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Transfer from a Lunar Distant Retrograde Orbit to Mars through Lyapunov Orbits

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Abstract

Because of the potential of the Lunar Gateway for cislunar operations, in particular missions to Mars, there is a growing interest in designing trajectories from lunar orbits. This work deals with the development of a low-energy transfer to Mars from a lunar Distant Retrograde Orbit, relying exclusively on the Circular Restricted Three-Body Problem (CR3BP). The full five-body problem (Moon-Earth-spacecraft-Sun-Mars) is decomposed into three different CR3BPs. In each of them, parts of the trajectory are built exploiting invariant manifolds stemming from Lyapunov orbits. These segments are then linked together successively, overlapping the three systems while considering the ephemerides. Dealing with date restrictions for the required configuration of the Earth and Mars, a period from 2027 to 2034 was investigated. Resulting trajectories demonstrate the convenience of the strategy: despite a long time of flight, this approach allows to design low-energy trajectories for cargo missions.

Keywords: CR3BP, Lyapunov orbits, DRO, invariant manifolds

Context

As the Lunar Gateway will pave the way for cislunar operations, the red planet is one of the main target of space agencies. Exploring and sending humans to Mars will call for cargo missions to deliver experiment and robotic devices and eventually build a permanent base. Maximizing the payload mass of spacecraft and following low-energy trajectories is key to making future missions possible. Relying on the Circular Restricted Three-Body Problem (CR3BP) instead of the usual Two-Body Problem opens approaches to develop energy efficient trajectories. There is a clear interest in developing such trajectories from lunar orbits. Among these, Distant Retrograde Orbits (DROs) present interesting features: they exhibit long-term stability and ease of access in terms of gravity. Investigated for the first time in 1969 by Hénon [1], DROs are a family of planar solutions of the CR3BP. With near-rectilinear halo orbits, one of which being planned to host the Lunar Gateway, they represent some of the most suitable orbits to locate space infrastructure such as a propellant depot, allowing a cargo mission to be refuelled [2]. Also, they could offer the possibility of assembling large spacecraft in orbit, thus removing launch vehicle constraints. Studies, namely by Murakami and Yamanake, already describe solutions to perform rendezvous between a future possible DRO station and visiting vehicles [3].

This study concerns the development of a trajectory from a lunar DRO to the vicinity of Mars. It relies exclusively on the CR3BP theory. It extends previous works on transfers conducted by overlapping two three-body problems. In this work, three three-body problems have been exploited: Earth-Moon-spacecraft, Sun-Earth-spacecraft and Sun-Mars-spacecraft. In each system, different segments of the trajectory are built by propagating the invariant manifolds inherent to the CR3BP theory. These segments are then linked together to obtain the full trajectory.

Problem Statement

In the CR3BP, two primary masses m_1 and m_2 revolve on a circular orbit around their center of mass and a third massless body moves under their mutual gravitational interaction. The equations of motion are expressed with respect to the synodic frame, a rotating reference frame centered at $(m_1,\ m_2)$ barycenter. The $\hat{x}$ axis is directed from m_1 to m_2, the $\hat{y}$ axis is in the plane of the primaries' orbit and $\hat{z}$ completes the right-handed triad. The frame coordinates are non-dimensional, based on the characteristic distance between the two primaries. So is the time, which is normalized by the period of m_2 in circular orbit around m_1, in such way that the universal constant is normalized to $G = 1$. The equations of motion involve the position of the spacecraft $\vec{R} = [x, y, z]^T$ and its velocity $\vec{V} = [\dot{x}, \dot{y}, \dot{z}]^T$, in terms of components in the synodic frame:

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} \qquad \ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y} \qquad \ddot{z} = \frac{\partial U}{\partial z} \tag{1}$$

where U is the pseudo-potential function and can be written as:

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{2}$$

where r_1 and r_2 are the distances of the primaries with respect to their center of mass and $\mu = m_2/(m_1 + m_2)$, the mass parameter, is this non-dimensional system's only parameter.

The CR3BP admits an integral of motion: the Jacobi constant C. The study of zero-velocity curves shows the existence of five equilibrium points, called Lagrange points: L_1, L_2, L_3 are collinear with the primaries; L_4 and L_5 are at the vertex of two equilateral triangles, generated with the primaries. These points correspond to the stationary points of the potential function, their location being given by $\nabla U = 0$. Around L_1 and L_2, there exist different families of periodic or quasi-periodic orbits that can be either stable or unstable. In the latter case, it is possible to develop manifolds, which are higher-dimensional surfaces that govern the asymptotic nature of the flow toward or away from a periodic orbit, depending on whether they are stable or unstable, respectively.

A family of unstable orbits is the set of Lyapunov orbits. They are CR3BP planar periodic solutions, which exist around both L_1 and L_2. They overlap another family of planar solutions, in geometry: the stable DROs. For a given DRO, it is possible to find tangential L_1 and L_2 Lyapunov orbits. Demeyer *et al.* [4], as well as Capdevilla *et al.* [5], have developed methods to exploit the stable manifolds of a tangent L_1 Lyapunov orbit to have access to the intended DRO. It seems possible to use a similar strategy also to leave a DRO. In addition, Lyapunov orbits can be used in interplanetary trajectories: studies have shown that unstable and stable manifolds associated with these orbits in two different three-body systems (with one common body) can be exploited for interplanetary transfer [6].

Strategy

To design the trajectory, two major simplifying hypothesis were formulated: the orbits of the planets around the Sun and the Moon's one around the Earth were considered circular and lying in the same plane, the ecliptic. The problem has been solved into three main steps.

- In the Earth-Moon system, a lunar DRO with a planar extension[1] $A_x \approx$ 70,000 km was chosen. This choice was inspired by the mission analysis made for the cancelled Asteroid Redirect Mission [7], yet the influence of A_x on the required ΔV is discussed later in this paper. The

[1] distance between the second primary, of x-coordinate $1-\mu$, and the intersection of the orbit with the $\hat{x}$ axis, towards the first primary

L_2 Lyapunov orbit tangential to the DRO was found, accepting a tolerance equal to 10^{-7} in the computation of the crossing point. Then, the associated unstable manifold was developed to leave this first system.

- In the Sun-Earth system, the transfer starts from a stable manifold that gives access to a L_2 Lyapunov orbit. Again, the unstable manifold stemming from this orbit was used to leave the system.
- In the Sun-Mars system, the purpose is to arrive at one of the orbits belonging to the L_1 Lyapunov family, exploiting again the associated stable manifold to have access. This orbit should be as near as possible to Mars to facilitate a final transfer to a Low Mars Orbit (LMO). For this reason, the chosen Lyapunov orbit has a planar extension of 400 millions kilometers.

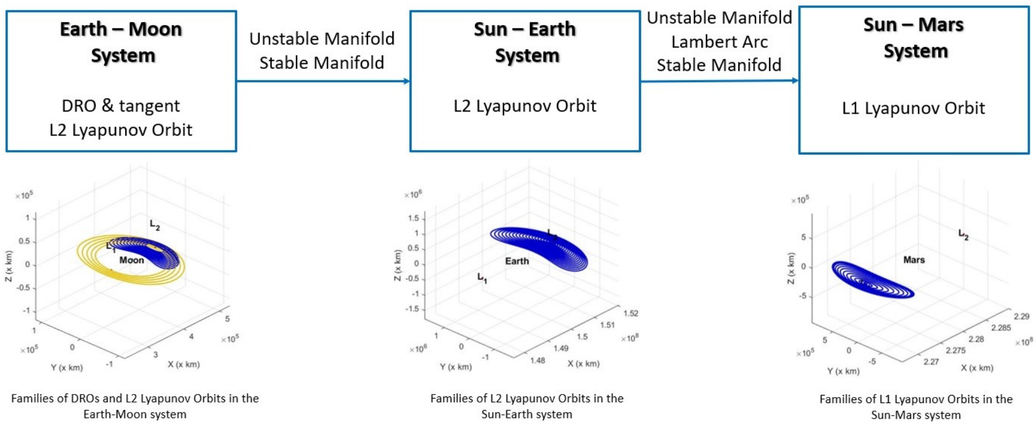

Fig. 1: Overview of the strategy.

It is possible to find an intersection between the unstable manifold of the Earth-Moon L_2 orbit and the stable one associated to the Sun-Earth L_2 orbit; thus, the transfer can be performed with a single impulsive burn. However, there is no link between the unstable manifold of the Sun-Earth L_2 orbit and the stable one of the Sun-Mars L_1 orbit. Following previous works conducted by Topputo *et al.* [6], the manifolds have been linked by a Lambert arc. This term refers to the solution of the Lambert's problem, concerned with the determination of the path between two points in space with a given time of flight (TOF). The transfer through the Lambert arc is the most expensive and its cost is highly dependent on time. For this reason, the transfer between the Sun-Earth system and the Sun-Mars system was built first, thus influencing the design of the full trajectory. The study was conducted for a period covering 2027 to 2034, considering a hypothetical timeline for the Lunar Gateway and the following cislunar operations.

Computations were carried out using Matlab, reusing and adapting codes developed in previous works [8]. The computation of the orbits and invariant manifolds relies on the differential correction method, whose methodology is detailed by Koon *et al.* [9] and Gordon [10]. The differential correction method is used also to solve the Lambert's problem, following the procedure by Topputo *et al.* [11].

Results

Trajectory computation results

The Earth-Moon L_2 Lyapunov orbit, tangent to the selected DRO (A_x = 70,000 km), is characterized by an estimated planar extension of 10,647 km and a Jacobi constant equal to C = 3.1678. The transfer from the DRO to this orbit requires an impulsion of ΔV = 0.3641 km/s.

The Sun-Earth L_2 Lyapunov orbit's size was treated as a parameter. The objective was to minimize the ΔV required for the transfer at the intersection point between its stable manifold and the unstable manifold of the Earth-Moon L_2 Lyapunov orbit. Only the orbits for which the intersection point exists were taken into consideration: performing a maneuver at such point is less costly than linking the manifolds through a conic. The existence of the intersection point and the required ΔV depend on the position of the Moon around the Earth, thus on time. The ΔV tends to increase as the Sun-Earth Lyapunov orbit's planar extension increases (Fig. 2).

There is no intersection between the unstable manifold of the Sun-Earth L_2 Lyapunov orbit and the stable manifold of the Sun-Mars L_1 Lyapunov orbit. Thus, a Lambert arc was generated. The times of extension of the manifolds and the TOF along the Lambert arc have been considered as variables. In fact, the energy required for this last transfer is largely influenced by the time at which it is performed. Starting this transfer, the angle θ defined by the Earth and Mars position vectors in the Sun-centered inertial frame, must verify: $\theta \in [150°; 160°]$ or $\theta \in [200°; 210°]$. Fulfilling this condition yields a value of ΔV that is comparable to results from other works regarding cargo missions (Fig. 3) [6][11][12].

Scheduling a departure in 2027, the selected transfer is summarized in Table 1 and shown on Fig. 3 and 5. Yet, the next occurrences of the favorable reciprocal position of the Earth and Mars allowed to investigate two other time windows: 2029-2031 and 2032-2034. The corresponding Sun-Earth L_2 Lyapunov orbits are reported in Table 2. Results show that there is no significant difference between the optimal transfers' costs in the different time windows (Fig. 6).

Departure	December 2027	
Lunar DRO	A_x = 70,000 km	C = 2.9405
Transfer to tangent Lyapunov orbit	ΔV = 0.3641 km/s	TOF = 0 s
Earth-Moon L_2 Lyapunov Orbit	A_x = 10,647 km	C = 3.1678
Earth-Moon to Sun-Earth system	ΔV= 0.3509 km/s	TOF = 340.52 days
Sun-Earth L_2 Lyapunov orbit	A_x = 56,327 km	C = 3.0009
Sun-Earth to Sun-Mars system	ΔV = 4.6887 km/s	TOF = 761.50 days
Sun-Mars L_1 Lyapunov orbit	A_x = 400,000 km	C = 3.0002
Total transfer	**ΔV = 5.4037 km/s**	**TOF = 1,102 days**

Table 1: Characteristics of the selected 2027-2029 transfer.

Period	A_x (km)	C
2029-2031	32,163	3.0009
2032-2034	68,408	3.0009

Table 2: Selected L_2 Lyapunov orbits for additional time windows.

Influence of the DRO extension

The influence the initial lunar DRO's size was analyzed by testing different planar extensions: the considered range, $A_x \in$ [40,000;80,000] km, corresponds to DROs for which there exists a tangent L_2 Lyapunov orbit. The optimal transfer was searched for each DRO. The results show how this parameter impact on costs is negligible (Fig. 7): the size of the DRO has no significant influence on the required energy. In terms of TOF, only approximately 100 days differentiate the shorter transfer from the longer one and this difference is not significant with respect to the total average transfer time.

Transfer to a Low Mars Orbit (LMO)

A Hohmann transfer was envisaged to join a Low Mars Orbit (LMO) from the selected L_1 Lyapunov orbit, to obtain an order of magnitude for the ΔV and TOF necessary to reach the red planet. For the calculations, the targeted LMO was arbitrarily chosen with an altitude of 1,000 km. Several strategies could be adopted; one possibility is to exploit a part of the unstable

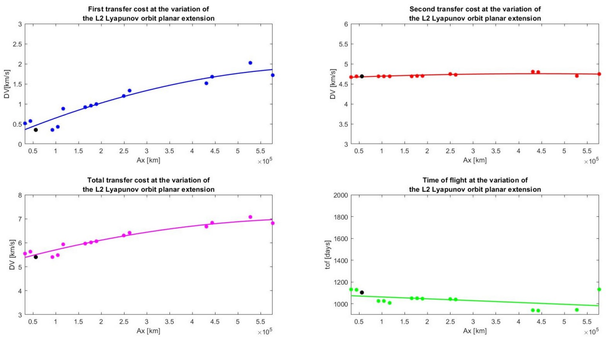

Fig. 2: Trajectory costs for different planar extensions of the Sun-Earth L_2 Lyapunov orbit.

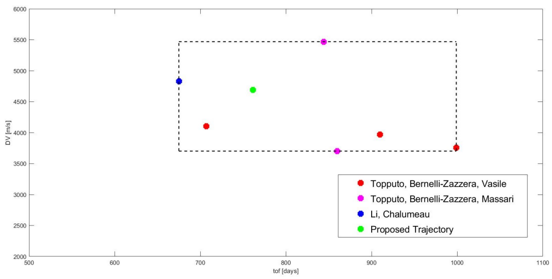

Fig. 3: Comparison of costs between the proposed solution and other studies (Sun-Earth to Sun-Mars transfer).

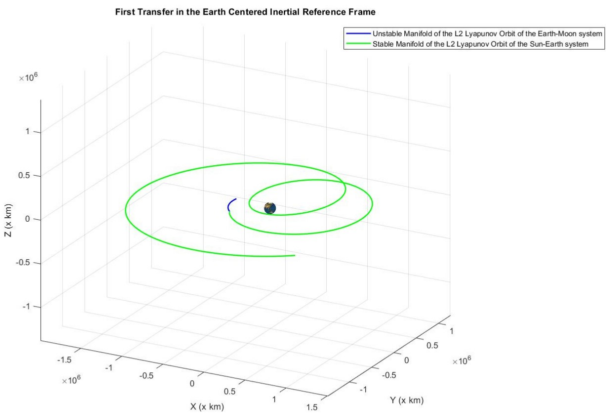

Fig. 4: Transfer between the Earth-Moon system and the Sun-Earth system, seen in the Earth Centered Inertial reference frame.

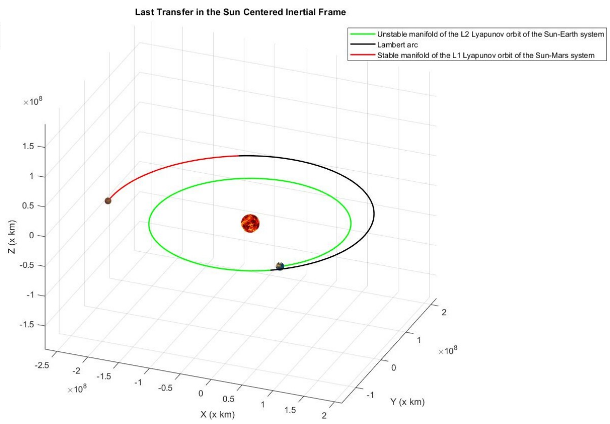

Fig. 5: Transfer between the Sun-Earth system and the Sun-Mars system, seen in the Sun Centered Inertial reference frame.

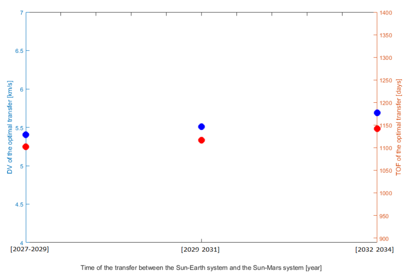

Fig. 6: Costs of the optimal trajectories at the variation of the analyzed time windows.

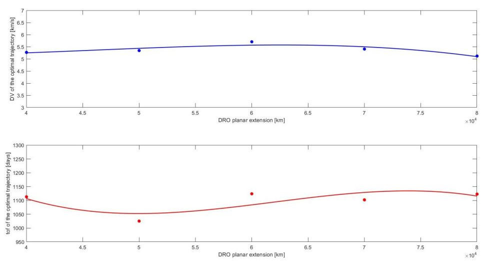

Fig. 7: Costs of the optimal trajectories at the variation of the initial DRO planar extension.

manifold stemming from the Lyapunov orbit and then engage the Hohmann transfer. A first ΔV at the selected point of the manifold is needed to join the osculating orbit. This departure point could be the periapsis, yet another point seems more interesting in terms of ΔV and TOF. Indeed, a departure at approximately 8% of the length of the manifold (propagated for one Mars period) gives the shortest TOF (375.83 days) and second smaller ΔV (1.5299 km/s).

Conclusion: Future Work

This paper demonstrates the feasibility and the convenience of relying exclusively on the CR3BP theory to design an interplanetary Moon-Mars transfer. The decomposition of the five-body problem into three three-body problems and the exploitation of Lyapunov orbits' invariant manifolds allow to find low-energy trajectories. The most critical segment of the transfer in terms of cost and choice of the most suitable time window is the one from the Sun-Earth system to the Sun-Mars system. The period from 2027 to 2034 was investigated, yielding three convenient time windows. On average, the required ΔV is 5.5 km/s and the TOF is 1,120.2 days.

Later on, further studies could be useful to bring about important improvements, such as introducing the eccentricity of the orbits and their inclination with respect to the ecliptic as perturbations, or adopting more refined techniques, like an optimization through a genetic algorithm to select the intermediate Lyapunov orbit and the Lambert arc.

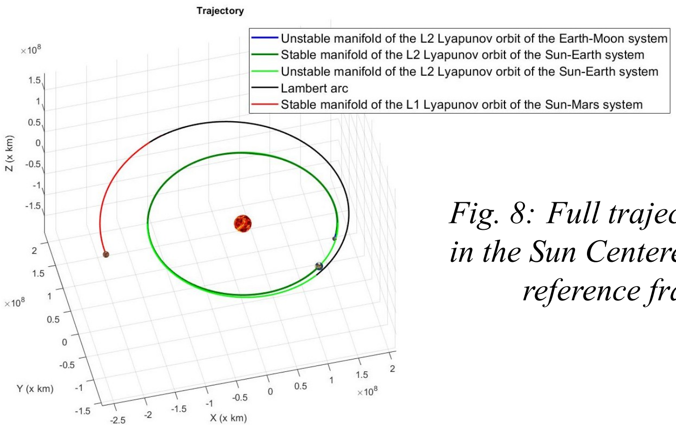

Fig. 8: Full trajectory, seen in the Sun Centered Inertial reference frame.

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