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To cite this version:
François Bailly, Justin Carpentier, Philippe Souères. Computational details for: "Optimal Estimation of the Centroidal Dynamics of Legged Robots". [Research Report] Rapport LAAS n° 21072, LAAS-CNRS; Université de Montréal. 2021. hal-03180052v4

HAL Id: hal-03180052
https://hal.archives-ouvertes.fr/hal-03180052v4
Submitted on 25 Mar 2021

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This document complements the paper entitled “Differential Dynamic Programming for Maximum a Posteriori Centroidal State Estimation of Legged Robots” [1]. The purpose of this work was to estimate the centroidal dynamics of legged robots by formulating a maximum a posteriori problem and solving it thanks to differential dynamic programming (DDP). In the following, the computations of the partial derivatives of the unoptimized value function ($Q_k$) are provided for the DDP algorithm. Then, the hypothesis about the $0$–mean property of the stochastic part of the dynamics is validated by Fig. [1] (result of the simulation ) which demonstrates that the DDP minimization of Eq.(11) does keep $\omega_k$ $0$–mean.

**APPENDIX I**

**PARTIAL DERIVATIVES OF $Q_k$**

- $Q_{xk} = \nabla_{x_k} l_k + \nabla_{x_k} V_{i+1}(f(x_k, \omega_k))$,
  
  \[
  \nabla_{x_k} l_k = 2 \partial (g(x_k) - y_k)^T \Sigma_k^{-1}(g(x_k) - y_k),
  \]

  \[
  \frac{\partial g(x_k)}{\partial x_k} = \frac{C(x_k)}{\partial x_k} x_k + C(x_k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2x_k & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \triangleq \tilde{C}(x_k),
  \]

  $Q_{xk} = 2\tilde{C}(x_k)^T \Sigma_k^{-1}(g(x_k) - y_k) + A^T \nu_x$.

- $Q_{\omega_k} = \nabla_{\omega_k} l_k + \nabla_{\omega_k} V_{i+1}(f(x_k, \omega_k))$,
  
  \[
  \nabla_{\omega_k} l_k = \frac{\partial |\omega_k|^2}{\partial \omega_k} \Sigma_k^{-1} \omega_k = 2\Sigma_k^{-1} \omega_k,
  \]

  $Q_{\omega_k} = 2\Sigma_k^{-1} \omega_k + B^T \nu_x$.

- $Q_{x}\omega_k = \nabla_{x_k}^2 l_k + \nabla_{x_k}^2 V_{i+1}(f(x_k, \omega_k))$,
  
  where, the element of $\nabla_{x_k}^2 l_k$ at the $i^{th}$ row and $j^{th}$ column is denoted by:
  
  \[
  [\nabla_{x_k}^2 l_k]_{ij} = \sum_{m=1}^{Y} \sum_{n=1}^{Y} \Sigma_{mn}^{-1} \left( \frac{\partial \tilde{C}^T}{\partial x_j} \partial g(x_k) \right)_{im},
  \]

  \[
  [\nabla_{x_k}^2 l_k]_{ij} = \tilde{C}^T \Sigma_k^{-1} \tilde{C}_{ij} + \sum_{m=1}^{Y} \sum_{n=1}^{Y} \partial \Sigma_k^{-1} \partial g(x_k)_{im} (g(x_k) - y_k)_n,
  \]

  \[
  [\nabla_{x_k}^2 l_k]_{ij} = \tilde{C}^T \Sigma_k^{-1} \tilde{C}_{ij} + \tilde{c}_{i,j} \Sigma_k^{-1} (g(x_k) - y_k)_n,
  \]

  where, $\tilde{c}_{i,j} \in \mathbb{R}^Y s$ is the stacked vector of $\frac{\partial \tilde{C}^T}{\partial x_j}$ for $m \in [1,Y]$.

- $Q_{x\omega_k} = 2\Sigma_k^{-1} \omega_k + B^T \nu_x$.

Fig. 1: Time evolution of the stochastic control inputs of the dynamics for the simulated walk of the HRP-2 robot.

REFERENCES