encourage authors to put in their efforts to produce better and quality publications.

To present the retraction model with control function, four possible control strategies are added to our model. The control variable u_1 is the reconsideration by objective reviewers, u_2 describes a second for the retracted paper to be reconsidered by another journal or publisher. The control variable u_3 stands for the fairness of the editorial board. The control variable u_4 is the extra opportunity given by the good editors for authors to defend themselves.

Our model with control functions can be modified as follows

$$\begin{aligned}
\dot{S}\left(t\right) &= \Lambda - \beta S\left(G_E + \tau B_E\right) + \kappa_5 B_E + u_2 D + u_4 R \qquad (91)\\
\dot{R}\left(t\right) &= \beta S\left(G_E + \tau B_E\right) - \left(1-\psi_1\right)\varphi_1 R - \left(1-\psi_2\right)\varphi_2 R - u_4 R\\
\dot{R_T}\left(t\right) &= \psi_1\varphi_1 R - \kappa_1 R_T - u_1 R\\
\dot{R_F}\left(t\right) &= \psi_2\varphi_2 R - \kappa_2 R_F + u_1 R + u_1 R\\
\dot{G_E}\left(t\right) &= \left(1-\psi_1\right)\varphi_1 R - \kappa_3 G_E + u_3 B_E\\
\dot{B_E}\left(t\right) &= \left(1-\psi_2\right)\varphi_2 R - \left(\kappa_4 + \kappa_5\right) B_E - u_3 B_E\\
\dot{D}\left(t\right) &= \kappa_1 R_T + \kappa_2 R_F + \kappa_3 G_E + \kappa_4 B_E - u_2 D.
\end{aligned}$$

The objective functional can be represented by;

$$\min_{(u_1,u_2,u_3,u_4)\in U} J\left(u_1,u_2,u_3,u_4\right) = \int_0^T \left(\begin{array}{c} c_1 R + c_2 B_E + c_3 R_T - c_4 G_E \\ +k_1 u_1^2 + k_2 u_2^2 + k_3 u_3^2 + k_4 u_4^2 \end{array}\right) dt \qquad (92)$$

on the set

$$U = \left\{\begin{array}{c} \left(u_1,u_2,u_3,u_4\right) \in L^\infty\left(0,T\right) \times L^\infty\left(0,T\right) \times L^\infty\left(0,T\right) \times L^\infty\left(0,T\right): \\ 0 \le u_1\left(t\right) \le \widetilde{u}_1, 0 \le u_2\left(t\right) \le \widetilde{u}_2, 0 \le u_3\left(t\right) \le \widetilde{u}_3, 0 \le u_4\left(t\right) \le \widetilde{u}_4, \end{array}\right\}. \qquad (93)$$

The parameters $c_1, c_2, c_3, c_4, k_1, k_2, k_3, k_4$ are the weighted parameters. The existence of the control functions[20] can be satisfied under the following conditions:

- The set of U is nonempty, convex, bounded and closed.
- The Lipschitz property of the state system is hold.
- The integrand of objective functional with respect to the controls is convex on the set U.

By the help of Pontryagin's Maximum Principle, we can write the Hamiltonian H given by

$$\begin{gathered}H=k_1u_1^2+k_2u_2^2+k_3u_3^2+k_4u_4^2+c_1R+c_2B_E+c_3R_T-c_4G_E\\+\lambda_1\left(\Lambda-\beta S\left(G_E+\tau B_E\right)+\kappa_5B_E+u_2D+u_4R\right)\\+\lambda_2\left(\beta S\left(G_E+\tau B_E\right)-\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R-u_4R\right)\\+\lambda_3\left(\psi_1\varphi_1R-\kappa_1R_T-u_1R\right)\\+\lambda_4\left(\psi_2\varphi_2R-\kappa_2R_F+u_1R\right)\\+\lambda_5\left(\left(1-\psi_1\right)\varphi_1R-\kappa_3G_E+u_3B_E\right)\\+\lambda_6\left(\left(1-\psi_2\right)\varphi_2R-\left(\kappa_4+\kappa_5\right)B_E-u_3B_E\right)\\+\lambda_7\left(\kappa_1R_T+\kappa_2R_F+\kappa_3G_E+\kappa_4B_E-u_2D\right).\end{gathered}$$

Then, we have the following necessary conditions

$$\begin{aligned}\frac{d\lambda_1}{dt}&=-\frac{\partial H}{\partial S}=-\left\{\left(\lambda_2-\lambda_1\right)\beta\left(G_E+\tau B_E\right)\right\}\\\frac{d\lambda_2}{dt}&=-\frac{\partial H}{\partial R}=-\left\{\begin{array}{c}\left(\lambda_1-\lambda_2\right)u_4+\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)\\c_1++\lambda_3\psi_1\varphi_1+\lambda_4\psi_2\varphi_2+\left(\lambda_4-\lambda_3\right)u_1\\+\lambda_5\left(1-\psi_1\right)\varphi_1+\lambda_6\left(1-\psi_2\right)\varphi_2\end{array}\right\}\\\frac{d\lambda_3}{dt}&=-\frac{\partial H}{\partial R_T}=-\left\{c_3-\lambda_3\kappa_1+\lambda_7\kappa_1\right\}\\\frac{d\lambda_4}{dt}&=-\frac{\partial H}{\partial R_F}=-\left\{-\lambda_4\kappa_2R_F+\lambda_7\kappa_2\right\}\\\frac{d\lambda_5}{dt}&=-\frac{\partial H}{\partial G_E}=-\left\{-c_4+\left(\lambda_2-\lambda_1\right)\beta S-\lambda_5\kappa_3+\lambda_7\kappa_3\right\}\\\frac{d\lambda_6}{dt}&=-\frac{\partial H}{\partial B_E}=-\left\{\begin{array}{c}c_2+\left(\lambda_2-\lambda_1\right)\beta S\tau+\lambda_5u_3\\-\lambda_6\left(\kappa_4+\kappa_5+u_3\right)+\lambda_7\kappa_4\end{array}\right\}\\\frac{d\lambda_7}{dt}&=-\frac{\partial H}{\partial D}=-\left\{\lambda_1u_2D-\lambda_7u_2\right\}\end{aligned}\tag{94}$$

with the transversality conditions $\lambda_k\left(t_f\right)=0$ for $k=1,2,3,4,5,6,7$ and control variables are given by

$$\begin{aligned}
u_1 &= \frac{R(t)(\lambda_3-\lambda_4)}{2k_1}\\
u_2 &= \frac{D(t)(\lambda_7-\lambda_1)}{2k_2}\\
u_3 &= \frac{B_E(t)(\lambda_6-\lambda_5)}{2k_3}\\
u_4 &= \frac{R(t)(\lambda_2-\lambda_1)}{2k_4}. \qquad (95)
\end{aligned}$$

Thus, the optimality conditions are given by

$$\begin{aligned}
u_1^* &= \min\left\{\widetilde{u}_1, \max\left\{0, \frac{R(t)(\lambda_3-\lambda_4)}{2k_1}\right\}\right\}\\
u_2^* &= \min\left\{\widetilde{u}_2, \max\left\{0, \frac{D(t)(\lambda_7-\lambda_1)}{2k_2}\right\}\right\}\\
u_3^* &= \min\left\{\widetilde{u}_3, \max\left\{0, \frac{B_E(t)(\lambda_6-\lambda_5)}{2k_3}\right\}\right\}\\
u_4^* &= \min\left\{\widetilde{u}_4, \max\left\{0, \frac{R(t)(\lambda_2-\lambda_1)}{2k_4}\right\}\right\}. \qquad (96)
\end{aligned}$$

7 Numerical solution of the model with classical derivative

We will now present Atangana-Seda scheme to solve for the suggested mathematical model for different differential operators. We start with classical case for numerical solution of retraction model

$$\begin{aligned}
\frac{d}{dt}S(t) &= \Lambda - \beta S(G_E+\tau B_E) + \kappa_5 B_E + \sigma_1 G_1(t,S) B_1\prime(t) \qquad (97)\\
\frac{d}{dt}R(t) &= \beta S(G_E+\tau B_E) - ((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2) R + \sigma_2 G_2(t,R) B_2\prime(t)\\
\frac{d}{dt}R_T(t) &= \psi_1\varphi_1 R - \kappa_1 R_T + \sigma_3 G_3(t,R_T) B_3\prime(t)\\
\frac{d}{dt}R_F(t) &= \psi_2\varphi_2 R - \kappa_2 R_F + \sigma_4 G_4(t,R_F) B_4\prime(t)\\
\frac{d}{dt}G_E(t) &= (1-\psi_1)\varphi_1 R - \kappa_3 G_E + \sigma_5 G_5(t,G_E) B_5\prime(t)\\
\frac{d}{dt}B_E(t) &= (1-\psi_2)\varphi_2 R - (\kappa_4+\kappa_5) B_E + \sigma_6 G_6(t,B_E) B_6\prime(t)\\
\frac{d}{dt}D(t) &= \kappa_1 R_T + \kappa_2 R_F + \kappa_3 G_E + \kappa_4 B_E + \sigma_7 G_7(t,D) B_7\prime(t).
\end{aligned}$$

After integrating above and putting Newton polynomial into these equations, we can solve our model as follows

$$
\begin{aligned}
S^{n+1} &= S^n + \left\{ \begin{array}{c} \frac{23}{12} S^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} S^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} S^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\} \\
&\quad +\sigma_1 \left\{ \begin{array}{c} \frac{5}{12} G_1 \left(t_{n-2}, S\right) \left(B_1 \left(t_{n-1}\right) - B_1 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_1 \left(t_{n-1}, S\right) \left(B_1 \left(t_n\right) - B_1 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_1 \left(t_n, S\right) \left(B_1 \left(t_{n+1}\right) - B_1 \left(t_n\right)\right) \end{array} \right\}
\end{aligned} \tag{98}
$$

$$
\begin{aligned}
R^{n+1} &= R^n + \left\{ \begin{array}{c} \frac{23}{12} R^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} R^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} R^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\} \\
&\quad +\sigma_2 \left\{ \begin{array}{c} \frac{5}{12} G_2 \left(t_{n-2}, R^{n-2}\right) \left(B_2 \left(t_{n-1}\right) - B_2 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_2 \left(t_{n-1}, R^{n-1}\right) \left(B_2 \left(t_n\right) - B_2 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_2 \left(t_n, R^n\right) \left(B_2 \left(t_{n+1}\right) - B_2 \left(t_n\right)\right) \end{array} \right\}
\end{aligned}
$$

$$
\begin{aligned}
R_T^{n+1} &= R_T^n + \left\{ \begin{array}{c} \frac{23}{12} R_T^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} R_T^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} R_T^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\} \\
&\quad +\sigma_3 \left\{ \begin{array}{c} \frac{5}{12} G_3 \left(t_{n-2}, R_T^{n-2}\right) \left(B_3 \left(t_{n-1}\right) - B_3 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_3 \left(t_{n-1}, R_T^{n-1}\right) \left(B_3 \left(t_n\right) - B_3 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_3 \left(t_n, R_T^n\right) \left(B_3 \left(t_{n+1}\right) - B_3 \left(t_n\right)\right) \end{array} \right\}
\end{aligned}
$$

$$
\begin{aligned}
R_F^{n+1} = R_F^n \quad &+ \left\{ \begin{array}{c} \frac{23}{12} R_F^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} R_F^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} R_F^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\} \\
&+\sigma_4 \left\{ \begin{array}{c} \frac{5}{12} G_4 \left(t_{n-2}, R_F^{n-2}\right) \left(B_4 \left(t_{n-1}\right) - B_4 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_4 \left(t_{n-1}, R_F^{n-1}\right) \left(B_4 \left(t_n\right) - B_4 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_4 \left(t_n, R_F^n\right) \left(B_4 \left(t_{n+1}\right) - B_4 \left(t_n\right)\right) \end{array} \right\}
\end{aligned}
$$

$$
\begin{aligned}
G_E^{n+1} &= G_E^n + \left\{ \begin{array}{c} \frac{23}{12} G_E^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} G_E^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} G_E^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\} \\
&\quad +\sigma_5 \left\{ \begin{array}{c} \frac{5}{12} G_5 \left(t_{n-2}, G_E^{n-2}\right) \left(B_5 \left(t_{n-1}\right) - B_5 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_5 \left(t_{n-1}, G_E^{n-1}\right) \left(B_5 \left(t_n\right) - B_5 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_5 \left(t_n, G_E^n\right) \left(B_5 \left(t_{n+1}\right) - B_5 \left(t_n\right)\right) \end{array} \right\}
\end{aligned}
$$

$$B_E^{n+1} = B_E^n + \left\{ \begin{array}{c} \frac{23}{12} B_E^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} B_E^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} B_E^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\}$$
$$+\sigma_6 \left\{ \begin{array}{c} \frac{5}{12} G_6 \left(t_{n-2}, B_E^{n-2}\right) \left(B_6 \left(t_{n-1}\right) - B_6 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_6 \left(t_{n-1}, B_E^{n-1}\right) \left(B_6 \left(t_n\right) - B_6 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_6 \left(t_n, B_E^n\right) \left(B_6 \left(t_{n+1}\right) - B_6 \left(t_n\right)\right) \end{array} \right\}$$

$$D^{n+1} = D^n + \left\{ \begin{array}{c} \frac{23}{12} D^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \Delta t \\ -\frac{4}{3} D^* \left(t_{n-1}, S^{n-1}, R^{n-1}, R_T^{n-1}, R_F^{n-1}, G_E^{n-1}, B_E^{n-1}, D^{n-1}\right) \Delta t \\ +\frac{5}{12} D^* \left(t_{n-2}, S^{n-2}, R^{n-2}, R_T^{n-2}, R_F^{n-2}, G_E^{n-2}, B_E^{n-2}, D^{n-2}\right) \Delta t \end{array} \right\}$$
$$+\sigma_7 \left\{ \begin{array}{c} \frac{5}{12} G_7 \left(t_{n-2}, D^{n-2}\right) \left(B_7 \left(t_{n-1}\right) - B_7 \left(t_{n-2}\right)\right) \\ -\frac{4}{3} G_7 \left(t_{n-1}, D^{n-1}\right) \left(B_7 \left(t_n\right) - B_7 \left(t_{n-1}\right)\right) \\ +\frac{23}{12} G_7 \left(t_{n-1}, D^{n-1}\right) \left(B_7 \left(t_{n+1}\right) - B_7 \left(t_n\right)\right) \end{array} \right\}.$$

8 Numerical solution of the model with the Atangana-Baleanu fractional derivative

To add into the mathematical model of retraction an effect of non-locality, especially a crossover behavior from stretched exponential to power law, the time derivative in the classical model is converted to the Atangana-Baleanu fractional derivative. The analysis of existence and uniqueness of the system solutions for this model will not be presented here. However, we will only present a numerical solution of the model using numerical method based on the step-Newton polynomial interpolation that was suggested by Atangana and Seda.

$$\begin{aligned} {}_0^{AB}D_t^\alpha S(t) &= \left(\Lambda - \beta S\left(G_E + \tau B_E\right) + \kappa_5 B_E\right) \\ {}_0^{AB}D_t^\alpha R(t) &= \left(\beta S\left(G_E + \tau B_E\right) - \left(\left(1-\psi_1\right)\varphi_1 + \left(1-\psi_2\right)\varphi_2\right) R\right) \\ {}_0^{AB}D_t^\alpha R_T(t) &= \left(\psi_1 \varphi_1 R - \kappa_1 R_T\right) \\ {}_0^{AB}D_t^\alpha R_F(t) &= \left(\psi_2 \varphi_2 R - \kappa_2 R_F\right) \\ {}_0^{AB}D_t^\alpha G_E(t) &= \left(\left(1-\psi_1\right)\varphi_1 R - \kappa_3 G_E\right) \\ {}_0^{AB}D_t^\alpha B_E(t) &= \left(\left(1-\psi_2\right)\varphi_2 R - \left(\kappa_4 + \kappa_5\right) B_E\right) \\ {}_0^{AB}D_t^\alpha D(t) &= \left(\kappa_1 R_T + \kappa_2 R_F + \kappa_3 G_E + \kappa_4 B_E\right). \end{aligned} \tag{99}$$

Above system can be solved by the following numerical scheme based on Newton polynomial

$$\begin{aligned} S^{n+1} &= \frac{1-\alpha}{AB(\alpha)} S^* \left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \\ &+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} S^* \left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \times \Pi \end{aligned} \tag{100}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} S^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ -S^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} S^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right) \\ -2S^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ +S^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Delta$$

$$\begin{aligned} R^{n+1} &= \frac{1-\alpha}{AB\left(\alpha\right)}R^{*}\left(t_{n},S^{n},R^{n},R_{T}^{n},R_{F}^{n},G_{E}^{n},B_{E}^{n},D^{n}\right) \\ &+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}R^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\times\Pi \end{aligned}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} R^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ -R^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} R^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right) \\ -2R^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ +R^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Delta$$

$$\begin{aligned} R_{T}^{n+1} &= \frac{1-\alpha}{AB\left(\alpha\right)}R_{T}^{*}\left(t_{n},S^{n},R^{n},R_{T}^{n},R_{F}^{n},G_{E}^{n},B_{E}^{n},D^{n}\right) \\ &+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}R_{T}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\times\Pi \end{aligned}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} R_{T}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ -R_{T}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} R_{T}^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right) \\ -2R_{T}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ +R_{T}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Delta$$

$$\begin{aligned} R_{F}^{n+1} &= \frac{1-\alpha}{AB\left(\alpha\right)}R_{F}^{*}\left(t_{n},S^{n},R^{n},R_{T}^{n},R_{F}^{n},G_{E}^{n},B_{E}^{n},D^{n}\right) \\ &+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}R_{F}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\times\Pi \end{aligned}$$

$$+\frac{\alpha(\Delta t)^{\alpha}}{AB(\alpha)\Gamma(\alpha+2)}\sum_{j=2}^{n}\left[\begin{array}{c} R_F^*\left(t_{j-1},S^{j-1},R^{j-1},R_T^{j-1},R_F^{j-1},G_E^{j-1},B_E^{j-1},D^{j-1}\right)\\ -R_F^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha(\Delta t)^{\alpha}}{2AB(\alpha)\Gamma(\alpha+3)}\sum_{j=2}^{n}\left[\begin{array}{c} R_F^*\left(t_j,S^j,R^j,R_T^j,R_F^j,G_E^j,B_E^j,D^j\right)\\ -2R_F^*\left(t_{j-1},S^{j-1},R^{j-1},R_T^{j-1},R_F^{j-1},G_E^{j-1},B_E^{j-1},D^{j-1}\right)\\ +R_F^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\end{array}\right]\times\Delta$$

$$\begin{aligned} G_E^{n+1} &= \frac{1-\alpha}{AB(\alpha)}G_E^*\left(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n\right)\\ &\quad+\frac{\alpha(\Delta t)^{\alpha}}{AB(\alpha)\Gamma(\alpha+1)}\sum_{j=2}^{n}G_E^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\times\Pi\end{aligned}$$

$$+\frac{\alpha(\Delta t)^{\alpha}}{AB(\alpha)\Gamma(\alpha+2)}\sum_{j=2}^{n}\left[\begin{array}{c} G_E^*\left(t_{j-1},S^{j-1},R^{j-1},R_T^{j-1},R_F^{j-1},G_E^{j-1},B_E^{j-1},D^{j-1}\right)\\ -G_E^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha(\Delta t)^{\alpha}}{2AB(\alpha)\Gamma(\alpha+3)}\sum_{j=2}^{n}\left[\begin{array}{c} G_E^*\left(t_j,S^j,R^j,R_T^j,R_F^j,G_E^j,B_E^j,D^j\right)\\ -2G_E^*\left(t_{j-1},S^{j-1},R^{j-1},R_T^{j-1},R_F^{j-1},G_E^{j-1},B_E^{j-1},D^{j-1}\right)\\ +G_E^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\end{array}\right]\times\Delta$$

$$\begin{aligned} B_E^{n+1} &= \frac{1-\alpha}{AB(\alpha)}B_E^*\left(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n\right)\\ &\quad+\frac{\alpha(\Delta t)^{\alpha}}{AB(\alpha)\Gamma(\alpha+1)}\sum_{j=2}^{n}B_E^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\times\Pi\end{aligned}$$

$$+\frac{\alpha(\Delta t)^{\alpha}}{AB(\alpha)\Gamma(\alpha+2)}\sum_{j=2}^{n}\left[\begin{array}{c} B_E^*\left(t_{j-1},S^{j-1},R^{j-1},R_T^{j-1},R_F^{j-1},G_E^{j-1},B_E^{j-1},D^{j-1}\right)\\ -B_E^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha(\Delta t)^{\alpha}}{2AB(\alpha)\Gamma(\alpha+3)}\sum_{j=2}^{n}\left[\begin{array}{c} B_E^*\left(t_j,S^j,R^j,R_T^j,R_F^j,G_E^j,B_E^j,D^j\right)\\ -2B_E^*\left(t_{j-1},S^{j-1},R^{j-1},R_T^{j-1},R_F^{j-1},G_E^{j-1},B_E^{j-1},D^{j-1}\right)\\ +B_E^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\end{array}\right]\times\Delta$$

$$\begin{aligned} D^{n+1} &= \frac{1-\alpha}{AB(\alpha)}D^*\left(t_n,S^n,R^n,R_T^n,R_F^n,G_E^n,B_E^n,D^n\right)\\ &\quad+\frac{\alpha(\Delta t)^{\alpha}}{AB(\alpha)\Gamma(\alpha+1)}\sum_{j=2}^{n}D^*\left(t_{j-2},S^{j-2},R^{j-2},R_T^{j-2},R_F^{j-2},G_E^{j-2},B_E^{j-2},D^{j-2}\right)\times\Pi\end{aligned}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} D^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ -D^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} D^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right) \\ -2D^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right) \\ +D^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right) \end{array}\right]\times\Delta$$

where

$$\begin{aligned}
\Pi &= \left[(n-j+1)^{\alpha}-(n-j)^{\alpha}\right], \qquad (101)\\
\Sigma &= \left[\begin{array}{c} (n-j+1)^{\alpha}(n-j+3+2\alpha) \\ -(n-j)^{\alpha}(n-j+3+3\alpha) \end{array}\right],\\
\Delta &= \left[\begin{array}{c} (n-j+1)^{\alpha}\left[\begin{array}{c} 2(n-j)^{2}+(3\alpha+10)(n-j) \\ +2\alpha^{2}+9\alpha+12 \end{array}\right] \\ -(n-j)^{\alpha}\left[\begin{array}{c} 2(n-j)^{2}+(5\alpha+10)(n-j) \\ +6\alpha^{2}+18\alpha+12 \end{array}\right] \end{array}\right].
\end{aligned}$$

9 Numerical solution of the fractional-stochastic model with the Atangana-Baleanu fractional derivative

More complex nonlocality could be added to the model. While the classical model predicts the future using only the initial condition and the model generator that is driven by the exponential function, such a model is known to be Markovian as it does not take into account memory effect [1,2]. Although the model containing the Atangana-Baleanu fractional derivative takes into account crossover effect, randomness is not considered here. To include into our model randomness, we add a stochastic component to the model with Atangana-Baleanu derivative [17]. Again, no different analysis will be done here, only, we will provide a numerical solution using a numerical scheme based on the Newton polynomial interpolation [18].

$$\begin{aligned}
{}_{0}^{AB}D_{t}^{\alpha}S(t) &= \left(\Lambda-\beta S\left(G_{E}+\tau B_{E}\right)+\kappa_{5}B_{E}\right)+\sigma_{1}S(t)\,B_{1}\prime(t) \qquad (102)\\
{}_{0}^{AB}D_{t}^{\alpha}R(t) &= \left(\beta S\left(G_{E}+\tau B_{E}\right)-\left(\left(1-\psi_{1}\right)\varphi_{1}+\left(1-\psi_{2}\right)\varphi_{2}\right)R\right)+\sigma_{2}R(t)\,B_{2}\prime(t)\\
{}_{0}^{AB}D_{t}^{\alpha}R_{T}(t) &= \left(\psi_{1}\varphi_{1}R-\kappa_{1}R_{T}\right)+\sigma_{3}R_{T}(t)\,B_{3}\prime(t)\\
{}_{0}^{AB}D_{t}^{\alpha}R_{F}(t) &= \left(\psi_{2}\varphi_{2}R-\kappa_{2}R_{F}\right)+\sigma_{4}R_{F}(t)\,B_{4}\prime(t)\\
{}_{0}^{AB}D_{t}^{\alpha}G_{E}(t) &= \left(\left(1-\psi_{1}\right)\varphi_{1}R-\kappa_{3}G_{E}\right)+\sigma_{5}G_{E}(t)\,B_{5}\prime(t)\\
{}_{0}^{AB}D_{t}^{\alpha}B_{E}(t) &= \left(\left(1-\psi_{2}\right)\varphi_{2}R-\left(\kappa_{4}+\kappa_{5}\right)B_{E}\right)+\sigma_{6}B_{E}(t)\,B_{6}\prime(t)\\
{}_{0}^{AB}D_{t}^{\alpha}D(t) &= \left(\left(1-\psi_{2}\right)\varphi_{2}R-\left(\kappa_{4}+\kappa_{5}\right)B_{E}\right)+\sigma_{7}D(t)\,B_{7}\prime(t).
\end{aligned}$$

The following numerical scheme with Newton polynomial is given by

$$
\begin{aligned}
S^{n+1} &= \frac{1-\alpha}{AB(\alpha)} S^*\left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \\
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} S^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \times \Pi \\
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} \sigma_1 G_1\left(t_{j-2}, S\right)\left(B_1\left(t_{j-1}\right) - B_1\left(t_{j-2}\right)\right) \times \Pi
\end{aligned} \tag{103}
$$

$$
\begin{aligned}
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{j=2}^{n} \left[\begin{array}{c} \sigma_1 G_1\left(t_{j-1}, S\right)\left(B_1\left(t_j\right) - B_1\left(t_{j-1}\right)\right) \\ -\sigma_1 G_1\left(t_{j-2}, S\right)\left(B_1\left(t_{j-1}\right) - B_1\left(t_{j-2}\right)\right) \end{array} \right] \times \Sigma \\
&+ \frac{\alpha (\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{j=2}^{n} \left[\begin{array}{c} \sigma_1 G_1\left(t_j, S\right)\left(B_1\left(t_{j-1}\right) - B_1\left(t_j\right)\right) \\ -2\sigma_1 G_1\left(t_{j-1}, S\right)\left(B_1\left(t_j\right) - B_1\left(t_{j-1}\right)\right) \\ +\sigma_1 G_1\left(t_{j-2}, S\right)\left(B_1\left(t_{j-1}\right) - B_1\left(t_{j-2}\right)\right) \end{array} \right] \times \Delta
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{j=2}^{n} \left[\begin{array}{c} S^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ -S^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Sigma \\
&+ \frac{\alpha (\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{j=2}^{n} \left[\begin{array}{c} S^*\left(t_j, S^j, R^j, R_T^j, R_F^j, G_E^j, B_E^j, D^j\right) \\ -2S^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ +S^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Delta
\end{aligned}
$$

$$
\begin{aligned}
R^{n+1} &= \frac{1-\alpha}{AB(\alpha)} R^*\left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \\
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} R^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \times \Pi \\
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} \sigma_2 G_2\left(t_{j-2}, R^{j-2}\right)\left(B_2\left(t_{j-1}\right) - B_2\left(t_{j-2}\right)\right) \times \Pi
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{j=2}^{n} \left[\begin{array}{c} R^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ -R^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Sigma \\
&+ \frac{\alpha (\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{j=2}^{n} \left[\begin{array}{c} R^*\left(t_j, S^j, R^j, R_T^j, R_F^j, G_E^j, B_E^j, D^j\right) \\ -2R^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ +R^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Delta
\end{aligned}
$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c}\sigma_{2}G_{2}\left(t_{j-1},R^{j-1}\right)\left(B_{2}\left(t_{j}\right)-B_{2}\left(t_{j-1}\right)\right)\\-\sigma_{2}G_{2}\left(t_{j-2},R^{j-2}\right)\left(B_{2}\left(t_{j-1}\right)-B_{2}\left(t_{j-2}\right)\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c}\sigma_{2}G_{2}\left(t_{j},R^{j}\right)\left(B_{2}\left(t_{j-1}\right)-B_{2}\left(t_{j}\right)\right)\\-2\sigma_{2}G_{2}\left(t_{j-1},R^{j-1}\right)\left(B_{2}\left(t_{j}\right)-B_{2}\left(t_{j-1}\right)\right)\\+\sigma_{2}G_{2}\left(t_{j-2},R^{j-2}\right)\left(B_{2}\left(t_{j-1}\right)-B_{2}\left(t_{j-2}\right)\right)\end{array}\right]\times\Delta$$

$$\begin{aligned}R_{T}^{n+1} &= \frac{1-\alpha}{AB\left(\alpha\right)}R_{T}^{*}\left(t_{n},S^{n},R^{n},R_{T}^{n},R_{F}^{n},G_{E}^{n},B_{E}^{n},D^{n}\right)\\&+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}R_{T}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\times\Pi\\&+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}\sigma_{3}G_{3}\left(t_{j-2},R_{T}^{j-2}\right)\left(B_{3}\left(t_{j-1}\right)-B_{3}\left(t_{j-2}\right)\right)\times\Pi\end{aligned}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c}R_{T}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right)\\-R_{T}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c}R_{T}^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right)\\-2R_{T}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right)\\+R_{T}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\end{array}\right]\times\Delta$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c}\sigma_{3}G_{3}\left(t_{j-1},R_{T}^{j-1}\right)\left(B_{3}\left(t_{j}\right)-B_{3}\left(t_{j-1}\right)\right)\\-\sigma_{3}G_{3}\left(t_{j-2},R_{T}^{j-2}\right)\left(B_{3}\left(t_{j-1}\right)-B_{3}\left(t_{j-2}\right)\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c}\sigma_{3}G_{3}\left(t_{j},R_{T}^{j}\right)\left(B_{3}\left(t_{j-1}\right)-B_{3}\left(t_{j}\right)\right)\\-2\sigma_{3}G_{3}\left(t_{j-1},R_{T}^{j-1}\right)\left(B_{3}\left(t_{j}\right)-B_{3}\left(t_{j-1}\right)\right)\\+\sigma_{3}G_{3}\left(t_{j-2},R_{T}^{j-2}\right)\left(B_{3}\left(t_{j-1}\right)-B_{3}\left(t_{j-2}\right)\right)\end{array}\right]\times\Delta$$

$$\begin{aligned}R_{F}^{n+1} &= \frac{1-\alpha}{AB\left(\alpha\right)}R_{F}^{*}\left(t_{n},S^{n},R^{n},R_{T}^{n},R_{F}^{n},G_{E}^{n},B_{E}^{n},D^{n}\right)\\&+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}R_{F}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\times\Pi\\&+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}\sigma_{4}G_{4}\left(t_{j-2},R_{F}^{j-2}\right)\left(B_{4}\left(t_{j-1}\right)-B_{4}\left(t_{j-2}\right)\right)\times\Pi\end{aligned}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} R_{F}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right)\\ -R_{F}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} R_{F}^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right)\\ -2R_{F}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right)\\ +R_{F}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\end{array}\right]\times\Delta$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} \sigma_{4}G_{4}\left(t_{j-1},R_{F}^{j-1}\right)\left(B_{4}\left(t_{j}\right)-B_{4}\left(t_{j-1}\right)\right)\\ -\sigma_{4}G_{4}\left(t_{j-2},R_{F}^{j-2}\right)\left(B_{4}\left(t_{j-1}\right)-B_{4}\left(t_{j-2}\right)\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} \sigma_{4}G_{4}\left(t_{j},R_{F}^{j}\right)\left(B_{4}\left(t_{j-1}\right)-B_{4}\left(t_{j}\right)\right)\\ -2\sigma_{4}G_{4}\left(t_{j-1},R_{F}^{j-1}\right)\left(B_{4}\left(t_{j}\right)-B_{4}\left(t_{j-1}\right)\right)\\ +\sigma_{4}G_{4}\left(t_{j-2},R_{F}^{j-2}\right)\left(B_{4}\left(t_{j-1}\right)-B_{4}\left(t_{j-2}\right)\right)\end{array}\right]\times\Delta$$

$$\begin{aligned} G_{E}^{n+1} &= \frac{1-\alpha}{AB\left(\alpha\right)}G_{E}^{*}\left(t_{n},S^{n},R^{n},R_{T}^{n},R_{F}^{n},G_{E}^{n},B_{E}^{n},D^{n}\right)\\ &+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}G_{E}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\times\Pi\\ &+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+1\right)}\sum_{j=2}^{n}\sigma_{5}G_{5}\left(t_{j-2},G_{E}^{j-2}\right)\left(B_{5}\left(t_{j-1}\right)-B_{5}\left(t_{j-2}\right)\right)\times\Pi\end{aligned}$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} G_{E}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right)\\ -G_{E}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} G_{E}^{*}\left(t_{j},S^{j},R^{j},R_{T}^{j},R_{F}^{j},G_{E}^{j},B_{E}^{j},D^{j}\right)\\ -2G_{E}^{*}\left(t_{j-1},S^{j-1},R^{j-1},R_{T}^{j-1},R_{F}^{j-1},G_{E}^{j-1},B_{E}^{j-1},D^{j-1}\right)\\ +G_{E}^{*}\left(t_{j-2},S^{j-2},R^{j-2},R_{T}^{j-2},R_{F}^{j-2},G_{E}^{j-2},B_{E}^{j-2},D^{j-2}\right)\end{array}\right]\times\Delta$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c} \sigma_{5}G_{5}\left(t_{j-1},G_{E}^{j-1}\right)\left(B_{5}\left(t_{j}\right)-B_{5}\left(t_{j-1}\right)\right)\\ -\sigma_{5}G_{5}\left(t_{j-2},G_{E}^{j-2}\right)\left(B_{5}\left(t_{j-1}\right)-B_{5}\left(t_{j-2}\right)\right)\end{array}\right]\times\Sigma$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c} \sigma_{5}G_{5}\left(t_{j},G_{E}^{j}\right)\left(B_{5}\left(t_{j-1}\right)-B_{5}\left(t_{j}\right)\right)\\ -2\sigma_{5}G_{5}\left(t_{j-1},G_{E}^{j-1}\right)\left(B_{5}\left(t_{j}\right)-B_{5}\left(t_{j-1}\right)\right)\\ +\sigma_{5}G_{5}\left(t_{j-2},G_{E}^{j-2}\right)\left(B_{5}\left(t_{j-1}\right)-B_{2}\left(t_{j-2}\right)\right)\end{array}\right]\times\Delta$$

$$
\begin{aligned}
B_E^{n+1} \quad = \quad & \frac{1-\alpha}{AB(\alpha)} B_E^*\left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \\
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} B_E^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \times \Pi \\
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} \sigma_6 G_6\left(t_{j-2}, B_E^{j-2}\right)\left(B_6(t_{j-1}) - B_6(t_{j-2})\right) \times \Pi
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{j=2}^{n} \left[\begin{array}{c} B_E^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ -B_E^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Sigma \\
& + \frac{\alpha (\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{j=2}^{n} \left[\begin{array}{c} B_E^*\left(t_j, S^j, R^j, R_T^j, R_F^j, G_E^j, B_E^j, D^j\right) \\ -2B_E^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ +B_E^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Delta
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{j=2}^{n} \left[\begin{array}{c} \sigma_6 G_6\left(t_{j-1}, B_E^{j-1}\right)\left(B_6(t_j) - B_6(t_{j-1})\right) \\ -\sigma_6 G_6\left(t_{j-2}, B_E^{j-2}\right)\left(B_6(t_{j-1}) - B_6(t_{j-2})\right) \end{array} \right] \times \Sigma \\
& + \frac{\alpha (\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{j=2}^{n} \left[\begin{array}{c} \sigma_6 G_6\left(t_j, B_E^j\right)\left(B_6(t_{j-1}) - B_6(t_j)\right) \\ -2\sigma_6 G_6\left(t_{j-1}, B_E^{j-1}\right)\left(B_6(t_j) - B_6(t_{j-1})\right) \\ +\sigma_6 G_6\left(t_{j-2}, B_E^{j-2}\right)\left(B_6(t_{j-1}) - B_6(t_{j-2})\right) \end{array} \right] \times \Delta
\end{aligned}
$$

$$
\begin{aligned}
D^{n+1} \quad = \quad & \frac{1-\alpha}{AB(\alpha)} D^*\left(t_n, S^n, R^n, R_T^n, R_F^n, G_E^n, B_E^n, D^n\right) \\
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} D^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \times \Pi \\
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+1)} \sum_{j=2}^{n} \sigma_7 G_7\left(t_{j-2}, D^{j-2}\right)\left(B_7(t_{j-1}) - B_7(t_{j-2})\right) \times \Pi
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{\alpha (\Delta t)^\alpha}{AB(\alpha)\Gamma(\alpha+2)} \sum_{j=2}^{n} \left[\begin{array}{c} D^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ -D^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Sigma \\
& + \frac{\alpha (\Delta t)^\alpha}{2AB(\alpha)\Gamma(\alpha+3)} \sum_{j=2}^{n} \left[\begin{array}{c} D^*\left(t_j, S^j, R^j, R_T^j, R_F^j, G_E^j, B_E^j, D^j\right) \\ -2D^*\left(t_{j-1}, S^{j-1}, R^{j-1}, R_T^{j-1}, R_F^{j-1}, G_E^{j-1}, B_E^{j-1}, D^{j-1}\right) \\ +D^*\left(t_{j-2}, S^{j-2}, R^{j-2}, R_T^{j-2}, R_F^{j-2}, G_E^{j-2}, B_E^{j-2}, D^{j-2}\right) \end{array} \right] \times \Delta
\end{aligned}
$$

$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{AB\left(\alpha\right)\Gamma\left(\alpha+2\right)}\sum_{j=2}^{n}\left[\begin{array}{c}\sigma_7 G_7\left(t_{j-1},D^{j-1}\right)\left(B_7\left(t_j\right)-B_7\left(t_{j-1}\right)\right)\\-\sigma_7 G_7\left(t_{j-2},D^{j-2}\right)\left(B_7\left(t_{j-1}\right)-B_7\left(t_{j-2}\right)\right)\end{array}\right]\times\Sigma$$
$$+\frac{\alpha\left(\Delta t\right)^{\alpha}}{2AB\left(\alpha\right)\Gamma\left(\alpha+3\right)}\sum_{j=2}^{n}\left[\begin{array}{c}\sigma_7 G_7\left(t_j,D^{j}\right)\left(B_7\left(t_{j-1}\right)-B_7\left(t_j\right)\right)\\-2\sigma_7 G_7\left(t_{j-1},D^{j-1}\right)\left(B_7\left(t_j\right)-B_7\left(t_{j-1}\right)\right)\\+\sigma_7 G_7\left(t_{j-2},D^{j-2}\right)\left(B_7\left(t_{j-1}\right)-B_7\left(t_{j-2}\right)\right)\end{array}\right]\times\Delta.$$

10 Numerical simulation

In this section, we present the numerical simulations for the suggested model with the Atangana-Baleanu fractional derivative. Here, firstly we consider deterministic model

$$\begin{aligned}
{}_0^{AB}D_t^{\alpha}S\left(t\right) &= \Lambda-\beta S\left(G_E+\tau B_E\right)+\kappa_5 B_E \qquad (104)\\
{}_0^{AB}D_t^{\alpha}R\left(t\right) &= \beta S\left(G_E+\tau B_E\right)-\left(\left(1-\psi_1\right)\varphi_1+\left(1-\psi_2\right)\varphi_2\right)R\\
{}_0^{AB}D_t^{\alpha}R_T\left(t\right) &= \psi_1\varphi_1 R-\kappa_1 R_T\\
{}_0^{AB}D_t^{\alpha}R_F\left(t\right) &= \psi_2\varphi_2 R-\kappa_2 R_F\\
{}_0^{AB}D_t^{\alpha}G_E\left(t\right) &= \left(1-\psi_1\right)\varphi_1 R-\kappa_3 G_E\\
{}_0^{AB}D_t^{\alpha}B_E\left(t\right) &= \left(1-\psi_2\right)\varphi_2 R-\left(\kappa_4+\kappa_5\right)B_E\\
{}_0^{AB}D_t^{\alpha}D\left(t\right) &= \kappa_1 R_T+\kappa_2 R_F+\kappa_3 G_E+\kappa_4 B_E
\end{aligned}$$

with the initial conditions

$$S\left(0\right)=2000, R\left(0\right)=0, R_T\left(0\right)=0, R_F\left(0\right)=0, G_E\left(0\right)=250, B_E\left(0\right)=200, D\left(0\right)=0. \qquad (105)$$

We depict simulations with the parameters

$$\begin{aligned}
\Lambda &= 1000, \beta=0.15, \tau=0.01, \psi_1=0.5, \varphi_1=0.25, \psi_2=0.2, \varphi_2=0.15,\\
\kappa_1 &= 0.1, \kappa_2=0.1, \kappa_3=0.3, \kappa_4=0.01, \kappa_5=0.2.
\end{aligned}$$

In Figures 47, 48, 49 and 50, numerical simulations for deterministic model are performed for different values of fractional orders.

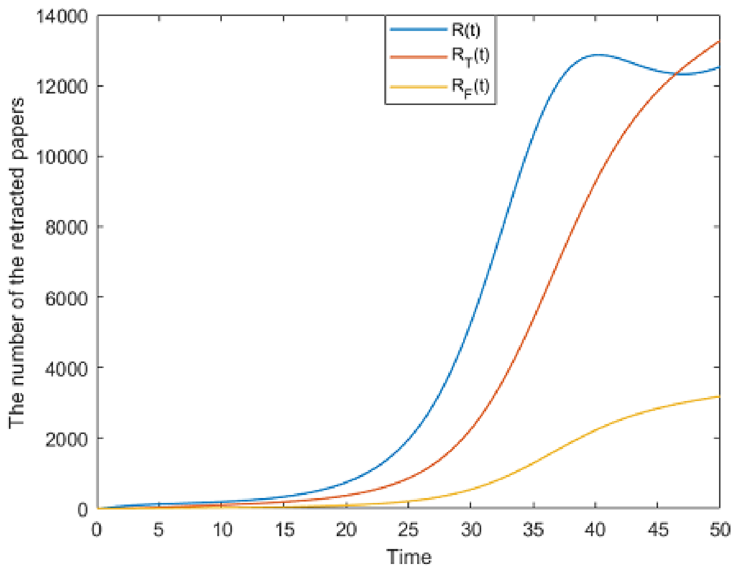

Figure 47. Retraction classes for suggested model for $\alpha = 1$.

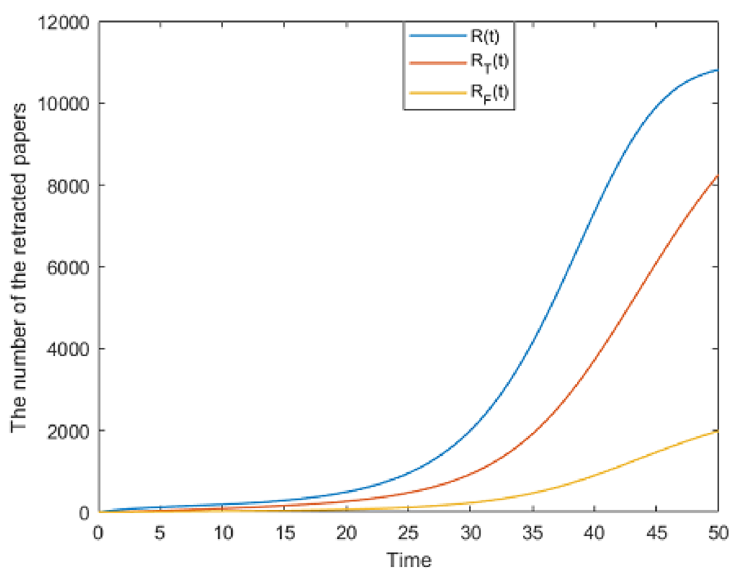

Figure 48. Retraction classes for suggested model for $\alpha = 0.92$.

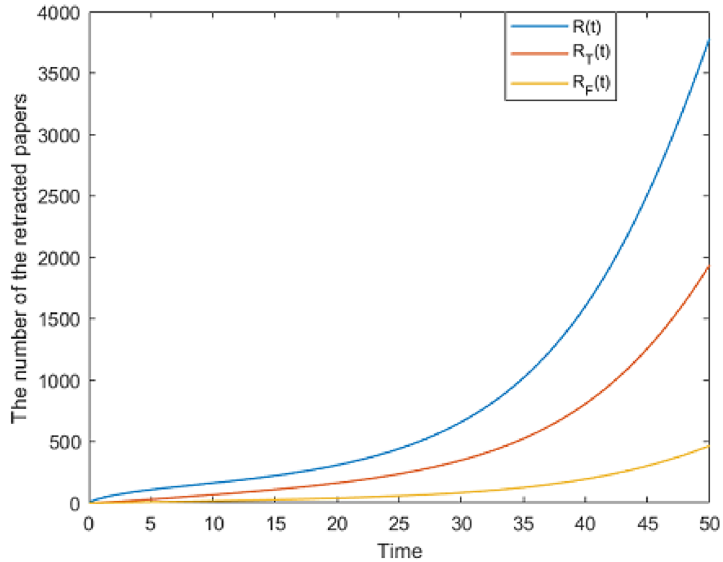

Figure 49. Retraction classes for suggested model for $\alpha = 0.81$.

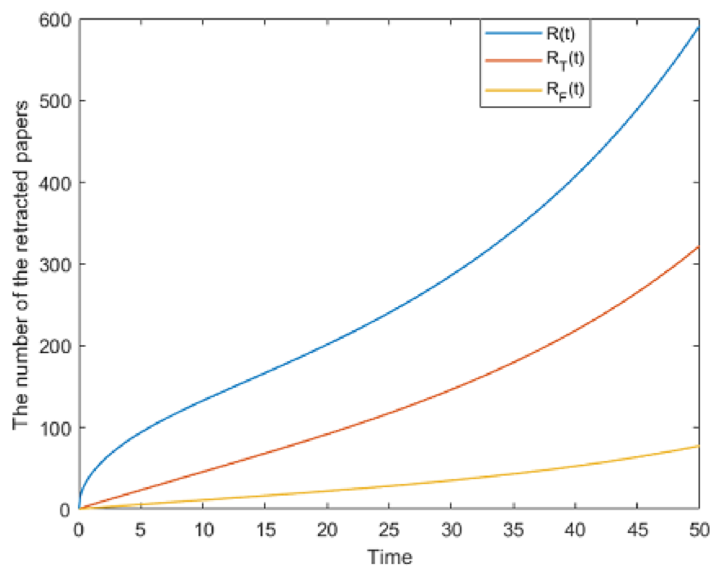

Figure 50. Retraction classes for suggested model for $\alpha = 0.68$.

To capture randomness, we consider the following fractional stochastic model with Atangana-Baleanu fractional derivative

$$\begin{aligned}
{}_{0}^{AB}D_t^{\alpha} S(t) &= \left(\Lambda - \beta S\left(G_E + \tau B_E\right) + \kappa_5 B_E\right) + \sigma_1 S(t)\, B_1\prime(t) \\
{}_{0}^{AB}D_t^{\alpha} R(t) &= \left(\beta S\left(G_E + \tau B_E\right) - \left(\left(1-\psi_1\right)\varphi_1 + \left(1-\psi_2\right)\varphi_2\right) R\right) + \sigma_2 R(t)\, B_2\prime(t) \\
{}_{0}^{AB}D_t^{\alpha} R_T(t) &= \left(\psi_1\varphi_1 R - \kappa_1 R_T\right) + \sigma_3 R_T(t)\, B_3\prime(t) \\
{}_{0}^{AB}D_t^{\alpha} R_F(t) &= \left(\psi_2\varphi_2 R - \kappa_2 R_F\right) + \sigma_4 R_F(t)\, B_4\prime(t) \\
{}_{0}^{AB}D_t^{\alpha} G_E(t) &= \left(\left(1-\psi_1\right)\varphi_1 R - \kappa_3 G_E\right) + \sigma_5 G_E(t)\, B_5\prime(t) \\
{}_{0}^{AB}D_t^{\alpha} B_E(t) &= \left(\left(1-\psi_2\right)\varphi_2 R - \left(\kappa_4 + \kappa_5\right) B_E\right) + \sigma_6 B_E(t)\, B_6\prime(t) \\
{}_{0}^{AB}D_t^{\alpha} D(t) &= \left(\left(1-\psi_2\right)\varphi_2 R - \left(\kappa_4 + \kappa_5\right) B_E\right) + \sigma_7 D(t)\, B_7\prime(t)
\end{aligned} \tag{106}$$

where initial data is as follows

$$S(0) = 2000, R(0) = 0, R_T(0) = 0, R_F(0) = 0, G_E(0) = 250, B_E(0) = 200, D(0) = 0. \tag{107}$$

Also the parameters are chosen as

$$\begin{aligned}
\Lambda &= 1000, \beta = 0.15, \tau = 0.01, \psi_1 = 0.5, \varphi_1 = 0.25, \psi_2 = 0.2, \varphi_2 = 0.15, \\
\kappa_1 &= 0.1, \kappa_2 = 0.1, \kappa_3 = 0.3, \kappa_4 = 0.01, \kappa_5 = 0.2, \sigma_1 = 0.01, \sigma_2 = 0.12, \\
\sigma_3 &= 0.11, \sigma_4 = 0.21, \sigma_5 = 0.014, \sigma_6 = 0.013, \sigma_7 = 0.011.
\end{aligned} \tag{108}$$

The numerical simulations for stochastic model are provided Figure in 51, 52, 53 and 54 for different fractional orders and density of randomness.

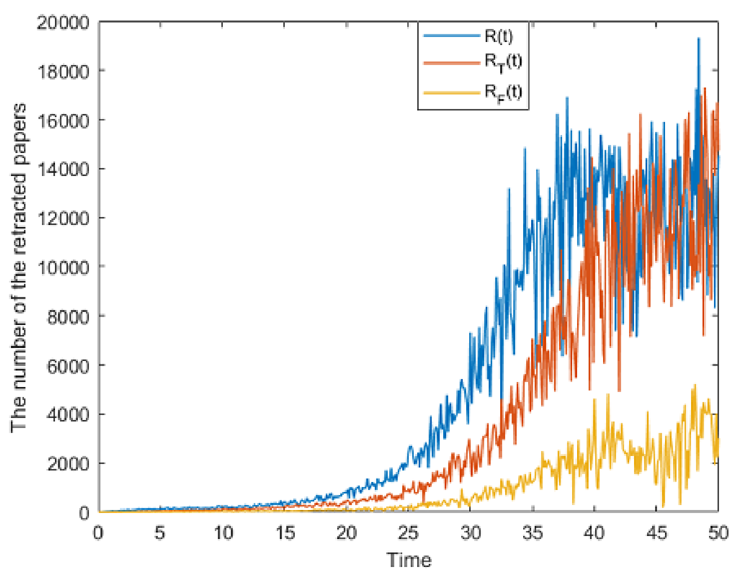

Figure 51. Retraction classes for suggested model for $\alpha = 1$ and $\sigma_2 = 0.12, \sigma_3 = 0.11, \sigma_4 = 0.21$.

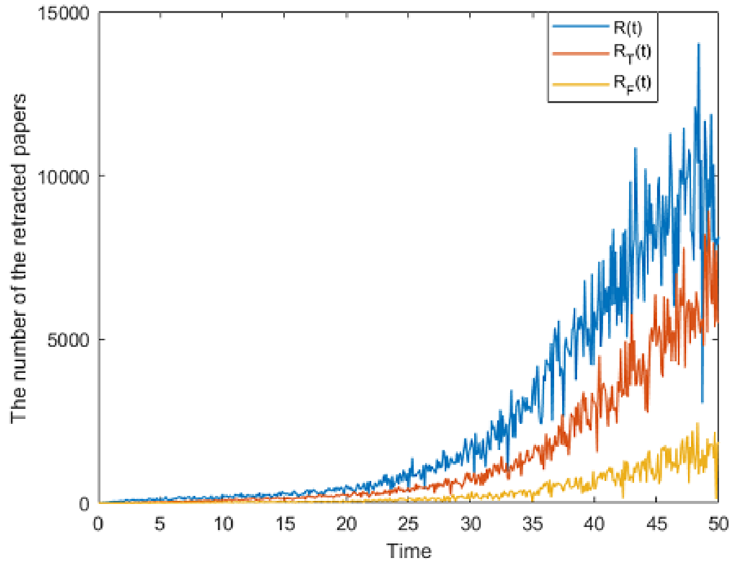

Figure 52. Retraction classes for suggested model for $\alpha = 0.92$ and $\sigma_2 = 0.12, \sigma_3 = 0.11, \sigma_4 = 0.21$.

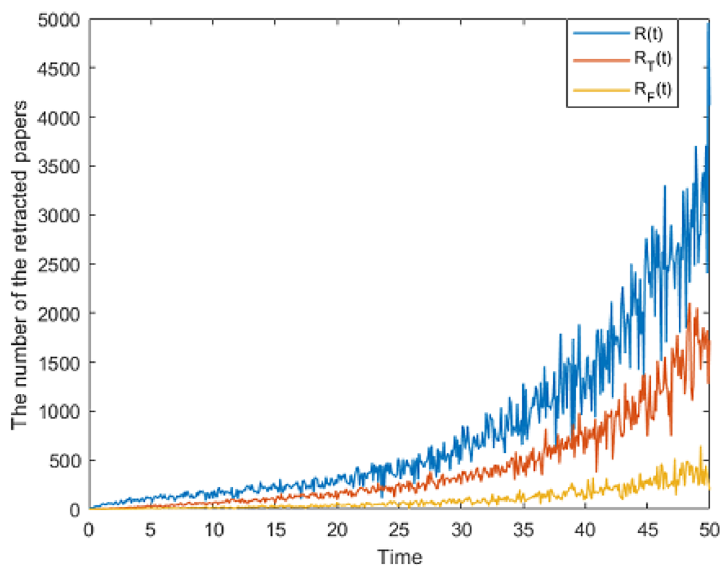

Figure 53. Retraction classes for suggested model for $\alpha = 0.81$ and $\sigma_2 = 0.12, \sigma_3 = 0.11, \sigma_4 = 0.21$.

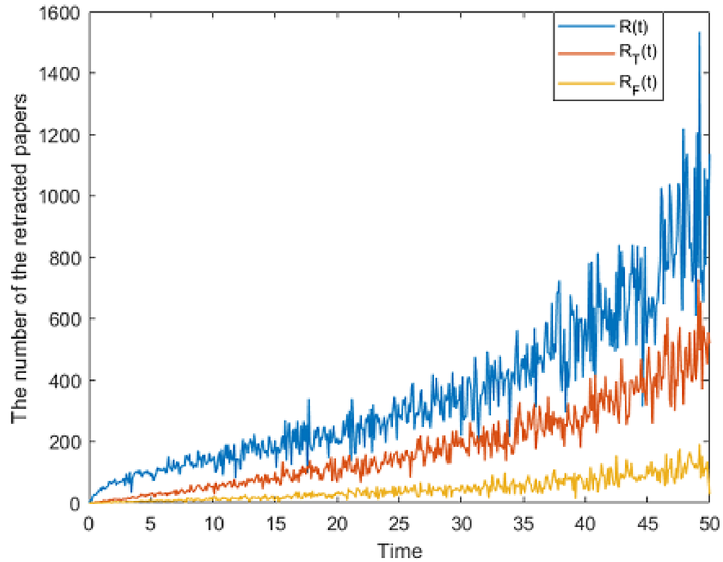

Figure 54. Retraction classes for suggested model for $\alpha = 0.68$ and $\sigma_2 = 0.12, \sigma_3 = 0.11, \sigma_4 = 0.21$.

11 Comparison between the retraction model and data

In this section, the compatibility of the model is tested by making a comparison between the proposed model and the data we have obtained from the Retraction Watch Database for some publishers. While the Atangana-Baleanu derivative model is discussed here, it can be easily seen from the simulations that the compatibility of the model is achieved successfully with the help of fractional derivatives. During the simulations, the data covers a period from 2000 to 2020, which corresponds to a total period of 21 years. For comparison of each case, we consider the following fractional stochastic model with Atangana-Baleanu fractional derivative

$$\begin{aligned}
{}_0^{AB}D_t^{\alpha} S(t) &= (\Lambda - \beta S (G_E + \tau B_E) + \kappa_5 B_E) + \sigma_1 S(t) B_1\prime(t) \\
{}_0^{AB}D_t^{\alpha} R(t) &= (\beta S (G_E + \tau B_E) - ((1-\psi_1)\varphi_1 + (1-\psi_2)\varphi_2) R) + \sigma_2 R(t) B_2\prime(t) \\
{}_0^{AB}D_t^{\alpha} R_T(t) &= (\psi_1 \varphi_1 R - \kappa_1 R_T) + \sigma_3 R_T(t) B_3\prime(t) \\
{}_0^{AB}D_t^{\alpha} R_F(t) &= (\psi_2 \varphi_2 R - \kappa_2 R_F) + \sigma_4 R_F(t) B_4\prime(t) \\
{}_0^{AB}D_t^{\alpha} G_E(t) &= ((1-\psi_1)\varphi_1 R - \kappa_3 G_E) + \sigma_5 G_E(t) B_5\prime(t) \\
{}_0^{AB}D_t^{\alpha} B_E(t) &= ((1-\psi_2)\varphi_2 R - (\kappa_4 + \kappa_5) B_E) + \sigma_6 B_E(t) B_6\prime(t) \\
{}_0^{AB}D_t^{\alpha} D(t) &= ((1-\psi_2)\varphi_2 R - (\kappa_4 + \kappa_5) B_E) + \sigma_7 D(t) B_7\prime(t).
\end{aligned} \tag{109}$$

To compare the model with Elsevier retraction data, we consider the following initial conditions

$$S(0) = 2000, R(0) = 6, R_T(0) = 0, R_F(0) = 0, G_E(0) = 250, B_E(0) = 200, D(0) = 0 \tag{110}$$

and the parameters

$$\begin{aligned}
\Lambda &= 600, \beta = 0.15, \tau = 0.21, \psi_1 = 0.5, \varphi_1 = 0.25, \psi_2 = 0.25, \varphi_2 = 0.45, \\
\kappa_1 &= 0.2, \kappa_2 = 0.4, \kappa_3 = 0.3, \kappa_4 = 0.01, \kappa_5 = 0.2, \sigma_1 = 0.13, \sigma_2 = 0.21, \\
\sigma_3 &= 0.15, \sigma_4 = 0.14, \sigma_5 = 0.02, \sigma_6 = 0.012, \sigma_7 = 0.05.
\end{aligned} \tag{111}$$

The numerical simulations for stochastic model are provided Figure in 55, 56 and 57 for different fractional order and density of randomness.

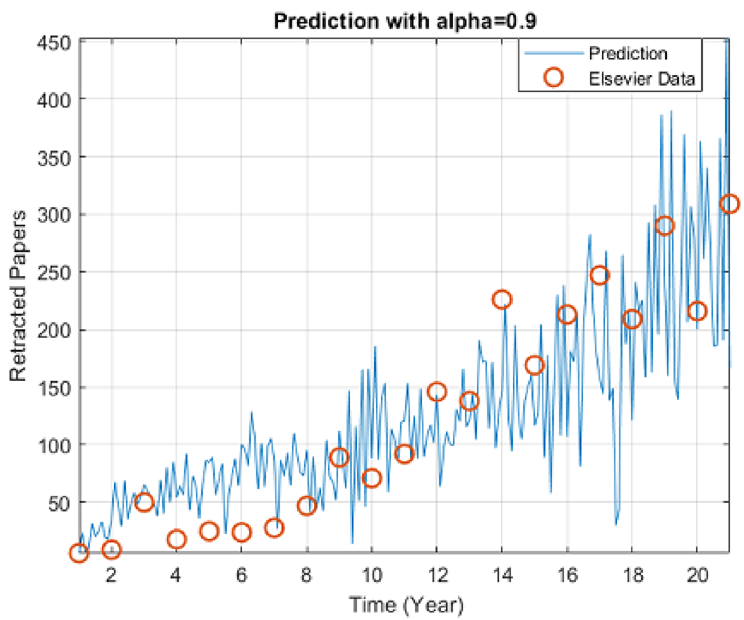

Figure 55. Comparison between model and Elsevier data for $\alpha = 0.9$ and $\sigma_2 = 0.21$.

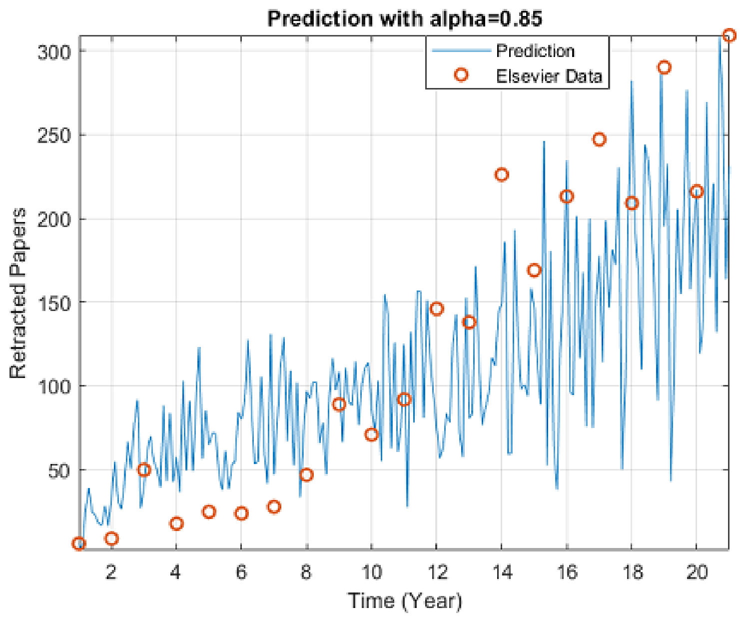

Figure 56. Comparison between model and Elsevier data for $\alpha = 0.85$ and $\sigma_2 = 0.21$.

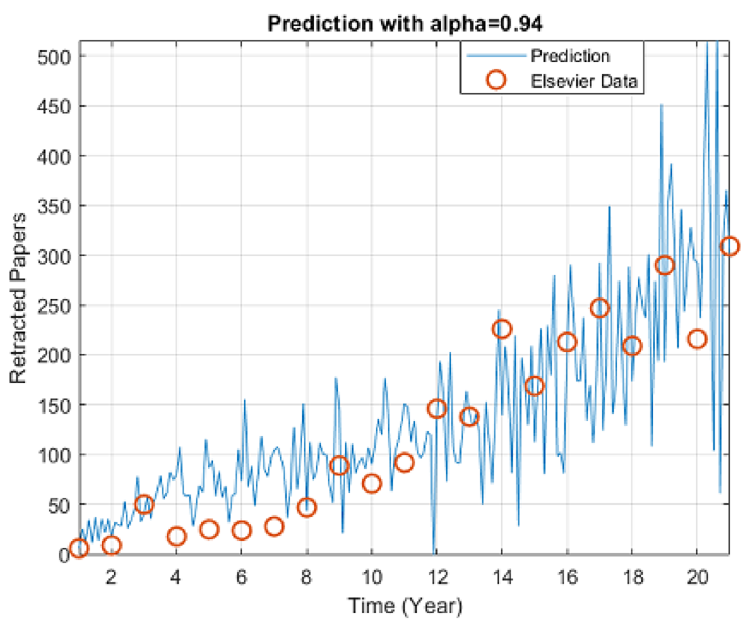

Figure 57. Comparison between model and Elsevier data for $\alpha = 0.94$ and $\sigma_2 = 0.21$.

For comparison between the model and Springer retraction data, we consider the following initial conditions

$$S(0) = 225, R(0) = 2, R_T(0) = 0, R_F(0) = 0, G_E(0) = 75, B_E(0) = 70, D(0) = 0 \tag{112}$$

and the parameters

$$\begin{aligned} \Lambda &= 2000, \beta = 0.15, \tau = 0.21, \psi_1 = 0.5, \varphi_1 = 0.25, \psi_2 = 0.25, \varphi_2 = 0.45, \\ \kappa_1 &= 0.2, \kappa_2 = 0.4, \kappa_3 = 0.3, \kappa_4 = 0.01, \kappa_5 = 0.2, \sigma_1 = 0.13, \sigma_2 = 0.21, \\ \sigma_3 &= 0.15, \sigma_4 = 0.14, \sigma_5 = 0.02, \sigma_6 = 0.012, \sigma_7 = 0.05. \end{aligned} \tag{113}$$

The numerical simulations for stochastic model are performed Figure in 58 and 59 for different fractional order and density of randomness.

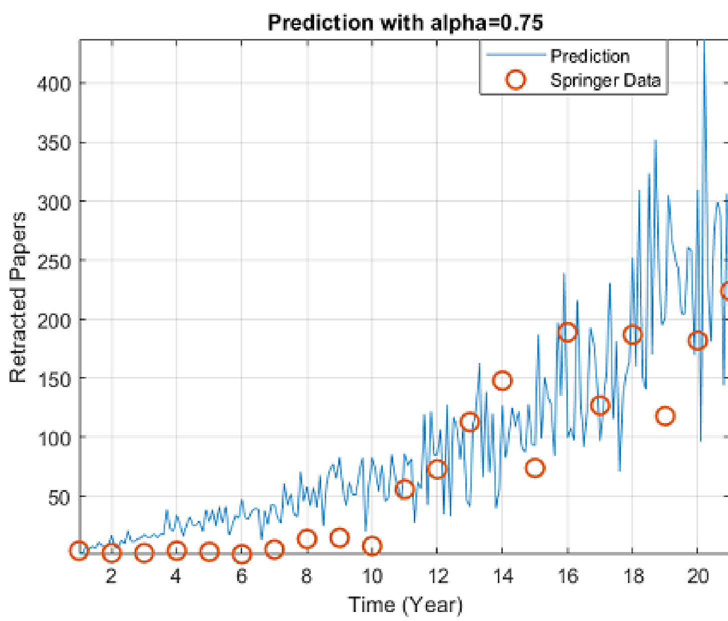

Figure 58. Comparison between model and Springer data for $\alpha = 0.75$ and $\sigma_2 = 0.21$.

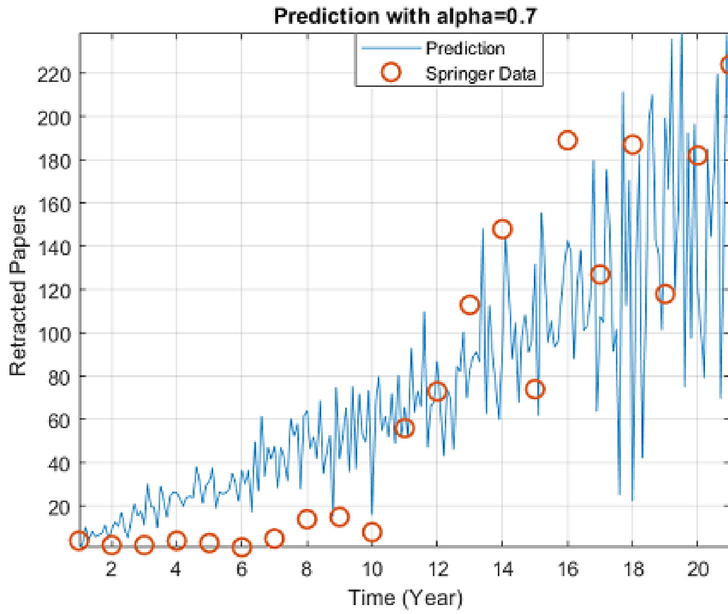

Figure 59. Comparison between model and Springer data for $\alpha = 0.7$ and $\sigma_2 = 0.21$.

To compare the model with Wiley retraction data, we consider the following initial conditions

$$S(0) = 125, R(0) = 2, R_T(0) = 0, R_F(0) = 0, G_E(0) = 75, B_E(0) = 70, D(0) = 0 \tag{114}$$

and the parameters

$$\begin{aligned} \Lambda &= 50, \beta = 0.15, \tau = 0.21, \psi_1 = 0.5, \varphi_1 = 0.25, \psi_2 = 0.25, \varphi_2 = 0.45, \\ \kappa_1 &= 0.2, \kappa_2 = 0.4, \kappa_3 = 0.3, \kappa_4 = 0.01, \kappa_5 = 0.2, \sigma_1 = 0.13, \sigma_2 = 0.21, \\ \sigma_3 &= 0.15, \sigma_4 = 0.14, \sigma_5 = 0.02, \sigma_6 = 0.012, \sigma_7 = 0.05. \end{aligned} \tag{115}$$

The numerical simulations for stochastic model are provided Figure in 60, 61 and 62 for different fractional order and density of randomness.

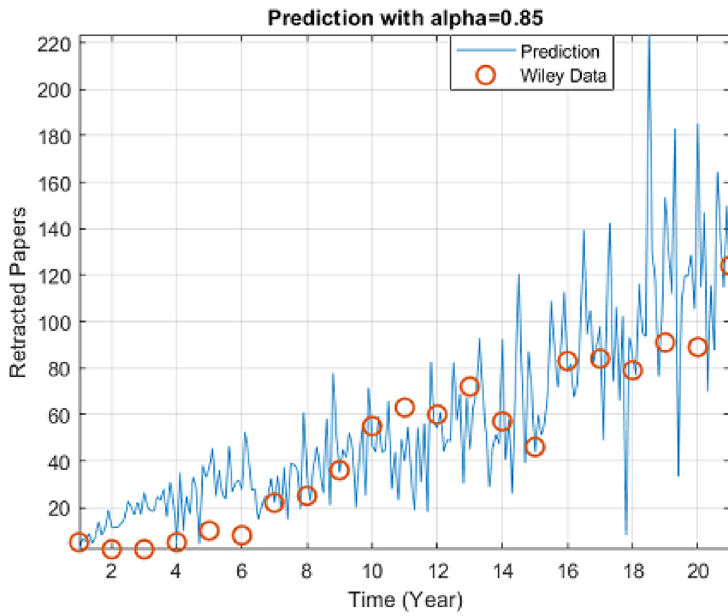

Figure 60. Comparison between model and Wiley data for $\alpha = 0.85$ and $\sigma_2 = 0.21$.

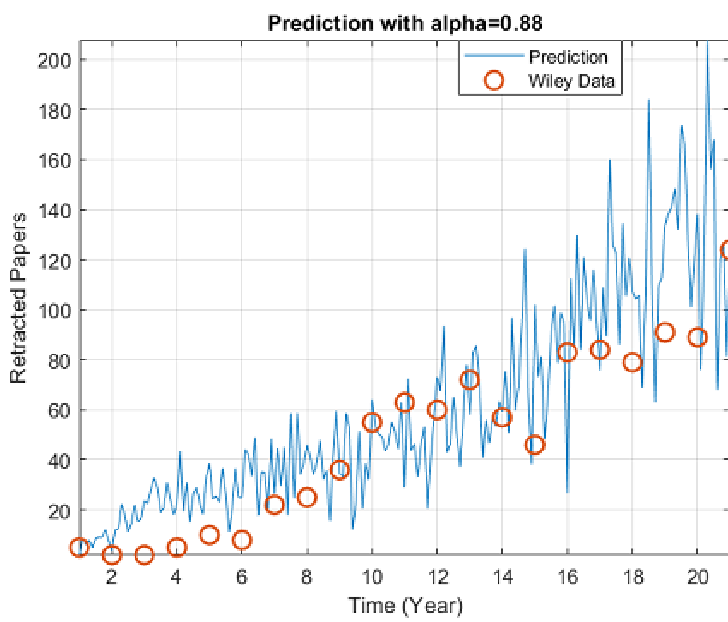

Figure 61. Comparison between model and Wiley data for $\alpha = 0.88$ and $\sigma_2 = 0.21$.

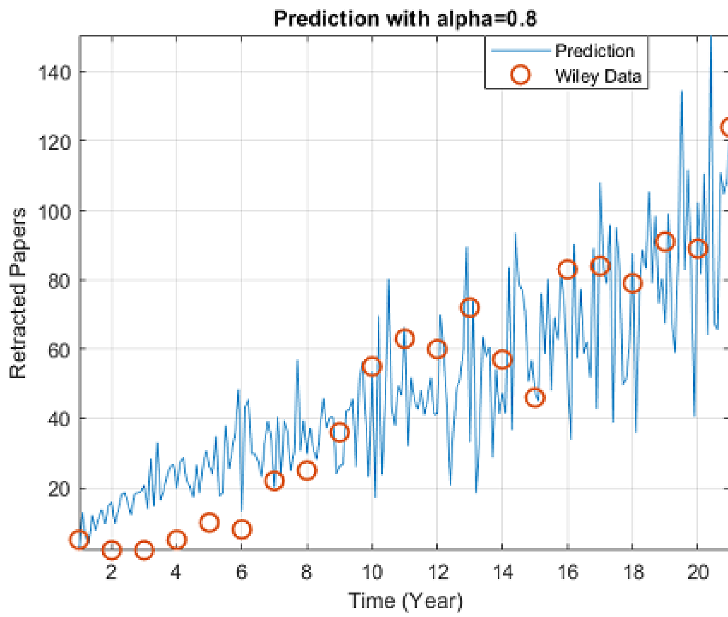

Figure 62. Comparison between model and Wiley data for $\alpha = 0.8$ and $\sigma_2 = 0.21$.

For comparison between the model and Taylor&Francis retraction data, the following initial conditions are chosen as

$$S(0) = 150, R(0) = 0, R_T(0) = 0, R_F(0) = 0, G_E(0) = 75, B_E(0) = 70, D(0) = 0 \tag{116}$$

and the parameters

$$\begin{aligned} \Lambda &= 20, \beta = 0.15, \tau = 0.21, \psi_1 = 0.5, \varphi_1 = 0.25, \psi_2 = 0.25, \varphi_2 = 0.45, \\ \kappa_1 &= 0.2, \kappa_2 = 0.4, \kappa_3 = 0.3, \kappa_4 = 0.01, \kappa_5 = 0.2, \sigma_1 = 0.13, \sigma_2 = 0.21, \\ \sigma_3 &= 0.15, \sigma_4 = 0.14, \sigma_5 = 0.02, \sigma_6 = 0.012, \sigma_7 = 0.05. \end{aligned} \tag{117}$$

The numerical simulations for stochastic model are depicted Figure in 63, 64 and 65 for different fractional order and density of randomness.

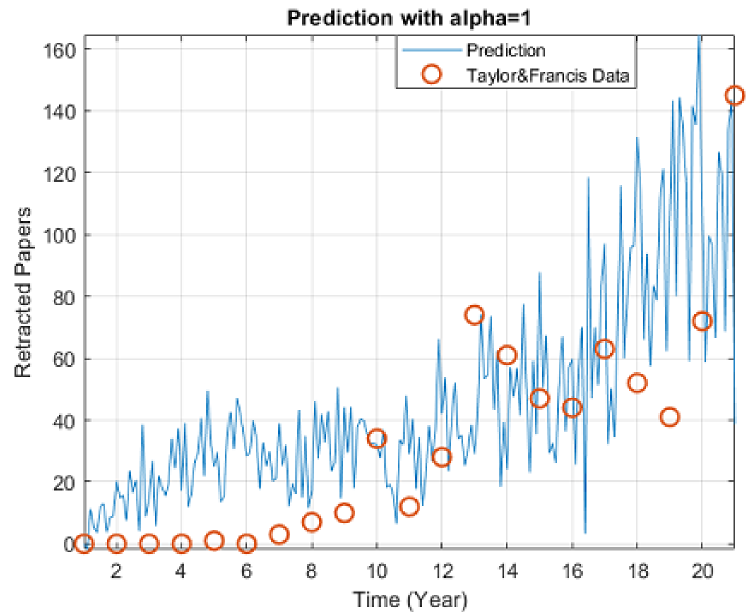

Figure 63. Retraction classes for suggested model for $\alpha = 1$ and $\sigma_2 = 0.21$.

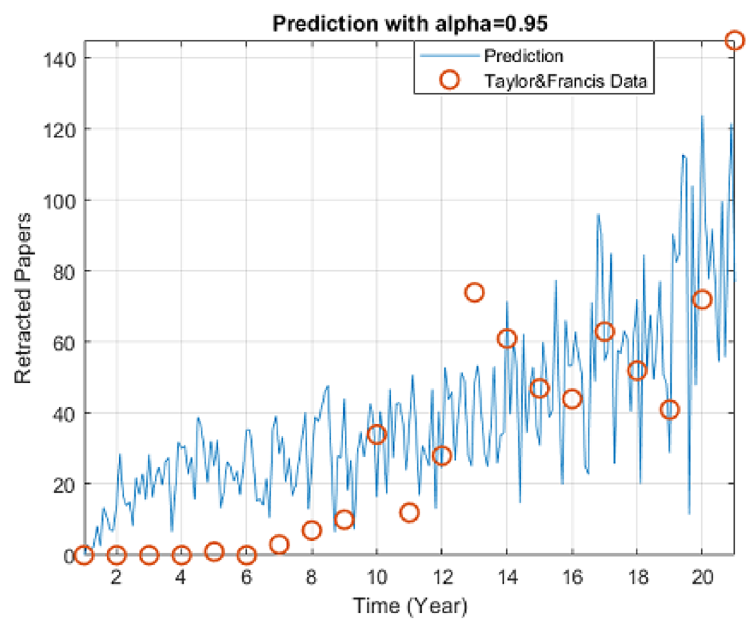

Figure 64. Retraction classes for suggested model for $\alpha = 0.95$ and $\sigma_2 = 0.21$.

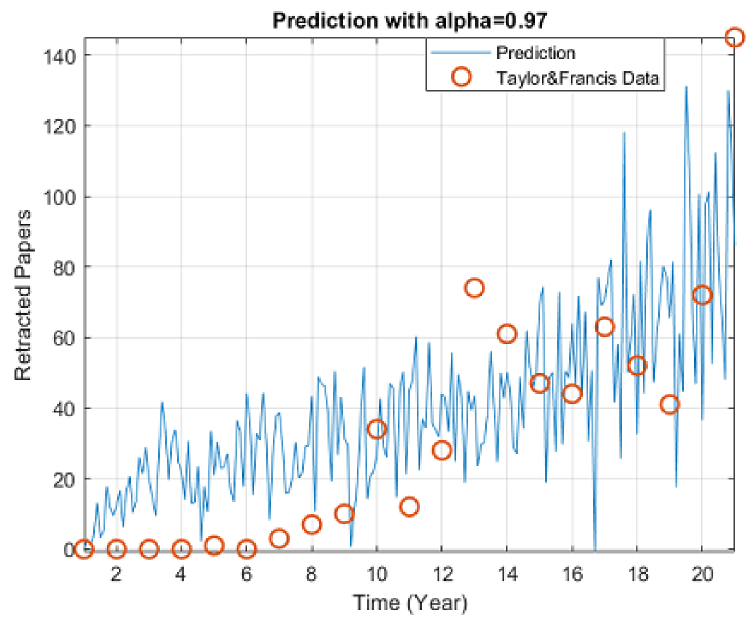

Figure 65. Retraction classes for suggested model for $\alpha = 0.97$and $\sigma_2 = 0.21$.

12 Conclusion and prediction

Retraction was introduced to protect the body of knowledge at least only correct results should be recorded, while erroneous results, plagiarized results and manipulated results should be removed from the database of collection of knowledge. However, in the last few decades, the numbers of yearly retracted papers have attracted attention of many actors including researchers, industries, medical bodies and publishers. An effort to record those retracted papers has led to a sponsored company that has been reporting information about retracted papers. However, it has been noticed that there are many papers that have been wrongly retracted for some different reasons. Few reasons beside those listed before, the retraction has nowadays become what is called in French "reglement de compte". A particular author can be targeted by a group of researchers because he has been listed on the prestigious list made by a fair, non-racial, non-discriminatory organization called Web of Science. This group will send comments to different journals with the aim to retract all highly cited papers, several other mechanisms are being used to reach their target, for example Pubpeer an international blog where anonymous individuals will post comments even if they are not expert in the field. With an accumulation of comments the journal is obligated to start an investigation, and sometimes this leads to retraction of the paper as the publisher wishes to protect the integrity of the journal by letting down the author that suffered to write the paper, while in the case of erroneous portion of the paper, a Corrigendum could have been submitted. We therefore prepared this paper of course we know that this paper could also join the class R_F and finally D however, at least to inform different actors involved in the process of publication and retraction how alarming the yearly numbers of new retractions are. It is a wake-up call for big publishers to revise their process of retraction. With the statistical analysis presented here and the predictions made from 2020 to 2050, of course we do not confirm that such predictions are absolutely true, however, at least they reveal an element of truth that should be considered seriously. Thus, few recommendations will be listed here, while we do not say these will help flatten the curve of retraction, at least only fairly retraction could be recorded. We now suggest a procedure that could be used to retract papers fairly.

1) The handle editor should be contacted then he should give a valid reason why he acceptedthe paper in the first place.

2) Reviewers that were involved in the process of peer review should be contacted and providea clear reason why they provided positive reports.

3) Their reasons could now be evaluated by the editor in chief of the journal together with thejournal manager and the publisher, at this point the comments sent by the reader can be comparedwith the reasons given by the editor and reviewers.

4) If the publisher, the journal manager and the editor-in-chief are not able to take a decision, then they should seek advice from a reliable and fair researcher that can evaluate the comments submitted by the reader, editor reasons, and reviewers reports.

5) Alternatively each publisher should set an independent team that reflects all the back-ground, at least a team comprising people from different continents, and genders that could be used to fairly settle the retraction matters.

6) Comments posted by anonymous individuals on Pub-Peer should not be considered, at least if the individual is not identified to be an expert in the field. For example, a Covid-19 paper published by a mathematician may not be understood by a researcher in the medical field therefore his comments on a mathematical paper should not be considered. Additionally to the above list, the author should be given the opportunity to re-publish his paper even after retraction, in fact having a paper retracted does not always mean the main idea developed in the paper is useless, the paper canbe revised, and resubmitted for publication. Therefore, we suggest that a willing publisher launch a new journal that will collect all retracted papers that have been corrected by the authors with an editorial board reflecting all disciplines.

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