A pocket-sized introduction to acoustics

Prof. Dr. Keith Attenborough
Dr. Michiel Postema

Department of Engineering
The University of Hull

ISBN 978-90-812588-2-1

Publisher: Michiel Postema, Bergschenhoek
Printed in England by The University of Hull

Typesetting system: LaTeX 2_ε

Contents

1 Acoustics and ultrasonics

Sound waves are a form of mechanical vibration. They correspond to particle displacements in the matter. Unlike electromagnetic waves, which can propagate in vacuum, sound waves need matter to support their propagation: a solid, a liquid, or a gas. The ear is an excellent acoustic detector in air but its sensitivity is limited to an interval between 20 Hz and 20 kHz. Audio frequency sound is essential in communication and entertainment. The acoustics of buildings, particularly concert halls, has been the subject of considerable study. Unwanted audio-frequency sound is called noise. The study of noise and noise control is an important part of engineering.

Ultrasound refers to sounds and vibrations at frequencies above this upper limit of 20 kHz to values that can reach 1 GHz. Consequently, ultrasonics involve higher frequencies and smaller wavelengths than audio acoustics. Conversely, infrasound involves sounds and vibrations at low frequencies (below 20 Hz) and long wavelengths. Because the physiological sensation of sound has disappeared at these frequencies, our perceptions of infrasound and ultrasound are different. Ultrasonic waves in fluids and solids are used for non-destructive evaluation. The general principle is to excite and detect a wave at ultrasonic frequencies and to deduce information from the signals detected. Example applications include the detection of flaws and inhomogeneities in solids, SONAR, SODAR, medical imaging, and acoustic microscopy.

For applications of acoustics, ultrasonics, and noise control, it is important to have a good understanding of wave propagation in infinite fluids and solids. The mathematical formulation of wave propagation in solids involves the use of the concepts and principles of 3-dimensional stress and strain analysis. But we start by outlining related concepts concerning the vibration of a single degree of freedom mass-on-a-spring system.

2
Mass on a spring

Consider a mass m attached by a massless elastic spring to a motionless vertical wall and resting on top of a horizontal frictionless surface. In the equilibrium situation, the distance between the centre of mass and the wall is x_0. The mass is pulled back over a distance Δx by a force F. Then, for a stationary mass,

$$F = s\,\Delta x, \tag{2.1}$$

where s is the stiffness of the spring. After release, we can define the excursion $x(t)$ around the equilibrium by

$$m\,\ddot{x} = -s\,x \tag{2.2}$$

or

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \omega_0^2\,x = 0, \tag{2.3}$$

where

$$\omega_0 = \sqrt{\frac{s}{m}} \tag{2.4}$$

is the natural (or resonance) frequency of the system in radians per time unit, and

$$\omega_0 = 2\pi f_0, \tag{2.5}$$

where f_0 is the resonance frequency in cycles per time unit.

Consider the following solution of (2.3): $x(t) = a_0 \cos(\omega_0 t) + a_1 \sin(\omega_0 t)$. Since $\max(x) = \Delta x$ when $t = 0$, $a_0 = \Delta x$. Also, since $\dot{x} = 0$ when $t = 0$, $a_1 = 0$.

Here, the period is given by

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{s}}. \tag{2.6}$$

3
Wave equation in fluid

Consider an infinitesimal element of length dx and cross-section dS. Let's assume the element is rigid in all directions except for the x-direction. Also, let's define a longitudinal compressive sound wave traveling in the x-direction. Suppose that the centre of the element is displaced by a distance u as a result of a sound pressure p. Then, the displacement of the boundaries is $\left(u + \frac{\partial u}{\partial x}\frac{\mathrm{d}x}{2}\right)$ and $\left(u - \frac{\partial u}{\partial x}\frac{\mathrm{d}x}{2}\right)$, respectively. The difference in volume, therefore, is

$$\left[\left(u - \frac{\partial u}{\partial x}\frac{\mathrm{d}x}{2}\right) - \left(u + \frac{\partial u}{\partial x}\frac{\mathrm{d}x}{2}\right)\right]\mathrm{d}S = -\frac{\partial u}{\partial x}\,\mathrm{d}x\,\mathrm{d}S. \tag{3.1}$$

The volumetric strain

$$\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z = -\frac{\partial u}{\partial x}\frac{\mathrm{d}x\,\mathrm{d}S}{\mathrm{d}x\,\mathrm{d}S} = -\frac{\partial u}{\partial x}. \tag{3.2}$$

By definition, the bulk modulus of elasticity κ is given by

$$\kappa = \frac{\text{stress}}{\text{strain}} = \frac{p}{\Delta} = -\frac{p}{\partial u/\partial x} \tag{3.3}$$

or

$$p = -\kappa\,\frac{\partial u}{\partial x}. \tag{3.4}$$

The difference in deformation is caused by a difference in the sound pressure is $p(u - \frac{1}{2}\,\mathrm{d}x)$ at one side of the element and $p(u + \frac{1}{2}\,\mathrm{d}x)$ at the other side. Hence, the force acting on the element is expressed by

$$-\left[p(u + \frac{1}{2}\,\mathrm{d}x) - p(u - \frac{1}{2}\,\mathrm{d}x)\right]\,\mathrm{d}S = -\frac{\partial p}{\partial x}\,\mathrm{d}x\,\mathrm{d}S. \tag{3.5}$$

This force accelerates the mass $\rho\,\mathrm{d}x\,\mathrm{d}S$ with ρ being the density. The acceleration equals the second time derivative of the displacement in the x-direction u. Therefore, the following balance must hold:

$$-\frac{\partial p}{\partial x}\,\mathrm{d}x\,\mathrm{d}S = \rho\,\frac{\partial^2 u}{\partial t^2}\,\mathrm{d}x\,\mathrm{d}S, \tag{3.6}$$

which gives the equation of motion

$$\frac{\partial p}{\partial x} = -\rho\,\frac{\partial^2 u}{\partial t^2}. \tag{3.7}$$

Substituting (3.4) for p into (3.7) results in

$$-\kappa\,\frac{\partial^2 u}{\partial x^2} = -\rho\,\frac{\partial^2 u}{\partial t^2} \tag{3.8}$$

or

$$\frac{\partial^2 u}{\partial t^2} = \frac{\kappa}{\rho}\,\frac{\partial^2 u}{\partial x^2}, \tag{3.9}$$

which is the displacement wave equation of a periodic fluctuation u in the x-direction at a speed

$$c = \sqrt{\frac{\kappa}{\rho}}. \tag{3.10}$$

Taking the partial derivative to x of (3.7) gives

$$\frac{\partial^2 p}{\partial x^2} = -\rho\,\frac{\partial^3 u}{\partial x\,\partial t^2}, \tag{3.11}$$

whereas taking the second derivative to t of (3.4) gives

$$\frac{\partial^2 p}{\partial t^2} = -\kappa\,\frac{\partial^3 u}{\partial x\,\partial t^2}. \tag{3.12}$$

Combining (3.11) and (3.12) gives the linear wave equation

$$\frac{\partial^2 p}{\partial t^2} = \frac{\kappa}{\rho}\,\frac{\partial^2 p}{\partial x^2} = c^2\,\frac{\partial^2 p}{\partial x^2}. \tag{3.13}$$

4
Sound speed in air

For an adiabatic change in an ideal gas

$$PV^{\gamma} = \text{constant}, \tag{4.1}$$

where P is the absolute pressure, V the volume of an element, and γ the ratio of specific heats $\frac{C_p}{C_v}$. Differentiating to V gives

$$V^{\gamma}\frac{dP}{dV} + \gamma P V^{\gamma-1} = 0. \tag{4.2}$$

Dividing this through by $V^{\gamma-1}$ gives

$$-\frac{V}{dV}\,dP = \gamma P. \tag{4.3}$$

The volumetric strain is

$$\Delta = -\frac{dV}{V} \tag{4.4}$$

and the stress associated with it is

$$p = dP. \tag{4.5}$$

Hence, using (3.3), (4.3) can be written as

$$\kappa = \gamma P. \tag{4.6}$$

Therefore, the speed of sound in air

$$c = \sqrt{\frac{\kappa}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}. \tag{4.7}$$

In an ideal gas,

$$PV = nR\mathcal{T}, \tag{4.8}$$

where n is the amount of substance, R is the gas constant, and $\mathcal{T}$ is the absolute temperature. Replacing n by $\frac{m}{M}$ and R by $M\bar{R}$, in which M is the molar mass and $\bar{R}$ the specific gas constant, we obtain

$$PV = m\bar{R}\mathcal{T} = \rho V\bar{R}\mathcal{T}, \tag{4.9}$$

or

$$\frac{P}{\rho} = \bar{R}\mathcal{T}, \tag{4.10}$$

Thus, for an ideal gas,

$$c = \sqrt{\gamma\bar{R}\mathcal{T}}. \tag{4.11}$$

5

Solutions of the 1-D wave equation

Consider the wave equation (3.13)

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \tag{5.1}$$

and a solution of the form

$$p = f_1(ct - x) + f_2(ct + x). \tag{5.2}$$

Then,

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(f_1'' + f_2''\right) \tag{5.3}$$

and

$$\frac{\partial^2 p}{\partial x^2} = f_1'' + f_2''. \tag{5.4}$$

For the displacement equation (3.9), a similar solution is considered:

$$u = g_1(ct - x) + g_2(ct + x). \tag{5.5}$$

If we concentrate on the wave traveling in the positive direction (progressive wave), described by g_1, for the moment ignoring g_2, at $t = 0$, $u_0 = g_1(-x)$. Also, at $t = 1$, $u_1 = g_1(c - x)$. The displacement u_1 caused by p_1 must have the same form as the displacement u_0 caused by p_0, only at a distance $1c$ further on. Hence,

$$g_1(-x_0) = g_1(c - x_1) \tag{5.6}$$

and thus

$$-x_0 = c - x_1 \tag{5.7}$$

or

$$x_1 = x_0 + c. \tag{5.8}$$

For the wave traveling in the negative direction (regressive wave), described by g_2, the corresponding equations are

$$g_2(x_0) = g_2(c + x_1) \tag{5.9}$$

and

$$x_1 = x_0 - c. \tag{5.10}$$

Consider a plane single-frequency (monotonous) progressive wave, for which the pressure deviation from the ambient constant value is described by

$$p = p_0 \cos\left[\omega\left(t - \frac{x}{c}\right)\right] = p_0 \cos\left(\omega t - kx\right). \tag{5.11}$$

Here, p_0 is the acoustic pressure amplitude and

$$k = \frac{\omega}{c} \tag{5.12}$$

is the wave number or propagation constant. Consider the spatial distribution at a fixed time $\omega t =$ constant. Then, if x is equal to the wavelength λ, kx must equal 2π, *i.e.*,

$$k = \frac{2\pi}{\lambda}, \tag{5.13}$$

from which it follows that

$$c = f\lambda. \tag{5.14}$$

6
Energy

The potential energy in a sound wave is

$$E_{\mathrm{P}} = -\int p\,\mathrm{d}V. \tag{6.1}$$

Note the negative sign because a pressure increase causes a volume decrease. Since

$$V = V_0 + \mathrm{d}V = V_0 - V_0\,\frac{\mathrm{d}p}{\kappa}, \tag{6.2}$$

$$\mathrm{d}V = -V_0\,\frac{\mathrm{d}p}{\kappa} = -V_0\,\frac{\mathrm{d}p}{\rho c^2}. \tag{6.3}$$

Therefore,

$$E_{\mathrm{P}} = \int \frac{pV_0}{\rho c^2}\,\mathrm{d}p = \frac{p^2 V_0}{2\rho c^2}. \tag{6.4}$$

The kinetic energy

$$E_{\mathrm{K}} = \frac{1}{2}\rho V_0 \nu^2. \tag{6.5}$$

Here, ν is the particle velocity. To find a relationship between p and ν, we combine equations (3.4) and (3.10):

$$p = -\rho c^2 \frac{\partial u}{\partial x}. \tag{6.6}$$

Consider (5.11) and (5.12). Since u has the form

$$u = u_0 \cos\left(\omega t - \frac{\omega}{c} x\right), \tag{6.7}$$

(6.6) can we rewritten as

$$p = -\rho c \omega\, u_0 \sin\left(\omega t - \frac{\omega}{c} x\right) \tag{6.8}$$

and

$$\nu = \frac{\partial u}{\partial x} = -\omega\, u_0 \sin\left(\omega t - \frac{\omega}{c} x\right). \tag{6.9}$$

These result in

$$p = \rho c \nu. \tag{6.10}$$

Therefore,

$$E_{\mathrm{K}} = \frac{p^2 V_0}{2\rho c^2} \tag{6.11}$$

and the total energy

$$E_{\mathrm{T}} = E_{\mathrm{P}} + E_{\mathrm{K}} = \frac{p^2 V_0}{\rho c^2}. \tag{6.12}$$

The energy per unit volume is defined by

$$\mathcal{E} = \frac{E_{\mathrm{T}}}{V} = \frac{p^2}{\rho c^2}. \tag{6.13}$$

The energy which, at any instant, is contained in a column of unit cross sectional area and of length $\mathrm{d}t$ is $\mathcal{E} c\, \mathrm{d}t$. Therefore, the average flow of energy, the intensity

$$I = \mathcal{E} c = \frac{p^2}{\rho c}. \tag{6.14}$$

7

Point and line sources

The sound power of a source is the rate at which the source produces sound energy. It is an intrinsic property. If the power passing through an area S is W, the intensity can also be defined as the power per unit area:

$$I = \frac{W}{S}. \tag{7.1}$$

For a point source, the intensity of the sound in point at distance r from the source is

$$I = \frac{W}{4\pi r^2}. \tag{7.2}$$

Thus,

$$I \propto \frac{1}{r^2}. \tag{7.3}$$

Also, quoting (6.14),

$$I \propto p^2. \tag{7.4}$$

Therefore,

$$p \propto \frac{1}{r}. \tag{7.5}$$

The complex representation (*cf.* Chapter 10) of the acoustic pressure is

$$p = \frac{p_0}{r}\, e^{j(\omega t - kr)}. \tag{7.6}$$

The spherical wave equation is given by

$$\frac{\partial^2}{\partial t^2}(rp) = \frac{\kappa}{\rho}\frac{\partial^2}{\partial r^2}(rp). \tag{7.7}$$

For a cylindrical wave,

$$I = \frac{W}{2\pi r}. \tag{7.8}$$

Hence,

$$p \propto \frac{1}{\sqrt{r}}. \tag{7.9}$$

The acoustic pressure is

$$p = p_0\, J_0(kr)\, e^{j\omega t}, \tag{7.10}$$

Where J_0 is the cylindrical Bessel function or order zero of the first kind. The cylindrical wave equation is given by

$$\frac{\partial^2 p}{\partial t^2} = \frac{\kappa}{\rho}\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right). \tag{7.11}$$

Doppler effect

If the sound source emitting at frequency f is moving at a velocity ν_s towards its audience at rest, the wavelength is reduced to

$$\lambda' = (c - \nu_s)\, T. \tag{7.12}$$

Hence, the frequency experienced is

$$f' = \frac{f}{1 - \frac{\nu_s}{c}}. \tag{7.13}$$

If the source moves away from the audience, ν_s is negative. If the source moves at supersonic speed ($\nu_s > c$), the wavefront has the shape of a cone with an aperture

$$\sin\frac{\theta}{2} = \frac{c}{\nu_s}. \tag{7.14}$$

If the audience is moving at a velocity ν_a towards the sound source, the frequency experienced is

$$f' = \frac{c + \nu_a}{\lambda} = \left(1 + \frac{\nu_a}{c}\right) f. \tag{7.15}$$

8
Root-mean-square pressure

The root-mean-square pressure $p_{\rm rms}$ is defined by

$$p_{\rm rms}^2 = \frac{1}{T}\int_0^T p^2 \, \mathrm{d}t = \frac{1}{T}\int_0^T p_0^2 \, \cos^2(\omega t) \, \mathrm{d}t, \tag{8.1}$$

where T is the period. Using $\cos 2\theta = 2\cos^2\theta - 1$, this simplifies to

$$p_{\rm rms}^2 = \frac{1}{T}\int_0^T \frac{p_0^2}{2} \left(\cos 2\omega t + 1\right) \mathrm{d}t = \frac{p_0^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{p_0^2}{2}. \tag{8.2}$$

Thus,

$$p_{\rm rms} = \frac{p_0}{\sqrt{2}} \approx 0.707 p_0 \tag{8.3}$$

for a monotonous wave. In general,

$$\frac{\text{pressure amplitude}}{\text{root-mean-square pressure}} = \frac{p_0}{p_{\rm rms}} = \text{crest factor}. \tag{8.4}$$

9
Superposition of waves

Consider, from (5.11) and (5.12), a sound wave consisting of two different frequencies and amplitudes

$$p = p_{0,1} \cos\left[\omega_1 \left(t - \frac{x}{c}\right)\right] + p_{0,2} \cos\left[\omega_2 \left(t - \frac{x}{c}\right)\right]. \tag{9.1}$$

Then,

$$\begin{aligned} p^2 &= p_{0,1}^2 \cos^2\left[\omega_1 \left(t - \frac{x}{c}\right)\right] + p_{0,2}^2 \cos^2\left[\omega_2 \left(t - \frac{x}{c}\right)\right] \\ &\quad + 2 p_{0,1} p_{0,2} \cos\left[\omega_1 \left(t - \frac{x}{c}\right)\right] \cos\left[\omega_2 \left(t - \frac{x}{c}\right)\right]. \end{aligned} \tag{9.2}$$

Using $\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$, this becomes

$$\begin{aligned} p^2 &= p_{0,1}^2 \cos^2\left[\omega_1 \left(t - \frac{x}{c}\right)\right] + p_{0,2}^2 \cos^2\left[\omega_2 \left(t - \frac{x}{c}\right)\right] \\ &\quad + p_{0,1} p_{0,2} \cos\left[\left(t - \frac{x}{c}\right)(\omega_1 + \omega_2)\right] \\ &\quad + p_{0,1} p_{0,2} \cos\left[\left(t - \frac{x}{c}\right)(\omega_1 - \omega_2)\right]. \end{aligned} \tag{9.3}$$

Thus,

$$p_{\mathrm{rms}}^2 = \frac{1}{T}\int_0^T p^2 \, \mathrm{d}t = p_{1,\mathrm{rms}}^2 + p_{2,\mathrm{rms}}^2. \tag{9.4}$$

This implies, that for waves with different frequencies but the same amplitude

$$p_{\mathrm{rms}}^2 = 2p_{1,\mathrm{rms}}^2 = 2p_{2,\mathrm{rms}}^2. \tag{9.5}$$

For a band of frequencies,

$$p_{\mathrm{rms}}^2 = \int p_{f,\mathrm{rms}}^2 \, \mathrm{d}f. \tag{9.6}$$

If the waves have the same frequency, the phase becomes in important. Consider the added waves

$$p = p_{0,1}\cos\left(\omega t - kx + \phi_1\right) + p_{0,2}\cos\left(\omega t - kx + \phi_2\right). \tag{9.7}$$

Then,

$$p_{\mathrm{rms}}^2 = p_{1,\mathrm{rms}}^2 + p_{2,\mathrm{rms}}^2 + 2p_{1,\mathrm{rms}}p_{2,\mathrm{rms}}\cos\left(\phi_1 - \phi_2\right). \tag{9.8}$$

Obviously, for waves of the same amplitude and phase,

$$p_{\mathrm{rms}} = 2p_{1,\mathrm{rms}} = 2p_{2,\mathrm{rms}}, \tag{9.9}$$

whereas for waves of the same amplitude but opposite phase,

$$p_{\mathrm{rms}} = 0. \tag{9.10}$$

10

Complex representation of a plane, harmonic wave

Both $p_0 \cos(\omega t - kx)$ and $p_0 \cos(\omega t - kx - \frac{1}{2}\pi) = p_0 \sin(\omega t - kx)$ satisfy the wave equation. Since $e^{\frac{j\pi}{2}} = j$, their complex sum also satisfies the wave equation:

$$p_0 \cos(\omega t - kx) + jp_0 \sin(\omega t - kx) = p_0 e^{j(\omega t - kx)}. \tag{10.1}$$

The first time derivative of (3.4) is known as the continuity equation:

$$\frac{\partial p}{\partial t} = -\kappa \frac{\partial u}{\partial x \partial t} = -\kappa \frac{\partial \nu}{\partial x}. \tag{10.2}$$

From (10.1), it is evident that

$$\frac{\partial p}{\partial t} = j\omega p, \tag{10.3}$$

whereas

$$-\frac{\partial \nu}{\partial x} = jk\nu = \frac{j\omega}{c}\nu. \tag{10.4}$$

Combining these gives

$$p = \frac{\kappa}{c}\nu = \frac{\rho c^2}{c}\nu = \rho c \nu, \tag{10.5}$$

which is the same result as (6.10). Hence, the impedance Z can also be expressed in terms of p and ν:

$$Z = \rho c = \frac{p}{\nu}. \tag{10.6}$$

Standing waves

Consider two waves of same amplitude moving in opposite directions:

$$p_1 = p_0 e^{j(\omega t - kx)} \tag{10.7}$$

and

$$p_2 = p_0 e^{j(\omega t + kx)}. \tag{10.8}$$

Then

$$p_1 + p_2 = p_0 e^{j\omega t}\left(e^{-jkx} + e^{jkx}\right) = 2p_0 e^{j\omega t}\cos kx. \tag{10.9}$$

This is the expression for a standing wave with nodes at $x = i\lambda$ and antinodes at $x = (i + \frac{1}{2})\lambda$, where $i \in \mathbb{Z}$.

11
Decibel scale

The threshold of normal hearing at 1 kHz p_h is 2×10^{-5} Pa. The threshold of pain at 1 kHz is 20 Pa. These extremes represent a hearing range factor of 10^6 in Pa. The intensity equivalent of the threshold of hearing is

$$I_0 = \frac{p^2}{\rho c} \approx \frac{4 \times 10^{-10}}{4 \times 10^2} = 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}. \tag{11.1}$$

The power equivalent to the threshold of hearing is taken $W_0 = 10^{-12}$ W. These thresholds are used as reference quantities when defining sound levels on logarithmic scales:

$$\begin{aligned}
\text{sound power level}\quad \mathrm{SWL} &= 10\log_{10}\frac{W}{W_0}\ \text{dB re } W_0;\\
\text{intensity level}\quad \mathrm{IL} &= 10\log_{10}\frac{I}{I_0}\ \text{dB re } I_0;\\
\text{sound pressure level}\quad \mathrm{SPL} &= 10\log_{10}\frac{p^2}{p_\mathrm{h}^2}\\
&= 20\log_{10}\frac{p}{p_\mathrm{h}}\ \text{dB re } p_\mathrm{h}.
\end{aligned} \tag{11.2}$$

For air at room temperature (20 °C) and normal pressure (1.01×10^5 Pa), the density $\rho = 1.21$ kg m^{-3} and the sound speed $c = 343$ m s^{-1}. Hence, the acoustic impedance

$$Z = \rho c = 415\,\mathrm{kg\,m^{-2}\,s^{-1}} = 415\,\mathrm{rayls}. \tag{11.3}$$

Then, the resulting relative intensity

$$\frac{I}{I_0} = \frac{p^2}{ZI_0} = \frac{p^2}{415 \times 10^{-12}} = \left(\frac{p}{2.038 \times 10^{-5}}\right) = \left(\frac{p}{p_\mathrm{h}}\right)^2 \left(\frac{1}{1.019}\right)^2, \tag{11.4}$$

so that the intensity level

$$\begin{aligned} \mathrm{IL} &= 10\log_{10}\frac{I}{I_0} = 10\log_{10}\left(\frac{p}{p_\mathrm{h}}\right)^2 + 10\log_{10}\left(\frac{1}{1.019}\right)^2 \\ &= 20\log_{10}\frac{p}{p_\mathrm{h}} - 20\log_{10}1.019 = \mathrm{SPL} - 0.160\ \mathrm{dB}. \end{aligned} \tag{11.5}$$

Hence, the difference between the sound pressure level and the intensity level is negligible.

Propagation from a point source

The acoustic power of a point source in a lossless medium is

$$W = 4\pi r^2 I, \tag{11.6}$$

so that

$$W \propto 4\pi r^2 p^2 \tag{11.7}$$

and

$$W_0 \propto p_\mathrm{h}^2. \tag{11.8}$$

It follows that the sound power level relates to the sound pressure level

$$\begin{aligned} \mathrm{SWL} &= 10\log_{10}\frac{W}{W_0} = 10\log_{10}\frac{4\pi r^2 p^2}{p_\mathrm{h}^2} \\ &= 20\log_{10}\frac{p}{p_\mathrm{h}} + 10\log_{10}4\pi r^2 = \mathrm{SPL} + 20\log_{10} r + 11\ \mathrm{dB} \end{aligned} \tag{11.9}$$

or

$$\mathrm{SPL} = \mathrm{SWL} - 20\log_{10} r - 11\ \mathrm{dB}. \tag{11.10}$$

On hard ground,

$$I = \frac{W}{2\pi r^2}, \tag{11.11}$$

resulting in

$$\mathrm{SWL} = \mathrm{SPL} + 20\log_{10} r + 8 \text{ dB} \tag{11.12}$$

or

$$\mathrm{SPL} = \mathrm{SWL} - 20\log_{10} r - 8 \text{ dB}. \tag{11.13}$$

Similarly, at a junction between a floor and a wall,

$$I = \frac{W}{\pi r^2}, \tag{11.14}$$

resulting in

$$\mathrm{SWL} = \mathrm{SPL} + 20\log_{10} r + 5 \text{ dB} \tag{11.15}$$

or

$$\mathrm{SPL} = \mathrm{SWL} - 20\log_{10} r - 5 \text{ dB}. \tag{11.16}$$

In general,

$$\mathrm{SPL} = \mathrm{SWL} + 10\log_{10} \frac{Q}{4\pi r^2}, \tag{11.17}$$

where Q is the directivity factor.

Distance doubling

Consider the sound pressure levels SPL_1 at a distance r_1 and SPL_2 at a distance $r_2 = 2r_1$ from the source. Then, for a point source,

$$\mathrm{SPL}_1 = \mathrm{SWL} - 20\log_{10} r_1 - 11 \text{ dB} \tag{11.18}$$

and

$$\begin{aligned} \mathrm{SPL}_2 &= \mathrm{SWL} - 20\log_{10} 2r_1 - 11 \text{ dB} \\ &= \mathrm{SWL} - 20\log_{10} 2 - 20\log_{10} r_1 - 11 \text{ dB} \\ &= \mathrm{SPL}_1 - 20\log_{10} 2 \\ &= \mathrm{SPL}_1 - 6 \text{ dB}. \end{aligned} \tag{11.19}$$

For a line source,

$$\frac{I}{I_0} = \frac{W}{W_0}\frac{1}{2\pi r}. \tag{11.20}$$

Therefore,

$$\begin{aligned}\mathrm{SPL} \approx \mathrm{IL} &= 10\log_{10}\frac{W}{W_0} - 10\log_{10} r - 10\log_{10} 2\pi \\ &= \mathrm{SWL} - 10\log_{10} r - 8 \text{ dB}.\end{aligned} \tag{11.21}$$

In this case,

$$\mathrm{SPL}_1 = \mathrm{SWL} - 10\log_{10} r_1 - 8 \text{ dB} \tag{11.22}$$

and

$$\begin{aligned}\mathrm{SPL}_2 &= \mathrm{SWL} - 10\log_{10} 2r_1 - 8 \text{ dB} \\ &= \mathrm{SWL} - 10\log_{10} 2 - 10\log_{10} r_1 - 8 \text{ dB} \\ &= \mathrm{SPL}_1 - 10\log_{10} 2 \\ &= \mathrm{SPL}_1 - 3 \text{ dB}.\end{aligned} \tag{11.23}$$

12
Sound perception

Consider Appendix B. Each contour line connects the sound pressure levels which appear to the average listener to be equal for different frequencies. Equal loudness contours are labeled by SPL value at 1,000 Hz, usually expressed in phons.

The level below which we cannot hear is called the threshold of audibility or the minimum audible field. The other extreme, the limit above which sound becomes painful, is called the threshold of feeling.

The frequencies of speech lie between 200 and 10,000 Hz. For speech to be understood, the range from 1,000 Hz to 2,500 Hz is necessary. The lower frequencies are useful to recognize the speaker's voice. Music extends from 20 Hz to 4,000 Hz.

Octave bands

Consonance means, that two tones of different pitch simultaneously played to an audience sound pleasant, whereas dissonance means that they sound unpleasant. Consonance can be explained by the level of coincidence of the overtones of the two tones played. The interval that shows the highest degree of consonance is indicated by the term octave. A shift in octave is equal to a doubling in frequency.

Tones with an octave distance are indicated with the same letter. An octave can be subdivided into 12 semitones (*cf.* a piano keyboard). Other intervals and their frequency ratios are shown in Table 12.1. The pure tones are derived from these intervals.

Table 12.1: Definitions of some intervals in music.

Interval	# semitones	Frequency ratio
Perfect octave	12	2:1
Perfect fifth	7	3:2
Perfect fourth	5	4:3
Major third	4	5:4
Minor third	3	6:5
Major second	2	9:8
Minor second	1	16:15

Playing a middle C on a piano corresponds to a pitch of 256 Hz. Shifting by a perfect fourth (F) and then a perfect fifth (C′) gives the expected octave shift

$$f' = 256 \cdot \frac{4}{3} \cdot \frac{3}{2} = 256 \cdot \frac{2}{1} = 512 \text{ Hz}. \tag{12.1}$$

However, shifting by a major third (E), another (A♭), and another (C′) gives the rather unexpected

$$f' = 256 \left(\frac{5}{4}\right)^3 = 256 \cdot \frac{125}{64} \neq 512 \text{ Hz}. \tag{12.2}$$

Hence, musical instruments have to be tuned using deviations from the pure tone scales to compensate for this discrepancy.

For many practical purposes in acoustics, such as audiology, we define octave bands. Although these might be chosen arbitrarily, the most common series, expressed by mid-frequencies, is shown in Table 12.2.

Frequency weighting

The A-weighted sound level is the level, in dB re 20 μPa, of a sound impinging upon a microphone that has been electronically altered with a weighting network

Table 12.2: Octave bands.

Band #	Mid-frequency (Hz)
1	63
2	125
3	250
4	500
5	1,000
6	2,000
7	4,000
8	8,000

(filter) whose frequency response is called the A-weighting curve. The curve, shown in Fig. 12.1, approximates an inverse of the equal loudness contour at 40 phons. Clearly, the A-weighting curves attenuates low and very high audible frequencies. Generally, the A-weighted sound level, which is sometimes expressed in dB(A), is a good indication of the loudness of most common noises.

A broad-band sound or noise is one that has an equally-distributed sound pressure level over all frequencies of interest. A narrow-band sound or noise has its the main energy in a narrow frequency range. Tonal sound means that it is possible to pick out discrete frequencies or pure tones in its spectrum.

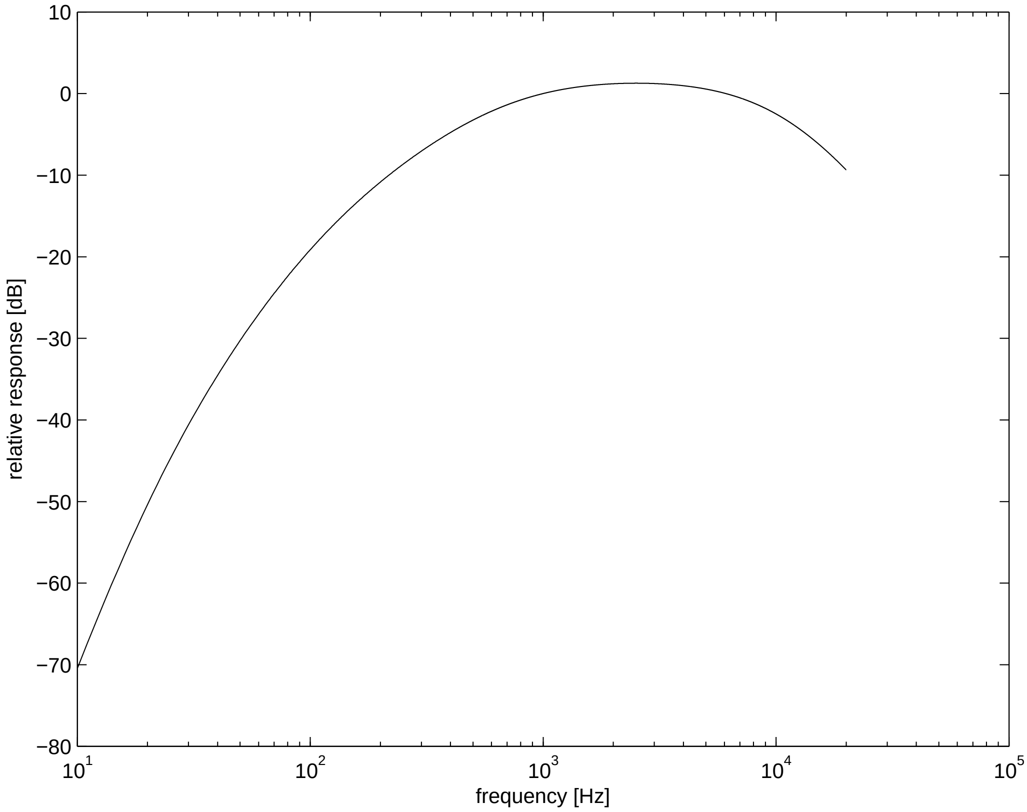

Figure 12.1: A-weighting curve.

13 Vectorial notation of the wave equation

The 3-dimensional wave equation in terms of pressure is given by

$$\ddot{\mathbf{p}} = c^2 \, \nabla^2 \mathbf{p}. \tag{13.1}$$

where $c = \sqrt{\frac{\text{modulus of elasticity}}{\text{density}}}$ is the phase velocity,

$$\nabla = \begin{pmatrix} \dfrac{\partial}{\partial x} \\[2ex] \dfrac{\partial}{\partial y} \\[2ex] \dfrac{\partial}{\partial z} \end{pmatrix}, \tag{13.2}$$

and

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \tag{13.3}$$

In vector notation,

$$\mathbf{p} = p_0 e^{j(\omega t - \mathbf{k} \cdot \mathbf{r})}, \tag{13.4}$$

where $\mathbf{k}$ is the wave vector and $\mathbf{r}$ is the distance vector. Note that

$$k = |\mathbf{k}|. \tag{13.5}$$

Rewriting the wave equation in terms of a particle displacement vector **u** gives

$$\ddot{\mathbf{u}} = c^2 \, \nabla^2 \mathbf{u}. \tag{13.6}$$

Here,

$$\mathbf{u} = u_0 e^{j(\omega t - \mathbf{k} \cdot \mathbf{r})}. \tag{13.7}$$

14

Plane waves in isotropic media

So far, we have dealt with compressive (longitudinal) waves only, which cause displacement parallel to the direction of propagation. Shear (transverse) waves cause displacement perpendicular to the direction of propagation. Since fluids cannot support shear stresses, shear waves can only propagate through solids.

In elastic media, the equation of motion (3.7) is rewritten in terms of the stress tensor:

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{pmatrix} = -\rho \begin{pmatrix} \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial^2 w}{\partial t^2} \end{pmatrix}. \tag{14.1}$$

Let's restate Hooke's law:

$$[\sigma] = 2G[\varepsilon] + \lambda\Delta[U], \tag{14.2}$$

where G is the shear modulus, $\lambda = \kappa - \frac{2}{3}G$ is Lamé's constant, and

$$[\varepsilon] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{yx} & \frac{1}{2}\gamma_{zx} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{zy} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}, \tag{14.3}$$

in which the strains and shear strains are given by

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}; & \varepsilon_y &= \frac{\partial v}{\partial y}; & \varepsilon_z &= \frac{\partial w}{\partial z}; \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}; & \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}; & \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}. \end{aligned} \tag{14.4}$$

Inserting Hooke's law into (14.1) yields the wave equation for isotropic solids. In vector notation:

$$(\lambda + 2G)\,\nabla(\nabla \cdot \mathbf{u}) - G\,\nabla \times \nabla \times \mathbf{u} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}. \tag{14.5}$$

In an isotropic solid, the vector displacement of matter $\mathbf{u}$ can be written as

$$\mathbf{u} = \nabla\phi + \nabla \times \psi. \tag{14.6}$$

The displacement involves a scalar potential ϕ and a vector potential ψ resulting from the fact that the movement consists of a translation and a rotation. This means that the wave equation can be split into two equations. One corresponds to the propagation of a compressional (longitudinal) wave, while the other corresponds to a shear (transverse) wave. In terms of the potentials the two equations are:

$$\nabla^2\phi = \frac{1}{c_\mathrm{p}^2}\frac{\partial^2\phi}{\partial t^2} \tag{14.7}$$

and

$$\nabla^2\psi = \frac{1}{c_\mathrm{s}^2}\frac{\partial^2\psi}{\partial t^2}, \tag{14.8}$$

where c_p and c_v are the phase velocities of the compressional wave and the shear wave, respectively. They are characteristic for the material and given by

$$c_\mathrm{p} = \sqrt{\frac{\lambda + 2G}{\rho}} \tag{14.9}$$

and

$$c_\mathrm{v} = \sqrt{\frac{G}{\rho}}. \tag{14.10}$$

Waves in fluids (special case of a solid)

In fluids, no shear deformation can occur. Hence, the shear modulus $G = 0$ and the stress tensor

$$[\sigma] = p[U]. \tag{14.11}$$

Thus, Hooke's law reduces to

$$[\sigma] = \lambda \Delta [U]. \tag{14.12}$$

Furthermore,

$$\kappa = \lambda. \tag{14.13}$$

Consequently, the speed of sound is given by

$$c_\mathrm{p} = \sqrt{\frac{\kappa}{\rho}}. \tag{14.14}$$

15
Mechanisms of wave attenuation

All media attenuate sounds, so that the amplitude u of a plane wave in the x-direction decreases exponentially with the distance as

$$u \propto e^{-\alpha x}, \tag{15.1}$$

where α is the attenuation coefficient. α is usually expressed in m^{-1} or in Neper/m. Since the acoustic power or the intensity are proportional to the squared amplitude, the corresponding attenuation coefficients become 2α, as shown in (15.10). The attenuation of a plane wave arises from the scattering of energy from the parallel beam by regular reflection, refraction and diffraction, and from absorption mechanisms as a result of which the mechanical energy is converted into heat.

Three main mechanisms of sound absorption can be identified:

1. Viscous damping. This corresponds to the friction associated with the relative motion of the particles.

   For a fluid, the viscous damping coefficient is given by

$$\alpha_{\mathrm{v}} = \frac{\omega^2 \eta}{2\rho c^3}, \tag{15.2}$$

where η is the viscosity of the fluid. In principle, a second viscosity coefficient can be defined but this is ignored here. Note that $\alpha_\mathsf{v} \propto \omega^2$. The dissipation mechanisms can also be accounted for in solids by considering the elastic moduli to be complex.

2. Thermal damping. Here, a fraction of the energy carried by the wave is converted into heat by thermoelasticity. Thus, energy is dissipated in the material by thermal conductivity.

   The mechanism of attenuation by thermal conductivity is related to the cycles of compression–dilatation associated with the passage of the wave. These cycles are responsible for the establishment of local thermal gradients within the material. Since the material is thermally conductive, the temperature tends towards uniformity within the material. This phenomenon contributes to the increase of the wave attenuation. The thermal damping $\alpha_\theta \propto \omega^2$, as well. The relative importance of the viscous/thermal attenuation depends on the state of matter. For gases,

$$\frac{\alpha_\theta}{\alpha_\mathsf{v}} \approx \frac{3}{8}, \tag{15.3}$$

   for liquids,

$$\frac{\alpha_\theta}{\alpha_\mathsf{v}} \approx 10^{-3}, \tag{15.4}$$

   and for solids,

$$\frac{\alpha_\theta}{\alpha_\mathsf{v}} \approx 0. \tag{15.5}$$

   This means that the viscous losses in gases are approximately three times more important than the thermal losses and in some cases thermal effects must be considered. In solids and liquids, the attenuation is mainly due to viscous friction and thermal effects can be ignored.

3. Molecular relaxation. In this case, the temperature or pressure variations associated with the passage of the wave is responsible for alterations in molecular energy level configurations.

   The cycles of compression–dilatation are also responsible for a modification of the molecular configuration of matter. During the passage of the wave, there may be a transition from one molecular configuration to another. Afterwards, the molecules can revert to their original configuration. This process transfers energy and is responsible for wave attenuation by molecular relaxation. A relaxation time is associated with this phenomenon.

In the linear regime, a solution of the wave equation can be written in a complex form as

$$u = u_0 e^{j(\omega t - kx)}. \tag{15.6}$$

where k is the complex wave number including the effect of sound attenuation

$$k = k_{\mathrm{Re}} - j\alpha. \tag{15.7}$$

Hence,

$$u = u_0 e^{-\alpha x} e^{j(\omega t - k_{\mathrm{Re}} x)} \tag{15.8}$$

and

$$p = p_0 e^{-\alpha x} e^{j(\omega t - k_{\mathrm{Re}} x)} \tag{15.9}$$

It can be shown that the formulae for the characteristic impedance, surface impedance, reflection and transmission coefficients introduced for real wave numbers later are valid when the complex wave number is used. Since $I \propto p^2$,

$$I = I_0 e^{-2\alpha x}. \tag{15.10}$$

Hence,

$$\begin{aligned} \mathrm{SPL} \approx \mathrm{IL} = 10 \log_{10} e^{-2\alpha x} &= 10 \cdot 0.434 \log_e e^{-2\alpha x} \\ &= -8.69\alpha x \text{ dB}, \end{aligned} \tag{15.11}$$

yielding an attenuation D of

$$D = -8.69\alpha \text{ dB m}^{-1}. \tag{15.12}$$

16

Sound propagation and attenuation outdoors

The propagation of sound outdoors is sensitive to effects of wind, temperature gradients, air turbulence, and humidity. The presence of wind means that the equilibrium position, about which air particles oscillate during the passage of a sound wave, is no longer stationary. Consequently, the sound speed has the wind speed superimposed on it. Because the wind is slowest near the ground where viscous drag is important, the effect of wind on sound speed increases with height and causes sound rays and wave fronts to curve. Upwind of the source, sound rays bend upwards away from the ground, creating a shadow zone near to the ground.

The sound speed increases with the temperature. The earth's surface is warmed by the sun. A hypothetical parcel of air next to the ground is warmed and rises as a result of becoming less dense. In rising, it moves to lower pressure and therefore expands. If it expands adiabatically, *i.e.*, it neither gains from nor loses energy to its surroundings, the energy for expansion comes from within the parcel and it cools. The calculated rate of cooling is approximately $1\,^{\circ}\mathrm{C}$ per 100 m increase in altitude. Thus, the temperature gradient approximates $-0.01\,^{\circ}\mathrm{C\,m^{-1}}$, which is known as the dry adiabatic lapse rate and is the rate in stable (windless) or neutral atmospheric conditions. In a meteorological neutral atmosphere, therefore, sound waves tend to bend away from the ground, creating a shadow zone near the ground. The edge of the shadow zone is defined by the limiting ray that just grazes the ground. For source close to the ground, the distance to the shadow zone x_s is approximated by

$$x_s = \sqrt{\frac{2c_0}{-\frac{dc}{dz}}} \left(\sqrt{h_s} + \sqrt{h_r} \right), \qquad (16.1)$$

where c_0 is the sound speed at ground level, h_s is the height of the sound source, and h_r is the height of the receiver. A superadiabatic lapse rate is favourable to noise reduction by distance alone.

Under inversion conditions, where temperature increases with height, the sound rays bend towards the ground. Temperature inversion can greatly increase traffic noise levels.

Attenuation of sound outdoors

The total attenuation of sound outdoors A_{tot} can be calculated by adding the various attenuation values to the attenuation due to the distance:

$$A_{tot} = A_{div} + A_{atm} + A_{ground} + A_{refl} + A_{refr} + A_{barrier} + A_{misc}, \qquad (16.2)$$

where

A_{div} is the attenuation due to the geometrical divergence. For a single point source, $A_{div} = 6$ dB per doubling distance (dd) as a result of the spherical spreading of the wavefronts, whereas for a line source such as a busy road, $A_{div} = 3$ dB per dd as a result of the cylindrical spreading of the wavefronts.

A_{atm} is the attenuation due to atmospheric absorption. Air itself is a sound-absorbing medium. The altered atmospheric absorption depends on temperature, frequency, and humidity. The reductions in sound level due to alterations in air humidity are small, being appreciable only for frequencies above 1 kHz. A_{atm} at a temperature of 20 °C can be estimated crudely from

$$A_{atm} \approx \left(7.4 \times 10^8 \text{ s}^2 \text{ m}^{-1}\right) \frac{f^2 r}{\mathrm{RH}} \text{ dB}, \qquad (16.3)$$

where f is the centre frequency of the band, r is the distance between source and receiver, and RH is the relative humidity. For temperatures other than 20 °C, assuming RH = 50%, a crude approximation is

$$\mathsf{A}_{\mathrm{atm},\mathcal{T}} = \frac{\mathsf{A}_{\mathrm{atm}}}{1 + \beta\,\Delta\mathcal{T}\,f}\,, \tag{16.4}$$

where $\beta = 4 \times 10^{-6}\,\mathrm{s\,K^{-1}}$ and $\Delta\mathcal{T}$ is the temperature difference from 20 °C.

$\mathsf{A}_{\mathrm{ground}}$ is the attenuation due to the ground effect. The presence of an acoustically hard ground increases the sound level by giving a source near the ground an effective directionality. However, the resulting increase is complicated by the interference of direct waves and reflected waves. Particularly in porous ground, the main interference effect can occur at useful frequencies from the noise control point of view. The ground effect refers to a phenomenon by which noise in a certain frequency range is reduced compared with that over acoustically hard ground. The bending of sound rays as a result of wind and temperature gradients affects the frequencies at which the ground effect occurs.

Keast has devised a method for predicting the extra soft-ground attenuation.

$$f_{\mathrm{max}} = \frac{1500\ \mathrm{m\,s^{-1}}}{h_{\mathrm{sr}}\ \log_{10}\dfrac{r}{0.3\ \mathrm{m}}}\,, \tag{16.5}$$

where h_{sr} is the mean height of the source-to-receiver path and r is the distance from source to receiver. $\mathsf{A}_{\mathrm{ground}}$ in the octave band containing f_{max} is calculated from

$$\mathsf{A}_{\mathrm{ground}} = 15\log_{10}\left(0.065\frac{r}{h_{\mathrm{sr}}}\right). \tag{16.6}$$

If $0.065\frac{r}{h_{\mathrm{sr}}} < 1$, we take $\mathsf{A}_{\mathrm{ground}} = 0$. In the two octave bands adjacent to that of maximum attenuation, the attenuation is half the maximum value; in all other frequency bands it is zero.

$\mathsf{A}_{\mathrm{refl}}$ is the attenuation due to reflection from nearby vertical surfaces. Generally, reflections result in higher sound levels. Therefore, $\mathsf{A}_{\mathrm{refl}}$ will be negative.

$\mathsf{A}_{\mathrm{refr}}$ is the attenuation due to upward refraction. In inverse conditions, $\mathsf{A}_{\mathrm{refr}}$ is negative.

A_{barrier} is the attenuation due to diffraction by barriers. Assuming an infinitely long barrier,

$$A_{\text{barrier}} = 10\log_{10}(3 + 20N), \tag{16.7}$$

where N is the Fresnel number defined by

$$N = \frac{\delta}{\lambda}, \tag{16.8}$$

in which δ is the additional path between source and receiver, normally calculated by the Pythagorean theorem.

A_{misc} is the attenuation due to miscellaneous factors. It represents the effects of meteorological conditions other than refraction, trees and shrubs, and buildings or elements of buildings.

The main effects of atmospheric turbulence are to reduce the ground effect and to scatter sound into the shadow zones. Turbulence may also cause the noise level to fluctuate, particularly at high frequencies.

17
Derivation of Snell's law

Huygens' principle, that each point on a wavefront can be treated as a secondary source, is consistent with the period T being constant between media. Since

$$T = \frac{1}{f}, \tag{17.1}$$

the frequency f does not change with a change of media.

Consider oblique incidence of a wavefront on a plane interface, as shown in Fig. 17.1. The angle of incidence relative to the normal of the interface is θ_i and the angle of transmission is θ_t. The sound speed of the incident wavefront is c_1 and of the transmitted wavefront c_2. We chose such equidistant rays through A, B, C, ..., so that it takes exactly T for the wavefront to advance from (A,B,...) to (A',B',...), and again T to advance from (A',B',...) to (A",B",...). Therefore, the distance AA' = A'A" = BB' $= c_1 T = \lambda_1$, whereas B'B" $= c_2 T = \lambda_2$. The shared hypotenuse of the triangles AB'A" and B'A"B" is denoted by d, so that

$$d \sin \theta_\mathrm{i} = c_1 T \tag{17.2}$$

and

$$d \sin \theta_\mathrm{t} = c_2 T, \tag{17.3}$$

from which it follows, that

$$\frac{\sin \theta_\mathrm{i}}{\sin \theta_\mathrm{t}} = \frac{c_1}{c_2} = \frac{\lambda_1}{\lambda_2} = \frac{k_\mathrm{t}}{k_\mathrm{i}} \tag{17.4}$$

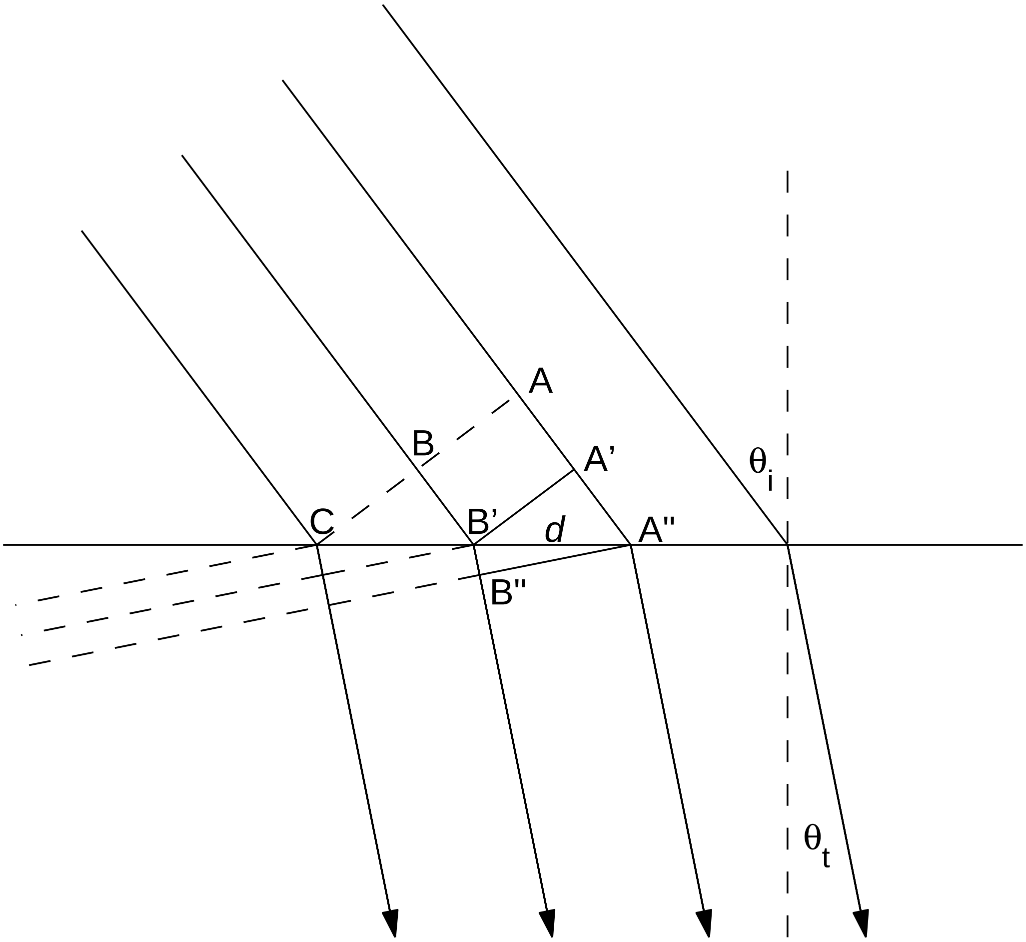

Figure 17.1: Incident rays on a plane interface.

or

$$k_\mathrm{i} \sin\theta_\mathrm{i} = k_\mathrm{t} \sin\theta_\mathrm{t}. \tag{17.5}$$

This is known as Snell's law.

Critical angle

In case $c_2 > c_1$, there exists a critical angle of θ_i at which transmission occurs parallel to the interface, *i.e.*,

$$\sin\theta_\mathrm{t} = 1 \tag{17.6}$$

or

$$\cos\theta_\mathrm{t} = 0. \tag{17.7}$$

Since

$$\cos\theta_\mathrm{t} = \sqrt{1-\sin^2\theta_\mathrm{t}} = \sqrt{1-\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_\mathrm{i}}. \tag{17.8}$$

The critical incidence angle is found at

$$\theta_\mathrm{i} = \arcsin\frac{c_2}{c_1}. \tag{17.9}$$

The index of refraction is defined by

$$\mathrm{n} = \frac{c_1}{c_2}. \tag{17.10}$$

For n$\gg$ 1, according to 17.8,

$$\cos\theta_\mathrm{t} \approx 1 \tag{17.11}$$

and thus

$$\theta_\mathrm{t} \approx 0. \tag{17.12}$$

This means that the refracted sound travels in the direction of the normal to the surface, whatever the angle of the incident sound is. The surface is then said to be locally reacting or to satisfy and impedance condition.

18

Reflection and transmission of waves on a plane fluid–fluid interface

Consider oblique incidence on a plane fluid–fluid interface, resulting in reflected and transmitted waves. The fluids are characterised by their respective densities and sound speeds c_1 and c_2. It is convenient to refer to the products of these quantities, the characteristic or wave impedances Z_1 and Z_2. These represent the ratios between pressure and velocity at any point in the wave. At a given angle of incidence θ_i relative to the normal of the interface, the angles of reflection θ_r and transmission θ_t are found by writing Snell's law of conservation of the components of the wave numbers along the interface:

$$k_\mathrm{i} \sin\theta_\mathrm{i} = k_\mathrm{r} \sin\theta_\mathrm{r} = k_\mathrm{t} \sin\theta_\mathrm{t}, \tag{18.1}$$

where the wave numbers of the incident wave k_i and of the reflected wave k_r

$$k_\mathrm{i} = k_\mathrm{r} = \frac{\omega}{c_1} \equiv k_1 \tag{18.2}$$

and the wave number of the transmitted wave

$$k_\mathrm{t} = \frac{\omega}{c_2} \equiv k_2. \tag{18.3}$$

This implies that

$$\theta_\mathrm{i} = \theta_\mathrm{r} \equiv \theta_1 \tag{18.4}$$

and

$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2}, \tag{18.5}$$

where $\theta_2 \equiv \theta_\mathrm{t}$.

Reflection and transmission coefficients

Two basic continuity conditions must be satisfied at the interface ($x = 0$) at a given time ($t =$ constant):

1. Continuity of velocity: The normal component of the particle velocity must be equal on either side of the interface:

$$\nu_\mathrm{i}\cos\theta_1 - \nu_\mathrm{r}\cos\theta_1 = \nu_\mathrm{t}\cos\theta_2. \tag{18.6}$$

2. Continuity of pressure: The pressure variations must be equal on either side of the interface:

$$p_\mathrm{i} + p_\mathrm{r} = p_\mathrm{t}, \tag{18.7}$$

where

$$\begin{aligned} p_\mathrm{i} &= A_1\, e^{-jk_1x}; \\ p_\mathrm{r} &= B_1\, e^{jk_1x}; \\ p_\mathrm{t} &= A_2\, e^{-jk_2x}. \end{aligned} \tag{18.8}$$

Since $x = 0$,

$$\begin{aligned} p_\mathrm{i} &= A_1; \\ p_\mathrm{r} &= B_1; \\ p_\mathrm{t} &= A_2. \end{aligned} \tag{18.9}$$

Thus, (18.7) reduces to

$$A_1 + B_1 = A_2. \tag{18.10}$$

Using (10.6), (18.6) becomes

$$\frac{A_1}{Z_1}\cos\theta_1 - \frac{B_1}{Z_1}\cos\theta_1 = \frac{A_2}{Z_2}\cos\theta_2. \tag{18.11}$$

The pressure reflection coefficient is defined by

$$\mathrm{R} = \frac{B_1}{A_1} \tag{18.12}$$

and the transmission coefficient by

$$\mathrm{T} = \frac{A_2}{A_1}. \tag{18.13}$$

Combining (18.10) and (18.11) yields

$$\mathrm{R} = \frac{\frac{Z_2}{\cos\theta_2} - \frac{Z_1}{\cos\theta_1}}{\frac{Z_2}{\cos\theta_2} + \frac{Z_1}{\cos\theta_1}} \tag{18.14}$$

and

$$\mathrm{T} = \frac{2\frac{Z_2}{\cos\theta_2}}{\frac{Z_2}{\cos\theta_2} + \frac{Z_1}{\cos\theta_1}}. \tag{18.15}$$

Normal incidence

At normal incidence, the cosines are equal to 1 and the expressions for R and T are simplified to

$$\mathrm{R} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{18.16}$$

and

$$\mathrm{T} = \frac{2Z_2}{Z_2 + Z_1}. \tag{18.17}$$

A good transmission means a low reflection coefficient. The sound power reflection coefficient is given by

$$\mathrm{R}^2 = \left(\frac{B_1}{A_1}\right)^2 = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2, \tag{18.18}$$

whereas the sound power transmission coefficient is given by

$$\mathrm{T}^2 = \left(\frac{A_2}{A_1}\right)^2 \frac{Z_1}{Z_2} = \frac{4Z_2Z_1}{(Z_2 + Z_1)^2}. \tag{18.19}$$

Adding these two yields the logical result

$$\mathrm{R}^2 + \mathrm{T}^2 = 1. \tag{18.20}$$

The energy loss when passing from steel to air is 99.96%. However, the transmission can be improved by interposing a layer of material on the interface. The idea is to reduce the difference of impedance by inserting an intermediate value for the impedance.

Absorption coefficient

The absorption coefficient is related to the reflection coefficient by

$$\mathrm{A} = 1 - |\mathrm{R}|^2. \tag{18.21}$$

This parameter is extensively used in engineering. Since the phase of R is not involved, A does not carry as much information as the impedance or the reflection coefficient.

Normal incidence on a wall (two fluid–fluid boundaries)

Consider normal incidence on a system of two plane interfaces, at $x = 0$ and $x = l$, respectively. At the first interface, the basic continuity conditions are formulated as

$$A_1 + B_1 = A_2 + B_2 \tag{18.22}$$

and

$$\frac{A_1}{Z_1} - \frac{B_1}{Z_1} = \frac{A_2}{Z_2} - \frac{B_2}{Z_2}. \tag{18.23}$$

At the second interface, the basic continuity conditions are formulated as

$$A_2 e^{-jk_2 l} + B_2 e^{jk_2 l} = A_3 e^{-jk_2 l} \tag{18.24}$$

and

$$\frac{A_2 e^{-jk_2 l}}{Z_2} - \frac{B_2 e^{jk_2 l}}{Z_2} = \frac{A_3 e^{-jk_2 l}}{Z_3}. \tag{18.25}$$

This system of equations can be solved to give the following expression for the power transmission coefficient:

$$\mathrm{T}^2 = \left(\frac{A_3}{A_1}\right)^2 \frac{Z_1}{Z_3} = \frac{4Z_3Z_1}{(Z_3+Z_1)^2\cos^2 k_2l + \left(Z_2 + \frac{Z_1Z_3}{Z_2}\right)^2 \sin^2 k_2l}. \tag{18.26}$$

Impedance of a rigid-backed fluid layer

If the third medium is rigid, the condition of pressure continuity at the second interface is formulated as

$$A_2e^{-jk_2l} + B_2e^{jk_2l} = 0, \tag{18.27}$$

whereas the condition of velocity continuity does not hold, since $Z_3 = 0$.

$$\frac{A_2e^{-jk_2l}}{Z_2} - \frac{B_2e^{jk_2l}}{Z_2} = 0. \tag{18.28}$$

Hence,

$$B_2 = -A_2e^{-2jk_2l}. \tag{18.29}$$

Substituting this in (18.22) and (18.23) yields

$$A_1 + B_1 = A_2\left(1 - e^{-2jk_2l}\right) \tag{18.30}$$

and

$$A_1 - B_1 = \frac{Z_1}{Z_2}A_2\left(1 + e^{-2jk_2l}\right), \tag{18.31}$$

respectively. The ratio of these equations is referred to as the relative surface impedance. It represents the ratio of the acoustic pressure to the particle velocity at $x = 0$.

$$Z_s = \frac{A_1+B_1}{A_1-B_1} = \frac{Z_2}{Z_1}\frac{1-e^{-2jk_2l}}{1+e^{-2jk_2l}} = \frac{Z_2}{Z_1}\frac{e^{jk_2l}-e^{-jk_2l}}{e^{jk_2l}+e^{-jk_2l}} \tag{18.32}$$

Note that

$$\cosh\phi = \frac{e^{\phi}-e^{-\phi}}{2} \tag{18.33}$$

and

$$\sinh\phi = \frac{e^{\phi}+e^{-\phi}}{2}. \tag{18.34}$$

Therefore, (18.32) can be reduced to

$$Z_{\mathsf{s}} = \frac{Z_2}{Z_1} \coth jk_2 l = Z_{\mathsf{c}} \coth jk_2 l, \tag{18.35}$$

where $Z_{\mathsf{c}} = \frac{Z_2}{Z_1}$ is the relative characteristic impedance of layer 2. Its inverse, $\beta_{\mathsf{c}} = \frac{1}{Z_{\mathsf{c}}}$ is the relative characteristic admittance.

19

Aspects of sound insulation

The requirements for good airborne sound transmission loss are mass and completeness. A practical form of the mass law gives

$$\mathrm{SRI_{av}} = 10 + 14.5 \log_{10} \sigma, \tag{19.1}$$

where $\mathrm{SRI_{av}}$ is the average transmission loss (or sound reduction) over frequencies from 250 Hz to 1,000 Hz and σ is the mass per unit area. A single-brick-wide (230 mm thick) brick wall has $\sigma = 415\ \mathrm{kg\,m^{-2}}$, giving an average insulation of 48 dB.

The sound reduction index SRI or transmission loss is defined by

$$\mathrm{SRI} = 10 \log_{10} \frac{1}{\mathrm{T}^2}. \tag{19.2}$$

Transmission loss of a wall

For an infinite fluid or quasi-fluid layer between regions of the same fluid, *i.e.*, $\rho_3 c_3 = \rho_1 c_1$, (18.26) reduces to

$$\begin{aligned} \mathsf{T}^2 = \left(\frac{A_3}{A_1}\right)^2 &= \frac{4}{4\cos^2 k_2 l + \left(\frac{Z_2}{Z_1} + \frac{Z_1}{Z_2}\right)^2 \sin^2 k_2 l} \\ &= \frac{4}{4\cos^2 k_2 l + (Z_c + \beta_c)^2 \sin^2 k_2 l}. \end{aligned} \tag{19.3}$$

For concrete, $\rho_2 c_2 = 8.1 \times 10^6$ rayls. So, for a concrete wall,

$$Z_c = \frac{\rho_2 c_2}{\rho_1 c_1} = \frac{8.1 \times 10^6}{415} \approx 2 \times 10^4 \tag{19.4}$$

and

$$\beta_c \approx 5 \times 10^{-5}. \tag{19.5}$$

Since $Z_c \gg \beta_c$, we can neglect β_c in (19.3). Furthermore, at audio-frequencies, the $\cos^2$ term can be ignored and

$$\sin k_2 l \approx k_2 l. \tag{19.6}$$

Hence, 19.3 further reduces to

$$\mathsf{T}^2 \approx \frac{4\,\rho_1^2\, c_1^2}{k_2^2\, l^2\, \rho_2^2\, c_2^2}. \tag{19.7}$$

Replacing k_2 by $\frac{2\pi f}{c_2}$ and $\rho_2 l$ by σ yields

$$\mathsf{T}^2 \approx \left(\frac{415}{\pi}\right)^2 \frac{1}{\rho_2^2\, l^2\, f^2} = \frac{1.75 \times 10^4}{\sigma^2 f^2}. \tag{19.8}$$

Consequently, the transmission loss is

$$\mathsf{SRI} = -42.4 + 20 \log_{10} \sigma f. \tag{19.9}$$

This is the basis of the mass law: At a given frequency, doubling the mass leads to a 6 dB increase in transmission loss.

Average transmission loss of a composite wall

When calculating the sound insulation of a composite partition, *i.e.*, one with parts made from different materials, it is necessary to know the power transmission coefficient and area of each part. The average power transmission coefficient is then given by

$$\mathrm{T}_{\mathrm{av}}^2 = \frac{\sum_i \mathrm{T}_i^2 S_i}{\sum_i S_i}, \tag{19.10}$$

where S_i is the area and T_i^2 is the power transmission coefficient of material $i \in \mathbb{N}$. T_i^2 can be found from the reduction indices of the respective materials

$$\mathrm{T}_i^2 = 10^{\frac{\mathrm{SRI}_i}{10}}. \tag{19.11}$$

The transmission loss of the composite material is given by

$$\mathrm{SRI} = 10 \log_{10} \frac{1}{\mathrm{T}_{\mathrm{av}}^2}. \tag{19.12}$$

Variation of transmission loss with frequency

For a rectangular panel, supported but not clamped at its edges, the resonant frequencies f_N are given by

$$f_N = 0.45\, c_{\mathrm{L}}\, h \left[\left(\frac{N_x}{l_x} \right)^2 + \left(\frac{N_y}{l_y} \right)^2 \right], \tag{19.13}$$

where c_{L} is the speed of sound in the panel material, h is the thickness, l_x is the length, l_y is the width, and N_x, N_y are integer values $\in \mathbb{N}$ representing the resonant frequencies. $N_x = N_y = 1$ is the lowest resonant frequency. Varying the values of N_x and N_y gives those frequencies at which the panel resonates and where the transmission loss is poor.

The transmission loss of a simply-supported panel is also poor when the wavelength in air matches the wavelength of bending waves in the panel. Bending waves produce large deflections in the structure and cause considerable radiation of sound energy. The lowest frequency at which this matching occurs in any component is known as its critical or coincidence frequency, given by

$$f_{\mathrm{c}} = \frac{c^2}{1.8 h\, c_{\mathrm{L}}}. \tag{19.14}$$

Table 19.1: Mass per area times critical frequency for building materials.

Material	$\sigma \times f_c$ ($\mathrm{kg\,m^{-2}} \times \mathrm{kHz}$)
Plywood	17
Glass	35
Brick	49
Plasterboard	50
Concrete	100
Steel	150

Table 19.1 gives the product of the mass per unit area and the critical frequency for some building materials.

20
Room acoustics

A sound field in a room or enclosed space is uniform if the root-mean-square pressure is constant throughout the room volume. A sound field is diffuse if the acoustic intensity in waves arriving at any point in the room is independent of the direction of arrival of the wave. A reverberant sound field is a field in which all waves have undergone reflection at room surfaces.

Consider an enclosed space with a wall surface area S and an absorption coefficient of the walls

$$\mathrm{A} = 1 - |\mathrm{R}|. \tag{20.1}$$

For diffusive sound arriving at the surface, (6.14) changes to

$$I = \frac{\mathcal{E} c}{4} = \frac{p^2}{4\rho c}. \tag{20.2}$$

Thus,

$$\mathrm{IL} \approx \mathrm{SPL} - 6 \text{ dB}. \tag{20.3}$$

The reverberation time is defined as the time it takes for impulsive sound to decay by 60 dB in intensity. It is given by the Sabine formula

$$\mathrm{RT} = \frac{55.2\, V}{S\, \mathrm{A}\, c}, \tag{20.4}$$

where V is the volume of the room in m^3. The Sabine formula is only valid where the sound field is uniform, diffuse, and reverberant.

Consider a room in the form of a parallelepiped with dimensions l_x, l_y, and l_z. If the walls are rigid, the following boundary conditions hold:

$$\begin{aligned}
\frac{\partial p}{\partial x} &= 0 \quad \text{at } x = 0,\ x = l_x; \\
\frac{\partial p}{\partial y} &= 0 \quad \text{at } y = 0,\ y = l_y; \\
\frac{\partial p}{\partial z} &= 0 \quad \text{at } z = 0,\ z = l_z.
\end{aligned} \tag{20.5}$$

Consider the 3-D wave equation

$$\left(\nabla^2 + k^2\right) p = 0. \tag{20.6}$$

Let's assume a solution of the form

$$p = e^{j\omega t}\, X(x)\, Y(y)\, Z(z), \tag{20.7}$$

where

$$\begin{aligned}
X(x) &= A_x \cos k_x x \,+\, B_x \sin k_x x; \\
Y(y) &= A_y \cos k_y y \,+\, B_y \sin k_y y; \\
Z(z) &= A_z \cos k_z z \,+\, B_z \sin k_z z.
\end{aligned} \tag{20.8}$$

The boundary conditions are satisfied if

$$\begin{aligned}
X(x) &= A_x \cos \frac{\pi N_i x}{l_x}; \\
Y(y) &= A_y \cos \frac{\pi N_j y}{l_y}; \\
Z(z) &= A_z \cos \frac{\pi N_k z}{l_z},
\end{aligned} \tag{20.9}$$

where $N_i, N_j, N_k \in \mathbb{N}$. From (20.6),

$$k^2 = \left(\frac{\pi N_i}{l_x}\right)^2 + \left(\frac{\pi N_j}{l_y}\right)^2 + \left(\frac{\pi N_k}{l_z}\right)^2. \tag{20.10}$$

Hence,

$$f_{ijk} = \frac{c}{2}\sqrt{\left(\frac{\pi N_i}{l_x}\right)^2 + \left(\frac{\pi N_j}{l_y}\right)^2 + \left(\frac{\pi N_k}{l_z}\right)^2}. \tag{20.11}$$

Here, f_{ijk} are the eigenfrequencies or normal mode frequencies of the room. The sound pressure can be expressed as a summation over the eigenmodes:

$$p(x, y, z; t) = \sum_{i,j,k} p_{ijk}\, e^{j\omega_{ijk} t} \cos\frac{\pi N_i x}{l_x} \cos\frac{\pi N_j y}{l_y} \cos\frac{\pi N_k z}{l_z}. \tag{20.12}$$

Reverberation involves the decay of many such modes at different frequencies. In any frequency band the sound field is a combination of many modes. Each mode consists of a series of plane waves inclined at particular angles to the walls of the room, depending on values of N_i, N_j, N_k, ensuing to give an interference pattern of the form $\cos\frac{\pi N_i x}{l_x} \cos\frac{\pi N_j y}{l_y} \cos\frac{\pi N_k z}{l_z}$. The resulting sound field is uniform if there are many modes so that peaks and troughs in any mode are canceled.

A
Fourier transform

In acoustics, it is very useful to express a signal $s(t)$ by its frequency content. The (complex) spectral density is obtained by the Fourier transform:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} s(t)\, e^{-j\omega t}\, \mathrm{d}t. \tag{A.1}$$

A signal can be recovered by the inverse Fourier transform:

$$s(t) = \int_{-\infty}^{\infty} S(\omega)\, e^{j\omega t}\, \mathrm{d}\omega. \tag{A.2}$$

B

Equal loudness contours

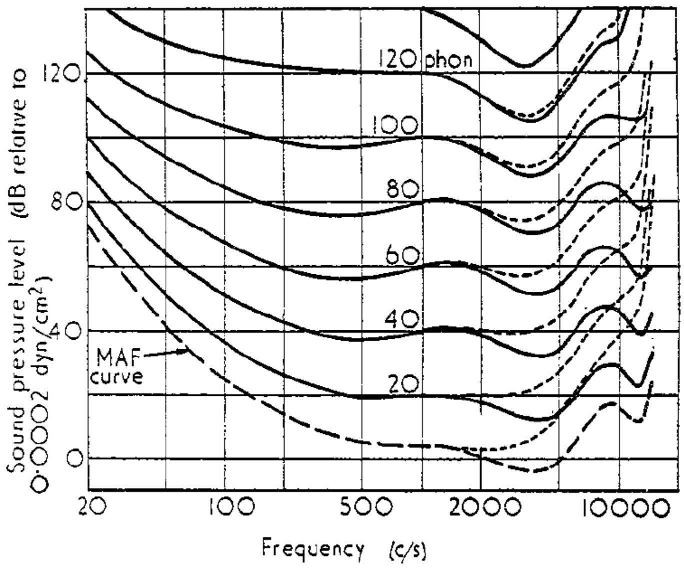

Fig. 8. Equal-loudness contours

——— age twenty years; – – – – age sixty years.

BRITISH JOURNAL OF APPLIED PHYSICS

Reprinted from Robinson DW, Dadson RS. A re-determination of the equal-loudness relations for pure tones. *Brit J Appl Phys.* **1956** 7(5):166–181. MAF stands for minimum audible field.

C
List of symbols

		M L T Θ dimensions
a_i	i^{th} coefficient	L
A_i	incident pressure amplitude in medium i at a given time	M L^{-1} T^{-2}
A_x	pressure amplitude in direction x	M L^{-1} T^{-2}
A_y	pressure amplitude in direction y	M L^{-1} T^{-2}
A_z	pressure amplitude in direction z	M L^{-1} T^{-2}
A	absorption coefficient	
$\mathrm{A}_{\mathrm{atm}}$	atmospheric absorption attenuation	
$\mathrm{A}_{\mathrm{atm},\mathcal{T}}$	atmospheric absorption attenuation at temperature $\mathcal{T}$	
$\mathrm{A}_{\mathrm{barrier}}$	attenuation due to diffraction by barriers	
$\mathrm{A}_{\mathrm{div}}$	attenuation due to geometrical divergence	
$\mathrm{A}_{\mathrm{ground}}$	attenuation due to ground effect	
$\mathrm{A}_{\mathrm{misc}}$	attenuation due to miscellaneous factors	
$\mathrm{A}_{\mathrm{refl}}$	attenuation due to reflection from vertical surfaces	
$\mathrm{A}_{\mathrm{refr}}$	attenuation due to upward refraction	

$\mathrm{A_{tot}}$	total attenuation	
B_i	reflected pressure amplitude in medium i at a given time	M L^{-1} T^{-2}
B_x	pressure amplitude in direction x	M L^{-1} T^{-2}
B_y	pressure amplitude in direction y	M L^{-1} T^{-2}
B_z	pressure amplitude in direction z	M L^{-1} T^{-2}
c	sound speed	L T^{-1}
c_0	sound speed at ground level	L T^{-1}
c_i	sound speed in medium i	L T^{-1}
c_L	sound speed in panel	L T^{-1}
c_p	phase velocity of a compressional wave	L T^{-1}
c_v	phase velocity of a transverse wave	L T^{-1}
d	shared hypotenuse	L
D	attenuation	L^{-1}
E_K	kinetic energy	M L^2 T^{-2}
E_P	potential energy	M L^2 T^{-2}
E_T	total energy	M L^2 T^{-2}
$\mathcal{E}$	energy per unit volume	M L^{-1} T^{-2}
f	frequency	T^{-1}
f'	experienced frequency	T^{-1}
f_0	resonance frequency	T^{-1}
f_c	coincidence frequency	T^{-1}
$f_i(x)$	pressure function i of x	M L^{-1} T^{-2}
f_i''	second partial derivative of f	
f_{ijk}	eigenfrequency	T^{-1}
f_max	frequency of maximal ground attenuation	T^{-1}
f_N	resonant frequency N	T^{-1}
F	force	M L T^{-2}
$g_i(x)$	displacement function i of x	L
G	shear modulus	M L^{-1} T^{-2}
h	thickness	L
h_r	height of receiver	L
h_s	height of sound source	L
h_sr	mean height of the source-to-receiver path	L
I	intensity	M T^{-3}
I_0	threshold of hearing intensity	M T^{-3}
IL	intensity level	
j	complex number with the property $j^2 = -1$	

J_0	Bessel function of order zero of the first kind	
k	wave number	L^{-1}
k_i	wave number in medium i	L^{-1}
k_i	wave number of incident wave	L^{-1}
k_r	wave number of reflected wave	L^{-1}
k_Re	real wave number	L^{-1}
k_t	wave number of transmitted wave	L^{-1}
$\mathbf{k}$	wave vector	L^{-1}
l	location of second interface	L
l_x	length	L
l_y	width	L
l_z	height	L
m	mass	M
M	molar mass	M
MAF	minimum audible field	
n	amount of gas	
n	refraction index	
N	Fresnel number	
N_i	integer values in direction i	
N_x, N_y	integer values $\in \mathbb{N}$	
p	acoustic pressure	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_0	pressure amplitude	$\mathrm{M\ L^{-1}\ T^{-2}}$
$p_{0,i}$	pressure amplitude of wave i	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_i	acoustic pressure of wave i	$\mathrm{M\ L^{-1}\ T^{-2}}$
$p_{i,\mathrm{rms}}$	root-mean-square pressure of wave i	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_i	pressure of incident wave	$\mathrm{M\ L^{-1}\ T^{-2}}$
$p_{f,\mathrm{rms}}$	root-mean-square pressure at frequency f	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_h	hearing threshold	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_r	pressure of reflected wave	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_rms	root-mean-square pressure	$\mathrm{M\ L^{-1}\ T^{-2}}$
p_t	pressure of transmitted wave	$\mathrm{M\ L^{-1}\ T^{-2}}$
$\mathbf{p}$	pressure vector	$\mathrm{M\ L^{-1}\ T^{-2}}$
$\ddot{\mathbf{p}}$	second time derivative of $\mathbf{p}$	$\mathrm{M\ L^{-1}\ T^{-4}}$
P	absolute pressure	$\mathrm{M\ L^{-1}\ T^{-2}}$
$\mathrm{d}P$	pressure difference	$\mathrm{M\ L^{-1}\ T^{-2}}$
Q	directivity factor	
r	distance	L
r_i	distance of location i	L

$\mathbf{r}$	distance vector	L
R	gas constant	M L^2 T^{-2} Θ^{-1}
$\bar{R}$	specific gas constant	L^2 T^{-2} Θ^{-1}
R	reflection coefficient	
R^2	power reflection coefficient	
RH	relative humidity	
RT	reverberation time	T
s	spring stiffness	M T^{-2}
$s(t)$	signal	
S	area	L^2
$S(\omega)$	spectral density	
dS	element cross-section	L^2
S_i	area i	L^2
SPL	sound pressure level	
SPL_i	sound pressure level at r_i	
SRI	sound reduction index	
$\mathrm{SRI}_{\mathrm{av}}$	average transmission loss	
SRI_i	reduction index of material i	
SWL	sound power level	
t	time	T
T	period	T
T_0	resonance period	T
T	transmission coefficient	
T^2	power transmission coefficient	
$\mathrm{T}^2_{\mathrm{av}}$	average power transmission coefficient	
T^2_i	power transmission coefficient i	
$\mathcal{T}$	absolute temperature	Θ
$\Delta\mathcal{T}$	temperature difference	Θ
u	displacement in the x-direction	L
u_0	initial displacement	L
u_i	displacement at time i	L
$\mathbf{u}$	displacement vector	L
$\ddot{\mathbf{u}}$	second time derivative of $\mathbf{u}$	L T^{-2}
U	unity	
v	displacement in the y-direction	L
V	volume	L^3
dV	volume difference	L^3

V_0	initial volume	L^3
w	displacement in the z-direction	L
W	sound power	$\mathrm{M\ L^2\ T^{-3}}$
W_0	threshold of hearing power	$\mathrm{M\ L^2\ T^{-3}}$
x	x-coordinate	L
$x(t)$	excursion	L
$\dot{x}$	first time derivative of x	$\mathrm{L\ T^{-1}}$
$\ddot{x}$	second time derivative of x	$\mathrm{L\ T^{-2}}$
$\mathrm{d}x$	element length	L
Δx	step size	L
x_0	initial distance	L
x_i	distance at time i	L
x_s	distance to shadow zone	L
$X(x)$	function of x	
y	y-coordinate	L
$Y(y)$	function of y	
z	z-coordinate	L
Z	acoustic impedance	$\mathrm{M\ L^{-2}\ T^{-1}}$
$Z(z)$	function of z	
Z_c	relative characteristic impedance	
Z_i	acoustic impedance of medium i	$\mathrm{M\ L^{-2}\ T^{-1}}$
Z_s	relative surface impedance	
α	attenuation coefficient	$\mathrm{L^{-1}}$
α_v	viscous damping coefficient	$\mathrm{L^{-1}}$
α_θ	thermal damping coefficient	$\mathrm{L^{-1}}$
β	$4 \times 10^{-6}\ \mathrm{s\,K^{-1}}$	$\mathrm{T\ \Theta^{-1}}$
β_c	relative characteristic admittance	
γ	ratio of specific heats $\frac{C_\mathrm{p}}{C_\mathrm{v}}$	
γ_{ij}	shear strain on face i in direction j	
δ	additional path	L
Δ	volumetric strain	
ε	strain	
ε_i	strain in direction i	
η	viscosity	$\mathrm{M\ L^{-1}\ T^{-1}}$
θ	aperture	
θ_i	angle of incidence in medium i	

θ_i	angle of incidence	
θ_r	angle of reflection	
θ_t	angle of transmission	
κ	bulk modulus	$\mathrm{M\ L^{-1}\ T^{-2}}$
λ	Lamé's constant	$\mathrm{M\ L^{-1}\ T^{-2}}$
$\mathrm{\lambda}$	wavelength	L
$\mathrm{\lambda}'$	reduced wavelength	L
$\mathrm{\lambda}_i$	wavelength in medium *i*	L
ν	particle velocity	$\mathrm{L\ T^{-1}}$
ν_a	velocity of the audience	$\mathrm{L\ T^{-1}}$
ν_i	phase velocity of incident wave	$\mathrm{L\ T^{-1}}$
ν_r	phase velocity of reflected wave	$\mathrm{L\ T^{-1}}$
ν_s	velocity of sound source	$\mathrm{L\ T^{-1}}$
ν_t	phase velocity of transmitted wave	$\mathrm{L\ T^{-1}}$
ρ	density	$\mathrm{M\ L^{-3}}$
ρ_i	density of medium *i*	$\mathrm{M\ L^{-3}}$
σ	stress	$\mathrm{M\ L^{-1}\ T^{-2}}$
σ_i	stress in direction *i*	$\mathrm{M\ L^{-1}\ T^{-2}}$
$\mathrm{\sigma}$	mass per area	$\mathrm{M\ L^{-2}}$
τ_{ij}	shear stress on face *i* in direction *j*	$\mathrm{M\ L^{-1}\ T^{-2}}$
ϕ	scalar potential	$\mathrm{L^2}$
ϕ_i	phase of wave *i*	
ψ	vector potential	$\mathrm{L^2}$
ω	angular frequency	$\mathrm{T^{-1}}$
ω_0	resonance angular frequency	$\mathrm{T^{-1}}$
ω_i	angular frequency *i*	$\mathrm{T^{-1}}$
ω_{ijk}	angular eigenfrequency	$\mathrm{T^{-1}}$
$\nabla\phi$	grad $\phi = \left(\frac{\partial\phi}{\partial x}\ \frac{\partial\phi}{\partial y}\ \frac{\partial\phi}{\partial z}\right)^\mathsf{T}$	$\mathrm{L^{-1}}$
$\nabla^2\phi$	$\nabla\cdot\nabla\phi = \mathrm{div\ grad}\ \phi$	
	$= \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$ (Laplacian)	$\mathrm{L^{-2}}$
$\nabla\cdot\mathrm{\phi}$	div $\mathrm{\phi}$	$\mathrm{L^{-1}}$
$\nabla\times\mathrm{\phi}$	curl $\mathrm{\phi}$	$\mathrm{L^{-1}}$

D
Bibliography

1. Breyer RT. *Nonlinear Acoustics*. Woodbury: Acoustical Society of America 1997.

2. Fagan MJ, Postema M. *Introduction to stress and strain analysis*. Bergschenhoek: Postema 2007.

3. Fletcher NH, Rossing TD. *The Physics of Musical Instruments*. 2nd ed. New York: Springer-Verlag 1998.

4. Landau LD, Lifshitz EM. *Theory of Elasticity*. 3rd ed. Oxford: Butterworth-Heinemann 1986.

5. Kuttruff H. *Acoustics: An introduction*. Abingdon: Taylor & Francis 2007.

6. Pain HJ. *The Physics of Vibrations and Waves*. 3rd ed. Chichester: John Wiley & Sons 1983.

7. Pierce AD. *Acoustics: An Introduction to Its Physical Principles and Applications*. 1994 ed. Melville: Acoustical Society of America 1994.

8. Porges G. *Applied Acoustics*. Los Altos: Peninsula Publishing 1977.

9. Lord Rayleigh. *The Theory of Sound*. Vol. 1, 2nd ed. Dover Publications 1945.

10. Lord Rayleigh. *The Theory of Sound*. Vol. 2, 2nd ed. Dover Publications 1945.

11. Wright MCM, Ed. *Lecture Notes on the Mathematics of Acoustics*. London: Imperial College Press 2005.

E
Biographies

Keith Attenborough, B.Sc., Ph.D., CEng, FIOA, FASA, holds a B.Sc.(Hons) (UCL) in physics and a Ph.D. (Leeds) in applied science. He is Emeritus Professor in Engineering at The University of Hull and Research Professor at the Open University. He has carried out studies on the linear and nonlinear acoustical characteristics of porous surfaces, acoustic-to-seismic coupling, sound propagation through suspensions and emulsions, acoustical methods for surveying soils, outdoor sound propagation, blast noise reduction, sonic boom propagation, passive acoustic tracking and ranging, nonlinear acoustic detection of buried objects, and on noise levels in the UK.

Prof. Attenborough is Editor-in-Chief for Applied Acoustics (Elsevier), Associate Editor for Acta Acustica United with Acustica, and Associate Editor for the Journal of the Acoustical Society of America. He has published 100 papers in refereed journals and more than 140 papers in conference proceedings. In 1996, he was awarded the Institute of Acoustics Rayleigh medal.

Michiel Postema, M.Sc., Dr., MIEEE, MIOA, holds an M.Sc. (Utrecht) in geophysics and a doctorate (Twente) in physics of fluids. He is lecturer in Engineering at The University of Hull. His particular expertise lies in analysing medical microbubble behaviour under ultrasound insonification and in high-speed photography. He also explores non-medical applications of bubbles and droplets in sound fields.

Dr. Postema is a member of the editorial board of Applied Acoustics (Elsevier). He has written 30 first-author publications.

Keith Attenborough

Michiel Postema