Price dynamics for Net electric energy metering on a distribution network: Ising's model approach

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Abstract— **The increased integration of decentralized renewable energies in distribution networks has brought different challenges in the analysis and planning of electrical networks. The network manager must have appropriate tools allowing rapid decision-making and fair invoicing of consumers or remuneration of producers. In this article, we introduce a novel approach based on the famous Ising's model in order to model the variation in the price of reactive energy circulating on a electric distribution network according to local interactions between subscribers but also according to the general condition of the network. A simulation on the CIGRE Test network allowed us to quantify the benefit that a network manager would have using the proposed billing strategy.**

Keywords: *Price dynamics, Energy, Ising model; Distribution network*

Highlight:

- Net electric energy metering
- Reactive power control
- Voltage regulation

I. Introduction

The increase in the integration of renewable energies into electricity grids is bringing a new dynamic to their operation. Subscribers who were until now passive in regulating the operation of the network have now become active and can supply or consume energy on the network [1,2].

A noteworthy effect of the integration of renewable energies is the significant increase in the exchange of reactive energy on the networks, thus pushing operators (DSO: Distribution System Operator) to review their billing models on the exchange of this energy. [3,4]. Maintaining the voltage on the nodes depends on the reactive energy compensation made by the active subscriber (agent), the DSO can pay or charge the agent depending on whether they supply or consumes reactive energy on the network.

In [5 - 7] we find several approaches to model the price of energy, and we notice that these studies are based either on a singular vision of the agent, or on a global vision of the network.

Several studies have studied economic phenomena in the light of statistical physics by building models that reveal the effects of local interactions of agents on the global structure of the market [8 - 11]. Cont and Bouchaud (CB) have created one of the simplest financial market models providing interesting predictions on the microscopic origin of macroscopic phenomena such as crashes or bubbles [12]. The structure of their model is close to a percolation model [13,14]. A reformulation of the CB model can be found in [13] including the notion of super-spin in order to study the evolution over time of the general structure of the market. While in [15] the CB model is modified to integrate a hierarchy of agents.

Another approach using statistical physics is introduced by Bornholdt [16,17]. Here he uses the well-known Ising model [18 - 20] in order to model the interaction between the closest agents and the action of the general structure of the market on a single agent. The Bornholdt model (BI: Bornholdt - Ising) validly predicted several stylized facts such as volatility or the distribution of relative price change [9]. Sieczka and Holyst modified Bornholdt's model in order to refine its predictions by changing the agent's dynamics, taking into account the agent's opinion [21].

We have found in the literature only a single application of Ising's model on electrical networks [23] which addresses the notion of flexibility.

In this article we reconcile these two previous visions by establishing a price model considering the interactions between agents that must lead to a global configuration on the network, we base ourselves on a model well known in statistical physics, namely the Ising's model.

In section 2 of this article, we will present the electrical network model, it will be followed in section 3 by the presentation of the active agent' operation and the billing mechanism. Section 4 will give the application of the BI model on the power grid. Then will come the results of the simulations in section 5 before concluding.

II. NETWORK STRUCTURE

We consider, an electrical distribution network having N-nodes in the form of an undirected graph $\mathcal{G} = (\mathcal{V}_N, \mathcal{L})$ where $\mathcal{V}_N$ is the set of nodes and $\mathcal{L}$ the set of lines between these nodes. The nodes are denominated by the index $i = 0, 1, \ldots, N-1$. If there is a line between nodes i and j, we will denote by $(i,j) \in \mathcal{L}$. The lines are considered for our study to have very close admittances so that we can consider that there is a basis such that $y_{ij} \approx 1\,pu, \forall (i,j) \in L$. This hypothesis is plausible for most urban distribution networks [26]. To each node i corresponds a vector $\underline{\mathcal{X}_i}$ giving the state of the node,

$$\underline{\mathcal{X}_i} = \begin{pmatrix} \mathcal{U}_i \\ \mathcal{Q}_i \end{pmatrix} \quad (1)$$

Where $\mathcal{U}_i$ and $\mathcal{Q}_i$ are respectively the voltage and the reactive power on node i.

From the Loadflow equations, we can notice a connection between voltage and reactive energy flow on a node [27,28]. So it can be said that reactive energy production causes an increase in voltage, while reactive energy consumption tends to lower voltage (Fig. 1). Thus an active agent can regulate the voltage on its node depending on whether they produce or consume a given amount of reactive energy (RPC method: Reactive Power control) [28].

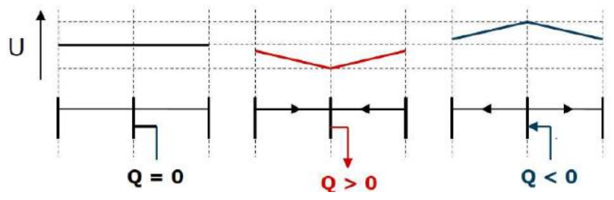

Fig. 1. Voltage variation depending on the direction of reactive energy [3]

III. THE ACTIVE AGENT

An active agent located at node i is a subscriber having the possibility of producing or consuming reactive energy on the network in order to maintain the voltage $(\mathcal{U}_i^k)$ at his node within a certain tolerance margin around a voltage of reference $(\mathcal{U}_r)$ fixed by the DSO with k which represents a time interval [3]. Its operation can be summarized as follows: if $\mathcal{U}_i^k < \mathcal{U}_r$ and $\mathcal{Q}_i < 0$ (producer), the DSO pays the agent for the amount of energy it provides while if $\mathcal{U}_i^k > \mathcal{U}_r$ et and $\mathcal{Q}_i < 0$, the agent is billed because the surplus reactive energy that they produce will destabilize the network, etc. In Fig. 2, the operating zones, namely the cost zone and the profit zone, are repeated. $\mathcal{U}_{tol}$ is the tolerance on the measurement of the voltage $\mathcal{U}_i^k$. An in depth explanation can be found in the work of Fetzner [3].

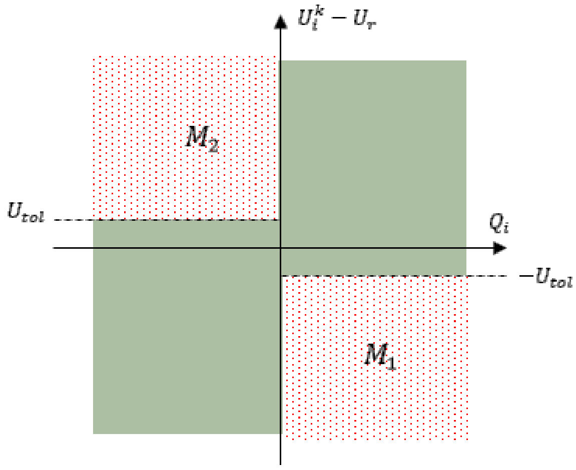

Fig. 2. Active agent operating zones [3]

The cost zone (in red on figure 2) is the zone $\mathcal{M} = \mathcal{M}_{1i} \cup \mathcal{M}_{2i}$ such that

$$\mathcal{M}_{1,i} \stackrel{\text{def}}{=} \{\underline{\mathcal{X}_i} \in \mathbb{R}^2 | \mathcal{Q}_i > 0,\ \mathcal{U}_i^k - \mathcal{U}_r < -\mathcal{U}_{tol}\} \quad (2)$$

and

$$\mathcal{M}_{2,i} \stackrel{\text{def}}{=} \{\underline{\mathcal{X}_i} \in \mathbb{R}^2\ |\mathcal{Q}_i < 0,\ \mathcal{U}_i^k - \mathcal{U}_r > \mathcal{U}_{tol}\} \quad (3)$$

If the agent's operating point is in zone $\mathcal{M}$, the agent must pay a price $\mathcal{C}(t)$ per unit of energy consumed, otherwise they are paid at price $\mathcal{B}(t)$ per unit of energy supplied. In this work we consider an identical pricing $\mathcal{P}(t)$ in both cases such $\mathcal{P}(t) = \mathcal{C}(t) = \mathcal{B}(t)$.

Thus the price that the agent pays or is paid is given by

$$price = \begin{cases} \mathcal{P}(t),\ \forall\, \underline{\mathcal{X}_i} \in \mathcal{M} \\ -\mathcal{P}(t),\ \forall \underline{\mathcal{X}_i} \notin \mathcal{M} \end{cases} \quad (4)$$

We can thus consider the agent as having two modes of operation namely buyer or seller. We will call the operating mode of the agent "spin" and we associate each spin with a sign. We retain the convention taken in [16] such that if the agent is in buyer mode his spin will be equal to +1, if he is in seller mode his spin will be -1. This consideration brings our system back to an Ising model [16 - 18].

IV. APPLICATION OF THE ISING MODEL TO THE DISTRIBUTION NETWORK

We will now formalize the process for establishing the spin of the agent according to its operating point. We consider a membership function $f_i(\underline{\mathcal{X}_i})$ such that

$$\begin{cases} f_i(\underline{\mathcal{X}_i}) < 0,\ \forall\, \underline{\mathcal{X}_i} \in \mathcal{M} \\ f_i(\underline{\mathcal{X}_i}) > 0,\ \forall\, \underline{\mathcal{X}_i}\ \notin \mathcal{M} \end{cases} \quad (5)$$

In [3], the author notices that the following function takes this property

$$f_i(\underline{\mathcal{X}_i}) = \frac{\mathcal{Q}_i}{|\mathcal{Q}_i|}(\mathcal{U}_i^k - \mathcal{U}_r) + \mathcal{U}_{tol} \quad (6)$$

The spin $\sigma_i(\underline{\mathcal{X}_i})$ which provides information on the operating mode of the agent is given by

$$\sigma_i(\underline{\mathcal{X}_i}) = \begin{cases} +1, & \forall\, \underline{\mathcal{X}_i} \in \mathcal{M}\ (buyer\ mode) \\ -1, & \forall\, \underline{\mathcal{X}_i} \notin \mathcal{M}\ (seller\ mode) \end{cases} \quad (7)$$

From relations (5) and (6), we can consider the spin as opposite of the sign of the function $f_i(\underline{\mathcal{X}_i})$ or

$$\sigma_i(\underline{\mathcal{X}_i}) = -\text{sign}(f_i(\underline{\mathcal{X}_i})) \quad (8)$$

From this relationship, it is easy to make a bridge between the electrical parameters of the network nodes and the configuration of an Ising model. We base ourselves on the work of Bornholdt [16, 17] who constructs an Ising model applicable to a financial market. We call configuration of the Ising model an element $\omega \in \Omega_N \overset{\text{def}}{=} \{-1,+1\}^{\mathcal{V}_N}$; Ω_N is the configuration space, a variable $\sigma_i: \Omega_N \to \{-1,+1\}$, $\sigma_i(\omega) \overset{\text{def}}{=} \omega_i$ is called spin at vertex i of the considered graph.

At each vertex i of the network graph, the quantity $\mathcal{H}_i(t)$ called local field is defined.

$$\mathcal{H}_i(t) = \sum_{i=1}^{N} J_{ij}\sigma_j - \alpha\sigma_i \left|\frac{1}{N}\sum_{i=1}^{N}\sigma_j\right| \quad (9)$$

From this expression $J_{ij} = 1\ \forall\ (i,j) \in L$ and $J_{ij} = 0$ otherwise. α is a parameter coupling the local field with the overall state of the network. We understand that the local field on an agent i depends on:

- the state of its direct neighbors, we can consider this to be a consequence of Millmann's theorem;
- the general state of the network, given by the parameter $\mathcal{D}(t)$ called magnetization

$$\mathcal{D}(t) = \frac{1}{N}\sum_{i=1}^{N}\sigma_j \quad (10)$$

The probability $\mu_{i,\beta}$ of local transition (tilting) of a spin as a function of the state of its neighbors defines the very dynamics of the spin, a review of the different possible dynamics can be found in [30]. In the BI model we consider a Heat –Bath dynamic.

$$\mu_{i,\beta}\big(\sigma_i = 1 \big| \sigma_j = \omega_j, \forall (i,j) \in \mathcal{L}\big) \overset{\text{def}}{=} \frac{1}{1+\exp(-2\beta\mathcal{H}_i(\omega))} \quad (11)$$

$\beta \in \mathbb{R}^+$ is called inverse temperature.

By the hypothesis of efficient financial markets, we can consider that the price $\mathcal{P}(t)$ is a function of the overall state of the market here quantified by the magnetization $\mathcal{D}(t)$ [13,16]. Sieczka and Holyst [21] then Denys, Gubiec and Kutner [31], consider the following expression to be a good approximation

$$\mathcal{P}(t) = P_0 \exp(\mathcal{D}(t)) \quad (12)$$

With P_0 the cost at equilibrium. We can consider the relative change in price over a period τ by

$$r(t) = \ln\frac{\mathcal{P}(t)}{\mathcal{P}(t-\tau)} = \mathcal{D}(t) - \mathcal{D}(t-\tau) \quad (13)$$

These two expressions define the whole dynamics of the price of energy. From the expression (12), we establish that the total cost of the flow of reactive energy on a network can be expressed at time t by

$$\mathcal{C}_{TOT}(t) = \sum_i^N |Q_i| \mathcal{P}(t)\sigma_i(t) \quad (14)$$

V. Results of simulation and discussion

We are going to test our model on the modified CIGRE test network (Fig. 3), this network has 18 active nodes, the parameters of this network are identical to those used in [28]. For simulation we allocate in a uniform random way to each node an active power between 18 and 4 kW under a power factor of 0.9. It is also considered that each agent can supply or consume only 10% of its nominal reactive power. The simulation is carried out over 24 hours in an interval of 15 minutes. The parameters α and $\beta \in \mathbb{R}^+$ of the Ising model are respectively 8 and 1 [16].

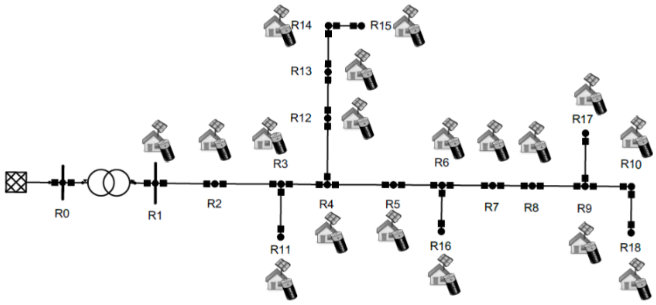

Fig. 3. The modified CIGRE test network [28]

The Heat-Bath algorithm is used for the simulation, it is a Monte-Carlo type algorithm. In this one we pass from one configuration ω to another configuration ω' as follows: we draw a number $u \in [0,1]$ according to a uniform probability, and a vertex $i \in \mathcal{V}_N$ also uniformly. We then ask

$$\omega'_j = \begin{cases} \omega_j\ si\ i \neq j \\ +1\ \ si\ \ u\ \leq \mu_{i,\beta}\big(\sigma_i = 1 \big| \sigma_j = \omega_j, \forall (i,j) \in \mathcal{L}\big) \\ -1\ \ si\ \ u\ > \mu_{i,\beta}\big(\sigma_i = 1 \big| \sigma_j = \omega_j, \forall (i,j) \in \mathcal{L}\big) \end{cases} \quad (15)$$

The simulation of the Ising model on the network randomly assigns different spins to the nodes of the network, our simulation being carried out over 24 hours at 15-minute intervals. We thus obtain 96 simulated states. We show in Fig. 4 three of these states of the network obtained during the simulation of the Ising model.

In Fig. 5, we show the variation of the magnetization $\mathcal{D}(t)$ over time, there is mainly an imbalance between the number of buyers and the number of sellers on the network. The effect of the variation of the magnetization on the price $\mathcal{P}(t)$ is perceptible, implying that if there are more buyers on the network the energy price increases and conversely if there are more sellers the price decreases.

We compare this price variation with a fixed reference price P_0 by simplification this price is maintained 1 monetary unit. The total cost of energy considering a dynamic price is lower, around -0.1769 currency unit than in the case of a fixed price where we have -0.6160 currency unit.

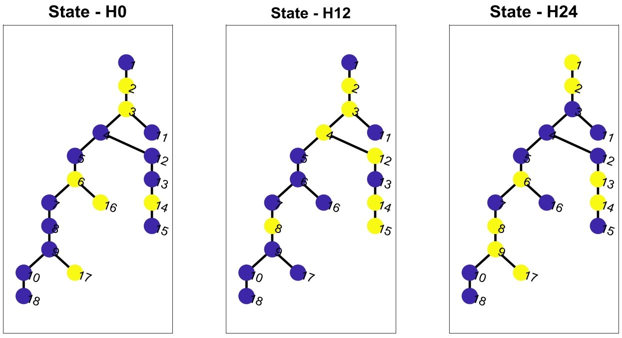

Fig. 4. Variation of the state of the nodes (agents) during the simulation of the Ising model

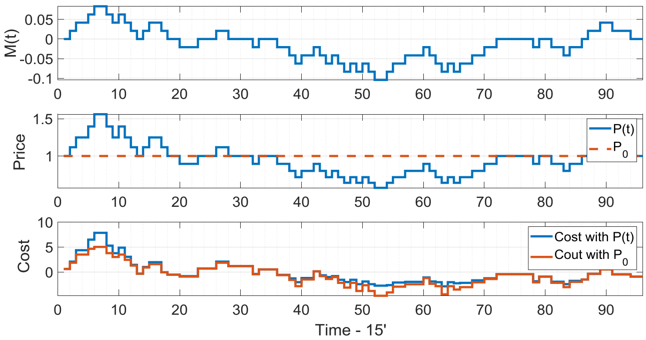

Fig. 5. Temporal variation of – magnetization – price – cost

In Fig. 6 we present the distribution of the relative price variation for different values of τ. We notice a flattening of the distribution following the increase in τ. This observation shows the validity of the model [8, 9, 13, 21, 31]. By considering τ ranging from 1 to 4 we analyze the price variation respectively over 15, 30, 45 and 60 minutes.

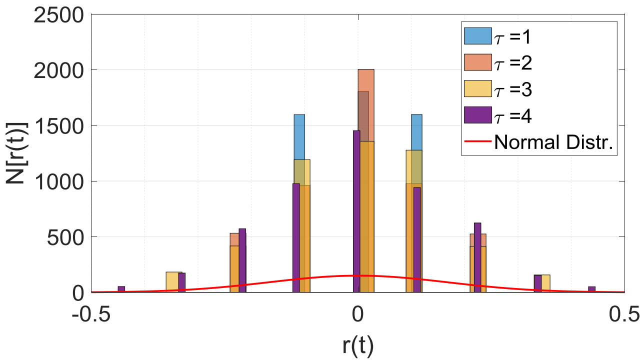

Fig. 6. Histogram of r (t) for τ 1 to 4

Fig. 7 shows a general overview of the simulation, showing the variation in the state of each agent, the variation in the power consumed as well as the variation in the cost / benefit of each agent. And some important observations can be drawn from these results presented in Fig. 5:

- We note from the proposed pricing method that the price remains higher than the fixed price if there are more buyers than sellers and it is lower than the fixed price if there are more sellers than buyers.
- This pricing increases the profits of the DSO and decreases losses.

VI. Conclusion

The simulation of an Ising model on the CIGRE test distribution network demonstrates that dynamic billing of the exchange of reactive energy between the active agents of the network and the DSO allows a reduction in the total cost of energy compared to fixed-price invoicing. The state of each agent as well as the general state of the network can be known through smart-meters, the DSO can thus adapt the price of energy over a given interval.

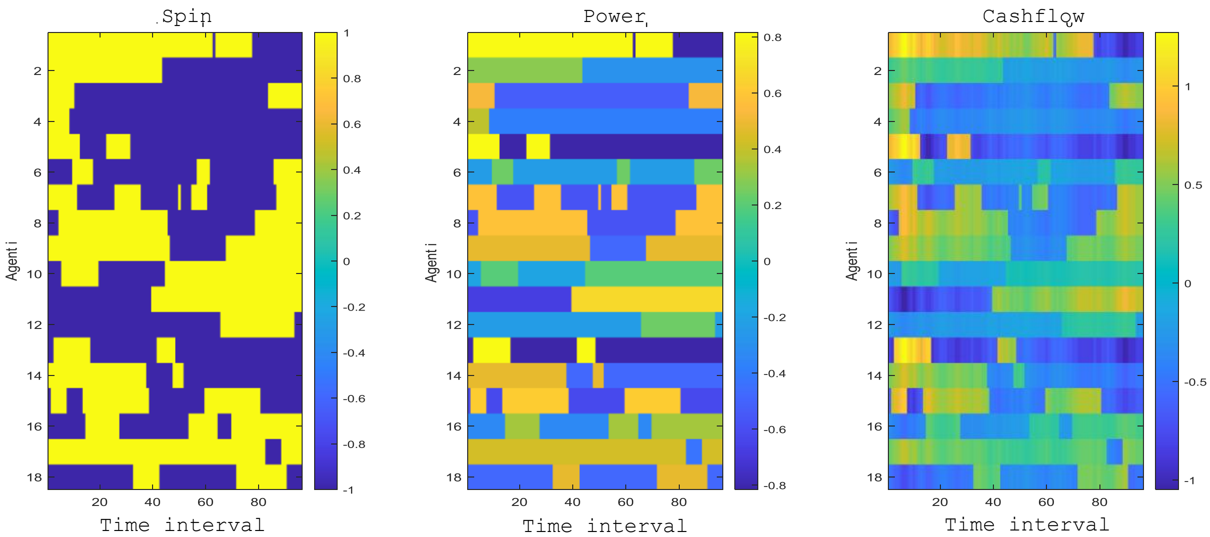

Fig. 7. Temporal variation for each agent - of the state - of the reactive power - cost / benefit

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