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Towards model-based approaches for musical instruments making : validation of the model of a Spanish guitar soundboard and characterization features proposal

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Abstract

Nowadays, the virtual prototyping method is widely used for industrial applications and can lead to a powerful tool for musical instruments making and conservation. Nevertheless, physics-based models of musical instruments are barely developed for this purpose and the confrontation between model predictions and experiments have been the focus of very few researches. The objective of this paper is to highlight the predictive capability of physics-based models in dynamic domain, even in presence of variable by nature material and climatic conditions. For this purpose, a finite element model of the soundboard of a Spanish guitar is developed for model validation purposes. The simulated modal bases are compared with experimental ones from a previous study. Screening and stochastic analyses are performed to rank which are, among material and climatic parameters, the most influential ones on the dynamics of guitar soundboard. Moreover, uncertainties are taken into account to evaluate the dispersion of the response for a given design, and simulations are validated facing experimental data. It is shown that specific elastic parameters of the wood (in longitudinal and radial directions and longitudinal-radial plane) of the top plate are mainly

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influential with regard to the dynamics of the soundboard, and the relative humidity changes have a non negligible impact. Moreover, test-model correlations have shown that a nominal model with average material parameters is able to predict the dynamical behaviour of a real braced soundboard with an average error on the first eight eigenfrequencies lower than 4%. In addition, when uncertainties are taken into account, the model is able to predict every experimental data. Finally, dynamic features like CFDAC and Fuzzy-FRFs are proposed in an innovative way in this application domain.

Keywords: Model validation, Uncertainty quantification, Modal analysis, Screening analysis, Musical acoustics, Guitar soundboard

Highlights

- Model validation of musical instrument part even in presence of strong uncertainties
- Fuzzy-FRF method illustrated to musical acoustics domain
- Screening analysis of influential material and climatic parameters on guitar soundboard dynamics for fixed geometry
- Similar impact of $\frac{E_L}{\rho}$, $\frac{E_R}{\rho}$, $\frac{G_{LR}}{\rho}$ and relative humidity on eigenfrequencies of guitar soundboard in free conditions
- Predominance of $\frac{E_L}{\rho}$, followed by $\frac{E_R}{\rho}$, $\frac{G_{LR}}{\rho}$ on eigenmode shapes

1. Introduction

The guitar, electric or acoustic, is one of the most popular musical instrument all over the world. It has been used in almost every musical style since the XX^{th} century and is often the first choice for musical learning. The guitar clusters a wide range of prices and qualities. One main family is considered here, the acoustic guitars. The acoustic guitars transfer the vibratory energy of the strings partly into an acoustic energy through their sounding box and the radiation of their flat parts, especially soundboards.

9 It is usually considered that the soundboard affects mainly the acoustic beha-
 10 viour of the guitars [1], [2]. The shape and thickness of the soundboard, as well
 11 as the characteristics of the braces (number, orientation, shapes) affect the me-
 12 chanical behaviour of the guitar in both the static and dynamic domain. In the
 13 static domain, the top plate has to sustain the strings tension, and the mecha-
 14 nical characteristics of both plate and braces affect the stiffness of the system.
 15 In the dynamical domain, the eigenmodes of the soundboard are also strongly
 16 linked with the mechanical behaviour of the parts. These different design and
 17 material choices, that depend on the instrument maker, impact the resonance
 18 modes and acoustical features of the instrument. However, these parameters
 19 are barely quantified and studied from an objective point of view, since the
 20 construction process is based mainly on tradition and empiricism.
 21 Generally, mechanically based works study acoustic guitars by experimental,
 22 analytical and numerical means, mainly focusing on modal parameters to esta-
 23 blish comparisons.

24

25 The experimental means have been used for decades to observe the resonance
 26 modes of the guitar soundboards, alone, or when coupled with the sides and the
 27 remaining parts. As an example, it has been used for the objective characte-
 28 risation of different guitar families, related to their soundboard braces pattern
 29 [3]. The experimental means are also useful for different purposes, such as the
 30 study of the global dynamics of the guitar and the radiated sound produced,
 31 dealing with a macro response of the instrument [4, 5]. This type of response
 32 contains a high amount of information, and it becomes hard, considering the
 33 total coupling of all the elements, to attribute the role of each part of the guitar
 34 (and the properties of its components) with respect to the observed response.

35 Specific numerical models related to guitar have been developped since 70's
 36 [6], up to a detailed model in [7]. More recently, the evolution of the compu-
 37 tational power enabled increasingly sophisticated models such as the complete
 38 channel of the production of sound, from the plucked guitar string to the radited
 39 sound [8]. One of the most complex model, mixing complete structure and fluid

40 structure interactions has been developed in [9, 10] and enabled the computation
 41 of the radiated sound around the instrument. In [11], a detailed vibroacoustic
 42 model of a Portuguese guitar has been compared, in a deterministic way, with a
 43 real instrument. Usually, models of soundboards include braces, that have also
 44 been studied separately in [12]. In addition to the computation of the modal
 45 basis of guitar soundboards, the models have been used to compute the bridge
 46 admittance of the guitar, in a similar way than the one considered for violins,
 47 and thus provided results that could be compared with easily measurable fea-
 48 tures of real guitars [13].

49
 50 So, historically, models were used as *a posteriori*, but, nowadays, increasin-
 51 gly for the prediction of complex structures. Physics-based modelling is used to
 52 predict the mechanical behaviour of complex virtual systems in the first steps
 53 of prototyping, and to quantify the variability of its behaviour, submitted to
 54 numerous unavoidable sources of uncertainties. Nevertheless, as the utilisation
 55 of the models concern more sophisticated structures, the models are still often
 56 unable to correctly predict their behaviour.

57
 58 Thus, in order to avoid issues inherent to model prediction errors, the verifica-
 59 tion and validation (V&V) process has been developed to assess the viability of a
 60 model, and its framework is detailed in [14] and the book [15]. The V&V method
 61 needs a large number of simulations to predict as much cases as possible, that
 62 are parts of the uncertainty domains. Its application on the vibratory behaviour
 63 of structures has been performed in [16, 17, 18]. The validation process aims at
 64 ensuring the reliability of the model when predicting the behaviour of a system.
 65 The predictions of the model (that can consist of numerous features) are com-
 66 pared to experimental data. The model is validated when the closeness between
 67 numerical and experimental results is below a tolerance level. The experiments
 68 make perfect sense when the material used exhibits a high variability, like the
 69 wood. Therefore, a higher number of experiments leads to an enhancement of
 70 the model reliability, through the validation criterion.

71

72 This paper aims at highlighting in an innovative way for this application
 73 domain the potential of numerical models and their relevance in regard with
 74 musical acoustic applications. Indeed, in the framework of the utilisation of nu-
 75 merical models for the musical instrument making, the reliability of the models
 76 have to be at first time assessed and quantified, which is the main objective
 77 of this paper. In this study, a model validation of a musical instrument part
 78 is proposed. Experiments have been carried out on five similar Spanish gui-
 79 tar soundboards at 15 different steps [19, 20]. Usually these steps performed
 80 by instrument makers are led by empiricism and traditions, and barely quanti-
 81 fied to highlight objective assessments. In the case of wooden parts of musical
 82 instruments, the variability of such material is high and inevitable. Thus, this
 83 aleatory uncertainty is taken into account with stochastic approaches. Sensiti-
 84 vity analysis have to be performed to evaluate the relative influence of material
 85 parameters. Screened material parameters will be implemented with uncertainty
 86 model. The climatic conditions are also a source of uncertainty that is modelled
 87 with probabilistic approaches.

88

89 In the next section, the model development and methods used, as well as the
 90 parameters of the material behaviour's law are given. The results give dynamic
 91 data, sensitivity analysis results and dispersion of the computed eigenfrequencies
 92 with comparison with experimental ones. The conclusion give the main results
 93 and advances of this paper.

94 **2. Model and methods**

95 *2.1. Experimental modal analysis of Spanish guitar soundboard*

96

97

98 The modal analysis of the guitar soundboard is the same as the one perfor-
 99 med in [19], the material and methods is based on this paper. The numerical

100 model and experimental data used correspond to the construction stage № 15 of
 101 the paper [19], where all the braces and reinforcements are glued on the sound-
 102 board, and the rosette has been glued in its cavity. So, this stage corresponds to
 103 the state of the soundboard before being glued on the sides of the guitar. The five
 104 soundboards tested are labelled Sb_{01} , Sb_{02} , Sb_{03} , Sb_{04} , Sb_{05} , and their masses
 105 are equal to 164, 166, 154, 178 and 170 g, respectively. The average mass of
 106 the soundboards is equal to 166.4 g. Only the out-of-plane motion of the sound-
 107 boards has been considered. The guitar soundboards have been tested under free
 108 boundary conditions. A unidirectional accelerometer (B&K 4518-003) has been
 109 used and glued near the future bridge location (at the point 69 on the figure
 110 1 (a)). An impact hammer (B&K 8204) has been used to excite the structure
 111 on the 99 points also displayed in the same figure. The test frequency ranges
 112 between 0 and 800 Hz, and the corresponding resolution is equal to 0.25 Hz.
 113 The vibration signals were measured and recorded as time series and processed
 114 into inertance FRF data. Both the applied excitation and the measured response
 115 were perpendicular to the soundboard. Signals were averaged two times for each
 116 measurement point. A modal analysis of the inertances has been performed to
 117 evaluate the modal basis of each soundboard. The modal analysis has led to
 118 the evaluation of a modal basis with eigenfrequencies and corresponding modal
 119 dampings for each modes. The experimental results obtained with this study,
 120 [19], for the first eight modes identified for the five soundboards are displayed
 121 in the table 3.

122 *2.2. Computer aided design and meshing of the soundboard*

123 The numerical model has been developed using the finite element method ba-
 124 sed on a Computer Aided Design (CAD). The CAD software used is SOLIDWORKS®.
 125 The software used for the pre-processing is PATRAN® and the solver is NASTRAN®.
 126 The figure 1 (b) represents the CAD of the soundboard and the nomenclature.
 127 The figure 2 (a) displays the finite element model mesh. The CAD is meshed
 128 using tetrahedral elements with quadratic interpolation. The interfaces between
 129 the soundboard and the different parts are considered as perfect and are mo-

130 delled with coincident nodes and equivalent faces. The finite element model
 131 contains 55000 elements and 104000 nodes and free boundary conditions are
 132 applied.

133 2.3. Material orientation

134 The material used is spruce, *Picea abies*, for all the braces, bars, patches and
 135 the soundboard. The material properties are taken from [21]. The values are
 136 given in the table 1. The parts are oriented according to the figure 2 (a). The
 137 specific elastic parameters are sampled. The density d_0 is also sampled according
 138 to variations evaluated in [21]. The temperature and relative humidity are also
 139 sampled as equi-probalistic approaches bounded between 25 and 85 % for RH
 140 and 15 to 35 °C for T.

141 As a second step, the moisture content is calculated from the RH and T
 142 values sampled, according to [22], given in the eq. 1 :

$$MC = 10 + 0.16 \times (RH - 50) - 0.03 \times (T - 21) \quad (1)$$

143 The density as a function of MC, ρ_{MC} , is then calculated from the value of
 144 MC, according to [23], given in the eq. 2 :

$$\rho_{MC} = \rho_0 \times (1 + 0.01 \times (MC - 10)) \quad (2)$$

145 The elastic constants values of E_L , E_R and G_{LR} depend on the relative hu-
 146 midity and, in a lesser degree, on the temperature. In order to implement this
 147 dependence, the values of the elastic properties are implemented as a function
 148 of RH and T, laws are taken from [24], given in the eq. 3 :

$$\begin{aligned} E_{LRHT} &= E_{L\rho}(1 - 0.0015 \times (RH - 50) - 0.0008 \times (T - 21)) \\ E_{RRHT} &= E_{R\rho}(1 - 0.005 \times (RH - 50) - 0.0025 \times (T - 21)) \\ G_{LRHT} &= G_{LR\rho}(1 - 0.007 \times (RH - 50)) \end{aligned} \quad (3)$$

149 When a set of elastic constants as a function of density ($\frac{E_i}{\rho}$ as an example) is
 150 sampled, it is multiplied by a sampled value of density ρ_i . The value of E_i is

then modified following eq. 3, with sampled values of RH and T. The sampled density is then also expressed as a function of the moisture content (eq. 2, given by 1), which gives ρ_{MC_i} . So, at the end of the sampling process, E_i , G_{ij} and ρ_{MC_i} are implemented in the numerical model, for the parts made of spruce.

Four sets of material parameters are considered dedicated to the soundboard, the braces, the bridge patch and the sound-hole reinforcement respectively. The material used for the parts is implemented to match the orientation of the wood in the reality. The wood samples are quarter-sawn and the dimensions are small enough to consider an orthotropic definition. Considering the coordinate frame represented in the figure 2 (b), R corresponds to the radial direction, L to the longitudinal direction, T to the tangential direction. A modal basis is computed with the nominal values given in the table 1. With these values, the mass of the model is estimated to 168 g and the model average mass of the real soundboards is equal to 166.4 g.

2.4. Model-based modal analysis

A modal analysis is computed with the numerical model. Starting at 1 Hz, the first 50 modes are computed, which lead to a bandwidth for the modes extraction between 1 and 1000 Hz. The frequency response functions (FRF) in acceleration, velocity and displacement (inertances, mobility and admittance), are computed on the 99 points of the experimental test, in the out-of-plane direction, Y. The driving force value is equal to 1 N, applied in the same direction on the point labelled 69 in the figure 1 (a). The computation of the FRF is made on the numerical nodes close to each experimental points. In the considered frequency bandwidth, relative humidity and strain levels it is hypothesised that the material exhibits a linear elastic behaviour. Thus the damping is applied a posteriori as a modal damping whose value corresponds to the mean modal damping measured on guitar soundboards in the considered bandwidth, with $\xi = 1.15\%$. This value is taken from the experimental part and given in the table 3.

181 2.5. Screening analysis

182 Sensitivity analyses are computed to evaluate the impact of the variability
 183 of inputs of the model with regard to eigenfrequencies and eigenvectors outputs
 184 $Y(X_i)$ matched using MAC criterion [25]. Finite difference analysis is used at
 185 first to roughly screen the material and climatic parameters as a function of their
 186 impact. For this purpose, a variation δX_i equal to 1 % is applied on each input
 187 parameter X_i one at a time. The sensitivity indicator is given by the eq. 4 [26] :

$$\phi_i = \frac{\frac{Y(X_1, \dots, X_i + \Delta X_i, \dots, X_n) - Y(X_1, \dots, X_i, \dots, X_n)}{Y(X_1, \dots, X_i, \dots, X_n)}}{\frac{\Delta X_i}{X_i}} \quad (4)$$

188 The morris screening analysis [27] is used to explore the input domain. The
 189 linear and coupling effects of the X_i are evaluated. For a number of n_p para-
 190 meters X_i in a n_p dimensions domain Ω , the domain is sampled in l levels. The
 191 values of the domain Ω are defined in order to depict all the values that the
 192 parameters can attain. The elementary effect of a parameter X_i in a sample X^j
 193 of the space is given by :

$$E_i^j = \frac{f(X^j \pm e_i) - f(X^j)}{\pm \Delta} \quad (5)$$

194 With e_i a unit vector and Δ a value taken in $\left\{ \frac{1}{p-1}, \dots, 1 - \frac{1}{p-1} \right\}$. The n_t
 195 trajectories will define the number of computations given by $(n_p + 1) \times (n_t) + 1$.
 196 For this computation, the number of levels is equal to 6, n_t is equal to 20 and
 197 n_p is equal to 12 after finite difference analysis first screening. This lead to 261
 198 computations.

199 2.6. Stochastic analysis

200 The uncertainty quantification is performed using Monte-Carlo sampling meth-
 201 thod. For the uncertainty quantification, 750 modal bases are computed. The
 202 comparison of the matched eigenmodes between nominal model results and the
 203 ones obtained with modified input parameters is performed. Material and clima-
 204 tic parameters are defined as equiprobabilistic distributions between upper and
 205 lower values given in the table 1. The matched eigenfrequencies are given as box

and whiskers plots and the experimental data are compared with the computed distributions. The error between numerical eigenfrequencies and averaged measured eigenfrequencies is compared with the relative standard deviation (RSD) of each numerical eigenfrequencies.

2.7. CFDAC

A frequency domain assurance criterion is used, based on the experimental and computed FRFs. This method can be regarded as the equivalent of the Modal Assurance Criterion in the FRF domain. The FDAC [28], adapted as Complex-FDAC, is expressed in the eq. 6 [29] :

$$CFDAC_{fg} = \frac{\left[\sum_{i=1}^N \sum_{j=1}^N h_{ij}(\omega_f) h_{ij}^d(\omega_g) \right]^2}{\left[\sum_{i=1}^N \sum_{j=1}^N h_{ij}(\omega_f) h_{ij}(\omega_f) \right] \left[\sum_{i=1}^N \sum_{j=1}^N h_{ij}^d(\omega_g) h_{ij}^d(\omega_g) \right]} \quad (6)$$

In this equation, i and j correspond to the excitation and measure FRF, h_{ij} refers to pristine state FRFs, h_{ij}^d refers to altered FRFs. f and g refer to each pair of spectral lines compared from the two sets of mobility functions. N is the number of sampled points in the specimen. CFDAC results in a complex two-dimensional array of dimension $N \times N$. Real and imaginary parts of the CFDAC are absolute-valued even when used for numeric computations. The CFDAC is performed between real soundboards and model nominal cases to observe the discrepancies in the behaviour of the soundboards.

2.8. Fuzzy-FRF analysis

The Fuzzy frequency response function (FUZZY-FRF) is computed using the 750 FRF that are computed during the uncertainty quantification process. For each case, admittance is computed. The FRF used are located at the same point than experimental ones. The 750 FRF are gathered in the same plot, and space of the plot is discretised in 2500×2500 subspaces. In each subspace, the number of lines passing through the subspace is evaluated. A color scale is applied to represent the amount of lines per subspace. This method is taken from [30, 31].

231 2.9. Modal overlap factor

232 The modal overlap factors (*MOF*) by third octave bands are calculated
233 using the eq 7

$$MOF = M_d \times \eta \times F_c \quad (7)$$

234 With M_d the modal density, given by the ratio between the number of modes
235 per third octave bands, for a central frequency F_c . η is the loss factor of the
236 system in the considered bandwidth and is equal to 2.3 %, two times the average
237 value of the modal damping, given in the table 3. Only one domain is considered,
238 the low-frequencies where the MOF value is comprised between 0 and 30 %.

239 3. Results

240 In this section the experimental and numerical results are given in different
241 subsections.

242 3.1. Experimental and numerical deformed shapes

243 The measures on the five soundboards have led to five experimental modal
244 bases. The values of the first eight eigenmodes frequencies and corresponding
245 modal dampings are given in the table 3. The first eight numerical eigenmodes
246 shapes and nominal eigenfrequencies are given in the figure 4. The eigenmodes
247 shape consist in torsion modes in the LR plane, flexure modes in the L and R
248 directions of the soundboard and mixed torsion and flexure modes. The mo-
249 dal overlap factor values are given in the table 2. Below 500 Hz, the modal
250 overlap factor is comprised between 2 and 28 % which corresponds to the low
251 frequency domain, where, mainly, the modal analysis is relevant. Above 500 Hz
252 the mid-frequencies domain is reached, which suggests an increase of discrepan-
253 cies between experimental and numerical results.

254 3.2. Co-located nominal FRF comparison

255 The co-located FRF in acceleration of the nominal model and the experi-
256 mental soundboards are given in the figure 3. It is shown that the level of the
257 dynamical responses of the experimental soundboards are close to the compu-
258 ted one up, and it is not possible to differentiate them based on the acceleration
259 level.

260 3.3. Comparison between numerical and experimental eigenfrequencies, deter- 261 ministic approach

262 The eigenfrequencies, as well as their mean and standard deviation, for the
263 first eight modes are given in the table 3. The average relative standard deviation
264 (RSD) of the experimental eigenfrequencies is equal to 4.3 % and the average
265 modal damping is equal to 1.15 % on the modes considered. The relative error
266 between nominal model eigenfrequencies and experimental ones is comprised
267 between -6.3 and 7.2 %.

268 3.4. Spectral correlation of numerical and experimental guitar soundboard

269 The figure 5 shows the frequency domain assurance criterion from 0 to 800
270 Hz for each case. For the first four soundboard, up to 600 Hz, the CFDACs
271 show a good correlation between experimental and numerical databases, which
272 corresponds to the domain of low-frequencies, as shown in the table 2. Moreo-
273 ver, the CFDAC matrix complex correlation is almost diagonal, which indicates
274 a good correspondence between stiffness and mass of the model and the real
275 soundboards.

276 3.5. Stochastic analysis results

277 The results of the uncertainty quantification study are given for the first eight
278 modes as box and whiskers plots on the figure 6 and mean, SD and RSD in the
279 table 4. In the figure 6, the in boxes vertical lines correspond to the median,
280 the lower and upper limits of the boxes correspond to the lower and upper
281 first quartile (25 percentiles) respectively and the limits of the left and right

whiskers correspond to 9 and 91 percentiles respectively. It is shown that all the experimental eigenfrequencies are comprised between the 9 and 91 percentiles. Moreover, most of the experimental eigenfrequencies are comprised in the second quartile (50 percentiles).

In the table 4, it is shown that the RSD of the first eight numerical eigenfrequencies is rather diffuse and comprised between 5.8 and 8.2 %, with an average value equal to 6.9 %.

3.6. Fuzzy-FRF of a free edges soundboard of Spanish guitar soundboard

The figure 7 (a) shows the Fuzzy-FRF evaluated with the database used for the uncertainty quantification, which corresponds to the co-located FRF in displacement of the 750 computations. This figure gives a display of the variability distribution of the FRF of the studied structures when undergoing material and climatic variations. The figure 7 (a) displays the number of FRF that are comprised in each discretised subspace of the plot. Each axis is divided 2500 times, which means that the figure displays 6.25×10^6 discretised squares. So, the sampling is equal to 0.32 Hz for the frequency axis and from 5.10^{-3} to $40 \text{ m.s}^{-2}.N^{-1}$ for the acceleration axis. The maximum number of FRF inside a square is equal to 100, at the lowest frequency where the dispersion is the lowest. This value is used as the maximum value for the color fringe and the lowest value 0 corresponds to the case where no FRF crosses a discretised square. The dispersion of the FRF increases above 100 Hz, and becomes rather diffuse above this value. The figure 7 (b) gives the min and max values of the FRF, represented as dashed black lines. The experimental FRF are also displayed in the figure 7 (b), and are generally comprised inside the area of presence probability, where the number of occurrences is higher than 30.

3.7. Screening analyses results

The ranking of the elementary effects of the material and climatic parameters is given for each modes in the figure 4. Depending on the eigenmode shape, the influential material parameters ranking varies, and is in accordance with the

finite differences sensitivity matrix given in the figure 8 (a). As a primary result of the sensitivity matrix, it is shown that the specific elastic parameters of the soundboard are the most influential. The specific elastic parameters, when their complete range is considered are more influential on the eigenfrequencies than the densities. The relative humidity impact on the eigenfrequencies is almost constant for each mode and corresponds to values comprised between 14 and 18 % of the total of the elementary effects.

The ranking of the material and climatic parameters for global dynamical behaviour of the first 20 modes is given in the figure 8 (a) for the eigenfrequencies and (b) for the eigenvectors. It is shown that the influence of each parameter varies according to the dynamical feature observed. Generally, in free conditions, the specific rigidity in the longitudinal direction of the soundboard plate is the most influential parameter (22 % of the total), followed by the specific shear rigidity in LR plane (19 %) and the specific radial rigidity (15 %). The impact of the density of the soundboard is similar to the impact of the density of the braces (9 and 8 %, respectively). The remaining specific elastic parameters of the braces are less influential, and correspond to 7 % for the braces L rigidity and 3 % for the LR shear elasticity of the braces. The RH has a strong impact on the eigenfrequencies, corresponding to 17 % of the total elementary effects. The effect of the RH is smaller on the eigenvectors; as its effect is global on every elastic and density parameter, it affects in a smaller way the eigenmode shapes. It has to be pointed out that, considering eigenvectors, the longitudinal specific modulus of the soundboard is mainly influential, up to two times the specific rigidities in R direction or LR plane.

4. Discussion

The results obtained lead to multiple discussions. First of all, the deterministic comparison has highlighted the good predictive capability of the model concerning the dynamical behaviour of Spanish guitar soundboards. It has been shown that the relative error between a nominal model (whose material and

climatic parameters are taken from the literature) and the average values of
 eigenfrequencies is close to 4%, which is close to the relative standard deviation
 of the experimental eigenfrequencies. When material and climatic uncertainties
 are taken into account, the model is able to predict every experimental frequen-
 cies, which are, in addition, comprised in the second quartile (50 percentiles)
 of the computed ones. The comparison of deterministic numerical and experi-
 mental FRF have shown that the discrepancies increase above 500 Hz which
 is close to the limit of the low frequencies domain given by the modal overlap
 factor. This result is in accordance with dynamics theory, and highlights the
 limits of a modal point of view for this type of study. The CFDAC compari-
 son between experimental and numerical FRF is innovative in this application
 domain, enables a global comparison and is more adapted in mid-frequencies
 range. The CFDAC correlation criterion is a relevant quantified global indicator
 of closeness between numerical and experimental data. The Fuzzy-FRF feature
 proposed here shows the dispersion of the model response for a fixed design.
 Thus, a guitar soundboard built with the same geometry and the same wood
 species will exhibit a dynamical behaviour that may be included in the Fuzz-
 FRF prediction. As shown in this study, experimental FRF were comprised in
 the fuzzy-FRF high plausibility area and inside the upper and lower bounds of
 the stochastic simulations, as shown in the figure 7 (b). This post-processing of
 the FRF in the case of an uncertainty quantification can be a useful tool for the
 decision-support in musical instruments making, and to confirm the relevance
 of a design change over the material and climatic variability impact.
 Based on the fact that the model was able to correctly predict dynamical beha-
 viour, it has been shown that the most influential parameters with regard to the
 eigenfrequencies and eigenvectors of a free conditions soundboard were mainly
 specific elastic parameters $\frac{E_L}{\rho}$, $\frac{E_R}{\rho}$ and $\frac{G_{LR}}{\rho}$ of the top plate soundboard, fol-
 lowed by the relative humidity and the density of the plate and braces. These
 results are similar to the one obtained considering a violin in [32]. It has been
 shown in [33] and [21] that studies on tonewood were most of the time focused
 on $\frac{E_L}{\rho}$, which is also correlated with high “quality” wood. Considering these new

371 results, it is clear that, as $\frac{E_R}{\rho}$ and $\frac{G_{LR}}{\rho}$ play an important role, the selection
 372 criteria and studies should also focus on these parameters.

373

374 Conclusion

375 In this paper, the comparison between experimental and numerical dynamical
 376 data have shown the capability of physic-based models to predict complex
 377 assemblies responses. It has been shown that model accuracy was maintained
 378 even in the presence of strong material and climatic uncertainties. This paper
 379 proposes new ways for the characterisation of musical instruments : the un-
 380 certainty quantification for a given geometry, the CFDAC between model and
 381 real instruments and the Fuzzy-FRF for the post processing of the study of the
 382 uncertainties effects on the dynamical behaviour of musical instruments. This
 383 is a first and innovative step in the validation process of physics-based models
 384 of musical instruments, which, associated with the different dynamical features
 385 proposed for this domain can be used for design and restoration purposes of
 386 musical instruments. Moreover, the results obtained have questioned the cur-
 387 rent selection criterion for spruce tonewood, and shown that specific moduli
 388 other than $\frac{E_L}{\rho}$ were also significant on the dynamics of the guitar soundboard
 389 in free conditions, which is its main boundary conditions during making steps.
 390 These results are relevant in the low frequency domain, where modal analysis is
 391 effective. In order to manage with higher domains, such as mid-frequencies and
 392 high frequencies domains, large frequency bandwidth descriptors [34], such as
 393 mean-values approaches should be considered, and have already proved useful
 394 for acoustic guitars [13], violins [35] and composite plates [36]. Modal approaches
 395 and medium and high frequencies methods would provide, through model vali-
 396 dation process, reliable large band datasets of musical instruments behaviours,
 397 even in presence of strong uncertainties.

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FIGURE 1: (a) location of the experimental measurement points and FRF synthesis nodes used in [19] experiments; (b), Computer Aided Design and nomenclature of Spanish guitar soundboard.

FIGURE 2: (a) finite element model of the guitar soundboard; (b) orientation of the material, L : longitudinal, R : radial, T : tangential.

FIGURE 3: Inertances of the colocated measured and excitation point for both model and experimental FRF.

TABLE 1: Material properties of spruce implemented in the numerical model; italic from [21] at $MC = 10\%$. Remaining values from [37] and [38]. Loss factors values are given but have not been implemented for undamped modal analysis.

Parameter	Mean value	Min. value	Max. value
$\frac{E_L}{\rho}$ (MPa g ⁻¹ cm ⁻³)	<i>29000</i>	<i>20590</i>	<i>35380</i>
$\frac{E_R}{\rho}$ (MPa g ⁻¹ cm ⁻³)	<i>2280</i>	<i>1460</i>	<i>3810</i>
$\frac{E_T}{\rho}$ (MPa g ⁻¹ cm ⁻³)	1480	1300	1660
ν_{LR} (-)	0.37	-	-
ν_{RT} (-)	0.48	-	-
ν_{TL} (-)	0.02	-	-
η_L (%)	<i>0.73</i>	<i>0.09</i>	<i>0.12</i>
η_R (%)	<i>1.7</i>	<i>0.17</i>	<i>0.1</i>
η_{LR} (%)	<i>1.2</i>	<i>0.2</i>	<i>0.17</i>
$\frac{G_{LR}}{\rho}$ (MPa g ⁻¹ cm ⁻³)	<i>1850</i>	<i>1295</i>	<i>2442</i>
$\frac{G_{RT}}{\rho}$ (MPa g ⁻¹ cm ⁻³)	<i>100</i>	<i>74</i>	<i>150</i>
$\frac{G_{TL}}{\rho}$ (MPa g ⁻¹ cm ⁻³)	<i>1910</i>	<i>1070</i>	<i>2750</i>
Density (g cm ⁻³)	<i>0.44</i>	<i>0.39</i>	<i>0.51</i>
Relative humidity (%)	<i>50</i>	<i>20</i>	<i>85</i>
Temperature (°C)	<i>21</i>	<i>15</i>	<i>35</i>

TABLE 2: Values of modal overlap factor for corresponding third octaves bands.

Third octave band	M.O.F. (%)	Domain
40	8	L.F.
50	2	L.F.
63	2	L.F.
80	14	L.F.
100	4	L.F.
125	10	L.F.
160	7	L.F.
200	9	L.F.
250	18	L.F.
315	28	L.F.
400	22	L.F.
500	34	M.F.
630	52	M.F.

FIGURE 4: Left side, frequencies and deformed shapes of the first eight computed modes of the spanish guitar soundboard, in free-free conditions. The red color represents the highest eigenvectors values. Right side, elementary effect ranking for each parameter using Morris sensitivity Method.

TABLE 3: Results for the first eight modes measured and computed. Mean, standard deviation (SD) and relative standard deviation (RSD) of the measured eigenfrequencies, as well as the average modal damping are given.

Mode	Num. (Hz)	Sb_{01} (Hz)	Sb_{02} (Hz)	Sb_{03} (Hz)	Sb_{04} (Hz)	Sb_{05} (Hz)	Av. exp (Hz)	SD. exp (Hz)	RSD. exp (%)	Num. vs Exp. error (%)	Average ξ (%)
1	42.7	42.4	45.2	37	38.2	37.8	40.1	2.9	7.3	6.5	0.6
2	66.7	66.3	72.3	62	64.5	56.9	64.4	4.0	6.1	3.6	0.7
3	82.3	77.2	77.8	74.4	77	75.8	76.4	1.1	1.4	7.2	0.7
4	114.1	114	120.9	106.2	110.5	117	113.7	4.3	3.8	0.3	1.1
5	131.6	133	134	121	123	140	130.2	6.6	5.0	1.1	1.2
6	176.4	185	192	170	-	199	186.5	9.0	4.8	-5.7	1.3
7	206.6	208	207	202	-	214	207.8	3.3	1.6	-0.7	1.7
8	235.6	242	245	253	237	275	250.4	10.9	4.3	-6.3	2.0
Mean	-	-	-	-	-	-	-	5.2	4.3	3.9 (abs.values)	1.15

TABLE 4: Mean, SD and RSD of the matched numerical eigenfrequencies.

Mode	Mean (Hz)	SD (Hz)	RSD (%)
1	42	2.8	6.7
2	66.6	4.9	7.3
3	80.7	4.7	5.8
4	112.8	7.7	6.8
5	133.1	10.2	7.7
6	177.1	14.6	8.2
7	201.4	11.8	5.9
8	233.6	6.3	7.0
Mean	-	-	6.9

FIGURE 5: CFDAC matrix (magnitude) between the measured (ordonate) and computed with the nominal model (abscissa) FRF for each soundboard.

FIGURE 6: Box and whiskers plots of the values of the first eight matched eigenfrequencies computed by the numerical model of soundboard, comparison with experimental eigenfrequencies values. The vertical lines of the boxes correspond, from left to right, to the first quartile (25th percentiles) the median and the third quartile. The limits of the left and right whiskers correspond to the 9th and 91th percentiles, respectively.

FIGURE 7: (a) fuzzy-FRF of the co-located admittances from the uncertainty quantification computations, the total number of runs is equal to 750; (b) statistical treatment of the FRFs, dashed black line, upper and lower limits of the stochastic computations.

FIGURE 8: Finite difference sensitivity matrix for the first 30 modes (a); relative elementary effects of the material and climatic parameters in regard with eigenfrequencies (b) and eigenvectors (c).

(a)

(b) Eigenfrequencies

(a) Eigenvectors