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QUake-MD: open source code to Quantify Uncertainties in Magnitude–Depth estimates of earthquakes from macroseismic intensities

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Abstract

This paper presents a tool to quantify uncertainties in magnitude-depth (M-H) estimates for earthquakes associated with macroseismic intensity data. The tool is an open-source code written in python and named *QUake-MD* (**Q**uantifying **U**ncertainties in earthquakes' **M**agnitude and **D**epth). In *QUake-MD* uncertainties are propagated from the individual intensity data point (IDP) to the final Magnitude (M) / Depth (H) / epicentral intensity (I_0) solution. It also accounts for epistemic uncertainties associated to the use of different intensity prediction equations (IPE). For each IPE, *QUake-MD* performs a sequential least square inversion process to estimate the central M-H value. *QUake-MD* then explores the uncertainties around this central M-H solution by constructing a probability density function (PDF) constrained to be consistent with the range of plausible epicentral intensity I_0 , a plausible depth range and IDP uncertainties. The resulting PDF of all IPEs provided to *QUake-MD* are then stacked to obtain a final PDF of

possible $M/H/I_0$ solutions representative of both data quality and IPE epistemic uncertainties. This tool is geared towards end-users that would like to grasp a more complete understanding of the uncertainties associated with historical earthquake parameters, beyond the classical standard deviation values proposed today in parametric earthquake catalogues. We apply QUake-MD to two events of the SISFRANCE macroseismic database to illustrate the challenges involved in building realistic spaces of $M/H/I_0$ solutions reflecting the quality of the data and the epistemic uncertainties in IPEs.

Introduction

In regions of moderate seismic activity, instrumentally recorded earthquakes are not sufficient to assess seismic hazard. Thanks to the precious research of historians, numerous historical sources found in the archives could be translated into macroseismic intensity data points (IDP) through the use of an intensity scale, such as the Medvedev-Sponheuer-Karnik scale (MSK). Estimates of magnitude and depth from macroseismic data date back to the late 20th century (Kövesligethy, 1907; Gutenberg and Richter, 1942; Sponheuer, 1960). Intensity prediction equations (IPE) are used to estimate those parameters. IPEs are calibrated on earthquake with both IDPs and instrumental magnitude and in some cases depth. One of the most common mathematical formulation of IPE is shown in equation (1) (Ambraseys, 1985; Beauval *et al.*, 2010; Boyd and Cramer, 2014; Quadros *et al.*, 2019):

$$I = C_1 + C_2 M + \beta \log(D_{hypo}) + \gamma D_{hypo} \quad (1)$$

Where I , M , C_1 , C_2 , β , γ and D_{hypo} are respectively the intensity measure, the magnitude, the

44 magnitude coefficients, the attenuation of intensity coefficients and the hypocentral distance
45 associated to the intensity measure. As a reminder, because IPEs do not generally account
46 explicitly for site effects, it is necessary to select IDPs not affected by site effects for the
47 calibration of IPEs.

48 In recent years, an effort has been sought to improve the quantification of uncertainties in
49 parametric earthquake catalogues. Gasperini et al. (2010), for example, propose to quantify
50 uncertainties in magnitude estimates based on the number of data in the calibration process
51 and in the application process ; Stucchi et al (2013) propose to make weighted averages of
52 existing parametric catalogue and computed weighted uncertainties; Traversa et al (2018)
53 propose to incorporate epistemic uncertainties of IPE. Irrespective of the methodology, in
54 these approaches, uncertainties are represented in the resulting earthquake parametric
55 catalogues as standard deviation values.

56 Bakun and Scotti (2006) proposed an objective method to quantify the impact of location
57 uncertainty on magnitude by combining Bakun and Wentworth's (1997) and the bootstrap re-
58 sampling (Efron, 1982) techniques. This method goes one step beyond classical methods by not
59 only spatially quantifying uncertainties in the location and thus the magnitude of historical
60 earthquakes but also by propagating epistemic and aleatory uncertainties in the model.

61 However, this approach requires making reasonable hypothesis on the depth of the earthquake
62 under consideration.

63 Complementary to Bakun and Scotti (2006), the methodology proposed here assumes, in its
64 present version, that the epicentral location is known and rather attempts to invert the IDPs for

depth as well as magnitude. The methodology, hereafter named QUake-MD, for **Quantifying Uncertainties for earthquake Magnitude and Depth** estimates, aims thus at providing end-users with a space of weighted magnitude (M), depth (H) and epicentral intensity (I_0) solutions for historical earthquakes reflecting epistemic uncertainties of IPE and quality of IDPs.

Inspired by the original work of Baumont and Scotti (2008) we here revisit their approach which integrates the use of different IPEs and of macroseismic data uncertainties into the estimate of historical earthquake parameters. The use of different IPEs aims to represent the epistemic uncertainty inherent to IPEs calibration.

In the first part of the paper, we briefly present the QUake-MD methodology beginning with the data quality integration and followed by the integration of IPE epistemic uncertainty. Then, in the second part we apply QUake-MD to two example earthquakes (an instrumental and an historical one) to illustrate the challenges involved in building realistic spaces of $M/H/I_0$ solutions reflecting the quality of the data and the epistemic uncertainties in IPEs.

QUake-MD is provided with a graphical user interface to facilitate, in the pre-processing stage, the visualization of the macroseismic field and the setting of QUake-MD modeling options (i.e. depth limits of the space of solution and intensity values used).

Methodology

The QUake-MD methodology is shown in the flowchart presented in Figure 1. The parameters required to run QUake-MD are IDPs with associated quality, I_0 with associated quality, an epicenter location, depth limits for the depth inversion and at least one IPE. IPEs should have

the mathematical formulation shown in equation (1). Given these input parameters QUake-MD provides a weighted $M/H/I_0$ space of solutions reflecting the intensity data uncertainties and IPEs epistemic uncertainties, when more than one IPE is considered.

Uncertainties that the expert have in the estimate of the intensity values are sometimes provided in macroseismic database in terms of quality factors (e.g. in SISFRANCE IDP quality range from A-certain to C-uncertain; for I_0 additional quality classes are defined here when there is no data in a radius greater than 30 km). To exploit this information, in Quake-MD the quality factors are transformed in numerical values that allow to weight each IDP and the I_0 in the computation of M , H parameters as well as to define the space of possible solutions associated with I_0 . The numerical value associated to I_0 called hereafter σ_{I_0} aims to represent the standard deviation of I_0 . The numerical values proposed in Table 1 and Table 2 are for illustrative purposes and can be easily changed by the end-user. Uncertainty in epicentral location is not yet included in QUake-MD. End-users can re-run the program using different epicentral locations (derived by other methods, e.g. BOXER (Gasperini *et al.*, 2010), or by other expert judgment).

Integrating the quality of the macroseismic field

The first step in QUake-MD is to bin the IDP (Figure 1). When it comes to applying IPE to historical events, data is often affected by lack of intensity values in the lower range, which limits the range over which the binning can be performed without introducing biases. In such cases only the ‘intensity-level binning’ strategy can be applied. Indeed, Bakun and Scotti (2006)

have shown that ‘intensity-level binning’ and ‘distance binning’ result in significantly different descriptions of the attenuation of intensity. They conclude that intensity-level binning is less biased by the heterogeneous distribution, quality and completeness of macroseismic data, and thus provides better intensity attenuation. Currently, in QUake-MD only one intensity binning strategy, called hereafter RAVG (for radius average) strategy is implemented. The RAVG strategy computes for each intensity level the weighted mean of the hypocentral distance (geometric mean), considering the weights attributed to each IDP. The associated weighted standard deviation in intensity is also computed for each bin and used in the following as representing the standard deviation σ_I associated to the intensity bins (see Figure 2). For one IPE, QUake-MD then performs a sequential least square inversion process to estimate M-H on the binned IDPs and the I_0 value. The I_0 value provided by the end-user is considered in the inversion of M and H as an additional data. In many cases, I_0 is an expert opinion value and not based on data available at the epicenter. Thus its value is also inverted with M and H in the inversion process. Details are explained in the electronic supplement material.

The weighted standard deviation associated to each intensity bin σ_I is used in two different ways in the inversion process. Firstly, σ_I is used to attribute a weight to each intensity bin, which is equal to the square of the inverse of σ_I . The M-H solution computed by the iterative process integrates then the IDP quality through the use of a weight based on σ_I . Secondly, σ_I is used to compute the standard deviations associated to M and H, respectively σ_M and σ_H which are the root mean square of the posterior covariance matrix (see electronic appendices for more details).

128 The inversion procedure ends when the M and H values stabilize (see electronic appendices).
129 The corresponding M, H, σ_M and σ_H are values classically fed to parametric earthquake
130 catalogues (Stucchi *et al.*, 2013; Rovida *et al.*, 2019).

131 QUake-MD allows the user to go one step further by considering the ensemble of M-H couples
132 with their associated weight given by the product of the value of the Gaussian distribution of σ_M
133 and σ_H . To be consistent with the initial hypothesis, for each M-H couple, the corresponding l_0 is
134 computed and filtered in order to comply with an a-priori $l_0 \pm 2\sigma_{l_0}$ limit (Table 2, Figure 3 (b),
135 Figure 3 (c)) and the H $\pm$ limits defined by the user. After filtering, the weights associated with
136 the space of plausible M/H/ l_0 solutions are normalized to 1. The computed l_0 is kept and
137 associated to each remaining M-H couple to define a three dimensional space of solutions in
138 M/H/ l_0 .

139 The exploration of plausible M-H couples is set at $2\sigma_H$ and $2\sigma_M$ in QUake-MD. This value can be
140 easily adjusted in the source code.

141 **Integrating IPE epistemic uncertainties**

142 Calibration of an IPE is quite challenging because of the numerous uncertainties implied in the
143 process. The robustness of an IPE depends on the quality of the calibration data set: the
144 instrumental magnitude and depth need to be well constrained, the magnitude range as large
145 as possible and the IDP dataset sufficiently well informed (number of IDPs, quality of the IDP,
146 good geographical repartition of the IDPs, epicentral distance range of the IDP). The uncertainty
147 propagation strategy adopted in QUake-MD allows the user to test the sensitivity of M/H/ l_0
148 solutions to different IPEs and decide how to best represent the epistemic uncertainties due to

the uncertain nature of the IPE calibration process. In order to do this, QUake-MD stacks the weighted space of solutions resulting from each IPE. The user can choose to weight equally each IPE or to weight IPEs based on intensity prediction performance of the IPE, its magnitude prediction performance, number of data used in the IPE calibration, its magnitude range used for the calibration earthquakes, etc. A parallel can be made between the weights associated to the IPEs and the weights associated to GMPEs in logic trees used in probabilistic seismic hazard assessment (Scherbaum *et al.*, 2009).

The impact of the use of different IPEs is illustrated in part 2, (Figure 5 and Figure 7).

Applications

We will now illustrate QUake-MD through two Pyreneans earthquakes located in the South-West of France, close to the Spanish border (Figure 4). The two earthquakes were chosen to illustrate the impact of (i) the choice of IPEs when considering a good macroseismic data set available for instrumental earthquakes and (ii) the choice of the depth-range when considering sparse data sets typically available for historical events. Epicentral parameters (location and intensity) and IDPs are provided by the SisFrance 2016 macroseismic intensity database (Scotti *et al.*, 2004). In the SisFrance database, each IDP has an associated quality factor: A (Reliable), B (Fair) or C (Uncertain). Weights associated to the quality factors are shown in Table 1. The SisFrance database contains half-degree intensity values. We decided here to keep them in the QUake-MD application.

Input data

The first example is based on the 29th February 1980, Mw 5 (Cara *et al.*, 2015) Arudy earthquake IDP dataset. This is one of the best studied events of the SisFrance database, with 1020 IDPs and a felt radius of ~ 400 km. SisFrance attributes a quality factor A to the epicentral intensity estimate ($I_0 = \text{VII-VIII}$). In QUake-MD this translates (Table 2) into an allowed range of I_0 solutions for the Arudy earthquake between VI-VII and VIII-IX, considering a two sigma uncertainty.

The second example is based on the 21th June 1660 Bigorre earthquake IDP dataset, characterized by 61 IDP and a felt radius of circa 400 km. A quality factor C is associated to the epicentral intensity that SisFrance estimates at VIII-IX. However, given the numerous (11) IDPs present in a 10 km radius around the SISFRANCE epicenter, we believe that in this case the uncertainty to consider for I_0 solutions should be between VII-VIII and IX-X, rather than between VI-VII and X-XI as Table 2 would suggest.

It should be noticed that for both earthquakes, the dataset is affected by national borders. This kind of dataset is representative of French damaging earthquakes: indeed most of French damaging earthquakes occurred near the French border or near the coastlines. We assume that the available IDPs are representative of the entire macroseismic field.

To estimate magnitudes and depths for these two earthquakes, two IPEs from Bakun and Scotti (2006) (hereafter BS2006), “Pyrenean-Provence” and “Southern France” and 16 “RAVG” IPEs for metropolitan France and high attenuation area from Baumont et al (2018) (hereafter BA2018) are selected. These IPEs are calibrated in Mw and are applicable to metropolitan France. It

should be underlined that the selected IPEs use the formulation of Equation 1 based on the RAVG binning strategy, the only one implemented so far in QUake-MD. In this example the same weight is attributed to the two IPEs approaches which implies that each BS2006 IPE is attributed a weight of $(1/2) \times (1/2) = 0.25$ and each BA2018 IPEs is attributed a weight of $(1/2) \times (1/16) = 0.03125$.

Depth is inverted between 1 and 25 km.

Results

In the pre-processing stage, Quake-MD allows the user to first visually analyze the fit between IPEs predictions and binned intensity data and the intrinsic trade-offs between $M/H/I_0$ central solutions (Figure 5). Colors of the predicted intensities curves help linking IPEs and H solutions.

For the Arudy case, one can notice that all BA2018 IPEs fit the binned data and that the computed magnitude does not vary much as a function of the different IPEs as opposed to H. Trade-offs between the H/I_0 and γ values (cf Equation 1) considered in the BA2018 IPE are also visible: higher γ values lead to lower I_0 values and deeper H. For some BA2018 IPEs, the predicted I_0 falls outside of the admissible I_0 limits. For these IPEs, only the M-H values consistent with the a-priori $I_0 \pm 2\sigma_{I_0}$ limits will be retained in the space of solutions. The two BS2006 IPEs, on the other hand, predict similar $M/H/I_0$ values.

In QUake-MD it is possible to select the level of intensities that the end-user wants to use in the inversion. In the case of the Arudy IDPs, for example, clearly the levels below intensity III lack data in the far field. Those intensity levels are thus removed from the inversion process.

The inversion of the Arudy earthquake IDPs based on the given inputs leads to a magnitude

210 estimate around M 5.2, a depth around 10 km and an I_0 of VII-VIII for the BS2006 IPEs and a
 211 magnitude around M 5.1 associated to a depth between 3 and 7 km and an I_0 of VIII for the
 212 BA2018 IPEs (Figure 5, lower figures). Adding the intensity data uncertainties leads to a wider
 213 space of M/H/ I_0 solutions (Figure 6) ranging between 2 and 15 km in depth and 4.8 and 5.5 in
 214 magnitude. Interestingly, the instrumental magnitude and depth values provided in the SI-Hex
 215 catalogue for this earthquake are Mw 5 and 5 km respectively (Cara *et al.*, 2015). In this case,
 216 the macroseismic field is spatially sufficiently well informed for the inversion to converge
 217 towards an H solution that is not influenced by the imposed depth constraints (fixed here at
 218 between 1 and 25 km). However, the estimated depth will strongly depend on the choice of IPE.

219 In the case of the Bigorre historic earthquake, the macroseismic data set suffers from lack of
 220 data in the far field below intensity level V, thus no binned intensity is provided below this
 221 value. All IPEs predict well the binned intensities (Figure 7 (b), solid lines) with I_0 values very
 222 close to the central value provided by Sisfrance. Effects of the γ value (Equation 1) are clearly
 223 visible on (Figure 7 (b)) and correlate with H estimates: higher γ values requiring deeper H and
 224 somewhat slightly higher magnitudes. Depth solutions range between 15 km and 25 km. For
 225 this dataset, the VIII intensity bin is playing a very strong role in the inversion. With its
 226 associated small σ_I at an epicentral distance of 12 km it is forcing the inversion deeper than 12
 227 km. In such cases, depth constraints may need to be imposed by the user based on additional
 228 knowledge. A statistical analysis of depth estimates provided by the OMP (Observatoire Midi
 229 Pyrénées) and reported in the SI-Hex catalogue (Cara *et al.*, 2015), indicates that in this region
 230 the maximal depth of instrumental earthquakes is around 11 km, a depth value representative
 231 of the deeper quartile of the OMP depth distribution.

Clearly, assuming an 11 rather than a 25 km depth limit has strong impact on the QUake-MD space of solutions (Figure 7). In a case of a maximal limit of 25 km (green-blue color scale on Figure 7 (a)), the $M/H/l_0$ space solutions ranges between 10 and 25 km for depth and between 6.1 and 7.2 for magnitude with the highest density of solutions between 6.5 and 7.2 in magnitude and between 20 km and 25 km in depth. In the case of a maximal depth limit of 11 km (copper color scale on Figure 7 (a)), the $M/H/l_0$ space of solutions ranges between 6.0 and 7.0 for magnitude, with higher density magnitude values between 6.2 and 6.8 (Figure 7(a)). Thus a shallower maximal depth inversion limit decreases the magnitude by 0.5 units (for the highest density of solutions). However the IPEs predict well the observed data with both a maximal depth limit of 11 km (dashed lines on Figure 7 (b)) and a maximal depth limit of 25 km (solid lines on Figure 7 (b)).

Discussion

The inversion methodology implemented in QUake-MD points out the importance of the choice of the depth limits used in the inversion, the choice of IPEs and of the propagation of uncertainties associated with the IDPs. Although alternative modelling options that could lead to a more reduced space of solutions could be sought for and implemented in QUake-MD, the examples considered here have shown that uncertainties in IDP and IPE have a major impact on the resulting $M/H/l_0$ space of solutions, even for such relatively well known events.

In reality, most earthquakes are informed by even less IDPs than those provided for Arudy and Bigorre. Considering the Sisfrance database, for example, 46% of the events do not have an estimate for l_0 ; of the remaining events, 61 % have less than 5 IDPs. For such events, strong

hypothesis need to be made for the estimate of the magnitude, such as attributing an intensity value to felt observations or actually assuming a fixed depth. For such events, the associated uncertainty in $M/H/I_0$ estimates is therefore even higher in comparison with the already high uncertainties associated to “good” events.

Last but not least, for many events, the epicentral location could also be questioned. In future versions of QUake-MD, strategies for the exploration of the epicentral location will be implemented. For the time-being, alternative published methodologies (Bakun and Scotti (2006), BOXER, Gasperini *et al.* (2010)) could be used to feed the Quake-MD methodology and stack the PDFs in a post-processing phase.

The impact of such uncertainties on seismic hazard assessments (deterministic and probabilistic) is the objective of future work.

Conclusion

Quantifying uncertainties in historical earthquake magnitude and depth estimates is a fundamental step in seismic hazard assessment and yet a highly underestimated task. The aim of the QUake-MD tool developed here is to provide parametric earthquake catalogue users with a tool to check the quality of their catalogues. To do so, QUake-MD propagates uncertainties in IDPs and IPEs using an iterative least-square inversion process. In the presence of a well-informed macroseismic field, such as the Arudy 1980 event, the inversion process provides reasonably well constrained $M/H/I_0$ values. When the macroseismic field is less well-informed, such as for the Bigorre 1690 event, the heterogeneous quality of individual intensity

bins does not allow constraining $M/H/I_0$ solutions and may introduce a strong bias that needs to be dealt with by using additional information on the most likely depth interval. It is hoped that a better quantification of uncertainties, such as the one proposed with the QUake-MD methodology, may lead to a better understanding of the underlying differences between parametric earthquake catalogues.

Data and resources

The QUake-MD tool, the user-manual and the examples provided in this paper are available at the following github site <https://github.com/ludmilaprvt/QUake-MD> and can be used under the license GNU Lesser General Public License V2.1. This open-source code can be freely modified by the user as long as the modified code is also available with the same license. Questions about QUake-MD can be asked on the issue tool of github. We hope to be able to answer the questions and to improve the points for which users identified shortcomings of QUake-MD. The details of the QUake-MD GUI are explained in the user manual (provided with the code: <https://github.com/ludmilaprvt/QUake-MD>).

The SisFrance database used for the application examples can be accessed here:

<http://www.sisfrance.net/>.

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Tables

Table 1 : IDP quality Factor and associated weights

IDP quality Factor	Weight
A (Reliable)	4
B (Fair)	3
C (Uncertain)	2

Table 2 : I0 quality factor and associated σ_{I0} .

I0 quality factor	σ_{I0}
A	0.5
B	0.5
C	0.5
E	0.75

List of Figures captions

Figure 1: Flowchart of the QUake-MD methodology

Figure 2: Illustration of the RAVG binning strategy for the 1980 Arudy earthquake (Pyrenees, South-West France). A weighted mean distance is computed for each intensity bin (diamonds) as well as an associated uncertainty that depends on the number of individual data points (blue circles). See text for more explanations

Figure 3: (a), initial space of M-H solutions delimited by $2\sigma_H$ and $2\sigma_M$. (b), the space of solutions tagged in violet and red are incompatible with the admissible I_0 values. (c), the space of M-H solution that is compatible with the I_0 and its uncertainties given by the end-user.

Figure 4: Locations (stars) of the Bigorre (1660) and the Arudy (1980) earthquakes and their associated IDPs (colored circles in MSK intensity scale) as provided in the SisFrance 2016 database.

Figure 5: Arudy (1980) earthquake: IPEs (curves color-coded by depth) fit to the binned IDPs (diamonds in (a) and (b) figures) and associated M-H central solutions of the inversions (dots color-coded by depth in lower figures). Grey points correspond to the IDP, the pink band corresponds to the I_0 uncertainty range used to filter the space of M-H solutions and the red dots, the I_0 value at the end of the inversion process for each IPE (for IPEs predicting I_0 outside of the accepted boundaries, no red dots are shown). Depth was inverted considering a 1 to 25 km plausible range. (a) and (c): output of the BA2018 IPEs. (b) and (d): output of the BS2006 IPEs. (c) and (d): M-H central solutions for each IPE.

Figure 6: The M/H/ I_0 space of solutions based for the Arudy 1980 earthquake considering in QUake-MD 18 weighted IPEs (BS2006 and BA2018) and IDPs uncertainties. (a) the M-H view of the M/H/ I_0 space of solutions and (b) the I_0 -H view. Depth was inverted considering a 1 to 25 km plausible range.

Figure 7: (a): the M/H/ I_0 space of solutions based for the Bigorre 1660 earthquake considering in QUake-MD 18 weighted IPEs (BS2006 and BA2018) and IDPs uncertainties for a maximal limit depth of 25 km (violet-yellow color scale) and for a maximal limit depth of 11 km (copper color scale). (b): fit of the IPEs with the observed data. The solid lines represent the IPE predictions for a maximal depth limit of 25 km and the dashed lines the IPE predictions for a maximal depth limit of 11 km. (c): the M-H central solutions of the IPEs. The round markers represent the IPE solutions for a maximal depth limit of 25 km and the triangle markers represent the IPE solutions for a maximal depth limit of 11 km. The (b) and (c) figures share the same color scale, representing the depth central solution of the IPE.

Figures

Figure 1: Flowchart of the QUake-MD methodology

Figure 2: Illustration of the RAVG binning strategy for the 1980 Arudy earthquake (Pyrenees, South-West France). A weighted mean distance is computed for each intensity bin (diamonds) as well as an associated uncertainty that depends on the number of individual data points (blue circles). See text for more explanations

Figure 3: (a), initial space of M-H solutions delimited by $2\sigma_H$ and $2\sigma_M$. (b), the space of solutions tagged in violet and red are incompatible with the admissible I_0 values. (c), the space of M-H solution that is compatible with the I_0 and its uncertainties given by the end-user.

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Figure 4: Locations (stars) of the Bigorre (1660) and the Arudy (1980) earthquakes and their associated IDPs (colored circles in MSK intensity scale) as provided in the SisFrance 2016 database.

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Figure 6: The M/H/I0 space of solutions based for the Arudy 1980 earthquake considering in QUake-MD 18 weighted IPEs (BS2006 and BA2018) and IDPs uncertainties. (a) the M-H view of the M/H/I0 space of solutions and (b) the I0-H view. Depth

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430 Figure 7: (a): the M/H/I0 space of solutions based for the Bigorre 1660 earthquake considering in QUake-MD 18 weighted
431 IPEs (BS2006 and BA2018) and IDPs uncertainties for a maximal limit depth of 25 km (violet-yellow color scale) and for a
432 maximal limit depth of 11 km (copper color scale). (b): fit of the IPEs with the observed data. The solid lines represent the IPE
433 predictions for a maximal depth limit of 25 km and the dashed lines the IPE predictions for a maximal depth limit of 11 km.
434 (c): the M-H central solutions of the IPEs. The round markers represent the IPE solutions for a maximal depth limit of 25 km
435 and the triangle markers represent the IPE solutions for a maximal depth limit of 11 km. The (b) and (c) figures share the
436 same color scale, representing the depth central solution of the IPE.

Appendices

Inversion method

The general flowchart of the IPE inversion process used in QUake-MD is described in figure A1. Data used for the inversion process are the observed intensities grouped by intensity bins. Each observed intensity, called hereafter IDP, have an associated weight. This weight is used to compute the hypocentral distance and the intensity standard deviation σ_i associated to each intensity level. The intensity standard deviation σ_i associated is equal to the weighted standard deviation of the decimal logarithm IDP hypocentral distances of the intensity level multiplied by the absolute of -3.5 (default value, can be changed), which aims to represent a central value of the geometrical attenuation coefficient used in the QUake-MD mathematical formulation of the IPEs (see equation A1).

$$I = C_1 + C_2 M + \beta \log(D_{hypo}) + \gamma D_{hypo} \quad (\text{A1})$$

The weight associated to each intensity levels is equal to the square of the inverse of σ_i . The epicentral intensity I_0 is added to the binned intensity as a data point. Weight associated to I_0 is defined by:

$$W_{I_0} = AF \times \frac{0.5}{\sigma_{I_0}} \quad (\text{A2})$$

Where, σ_{I_0} is standard deviation associated to I_0 and AF an adaptation factor that insured that the weight on I_0 will not be too important compared to the other intensity levels. By default this AF is equal to 0.1.

Once the binned intensity data and associated σ_i prepared, H , I_0 and M are sequentially inverted through non-linear least-square and linear least square method for M , until stabilization of the results. Depth is inverted in a non-linear process through a Trust Region Reflective algorithm (Branch et al 1999).

457 Jacobian matrix used for the depth inversion is shown in Table A1 and based on equation A1. Depth
458 limits are provided by the user. Since l_0 is in many cases an expert opinion value and not based on data
459 available at the epicenter, l_0 is inverted along H and M. In this inversion l_0 is not considered as a data
460 point. Trust Region Reflective algorithm is also used for the l_0 inversion. Inequality constrains are equal
461 to $l_0 \pm 2\sigma_{l_0}$. Jacobian matrix used for the l_0 inversion is shown in Table A1 and based on equation A3.
462 The l_0 inversion is an option that can be easily discarded in in source code.

$$I = I_0 + \beta \log\left(\frac{D_{hypo}}{H}\right) + \gamma(D_{hypo} - H) \quad (\text{A3})$$

463 Then magnitude is inverted through a Levenberg-Marquardt algorithm (More, 1977). Jacobian matrix
464 used for the magnitude inversion is shown in Table A1 and based on equation A1.

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Figure A1: Flowchart of the application of one IPE to estimate H and M for historical earthquakes.

At least 3 iterations are performed for the sequential inversion of H, I_0 and M. The stabilization of the result is controlled by three convergence parameters defined by:

$$Cv_M = \sum_i^2 \frac{|M_{n-i} - M_{n-i-1}|}{2} \quad (\text{A4})$$

$$Cv_H = \sum_{i=0}^2 \frac{|H_{n-i} - H_{n-i-1}|}{3} \quad (\text{A5})$$

$$Cv_{I_0} = \sum_{i=0}^2 \frac{|I_{0n-i} - I_{0n-i-1}|}{3} \quad (\text{A6})$$

The iterations stop when Cv_M is lower than 1.10^{-2} , Cv_{I_0} lower than 1.10^{-2} and Cv_H lower than 5.10^{-2} . To avoid infinite loop, we arbitrarily assign the maximal number of iteration to 100.

Parameter	Bounds	G matrix
H	Yes	$G_H = \begin{pmatrix} \frac{H}{D_{\text{hypo}_1}} \left[\frac{\beta}{D_{\text{hypo}_1} \cdot \ln(10)} + \gamma \right] \\ \vdots \\ \frac{H}{D_{\text{hypo}_m}} \left[\frac{\beta}{D_{\text{hypo}_m} \cdot \ln(10)} + \gamma \right] \end{pmatrix}$
I_0	Yes	$G_{I_0} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$
M	No	$G_M = \begin{pmatrix} C_2 \\ \vdots \\ C_2 \end{pmatrix}$

Table A1 : Functions, options and G matrix used in the inversion process in the application of one IPE.

Standard deviation associated to central solutions of M and H inversions is retrieved from the square root of the diagonal elements of the parameter covariance matrix defined by $Variance = m^T C_d m$, with m the model matrix, here respectively H and M central solution and C_d , the covariance matrix associated to the binned intensity data and the additional I_0 data. C_d diagonal elements are equal to the square of σ_i and the standard deviation associated to I_0 (resulting from the I_0 inversion and based on the associated covariance matrix, with minimal value of 0.5) included. Other C_d elements are equal to zero.

Reference

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More, J. J. (1977). The Levenberg-Marquardt Algorithm: Implementation and Theory, *Numerical Analysis*, ed. G. A. Watson, Lecture Notes in Mathematics 630, Springer Verlag, pp. 105-116.