Ecole Temps Réel 2017 - Uniprocessor real-time scheduling

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***Abstract*—A real-time system is a computer system where it is just as important to compute a correct value as it is to compute this value at the right time. Such a system is usually modelled as a set of tasks that must satisfy real-time constraints (periods and deadlines mainly). Real-time scheduling consists in finding a task execution order that satisfies all real-time constraints.**

In this paper, we provide a brief overview of real-time scheduling on uniprocessor systems. We present classic scheduling policies and associated schedulability analyses. In addition, the paper gives some background on the relation between the classic real-time task model and dynamics of the modelled system. It also emphasizes the role of data-dependencies, how they are implemented and their impact on scheduling. [1], [2] have been great sources of inspiration for this paper.

I. Introduction

A real-time system is a computer system subject to a set of real-time constraints: not only does such a system need to compute the correct values, it must do so in a timely manner. For instance, let us consider the longitudinal flight control system of an airplane, such as [3]. This system controls the angle of the control surfaces of the plane based on the current state of the plane and on the altitude required by the pilot. An important real-constraint is that the time required for the system to adjust the control surfaces in reaction to a pilot order or to a gust of wind must respect a predefined delay, in order to ensure the plane stability. Similar constraints can be found in many other areas, such as for instance in assisted driving systems in the automotive domain, in nuclear plant surveillance systems, in video processing systems, ...

A real-time system is usually modelled as a set of tasks, each with its own dedicated functionality and real-time constraints. *Real-time scheduling* consists in finding an execution order for tasks, which satisfies all the task real-time constraints. This is a twofold problem. On one hand, we must choose a *scheduling policy* capable of satisfying the real-time constraints. General purpose scheduling policies (FIFO, Round-Robin, etc) are not well-adapted to real-time systems, since they usually focus on reducing the average response time of processes. In a real-time system, what matters is that tasks respect their real-time constraints in the worst-case. On the other hand, real-time systems are usually critical, so developers need a *schedulability analysis*, which ensures before execution that, for a given scheduling policy, the resulting execution will always respect the system real-time constraints.

In this paper, we focus on the scheduling of systems executing on a single computation core. The paper first details the classic way to model a real-time system (Section II). In addition to providing usual definitions on task real-time characteristics, it tries to relate these constraints to the dynamics of the system, and also gives a brief overview on how real-time tasks are usually implemented. Though the classic real-time task model assumes independent tasks, tasks are actually often related by data-dependencies. Section III discusses the modeling and implementation of such dependencies. Scheduling terminology and problem definition are provided in Section IV. Main results on uniprocessor schedulability analysis are provided in Section V, for fixed-task priority scheduling, and in Section VI, for fixed-job priority scheduling. Scheduling of dependent tasks is presented in Section VII.

II. Real-time tasks

Real-time systems can be separated into two sub-classes: event-driven systems and sampled systems. In an event-driven system, the system waits for an input event to occur and then computes its reaction. In a sampled system, the system acquires its inputs at regular intervals of time and computes its reaction for each sample of its inputs. While the event-driven model is potentially more expressive, sampled systems are usually easier to analyze and their behaviour is more predictable, especially concerning the real-time aspects. Furthermore, in many event-driven systems, the time that separates the occurrence of two events can be lower-bounded, in which case the scheduling problem becomes quite similar to that of sampled systems (see *sporadic tasks* below).

A. Real-time constraints

a) Period: A *periodic task* executes at regular intervals of time, as defined by its *period*. The period of a task is chosen as follows. On one hand, the period of a task must be below some bound, related to the inertia and physical characteristics of the device it controls. Above this value, the safety of the device (e.g. the stability of the airplane) is not ensured. This bound differs between the different physical devices of the controlled system. For instance, the propulsion devices of a space vehicle must be controlled very fast to ensure the stability of the vehicle, while the position of its solar panels can be controlled a little slower as a little energy loss has less dramatic impact on the vehicle. On the other hand, the period must be above some bound, below which the device will not be able to apply the commands fast enough or may get damaged. Of course, this lower bound also differs from one device to another. The period is usually chosen as close to the upper bound as possible, which spares unnecessary computations and thus enables the use of less powerful hardware, reduces energy consumption, and so on.

b) Deadline: The *deadline* of a task bounds the maximum time allowed between the task invocation and the task completion. In many real-time systems, it is equal to the task period, meaning that one invocation of a task must complete before the next invocation of that task. In some systems, the deadline can be lower than the period. While this may, in rare cases, correspond to a constraint related to the system dynamics, in most cases it is used as a way to artificially influence the system schedule, because tasks with shorter deadlines will usually be scheduled first (see scheduling techniques for dependent tasks in Section VII for instance). Systems where the deadline of a task can be higher than its period are quite uncommon, though they have been studied in the literature.

c) Offset: In some systems, *offsets* are associated to tasks. Instead of releasing tasks simultaneously at system start-up, tasks are released at the date specified by their offset. This parameter is rarely related to the system dynamics but rather to implementation concerns (again, see Section VII for instance).

d) WCET: Performing a schedulability analysis requires the knowledge of the execution time of each task. Since the exact execution time of a task can be very hard to predict, an approximate *worst-case execution time* (WCET) is usually considered. It is *pessimistic*, in the sense that actual execution time might be lower, but *safe*, in the sense that the actual execution time will definitely be lower. WCET is not a real-time constraint per se, since we do not need to force a task to execute for as long as its WCET. It is nonetheless essential for system analysis.

e) Task model: We will use the following notations. A real-time task is denoted $\tau_i(C_i, T_i, D_i O_i)$, where C_i is the WCET of τ_i, T_i is its activation period, D_i is its relative deadline and O_i its offset. In this paper, we assume that $D_i \leq T_i$ and $O_i \geq 0$, though more commonly it is assumed that $D_i = T_i$ and $O_i = 0$.

We denote $\tau_{i.k}$ the k^{th} ($k \geq 0$) invocation, or *job*, of τ_i. For periodic tasks, the job $\tau_{i.k}$ is released at time $o_{i.k} = O_i + kT_i$. For sporadic tasks, we have $o_{i.k} \geq O_i + kT_i$ instead. In both models, every job $\tau_{i.k}$ must be completed before its absolute deadline $d_{i.k} = o_{i.k} + D_i$. In the following, we will focus on periodic tasks. The *hyperperiod* of a task set, which we will denote HP, is defined as the least common multiple of the periods of the tasks.

Figure 1 illustrates task real-time parameters for a periodic task $\tau_i(3, 6, 4, 0)$. Gray rectangles correspond to the execution of τ_i, while hatched rectangles correspond to time intervals during which the system is busy executing another task. $\tau_{i.1}$ is preempted after it starts its execution to execute another task (about preemption, see Section IV). $\tau_{i.2}$ executes for less than the WCET (2 instead of 3). The jobs all respect the deadline constraints.

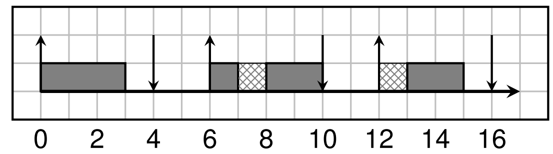


Fig. 1. Execution of a real-time task

B. Implementation

A real-time task is usually implemented using mechanisms akin to threads, provided by the (Real-Time) Operating System. While the exact code is dependent on the target OS, the global structure remains more or less the same. Algorithm 1 details the typical implementation of a periodic task in pseudo-C code, while Algorithm 2 illustrates how to invoke such tasks. Concerning Algorithm 1:

1) The task first goes through an initialization phase and then waits for its initial release. In RTOS derived from Linux (for instance RTAI [4] or the `ptask` API [5]), tasks start executing as soon as they are created, just like threads. Since creating all the tasks takes time, the programmer needs to implement in this phase mechanisms that synchronize the initial release of tasks (e.g. a synchronization barrier). In other RTOS, such as FreeRTOS for instance [6], tasks start only when the scheduler is explicitly invoked (`start_scheduler` in Algorithm 2), so only tasks with offsets need to explicitly wait for the initial release;
2) Once the initialization phase is complete, the task starts executing periodically. It repeats indefinitely the same behaviour: 1) execute the code corresponding to the actual functionality of the task; 2) wait for the next task activation (typically, wait for the next period).

Concerning Algorithm 2:

- The task creation primitive usually takes, at least, three parameters: 1) a pointer to the task function that defines the task functional behaviour (like `periodic_task1`); 2) the arguments (often empty) to pass to that function (here, to function `periodic_task1`); 3) the real-time parameters of the task;
- In a RTOS where tasks start executing as soon as they are created, we usually add an infinite idle loop (`while(1);`) at the end of the `main` function, because the termination of the `main` function causes the termination of the tasks. In other systems, the execution of tasks starts when the scheduler is explicitly invoked (`start_scheduler`).

Even though the task structure presented here is very common, RTOS usually allow more complex task implementations, where for instance the code executed by the task changes from one iteration to the next, where the period can be modified dynamically, etc. For instance, a common rookie mistake is to forget to wait for the next period at the end of a task iteration. This is in no way verified by the OS or by the compiler, even though this causes obvious problems at execution. So, it is up to the programmer to ensure that the program respects the task model that was used to perform the schedulability analysis.

RTOS that follow the OSEK standard [7], including Trampoline [8], opt for a more rigid approach where the task set and its real-time characteristics are defined statically in a dedicated OIL file [9]. The task implementation model is predefined and the user only provides the function corresponding to a single iteration of the task (the `task_functionality()` function in Algorithm 1), which is automatically repeated by the RTOS at each task invocation. This model is less permissive, but it prevents some design mistakes, such as those discussed earlier for instance (immediate loops or main function finishing and killing tasks).

Algorithm 1 Typical implementation of a periodic task

```
void* periodic_task1(void* args) {
  //initialization
  wait_for_release();

  while (1) {
    task_functionality();
    wait_next_activation();
  }
}
```

Algorithm 2 Typical program for invoking two tasks

```
int main() {
  // initialize real-time parameters params1
  create_task(periodic_task1,args1,params1);

  // initialize real-time parameters params2
  create_task(periodic_task2,args2,params2);


  // either start_scheduler(); or while(1);

  // This should never be reached
  return 1;
}
```

III. Data-dependencies

The seminal work on real-time scheduling of periodic tasks by Liu & Layland [10], and a very large portion of the real-time scheduling literature, assume real-time systems consisting of *independent* tasks, meaning that there is no a-priory ordering constraint relating tasks. Obviously though, tasks collaborate to compute the system outputs and as a result are related by data-dependencies, meaning that some tasks produce data used by some other tasks to perform their own computations.

A. Communication semantics

There are mainly two possible semantics for data-communications. With *register-based* communications, used for instance in the automotive domain [11], when a task executes it consumes the last value produced, not considering when it was produced. With such communications, the relative order between the task producing the data and the task consuming it remains unconstrained. On the contrary, with *causal* communications, the task producing the data must complete its execution before the task consuming it starts its execution. Such communications yield more deterministic systems and are thus preferred in critical systems, such as avionics systems for instance [3].

Causal communications introduce additional real-time constraints, called precedence constraints: a *precedence constraint* requires one task to execute before another. A *dependent task set* is modelled as a directed acyclic task graph $\mathcal{G} = (\mathcal{S}, \mathbb{E})$, where $\mathcal{S} = (\{\tau_i(C_i, D_i, T_i, O_i)\}_{1\leq i\leq n})$ is a set of tasks, as defined previously, and $\mathbb{E} \subseteq \mathcal{S}\times\mathcal{S}$. A precedence constraint, is denoted $\tau_i \rightarrow \tau_j$, with $\tau_i \rightarrow \tau_j \equiv (\tau_i, \tau_j) \in \mathbb{E}$. If $T_i = T_j$, we say that $\tau_i \rightarrow \tau_j$ is a *simple precedence constraints*. Otherwise we say that it is an *extended precedence constraint*.

B. Extended precedence constraints

In the general case, precedence constraints may relate tasks with different periods. Such a system is illustrated in Figure 2, which describes the tasks of the longitudinal flight control system of an aircraft (this description is based on [12]). This system controls the angle of the control surface ($order$) based on: the current surface angle ($angle$), the current aircraft vertical speed (vz), the current aircraft altitude ($altitude$) and the altitude required by the pilot ($required\ altitude$). The software architecture consists of six tasks that make up three computation chains:

- A fast chain operating at 30 ms from *EF* to *EL*, which controls the control surface according to a required angle;
- A multi-rate chain from *vzF* (period 30 ms) to *vzL* (period 40 ms), then to *EL* (period 30 ms), which controls the vertical speed;
- A multi-rate chain from *hF* (period 60 ms), to *hHL* (period 60 ms), to *vzL* (period 40 ms), to *EL* (period 30 ms), which controls the trajectory the plane follows to reach the altitude ordered by the pilot.

An extended precedence constraint $\tau_i \rightarrow \tau_j$ (where $T_i \neq T_j$) only imposes precedence constraints on a subset of the jobs of τ_i and τ_j. The system model must thus specify the set of pairs (p, q) such that $\tau_{i.p} \rightarrow \tau_{j.q}$.

In most real-time applications and design tools, such as Simulink [13], AADL [14] or Prelude [15] for instance, extended precedence constraints follow patterns repeated periodically on the tasks hyperperiod. Some examples of extended precedence constraints are illustrated in Figure 3 (see [16] for more details on extended precedence constraints models). The programmer can use the patterns of Figures 3(a), 3(b), to leave flexibility for the execution of the slow task: the slow task consumes values produced early, i.e. samples only the first out of 3 successive jobs of the producer, and produces values late, i.e. communicates with the last out of 3 successive jobs of

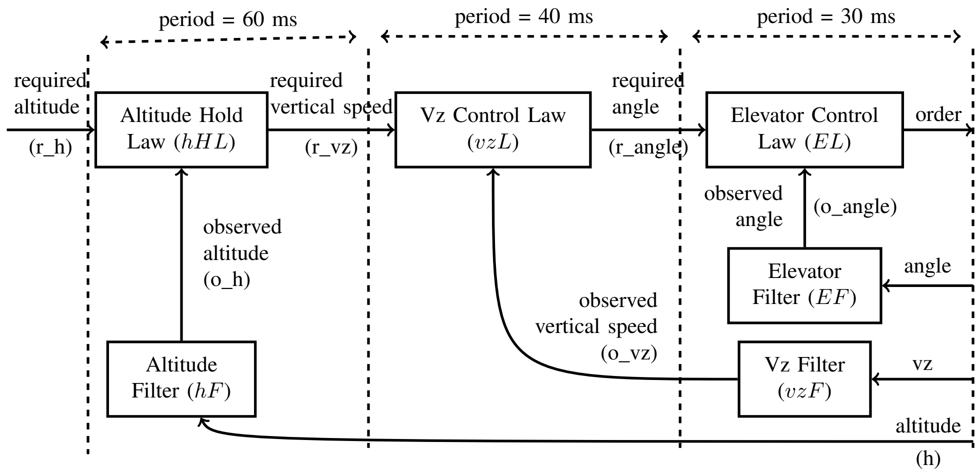
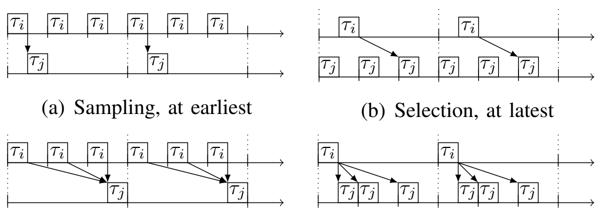


Fig. 2. Vertical speed control



(a) Sampling, at earliest

(b) Selection, at latest

(c) Array gathering

(d) Array scattering

Fig. 3. Extended precedence constraints

the consumer. The patterns of Figure 3(c) and Figure 3(d) correspond to classic signal processing patterns, where repetitive array computations are distributed between several repetitions of the same task: on one hand the slow task scatters the content of a big array between successive jobs of its consumer and on the other hand it gathers array fragments from its producer to construct a big array. These two patterns behave in a fashion similar to the `MPI_gather` and `MPI_scatter` primitives of the popular Message Passing Interface (MPI) API [17].

IV. Real-time scheduling: problem definition

As mentioned earlier, scheduling real-time systems requires to:

1) Design a scheduling policy that will decide how to order the execution of tasks;
2) Design a schedulability test that will ensure, before system execution, that the execution order produced by the scheduling policy will respect the real-time constraints.

A. Schedulability

The execution of a set of real-time tasks is controlled by a *scheduler* that decides at each instant which task to execute on the processor. For a given schedule (produced by the chosen scheduler), we let $e(\tau_{i.k})$ denote the start time of $\tau_{i.k}$ in the schedule and $s(\tau_{i.k})$ denote its completion time. The validity of a schedule is established as follows:

Definition 1. *A schedule is* feasible *if it respects the following constraints:*

$$\forall \tau_{i.p}, s(\tau_{i.p}) \geq o_{i.p} \quad (1)$$

$$\forall \tau_{i.p}, e(\tau_{i.p}) \leq d_{i.p} \quad (2)$$

$$\forall \tau_{i.p} \rightarrow \tau_{j.q}, e(\tau_{i.p}) \leq s(\tau_{j.q}) \quad (3)$$

A *scheduling policy* is a set of rules that dictates how the scheduler will choose which task to execute. A task set is *schedulable* by a given scheduling policy if and only if the schedule it produces is feasible. A *schedulability analysis* checks whether a given task set is schedulable by a given scheduling policy. This analysis is usually performed statically, that is to say before system execution. Except for simplified problems, schedulability analysis is NP-hard. Therefore, existing analyses are either exact and have exponential complexity, or are pessimistic and have a polynomial or pseudo-polynomial complexity. With a *sufficient* schedulability test, all task sets considered schedulable by the test are actually schedulable. With a *necessary* schedulability test, all task sets considered unschedulable by the test are in fact unschedulable. An exact test if both sufficient and necessary.

B. Scheduling policy classes

Scheduling policies can be separated in different classes of policies. A scheduling policy is *preemptive* if it allows interrupting a job during its execution and resuming it later (e.g. to execute a higher priority task). *Off-line scheduling* consists in computing a (cyclic) feasible schedule before system execution. In that case, the scheduler becomes a simple dispatcher that repeats indefinitely the off-line schedule during execution. With *on-line scheduling*, the scheduler computes the schedule as execution progresses, based on the chosen scheduling policy. Most on-line scheduling policies are *priority-based*, meaning that they only define how to assign priorities to tasks and that the scheduler then always chooses to execute the highest priority ready task.

A scheduling policy $\mathcal{P}$ is *optimal* within a certain class of policies if the following holds: if a task set is schedulable by some policy of this class, then it is schedulable by $\mathcal{P}$. For instance, we will see that the Deadline Monotonic policy is optimal within the class of fixed-task priority policies, for independent task sets without offsets. In this paper, we will focus on on-line priority based preemptive policies.

C. Fixed-tasks vs fixed-job priority policies

With a *fixed-task priority* scheduling policy (FTP), the priority of a task remains unchanged during the whole system execution. It is the most widely used class of policies for scheduling real-time systems and every RTOS provides support for it. With a *fixed-job priority* scheduling policy (FJP), the priority can differ between jobs of the same task, but remains unchanged for a given job. Liu and Layland [10] proposed the *rate-monotonic* (RM) fixed-task priority policy, where tasks with a shorter period are affected a higher priority and the *earliest-deadline-first* (EDF) fixed-job priority policy, where jobs with a shorter absolute deadline are affected a higher priority. RM is optimal within the class of fixed-task priority policies for periodic task sets with $T_i = D_i$ and $O_i = 0$. It can be extended to the *deadline-monotonic* policy (DM) [18], to schedule optimally a set of tasks with $D_i \leq T_i$ and $O_i = 0$. For the case where $O_i \geq 0$, an optimal algorithm based on DM was defined by Audsley in [19]. EDF is optimal in the class of fixed-job priority policies for scheduling a set of periodic tasks with $D_i \leq T_i$ and $O_i \geq 0$.

RM tends to be favored instead of EDF by real-time developers, at least for uniprocessor systems. A detailed comparison is available in [20], it can be summarized as follows:

- *Implementation*: With FTP, task priorities can be computed before run-time (either by the programmer or at start-up by the OS), while with FJP they must be computed by the scheduler at run-time, each time a task is released. Furthermore, RTOS always provide support for fixed-task priority scheduling, while it is not always the case for fixed-job priority scheduling;
- *Run-time overhead*: FJP requires to compute task priorities at run-time, which introduces a run-time overhead that does not exist with FTP. However, when context switches are taken into account, FJP introduces *less overhead* than FTP because the number of preemptions that occur with FJP is usually much lower than with FTP;
- *Task jitter*: The task jitter is the variation between the response times of different jobs of the task. FTP reduces the jitter of high priority tasks but increases that of low priority tasks, while FJP treats tasks more equally. As a result, when the utilization factor of the processor is high (i.e. when the processor has few idle times), the *average* task jitter of the task set is lower with FJP than with FTP;
- *Overloads*: When the total execution time demand of tasks exceeds the processor capacity, FTP causes less deadline misses than FJP. The problem with FJP is that if a task misses its deadline and carries on anyway, it causes other tasks to miss their deadlines (domino effect);
- *Processor utilization*: FJP enables better processor utilization (up to 100%) than FTP (around 70-80%), thus allowing more complex computations to be performed.

V. Schedulability with Fixed-task priority

A. Busy period

The concept of *busy period* plays a central role in the schedulability analysis of FTP. A *k-busy period* is a time interval where the processor is kept busy executing task of priority higher than or equal to k. This is illustrated in Figure 4. The step function corresponds to the cumulative demand for execution on the processor, also called the Demand Function. At time 0, three jobs are released so the Demand Function steps for an amount equal to the sum of the three jobs WCET. This starts the first $3-busy$ period of the system. This period finishes when the Demand Function meets the Time Function $w = t$ (at time t, the processor has executed t units of work). Such a meeting point is called an *idle time*: the processor remains idle until the next job release.

The *response-time* of a job is the time interval from the job release to the job completion. The worst-case response-time of a task is the greatest response-time of its jobs. A task-set is schedulable if and only the worst-case response-time of each task is lower than its deadline. The following theorem enables to relate response times to busy periods

Theorem 1 ([19], [21])**.** *The worst-case response time of a task of priority k is equal to the longest $k-$busy period.*

So, to check system schedulability, we can compute the longest k-busy period of each task τ_k and compare it to its deadline. A busy period always ends on the condition that the processor utilization ratio $U = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq 1$ (note that this is also a trivial necessary schedulability condition for FTP). In other words, if this condition is fulfilled, then there is at least one idle time. Let n be the lowest priority of the system, if $U = 1$ then the length of the longest $n-$busy period is equal to the hyperperiod of the task set (HP). Since HP is the least common multiple of the periods of the task set, HP is exponential with respect to the number of tasks of the task set.

In [22], authors propose a pseudo-polynomial solution for computing the length of the longest $i-$busy period B_i. It is computed as the least fixed-point of the following equation (where $hp(i)$ denotes the tasks with higher priority than τ_i):

$$B_{i,0} = C_i$$
$$B_{i,k+1} = C_i + \sum_{j \in hp(i)} \lceil \frac{B_{i,k}}{T_i} \rceil C_j$$

B. Schedulability tests

Response-time analysis (RTA) based on the busy period, as presented in the previous section, provides a feasibility test that is actually independent of the scheduling policy. Indeed, since it is a necessary and sufficient schedulability condition, then any **optimal** scheduling policy can rely on

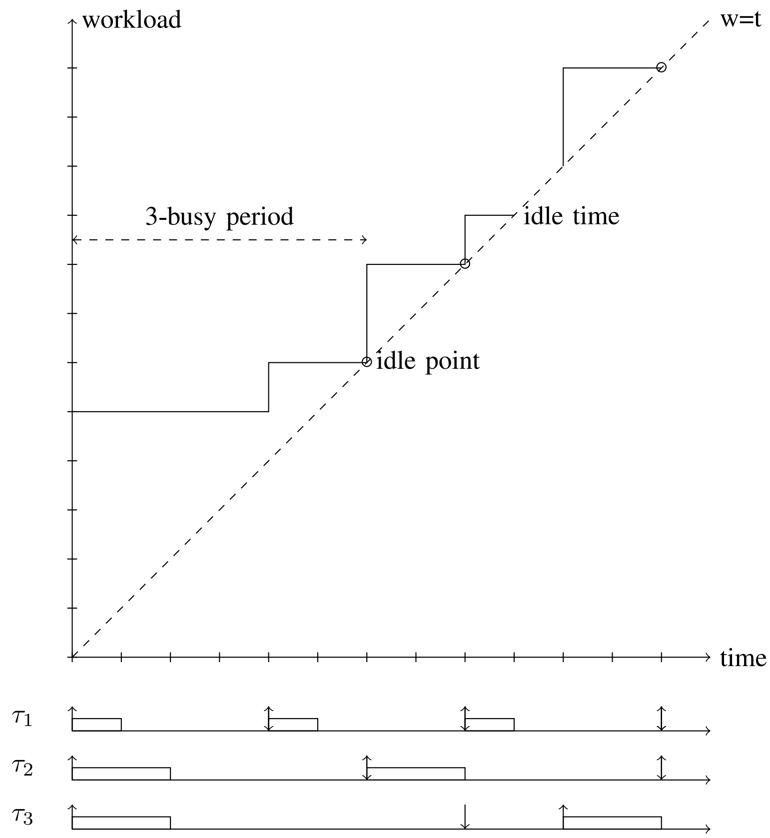


Fig. 4. Time demand function

this condition. Unfortunately, RTA does not have polynomial complexity. In this section we discuss some less complex specific schedulability tests.

For RM, the following sufficient schedulability test was proposed in [10]:

$$U = n(2^{1/n} - 1)$$

It is however quite pessimistic. It yields maximum processor utilization ratios between 80% (for smaller values n) and 70% (for greater values of n), while simulations show that the average bound is around 88% [23]. Tighter utilization bound tests have been proposed in [24], [25].

For DM, exact exponential schedulability tests have been proposed in [26], [27].

Le us now consider systems with offsets. A *critical instant* occurs when all the tasks are released simultaneously. It was proved in [19] that the longest busy period is initiated by the critical instant. For systems without offsets, this means at date 0. For systems with offsets however, the critical instant may not exist, which means that we need to study all the busy periods up to $max(O_i) + 2HP$ [18], [28].

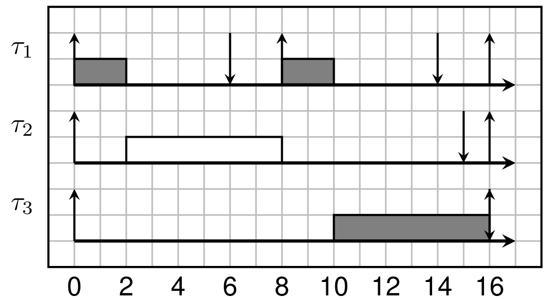
C. Shared resources

Using mutual exclusion mechanisms to handle shared resources in real-time application scheduled with FTP, via either mutexes or semaphores, introduces two well-known sources of bugs: priority inversion and scheduling anomalies.

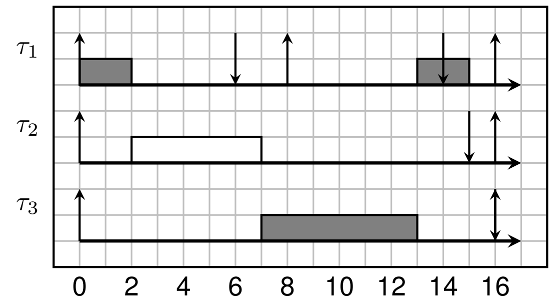
A *scheduling anomaly* occurs when the system is deemed schedulable by a schedulability test, but the schedule produced at execution is infeasible, due to a task executing for less than its WCET. This is illustrated in Figure 5. We consider a task set without offsets $\mathcal{S} = \{\tau_1(2, 8, 6), \tau_2(6, 16, 15), \tau_3(6, 16, 16)\}$ where tasks sharing a resource using mutexes are depicted in gray. When $C_2 = 6$, the schedule is feasible, while it is not when $C_2 = 5$ (τ_1 misses its deadline). This phenomenon is quite counter-intuitive, since here reducing an execution time produces a worse case. This implies that we cannot rely on a simple simulation to test the feasibility of a task set with shared resources.

A *priority inversion* occurs when a task is delayed by a lower priority task, even though it does not share a resource with it. This is illustrated in Figure 6. We consider a task set with offsets, where mutual exclusion sections are depicted in gray. In this example, τ_2 is running while the highest priority task τ_1 is waiting for τ_3 to complete its critical section, which causes τ_1 to miss its deadline.

An intuitive solution to prevent priority inversion is the Priority Inheritance Protocol (PIP) [29]: when a task gains control of a shared resource, it inherits the priority of the highest priority task that shares this resource, until completion of its critical section. This completely prevents priority inver-



(a) Simulation is feasible



(b) Execution is infeasible

Fig. 5. Scheduling anomaly

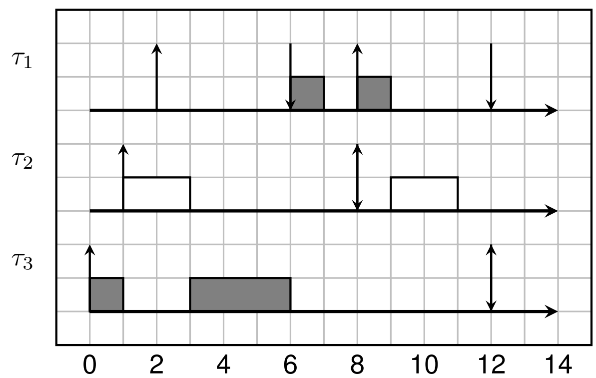


Fig. 6. Priority inversion

sions, however the number of times a task can be blocked when trying to enter its critical section can be quite high. Authors of [29] instead proposed the Priority Ceiling Protocol (IPCP, here we consider the "immediate" variation of the protocol), which reduces blocking times. Each resource has a *priority ceiling* equal to the highest priority amongst task using it. When a task executes, its priority becomes the maximum of the priority ceilings of the resources it uses. As a consequence, a task can only be blocked once, at the beginning of its execution. Indeed, to start executing all the resources it requires must be free, otherwise it is blocked (by higher priority tasks). IPCP is readily available in many real-time executives (e.g. Real-Time POSIX, OSEK/VDX, Real-Time Java).

VI. Schedulability with Fixed-job priority

A. Schedulability tests

When $D_i = T_i$ and $O_i = 0$, schedulability with EDF is a very simple problem: it was proved in [10] that $U \leq 1$ is an exact schedulability condition.

When $D_i \leq T_i$ and $O_i = 0$, the problem becomes much more difficult. Unfortunately, the worst-case response time does *not* occur at a critical instant, which makes response-time analysis for EDF quite difficult. Instead, schedulability tests for EDF rely on the concept of *processor demand* [30]. The processor demand $h(t)$, which represents the amount of work that must be completed before time t, can be computed as follows:

$$h(t) = \sum_{i=1}^{n} \lfloor \frac{t + T_i - D_i}{T_i} \rfloor C_i$$

As the load must never exceed the available processing time, schedulability can be stated as:

$$\forall t, h(t) \leq t$$

Schedulability analysis checks this condition on a limited number of values of t, by noting that: 1) $h(t)$ only increases at job deadlines; 2) there exists an upper bound L on the values of t that must be checked. Two formulas have been provided to compute L. The first bound L_a is computed as follows:

$$L_a = max\{D_1, \ldots, D_n, \frac{\sum_{i=1}^{n}(T_i - D_i)C_i/T_i}{1-U}\}$$

The second bound L_b is the least fixed-point of the following recurrence:

$$w^0 = \sum_{i=1}^{n} C_i$$
$$w^{j+1} = \sum_{i=1}^{n} \lceil \frac{w^j}{T_i} \rceil C_i$$

Schedulability can then be tested by verifying that $h(t) \leq t$ for each deadline in $L = min(L_a, L_b)$.

When $D_i \leq T_i$ and $O_i \geq 0$, a schedulability test was proposed in [31].

B. Shared resources

Several solutions have been proposed to support shared resources with FJP. An adapted version of PCP was described in [32], however it has high run-time overhead. A better version, which retains the properties of PCP, is available in [33].

VII. Schedulability of dependent task sets

A. Precedence encoding

An elegant solution for scheduling dependent tasks was presented in [34]. Authors proposed to *encode* precedence constraints in task real-time constraints. Let $preds(\tau_i)$ denote the set of all predecessors of τ_i and $succs(\tau_i)$ the set of its successors. For simple precedence constraints, the encoding consists in adjusting the release date and deadline of every task as follows:

$$O_i^* = \max(O_i, \max_{\tau_j \in preds(\tau_i)} (O_j^*)) \tag{4}$$

$$d_i^* = \min(d_i, \min_{\tau_j \in succs(\tau_i)} (d_j^* - C_j)) \tag{5}$$

The intuition is that a task must finish early enough for its successors to have sufficient time to complete before their own deadline. For each precedence constraint $\tau_i \rightarrow \tau_j$, Equation 4 ensures that τ_i is released before τ_j. Assuming that priorities are assigned based on deadlines (e.g. with EDF or DM), Equation 5 ensures that τ_i will have a higher priority than τ_j. As a consequence, τ_i will be scheduled before τ_j.

Theorem 2 ([34], [35])**.** *Let $\mathcal{S} = \{\tau_i(C_i, T_i, D_i, O_i)\}$ be a dependent task set with simple precedence constraints. Let $\mathcal{S}^* = \{\tau_i'(O_i^*, C_i, D_i^*, T_i)\}$ be a set of independent tasks such that O_i^* and D_i^* are given by the previous formulas:*

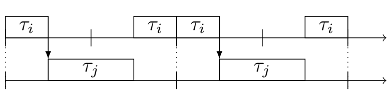
$\mathcal{S}$ is feasible if and only if $\mathcal{S}^*$ is feasible.

The consequence of this theorem is that we obtain a schedulability for dependent tasks with simple precedence constraints that works as follows ([34], [35], [16]):

1) Perform the precedence encoding;
2) Apply a schedulability test for an optimal scheduling policy for independent tasks (DM, Audsley's policy, or EDF) on the modified task set.

B. Extended precedence constraints

For extended precedence constraints, the encoding technique requires to assign different deadlines and offsets for different jobs of the same task. Let us for instance consider the tasks set $\mathcal{S}_1 = \{\tau_i, \tau_j\}$, with $(O_i = 0, C_i = 2, D_i = T_i, T_i = 4)$, $(O_j = 0, C_j = 4, D_j = 6, T_j = 8)$. The precedence pattern for $\tau_i \rightarrow \tau_j$ is defined informally as follows:



If we set the same adjusted relative deadline for all jobs of τ_i, that is $D_{i.n}^* = 2$ for $n \in \mathbb{N}$, then $\tau_{j.0}$ will miss its deadline (date 6). However, if we set $D_{i.n}^* = 2$ for $n \in 2\mathbb{N}$ and $D_{i.n}^* = 4$ for $n \in 2\mathbb{N} + 1$, $\mathcal{S}^*$ is schedulable with EDF (we obtain the schedule depicted above). Let $preds(\tau_{j.q})$ denote the predecessors of job$\tau_{j.q}$ and $succs(\tau_{i.p})$ denote its successors. The encoding rule becomes:

$$o_{j.q}^* = \max(o_{j.q}, \max_{\tau_{i.p} \in preds(\tau_{j.q})} (o_{i.p}^*)) \tag{6}$$

$$d_{i.p}^* = \min(d_{i.p}, \min_{\tau_{j.q} \in succs(\tau_{i.p})} (d_{j.q}^* - C_j)) \tag{7}$$

For schedulability analysis, extended precedence constraints usually follow patterns repeated on the tasks hyperperiod. So, for FTP, for each task we can basically take the maximum adjusted release date of its jobs over one hyperperiod and the minimum adjusted deadline [35]. For FJP, we can unfold the extended precedence graph on the hyperperiod of the tasks (as suggested in [36]) and apply the encoding on the unfolded graph, which only contains simple precedence constraints [16].

C. Note on synchronizations

When dealing with precedence constraints, one could expect problems similar to those encountered with shared resources. This is however not the case. First, concerning scheduling anomalies, let us consider our previous example of Figure 5. Figure 7 considers two different extended precedence constraints and shows that the anomaly does not occur anymore: the inversion between jobs of τ_1 and τ_3 does not happen when the execution time of $\tau_{2.0}$ is reduced. This is because the relative order of τ_1 and τ_3 remains unchanged due to the precedence constraint, no matter what the execution time of τ_2 may be.

Concerning priority inversion, let us consider our previous example of Figure 6. Figure 8 shows that the inversion does not occur anymore. Assume we have a precedence constraint $\tau_{3.0} \rightarrow \tau_{1.0}$. Due to the precedence encoding, the deadline of τ_3 is reduced, this prevents τ_2 from preempting it, which was the cause of the priority inversion previously. The inversion is avoided because, in a way, precedence encoding mimics Priority Inheritance: the predecessor task (τ_3) "inherits" the deadline of the successor task (τ_1).

VIII. Conclusion

This paper presented a quick overview of real-time uniprocessor scheduling. It does not aim at providing an exhaustive list of existing results on this topic, but instead focuses on the main ones. Furthermore, it tries to take the perspective of the programmer, instead of only considering purely theoretical aspects. For starters, it provides some insights on where real-time constraints come from during the design process. Then, it gives an overview of how to implement a multi-task real-time system. Finally, it focuses on the scheduling of dependent tasks, that is to say systems where data-communications play a non-negligible role.

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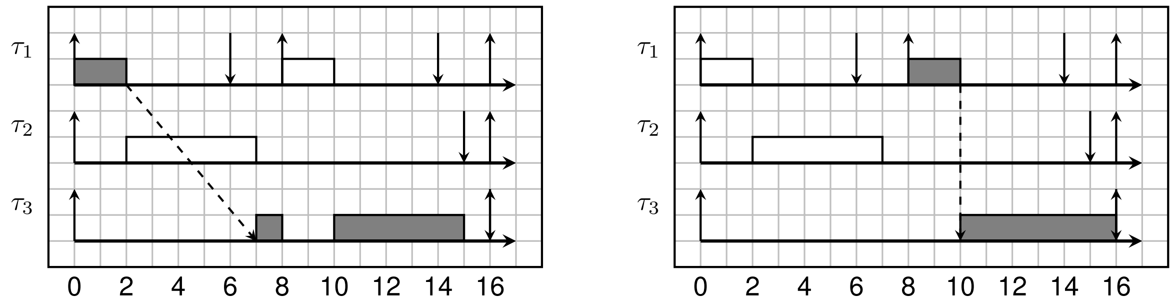

(a) $\tau_{1.0} \rightarrow \tau_{3.0}$ (b) $\tau_{1.1} \rightarrow \tau_{3.0}$

Fig. 7. No scheduling anomaly with precedence constraints

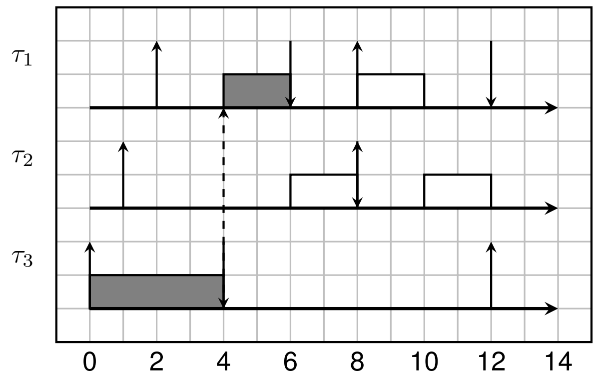

Fig. 8. No priority inversion with precedence encoding

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