Single Gain Super Twisting Algorithm Application to PMSM

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***Abstract*—In this paper, by means of a parametrization leading to a single gain, a tuning method is proposed to simplify the implementation of the well known Super Twisting Algorithm (STA). Furthermore, inspired by ideas from the high-gain observer design, and using a Lyapunov approach, the stability analysis of the closed-loop system is presented. Thanks to a simple Lyapunov function, sufficient condition are obtained to prove the finite-time convergence. Finally, the performances of the proposed strategy are illustrated experimentally on a Permanent Magnet Synchronous Motor (PMSM) using an industrial benchmark for generating the desired trajectories to be tracked, in presence of external disturbances.**

I. Introduction

Sliding mode theory has received a large interest in the last years. It is commonly used for the design of robust nonlinear observers or control laws. Indeed, sliding modes have interesting properties such as: disturbance and uncertainty compensation and the finite-time convergence. Moreover, the use of second-order sliding modes reduces the high-frequency commutations known as *chattering* [10]. According to the literature, sliding mode control has been successfully applied to in electrical engineering [28], [29], [31], [19], [32] and many others systems as mentioned in [15], [14], [33], [20], [6].

From a practical point of view, PMSM are widely used in the industry, due to the growing interest in electrical applications and for their attractive advantages compared with other electrical machines. For instance, PMSM are more robust than brushed DC motors and produce higher torque per volume. However, robust PMSM control is a challenge because such motors have parameters uncertainties, unknown load torque as well as noise measurement.

Several solutions based on sliding mode techniques have been proposed to control PMSM in presence of uncertainties and noise measurement and several control strategies based on backstepping design, passivity, neural-networks, H_∞ adaptive,control have been proposed to solve this control problem. In the literature, sliding mode control as been widely used for electro-mechanical systems to overcome such difficulties (see [30]), in which AC motor control is successfully applied. A particular attention has been paid to motor control, see for instances [8]. The sliding mode control and observation of PMSM in particular has been the subject of a large number of publications. In [30], [35], [34] control and observation using first order sliding mode have been proposed. On the other hand, in order to tackle the chattering problem, sigmoid function was introduced instead of the sign function in [11]. Then, the attention was focused on higher order sliding modes. Research can be found for control and observation using largely the STA in the case where position is measured [13], fault tolerant control [12] but above all sensorless control (without mechanical sensors) [9], [1], [5]. However, the introduction of high order sliding mode increases the number of gains to be tuned. To the knowledge of the authors, few methods for gain tuning are proposed in the literature [26], [23], [18], [3]. In contrast with the gain tuning proposed in the above references, in our method, the convergence time is directly related to the gain which allows a simple selection of the gain for a given convergence time. In addition the proof in our work is simplified because it is based on a standard Lyapunov function and this is made possible by using the high-gain theory.

In this paper, by means of a single parameter, a simple methodology to tune the gains of a STA [16] is proposed, in order to compensate for the effect of a perturbation and its time-derivative, which are assumed to be bounded. Furthermore, using a Lyapunov approach, sufficient conditions are given to guarantee the finite time convergence to zero, in presence of matched disturbances/perturbations. The stability analysis of this algorithm is inspired by ideas from high-gain theory. Experimental results are obtained under the action of the proposed control strategy in order to validate its performance and its easy implementation on a PMSM. Moreover, based on the Lyapunov approach, a Lyapunov function is proposed to analyze the convergence of the closed-loop system to zero [21]. Furthermore, instead of the two conditions required to design the gains of the classical STA [22], which are obtained using linear matrix inequalities techniques, in the proposed methodology, a single condition has to be verified.

On the other hand, inspired by the design of the high-gain observer given in [7], it is possible to derive an observer (controller) based on sliding mode techniques that shares all the appealing features of the high-gain concept. Following this ideas, the gains of the proposed STA are determined, first, from a parameterization in terms of a single parameter. Then, after a variable change, the resulting STA structure is equivalent to a high-gain observer design.

Then, the contribution of this paper can be summarized as follows:

i) Thanks to a parmetrization of the gains, in terms of only single parameter, a simple method to tune the gains of the STA is derived, reducing the number of gains to a single gain to be tuned instead of two gains required in the classic STA of [16]. Furthermore, STA is tuned through the choice of a single design parameter, satisfying a condition obtained from the stability analysis of the closed-loop system. More precisely, using a Lyapunov approach, sufficient conditions are obtained to ensure the finite time convergence towards zero (see tuning guidelines at the end of Section III).

ii) The performance of the proposed method is validated experimentally on a PMSM, where a speed control based on the proposed sliding mode observer-control strategy is implemented.

iii) The proposed methodology based on a single gain is very interesting from the engineering point of view since it reduces the computational effort (no LMIs are solved), and the ease of implementation

Finally, the proposed methodology can be easily be extended for sensorless control of machines based on STA.

This paper is organized as follows. In Section II, some preliminaries on high-gain observer design and the problem statement are introduced. The main result which concerns the design of the single gain Super Twisting algorithm and its convergence proof is given in Section III. Section IV is devoted to the application of the proposed algorithm to control a PMSM. Furthermore, experimental results to illustrating the performance of the proposed strategy are presented in Section V. Finally, some conclusions are given in Section VI.

LIST OF ACRONYMS

FOC Field Oriented Control
PMSM Permanent Magnet Synchronous Motor
STA Super Twisting Algorithm
STC Super Twisting Controller
STO Super Twisting Observer

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the class of nonlinear systems described by:

$$\dot{x}(t) = f(x(t),t) + g(x(t),t)u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the input, f and g are smooth uncertain functions, with $g(x(t),t) \neq 0$. For the controller design, a scalar sliding variable $s(t)$ can be defined so that $s(t)$ is considered as the output of the system (1) and the following assumption holds.

Assumption 1: System (1) admits a relative degree equal to one with respect to the sliding variable $s(t)$.

To synthesize the control law, taking the time derivative of $s(t)$ such that the sliding surface dynamics can be given by

$$\dot{s}(t) = \phi(x(t),t) + \varphi(x(t),t)u(t) + \rho(x(t),t) \tag{2}$$

where $s \in \mathbb{R}$, ϕ and φ are nominal nonlinear functions and ρ represents parametric uncertainties and external disturbances. These functions satisfy the following assumption.

Assumption 2: The functions $\phi : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}$ and $\varphi : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}$ are known and $\rho : \mathbb{R}^+ \to \mathbb{R}$ is differentiable and its derivative is unknown but bounded, i.e. there exists a known constant $d > 0$ such that for all $t \geq 0$ it follows that

$$|\dot{\rho}(x(t),t)| \leq d. \tag{3}$$

Notice that (3) is usually verified in practice because physical systems have bounded states. In the sequel, $\rho(t) = \rho(x(t),t)$ will be used.

Problem: Consider the linearizing state feedback u defined as :

$$u = \frac{1}{\varphi}\left[-\phi + w^{\lambda}_{\mathrm{STC}}\right], \tag{4}$$

Then, system (2) in closed-loop with the control (4) leads to

$$\dot{s}(t) = w^{\lambda}_{STC}(s(t)) + \rho(t) \tag{5}$$

Then, the control objective is to design a controller $w^{\lambda}_{\mathrm{STC}}$ based on STA in closed-loop with the system (5), such that the sliding condition holds, i.e. the sliding surface and its first-time derivative are equal to zero: $s(t) = \dot{s}(t) = 0$ in finite time for every disturbance $\rho(t)$ satisfying Assumption 2.

A. High-gain Concepts

We introduce the high-gain observer design for a class of uniformly observable nonlinear systems (observable for any input).

Consider system (1) with the measured output

$$y = h(x) \tag{6}$$

with $y \in \mathbb{R}$ and h a smooth known function, that can be transformed, by means of a transformation $\xi = T(x)$ into the following form

$$\begin{cases} \dot{\xi} &= A\xi + \Phi(\xi,u) \\ y &= C\xi \end{cases} \tag{7}$$

with $\xi \in \mathbb{R}^n$, and a function $\Phi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$.

Then, under the following assumptions it is possible to design an observer.

A1. The state $x(t)$ and the input $u(t)$ are bounded,. i.e. there exist compact sets X and U such that for all t , $x \in X$ and $u \in U$.

A2. The functions ϕ_i, which are the components of Φ, are Lipschitz with respect to $x(t)$ and uniformly with respect to $u(t)$.

A high-gain observer for system (7) is given by

$$\begin{cases} \dot{\hat{\xi}} &= A\hat{\xi} + \Phi(\hat{\xi},u) + \lambda\Delta_{\lambda}^{-1}S^{-1}C^TC(\hat{\xi} - \xi) \\ \hat{y} &= C\hat{\xi} \end{cases} \tag{8}$$

where $\lambda > 0$ is a real parameter and S is the unique solution of the following algebraic Lyapunov equation

$$S + A^TS + SA - C^TC = 0 \tag{9}$$

and $\Delta_\lambda = diag(1, \frac{1}{\lambda}, ..., \frac{1}{\lambda^{n-1}})$. Then we have the following result that provides the performance of the observer.

Theorem 1: Under Assumptions A1 and A2, system (8) is an exponential observer for system (7).

The proof of convergence of this observer is as follows. Let $e = \hat{\xi} - \xi$ be the estimation error. Then, the dynamics of the estimation error is given by

$$\dot{e} \;=\; (A - \lambda\Delta_\lambda^{-1}S^{-1}C^TC)e + \Phi(\hat{\xi}, u) - \Phi(\xi, u). \tag{10}$$

Using the following change of coordinates $\epsilon = \Delta_\lambda e$, and the identities $\Delta_\lambda A\Delta_\lambda^{-1} = \lambda A$ and $\Delta_\lambda^{-1}C = C$, then the dynamics of the estimation error can be rewritten as follows

$$\dot{\epsilon} \;=\; \lambda(A - S^{-1}C^TC)\epsilon + \Delta_\lambda(\Phi(\hat{\xi}, u) - \Phi(\xi, u)). \tag{11}$$

Considering the Lyapunov function $V(\epsilon) = \epsilon^TS\epsilon$, and taking the time derivative along the trajectories of (12), and from Assumption A2, it follows that

$$\begin{array}{rcl} \dot{V(\epsilon)} & = & -\lambda V(\epsilon) + 2\epsilon^TS\Delta_\lambda(\Phi(\hat{\xi}, u) - \Phi(\xi, u)) \\ & < & -(\lambda - k_1)V(\epsilon) \end{array} \tag{12}$$

where $k_1 = \dfrac{2\mu_1||S||}{\lambda_m(S)}$ with $\lambda_m(S)$ the minimum eigenvalue of S, and μ_1 the Lipschitz constant, i.e. $||\Phi(\hat{\xi}, u) - \Phi(\xi, u)|| < \mu_1||\epsilon||$.

III. Main result

Consider the classical STA (see [22]) given by

$$\left\{ \begin{array}{rcl} w_{\text{STC}}^\lambda(s) & = & -k_1|s|^{\frac{1}{2}}\text{sign}(s) + \zeta^\lambda \\ \dot{\zeta}^\lambda(s) & = & -k_2\text{sign}(s), \quad \zeta^\lambda(s(0)) = 0 \end{array} \right. \tag{13}$$

where k_1 and k_2 are the gains of the controller. The gains are selected using LMI techniques, and hence two inequalities must be satisfied (see [22] for more details).

The following parametrization is introduced

$$k_1 = 2\lambda, \quad k_2 = \frac{\lambda^2}{2}$$

where λ is a positive constant to be tuned.

Then, the proposed Super Twisting Controller (STC) w_{STC}^λ is given by

$$\left\{ \begin{array}{rcl} w_{\text{STC}}^\lambda(s) & = & -2\lambda|s|^{\frac{1}{2}}\text{sign}(s) + \zeta^\lambda \\ \dot{\zeta}^\lambda(s) & = & -\dfrac{\lambda^2}{2}\text{sign}(s), \quad \zeta^\lambda(s(0)) = 0. \end{array} \right. \tag{14}$$

Inspired by the high-gain observer design, given in Section II-A, the main result of the paper is established in the following theorem.

Theorem 2: Consider system (2) where Assumptions 1 and 2 are satisfied, in closed-loop with the control (4) and combined with STA (14). Then, the sliding surface s converges in finite-time to zero provided that λ is large enough.

Proof: Consider the following change of coordinates

$$\left\{ \begin{array}{l} \xi_1(t) = |s(t)|^{\frac{1}{2}}\text{sign}(s(t)) \\ \xi_2(t) = -\dfrac{\lambda}{2}\displaystyle\int_0^t \text{sign}(s(\tau))d\tau + \rho(t)/\lambda \end{array} \right. \tag{15}$$

whose dynamics is given by

$$\dot{\xi} = \frac{\lambda}{2|s|^{\frac{1}{2}}}\{(A - KC)\xi + \Delta_\lambda\Phi\} \tag{16}$$

where $\xi = \begin{bmatrix}\xi_1 & \xi_2\end{bmatrix}^T$, $A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, $C = \begin{bmatrix}1 & 0\end{bmatrix}$,

$$\Delta_\lambda\Phi = \begin{bmatrix}\dfrac{1}{\lambda} & 0 \\ 0 & \dfrac{1}{\lambda^2}\end{bmatrix}\begin{bmatrix}0 \\ 2|s|^{\frac{1}{2}}\dot{\rho}\end{bmatrix}.$$

where $K = S_o^{-1}C^T$ with S_o is a symmetric and definite positive matrix solution of the following Algebraic Lyapunov Equation:

$$S_o + A^TS_o + S_oA - C^TC = 0. \tag{17}$$

Then, S_o^{-1} is given by

$$S_o^{-1} = \begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}$$

it follows that $K = \begin{bmatrix}2 & 1\end{bmatrix}^T$.

Notice that system (16) has a structure similar to (11), then an analysis of convergence similar to the high-gain observer design can be applied (see [9]).

It is clear that system (16) is defined for $s \neq 0$, which is the case before reaching the sliding surface. In order, to analyze the convergence of the tracking error dynamics (16), consider the following candidate Lyapunov function

$$V(\xi) = \xi^TS_o\xi \tag{18}$$

which is quadratic in the new coordinates. Note that $V(\xi)$ is continuous and continuously differentiable everywhere except in a domain given by $\Psi = \{(\xi_1, \xi_2) \in \mathbb{R}^2 | \xi_1 = 0\}$ (see [21] for more details). Since the trajectories of the system cannot stay in the set Ψ before reaching the origin, the time derivative of $V(\xi)$ can be calculated in the usual way everywhere except when the trajectories intersect the set Ψ. If the trajectories reach the origin after a time T, then they will remain there.

The time derivative of $V(\xi)$ along the trajectories of (16) is

$$\dot{V}(\xi) = \frac{\lambda}{2|s|^{\frac{1}{2}}}[\xi^T(A - KC)^TS_o + S_o(A - KC))\xi + 2\xi^TS_o\Delta_\lambda\Phi.]$$

Using (17) and the definition of $K = S_o^{-1}C^T$, one gets

$$\dot{V}(\xi) = \frac{\lambda}{2|s|^{\frac{1}{2}}}[-\xi^TS_o\xi - \xi^TC^TC\xi + 2\xi^TS_o\Delta_\lambda\Phi]$$

or equivalently

$$\dot{V}(\xi) \;\leq\; \frac{\lambda}{2|s|^{\frac{1}{2}}}[-\xi^TS_o\xi + 2\xi^TS_o\Delta_\lambda\Phi]. \tag{19}$$

From (15), the inequality $|s|^{\frac{1}{2}} \leq ||\xi||$ holds, and from Assumption 2, it follows that $||\Delta_\lambda\Phi|| \leq 2d||\xi||/\lambda^2$. Then, substituting in the above inequalities in (19), we obtain

$$\dot{V}(\xi) \leq -\frac{\lambda}{2|s|^{\frac{1}{2}}}V(\xi) + \frac{4d}{2|s|^{\frac{1}{2}}\lambda}||S_o|| \cdot ||\xi||^2. \tag{20}$$

Taking into account the following inequality

$$\lambda_{min}(S_o)||\xi||^2 \leq V(\xi) \leq \lambda_{max}(S_o)||\xi||^2 \tag{21}$$

where $\lambda_{min}(S_o)$, $\lambda_{max}(S_o)$ are the minimum and maximum eigenvalues of S_o respectively, it follows that

$$\dot{V}(\xi) \leq -\frac{\lambda - \mu(\lambda)}{2|s|^{\frac{1}{2}}}V(\xi) \tag{22}$$

where $\mu(\lambda) = \dfrac{4d||S_o||}{\lambda \cdot \lambda_{min}(S_o)}$. From the inequality $|s|^{\frac{1}{2}} \leq ||\xi|| \leq \left\{ \dfrac{V(\xi)}{\lambda_{min}(S_o)} \right\}^{\frac{1}{2}}$, it follows that

$$\dot{V}(\xi) \leq -\gamma V^{\frac{1}{2}}(\xi) \tag{23}$$

where

$$\gamma = \frac{\lambda - \mu(\lambda)}{2\lambda_{min}^{-\frac{1}{2}}(S_o)}. \tag{24}$$

Choosing λ sufficiently large such that the inequality

$$\lambda > \mu(\lambda) \tag{25}$$

holds, then $\dot{V}(\xi)$ is definite negative, $V(\xi)$ is a Lyapunov function and the trajectories converge to zero in finite-time.

It is clear that the stability of the closed-loop system to zero is determined by only a single condition, which depends on the choice of the parameter λ.

For estimating the convergence time, note that the solution of the differential equation

$$\dot{v} = -\gamma v^{\frac{1}{2}}, \ v(0) = v_0 \tag{26}$$

is given by

$$v(t) = \left[v_0^{\frac{1}{2}} - \frac{1}{2}\gamma t \right]^2. \tag{27}$$

From the comparison principle [24], one has $V(\xi) < v(t)$ when $V(\xi(0)) < v_0$, then ξ converges to zero in finite-time and reaches that value at the instant T defined by

$$T = \frac{2V^{\frac{1}{2}}(\xi(0))}{\gamma}. \tag{28}$$

Thus, the states ξ_1 and ξ_2 converge to zero in finite-time. As a result, the surface s will converge to zero in finite-time.

□

Substituting γ in (28), it follows that

$$T = \frac{4V^{\frac{1}{2}}(\xi(0))\lambda_{min}^{-\frac{1}{2}}(S_o)}{\lambda - \mu(\lambda)} \tag{29}$$

Note that the STA (14) depends on λ only, which simplifies the tuning. From (29) it can be seen that choosing the gain λ sufficiently large, the convergence time of the algorithm is reduced. Note that this convergence time depends on the initial conditions through $\xi(0)$.

The choice of λ can be made as follows:

Choose $\lambda \geq \lambda_s$ where

$$\lambda_s = \sqrt{\frac{4d||S_0||}{\lambda_{min}(S_0)}}. \tag{30}$$

The value of S_0 is known, then $\lambda_s \approx 5.24\sqrt{d}$, where d is given in (3). This will ensure that γ defined in (24) is positive, and thus the whole system in closed-loop with STA is stable. The stability condition is more conservative than that proposed in [3] and references therein. Reducing this conservatism is an objective for a future work. Then we can increase λ in order to obtain the desired convergence time using (29). Note that increasing λ will also result in noise amplification so there is a trade-off between convergence speed and noise amplification.

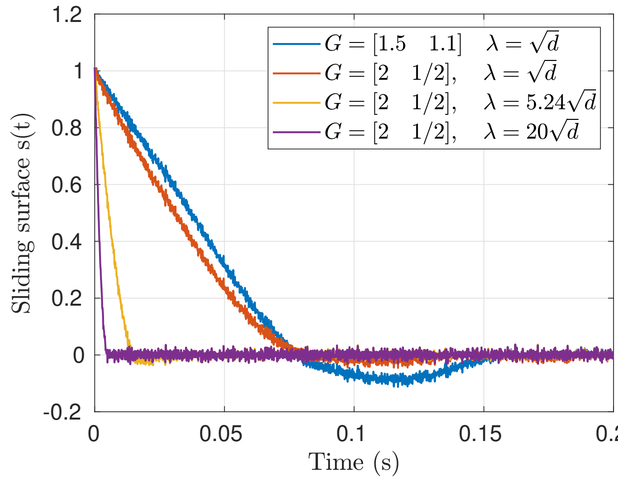


Fig. 1: Comparison with [17] and [18]

Numerical comparison with existing method: The proposed approach is compared with existing algorithm presented in [17] and [18]. This comparison is illustrated trough a numerical example.

Consider system (5) with:

$$\begin{array}{rcl} \rho(t) & = & p(t) + n(t), \\ p(t) & = & \sin(10t) + 1, \\ n(t) & \in & (0, 0.01^2). \end{array} \tag{31}$$

In [17] and [18] the STA is expressed as

$$\left\{ \begin{array}{rcl} \tilde{w}_{\text{STC}}^{\lambda}(s) & = & -g_1\lambda|s|^{\frac{1}{2}}\text{sign}(s) + \tilde{\zeta}^{\lambda} \\ \dot{\tilde{\zeta}}^{\lambda}(s) & = & -g_0\lambda^2 d \cdot \text{sign}(s) \end{array} \right. \tag{32}$$

where d is defined as in Assumption 2, and $G = [g_1 \quad g_0]$ are the tuning gains of the controller, using the values ($G = [1.5 \quad 1.1]$ and $\lambda = \sqrt{d}$). Notice that no conditions are given to increase the speed convergence and ensure the stability to zero.

From (31), the bound d is fixed as $d < 100$. Simulation results using four different gains are plotted in Fig. 1, as follows :

1) For $G = [1.5 \quad 1.1]$ and $\lambda = \sqrt{d}$: are selected as in [18]
2) For $G = [2 \quad 1/2]$ and $\lambda = \sqrt{d}$: this configuration uses λ from configuration 1 and the G proposed in this article. It shows that regardless of λ parameter the choice of the parameter G reduces the overshoot compared to configuration 1.
3) $G = [2 \quad 1/2]$ and $\lambda = 5.24\sqrt{d}$: this configuration corresponds to the proposed gain tuning with $\lambda = \lambda_s$ defined in (30).
4) $G = [2 \quad 1/2]$ and $\lambda = 20\sqrt{d}$: choosing $\lambda \geq \lambda_s$ increase convergence speed.

Finally, the simulation results have demonstrated the efficiency of the proposed approach compared with those proposed in [17] and [18]. Note that the implementation of these algorithms is similar but the novelty of the proposed algorithm lies in the form of the gain which allow to adjust the convergence rate.

IV. Observer based sliding mode control for PMSM

Permanent magnet synchronous motors have an important role in motion control applications in the low and medium power range. PMSM have desired features such as high torque to weight ratio, fast dynamical response. Traditionally this kind of motors have been controlled using the Field Oriented Control (FOC). However, accurate motor parameters and load conditions are necessary to guarantee good performance under disturbances. The parameter variations are a recognized problem and several robust control techniques have been proposed to overcome this difficulty. For instance adaptive control, backstepping control or those based on network-based control.

In this section, the proposed control algorithm is applied to a three-phase PMSM in order to track a desired reference speed under parametric uncertainties (load torque). The control algorithm is designed from a d-q model of the PMSM, which is obtained from Clarke and Park transformations [25].

A. PMSM Model in d-q coordinates

First, consider the mathematical model, described in d-q reference frame, of the PMSM:

$$\begin{cases} L\dfrac{di_d}{dt} = v_d - Ri_d + Lp\Omega i_q & (33) \\ L\dfrac{di_q}{dt} = v_q - Ri_q - Lp\Omega i_d - \phi_f p\Omega & (34) \\ J\dfrac{d\Omega}{dt} = p\dfrac{3}{2}\phi_f i_q - f\Omega - \tau_l & (35) \\ \dfrac{d\theta}{dt} = \Omega & (36) \end{cases}$$

where

Notations used for the PMSM	
R	stator resistance
L	stator inductance
ϕ_f	permanent-magnet flux linkage
i_d, i_q	stator $d-q$ currents
v_d, v_q	stator $d-q$ voltages
Ω	rotor mechanical speed
θ	rotor angular position
J	moment of inertia
f	viscous friction coefficient
p	number of pole pairs
τ_l	load torque

The control strategy is designed from the flatness property of the motor. Indeed, in [27] it was shown that PMSM are flat systems, where the flat outputs are the direct current i_d and the speed Ω. Then, to control the PMSM, it is necessary to design a control loop for the direct current i_d and another for the rotor speed Ω.

For the current loop, the sliding variable is chosen such that the system has relative degree equal to one, thus a STA can be used.

For the speed control loop, the sliding variable is selected, in terms of the rotor speed Ω and its derivative for the system to have relative degree equal to one with respect to the sliding variable. The motor acceleration is usually not available so a STA-based observer will be designed to overcome this difficulty.

Furthermore, the stability of the overall observer-controller scheme in closed-loop with the system is developed. The proposed control scheme is represented Fig. 2.

Assumption 3: The conditions under which the approach works are :

1) the currents i_d and i_q are measured,
2) the position θ and the velocity Ω are measured,
3) the torque τ is C^1 and its time derivative is bounded i.e.

$$\left|\frac{d\tau_l}{dt}\right| < \eta^*$$

where $\eta^* > 0$.

B. Direct current control

The following sliding surface

$$s_d = i_d - i_d^\star \tag{37}$$

where $i_d^\star$ is the direct current reference is defined. Now, taking the time derivative of s_d, it follows that

$$\dot{s}_d = \frac{1}{L}[-Ri_d + Lp\Omega i_q] + \frac{1}{L}v_d - \frac{di_d^\star}{dt}. \tag{38}$$

Choosing the control v_d as

$$v_d = Ri_d - Lp\Omega i_q + L\frac{di_d^\star}{dt} + Lw_{STC}^{\lambda_{i_d}}(s_d) \tag{39}$$

where the function w_{STC}^{λ} is defined in (14). Then, it follows that

$$\begin{cases} \dot{s}_d & = & -2\lambda_{i_d}|s_d|^{\frac{1}{2}}\operatorname{sign}(s_d) + \zeta^\lambda \\ \dot{\zeta}^{\lambda_{i_d}}(s_d) & = & -\dfrac{\lambda_{i_d}^2}{2}\operatorname{sign}(s_d), \quad \zeta^\lambda(s_d(0)) = 0 \end{cases} \tag{40}$$

with $\lambda_{i_d} > 0$.

Notice that the closed-loop system (40) is in the form of (5) with $\rho = 0$. As a consequence, Theorem 1 guarantees the existence of a sufficiently large λ_{i_d} in order to stabilize the surface s_d to zero.

C. Observer based speed control

For designing a speed control, taking the time derivative of (35) and substituting (34), one gets

$$\frac{d^2\Omega}{dt^2} = \frac{3p\phi_f}{2J}\frac{1}{L}[v_q - Ri_q - Lp\Omega i_d - \phi_f p\Omega] - \frac{f}{J}\frac{d\Omega}{dt} - \frac{1}{J}\frac{d\tau_l}{dt}. \tag{41}$$

Defining $x_\Omega = \begin{bmatrix}\Omega & \dot{\Omega}\end{bmatrix}^T$, the above equation can be written, in a state space representation, as follows

$$\begin{cases} \dot{x}_{\Omega,1} & = & x_{\Omega,2} \\ \dot{x}_{\Omega,2} & = & -\dfrac{f}{J}x_{\Omega,2} + \Gamma(v_q, i_d, i_q, \Omega) - \dfrac{1}{J}\dfrac{d\tau_l}{dt} \end{cases} \tag{42}$$

where

$$\Gamma(v_q, i_d, i_q, \Omega) = \frac{3p\phi_f}{2JL}[v_q - Ri_q - Lp\Omega i_d - \phi_f p\Omega] \tag{43}$$

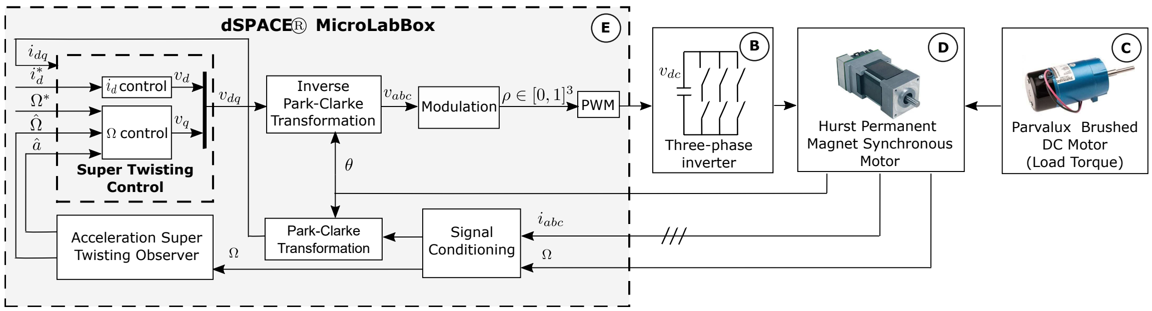


Fig. 2: Control Scheme

is an input-output injection term.

The control objective is to design a STA control for the speed loop to track a reference speed trajectory $\Omega^\star$, in spite of the disturbance due to the term $\dfrac{1}{J}\dfrac{d\tau_l}{dt}$.

To solve the speed tracking problem, let be $e_\Omega = \begin{bmatrix} x_{\Omega,1} - \Omega^\star & x_{\Omega,2} - \dot{\Omega}^\star \end{bmatrix}^T$, the tracking error vector, with $\Omega^\star$ is a smooth speed reference. Then, the tracking error dynamics is given by

$$\left\{ \begin{array}{rcl} \dot{e}_{\Omega,1} & = & e_{\Omega,2} \\ \dot{e}_{\Omega,2} & = & -\dfrac{f}{J}x_{\Omega,2} + \Gamma(v_q, i_d, i_q, \Omega) - \dfrac{1}{J}\dfrac{d\tau_l}{dt} - \ddot{\Omega}^\star. \end{array} \right. \tag{44}$$

Defining the following sliding surface :

$$s_\Omega = c_\Omega e_{\Omega,1} + e_{\Omega,2} \tag{45}$$

it follows that system (44) has a relative degree equal to 1 with respect to the s_Ω.

Since $e_{\Omega,2}$ is not available for measurement, then in order to implement a control STA it is necessary to estimate it.

1) Acceleration Super Twisting Observer (STO): A Super Twisting observer is designed to estimate the acceleration, in order to implement the speed controller.

Consider the following Super Twisting observer for system (42)

$$\left\{ \begin{array}{rcl} \dot{\hat{x}}_{\Omega,1} & = & \hat{x}_{\Omega,2} + 2\lambda_{\hat{a}}|e_{\hat{a},1}|^{\frac{1}{2}}\operatorname{sign}(e_{\hat{a},1}) \\ \dot{\hat{x}}_{\Omega,2} & = & -\dfrac{f}{J}\hat{x}_{\Omega,2} + \Gamma(v_q, i_d, i_q, \Omega) + \dfrac{\lambda_{\hat{a}}^2}{2}\operatorname{sign}(e_{\hat{a},1}) \end{array} \right. \tag{46}$$

with $\lambda_{\hat{a}} > 0$ and $e_{\hat{a}} = x_\Omega - \hat{x}_\Omega$. Then. the estimation error dynamics is given by

$$\left\{ \begin{array}{rcl} \dot{e}_{\hat{a},1} & = & e_{\hat{a},2} - 2\lambda_{\hat{a}}|e_{\hat{a},1}|^{\frac{1}{2}}\operatorname{sign}(e_{\hat{a},1}) \\ \dot{e}_{\hat{a},2} & = & -\dfrac{\lambda_{\hat{a}}^2}{2}\operatorname{sign}(e_{\hat{a},1}) - \dfrac{f}{J}e_{\hat{a},2} + \dot{\rho}_{\hat{a}} \end{array} \right. \tag{47}$$

where $\dot{\rho}_{\hat{a}}(t) = -\dfrac{1}{J}\dfrac{d\tau_l}{dt}$. Defining $s_{\hat{a}} = e_{\hat{a},1}$, one gets

$$\dot{s}_{\hat{a}} = w_{STC}^{\lambda_{\hat{a}}}(s_{\hat{a}}) - \frac{f}{J}\int_0^t e_{\hat{a},2}(\tau)d\tau + \rho_{\hat{a}}. \tag{48}$$

Notice that equation (48) is in the form of the general dynamics (5), with an extra term $-\dfrac{f}{J}\displaystyle\int_0^t e_{\hat{a},2}(\tau)d\tau$.

Noting that $-(f/J)e_{\hat{a},2}$ is a stabilizing term of the second equation in (47), it will only result in an extra negative term in the Lyapunov stability analysis.

As a consequence, form Assumption 2, it follows that the external torque derivative is bounded, then Theorem 1 ensures that for a sufficiently large gain $\lambda_{\hat{a}}$, $s_{\hat{a}}$ tends to zero in finite-time.

2) Speed control: Substituting the estimated acceleration in the sliding surface (45), it follows that:

$$\hat{s}_\Omega = c_\Omega e_{\Omega,1} + \hat{e}_{\Omega,2} \tag{49}$$

where $c_\Omega > 0$ and $\hat{e}_{\Omega,2} = \hat{x}_{\Omega,2} - \dot{\Omega}^\star$. Notice that $e_{\Omega,2} = x_{\Omega,2} - \dot{\Omega}^\star = e_{\hat{a},2} + \hat{e}_{\Omega,2}$, then (49) is given by

$$\begin{array}{rcl} \hat{s}_\Omega & = & c_\Omega e_{\Omega,1} + e_{\Omega,2} - e_{\hat{a},2} \\ & = & c_\Omega e_{\Omega,1} + \dot{e}_{\Omega,1} - e_{\hat{a},2}. \end{array} \tag{50}$$

It follows that

$$\dot{e}_{\Omega,1} = s_\Omega - c_\Omega e_{\Omega,1} + e_{\hat{a},2}. \tag{51}$$

Taking the time derivative of s_Ω, one gets

$$\begin{array}{rcl} \dot{\hat{s}}_\Omega & = & c_\Omega \dot{e}_{\Omega,1} + \dot{\hat{e}}_{\Omega,2} \\ & = & c_\Omega e_{\Omega,2} + \dot{\hat{x}}_{\Omega,2} - \ddot{\Omega}^\star \\ & = & c_\Omega(e_{\hat{a},2} + \hat{e}_{\Omega,2}) - \dfrac{f}{J}\hat{x}_{\Omega,2} + \Gamma(v_q, i_d, i_q, \Omega) \\ & & + \dfrac{\lambda_{\hat{a}}^2}{2}\operatorname{sign}(e_{\hat{a},1}) - \ddot{\Omega}^\star. \end{array} \tag{52}$$

Choosing the controller v_q as follows

$$\begin{array}{rcl} v_q & = & Ri_q + Lp\Omega i_d + \phi_f p\Omega + \dfrac{2JL}{3p\phi_f}\Big[-c_\Omega \hat{e}_{\Omega,2} + \dfrac{f}{J}\hat{x}_{\Omega,2} \\ & & -\dfrac{\lambda_{\hat{a}}^2}{2}\operatorname{sign}(e_{\hat{a},1}) + \ddot{\Omega}^\star + w_{STC}^{\lambda_\Omega}(s_\Omega) \Big] \end{array} \tag{53}$$

where the function w_{STC}^{λ} is defined in (14), $\lambda_\Omega > 0$. Then, substituting (53) in (52), it follows that

$$\left\{ \begin{array}{rcl} \dot{e}_{\Omega,1} & = & \hat{s}_\Omega - c_\Omega e_{\Omega,1} + e_{\hat{a},2}, \\ \dot{\hat{s}}_\Omega & = & c_\Omega e_{\hat{a},2} + w_{STC}^{\lambda_\Omega}(\hat{s}_\Omega). \end{array} \right. \tag{54}$$

3) STC based on STO: The system in closed-loop with the control (53) using the estimates given by the observer (47) composed by the STO can be represented as

$$\begin{array}{l} \Pi : \left\{ \begin{array}{rcl} \dot{e}_{\Omega,1} & = & \hat{s}_\Omega - c_\Omega e_{\Omega,1} + e_{\hat{a},2}, \\ \dot{\hat{s}}_\Omega & = & c_\Omega e_{\hat{a},2} + w_{STC}^{\lambda_\Omega}(\hat{s}_\Omega). \end{array} \right. \\ \Xi : \left\{ \begin{array}{rcl} \dot{e}_{\hat{a},1} & = & e_{\hat{a},2} - 2\lambda_{\hat{a}}|e_{\hat{a},1}|^{\frac{1}{2}}\operatorname{sign}(e_{\hat{a},1}), \\ \dot{e}_{\hat{a},2} & = & -\dfrac{\lambda_{\hat{a}}^2}{2}\operatorname{sign}(e_{\hat{a},1}) - \dfrac{f}{J}e_{\hat{a},2} + \dot{\rho}_{\hat{a}}. \end{array} \right. \end{array} \tag{55}$$

As already discussed before, since the estimation error of Ξ converges to zero in finite time, then it follows that $e_{\hat{a},1} =$

$e_{\hat{a},2} = 0$. Furthermore, if the observer gains are chosen such that observation error converges faster than the dynamics of system Π, then one gets:

$$\begin{aligned} \dot{e}_{\Omega,1} &= \hat{s}_\Omega - c_\Omega e_{\Omega,1}, \\ \dot{s}_\Omega &= w^{\lambda_\Omega}_{STC}(\hat{s}_\Omega). \end{aligned} \tag{56}$$

The second equation of (56) is the STA, thus from Theorem 1, and taking λ_Ω sufficiently large ensures that $\hat{s}_\Omega = \dot{\hat{s}}_\Omega = 0$ is achieved in finite-time. As a result, the dynamics of $e_{\Omega,1}$ will converge asymptotically to zero and the convergence rate will depend on the value of c_Ω.

Remark 1: The practical implementation of STA for sliding mode observer based approaches was studied in [2] where the strategy proposed here is discussed. It has the disadvantage that the control (53) contains the discontinuous term $-\frac{\lambda^2_{\hat{a}}}{2}\mathrm{sign}(e_{\hat{a},1})$ which is not ideal for practical implementation. The observer proposed in [2] Section IV is indeed more suitable for our application. However the gain tuning approach proposed here can not be applied for third order system yet. A perspective of this work is the extension of the proposed approach for higher order system and not only limited to STA.

In Table I, we recall the notation used for control and observer designs, and in Table III gathers the four gains used in the control design.

Loop	Description	Symbol
Current control	current reference	$i_d^\star$
	sliding surface for i_d current control	s_d
Acceleration observer	mechanical state (rotor speed and acc.)	x_Ω
	acceleration observer state	$\hat{x}_\Omega$
	acceleration observer error	$e_{\hat{a}}$
	sliding surface for acceleration observer	$s_{\hat{a}}$
Speed Control	speed reference	$\Omega^\star$
	trajectory tracking error with x_Ω	e_Ω
	trajectory tracking error with $\hat{x}_\Omega$	$\hat{e}_\Omega$
	sliding surface for trajectory tracking error	s_Ω

TABLE I: Notations

V. Experimental results

Experimental results are presented to illustrate the performance of the proposed control algorithm applied to the PMSM. The experimental test bench consists of two motors connected by their shafts. A PMSM is used as a motor while a DC motor is used as a generator and act as a load. The PMSM is a Hurst motor (60W, 24V, 3000 rpm) and the DC motor is a Parvalux Brushed DC Motor (90W, 24Vdc, 3000 rpm). The DC load was chosen to be able to reach the PMSM limits. Both motors are driven using a dSPACE® MicroLabBox. The test bench is represented in Fig. 3. The parameters of the PMSM have been identified using [4] and are given in Table II along with some experimental characteristics.

Note that for this experimentation, to show the robustness of the proposed approach, a low cost motor was used, with a 250-line incremental encoder only (to be compared with classical 12 bits encoder with 4096 lines). It follows that the information of the motor $d-q$ variables is not very accurate , since they are obtained using the measured position.

Parameter	Symbol	Value	Unit
Stator resistance	R	0.405	Ω
Stator inductance	L	$300 \cdot 10^{-6}$	H
Viscous friction coefficient	f	$1.044 \cdot 10^{-4}$	Nm/(rad/s)
Moment of inertia	J	$2.5908 \cdot 10^{-4}$	kg.m^2
Permanent-magnet flux linkage	Φ_f	$7.63 \cdot 10^{-3}$	Wb
Number of poles	p	5	
Maximum voltage	V_{MAX}	12	V
PWM Frequency	f_{PWM}	20	kHz
Dead time	T_m	$0.5 \cdot 10^{-6}$	s
Sampling period	T_s	$1 \cdot 10^{-4}$	s

TABLE II: Parameters of the PMSM and experimentation characteristics

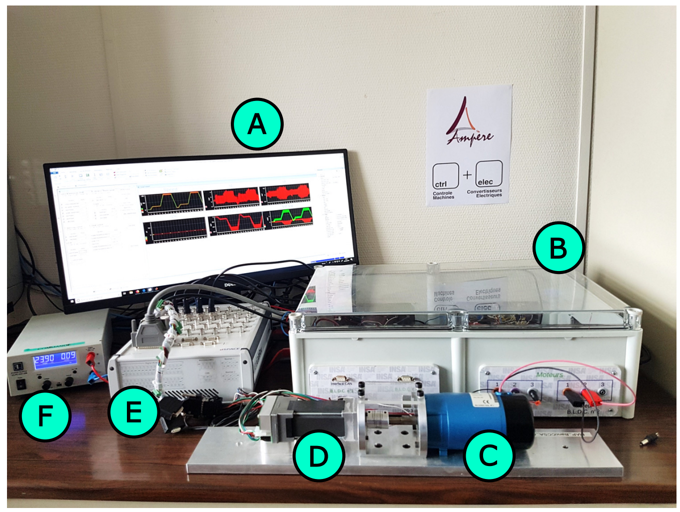


Fig. 3: Test Benchmark: A) GUI, B) Three-phase inverter, C) Load torque, D) PMSM, E) MicroLabBox, F) Power supply

Now, in order to illustrate the performance of the controller under different operation conditions: (low/high speed, with/without perturbation).

The speed and load torque references are given in Fig. 4, which correspond to a standard industrial benchmark [9]. As can be seen from Fig. 4, from 0 to 0.7 s the speed reference is equal to 0, from 0.7 to 4.2 s the control scheme is tested at low speed. From 5.7 to 8.6 s the proposed scheme is tested at high speed. For each speed level, a load torque is applied. The load is realized with the use of a current control on the DC motor. Note that the dynamics of the load torque has to be bounded.

Remark 2: In practice, the dynamics of physical signals is always bounded, then Assumption 2 is always verified. Notice that for the current and the speed loops, there is no perturbation as explained after (40) and (56), then the gains only need to be positive to guarantee the stability. For the observation loop, the bound d defined in (3) can be computed as $d = \frac{1}{J}\frac{\tau_{l_{max}}}{T_r}$, where $\tau_{l_{max}}$ is the maximum of the perturbation and T_r is the rising time of the DC motor used as a load. According to Fig. 4-bottom, $\tau_{l_{max}} = 0.25$ N.m and from the DC motor characteristics one gets $T_r \approx 100$ ms, then $\lambda_s \approx 515$ (see definition in (30)). Note that the chosen $\lambda_{\hat{a}}$ value (Table III) is smaller to limit the noise amplification, and it is reminded that the condition is conservative as explained at the end of

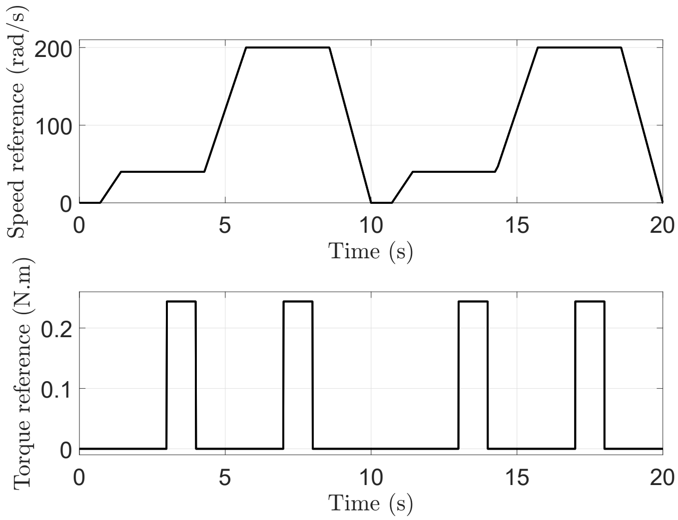


Fig. 4: Industrial test trajectories

Gain	Symbol	Value
d-axis current gain	λ_{i_d}	1500
Acceleration observer gain	$\lambda_{\hat{a}}$	100
Speed gain	λ_{Ω}	3000
Speed control surface constant	c_{Ω}	100

TABLE III: Controllers and observer tuning gains

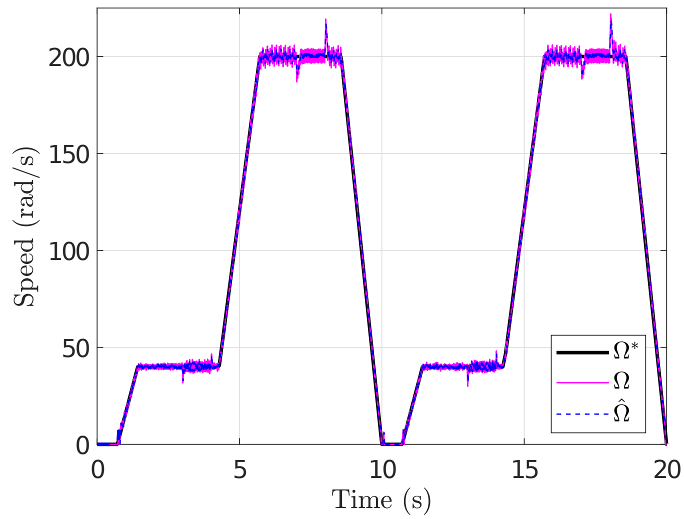
Section III that is why the system can be stable even if the condition $\lambda \geq \lambda_s$ is not verified.

The controller gains have been chosen as shown in Table III in order to achieve a good performance. It is important to recall that only four parameters are needed and that the tuning is simple, because each gain can be increased till getting the desired convergence time. Furthermore, one gain is used for the current control: λ_{i_d}, and two gains are necessary for the velocity control: c_{Ω}, $\lambda_{\hat{a}}$. Finally, the last gain is required for the observer: $\lambda_{\hat{a}}$. The number of gains for the single gain and classical Super Twisting Algorithm [16] is compared in Table IV. Note that for classical STA the number of gains is two instead of a single gain for the proposed approach. For the speed control, the extra gain comes from the computation of the sliding surface (49).

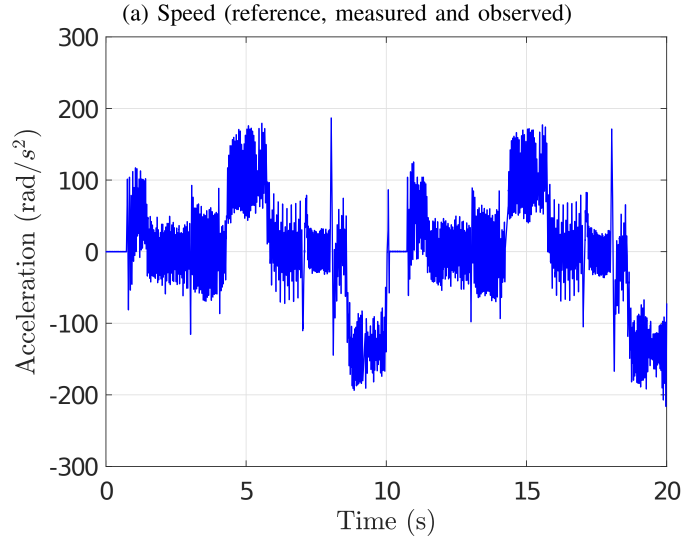
In Fig. 5a the speed reference, the measured speed as well as the estimated velocity used for the control are displayed and the observed acceleration is shown in Fig. 5b. The figure highlights very good reference tracking. The small overshoots on Fig. 5a are at the instant where the load torque is applied, it demonstrates that perturbation rejection is fast. Indeed the zoom of Fig. 5a, when the load is applied at high speed is represented in Fig. 6. In less than 0.15 seconds, the perturbation is rejected. Note that at high speed, the available voltage to produce torque to reject the perturbation is lower.

Controller/ Observer	Single gain STA	Classical STA
Current control	1	2
Speed control	2	3
Acceleration observer	1	2

TABLE IV: Comparison of number of gains



(a) Speed (reference, measured and observed)



(b) Estimated acceleration

Fig. 5: Speed and acceleration

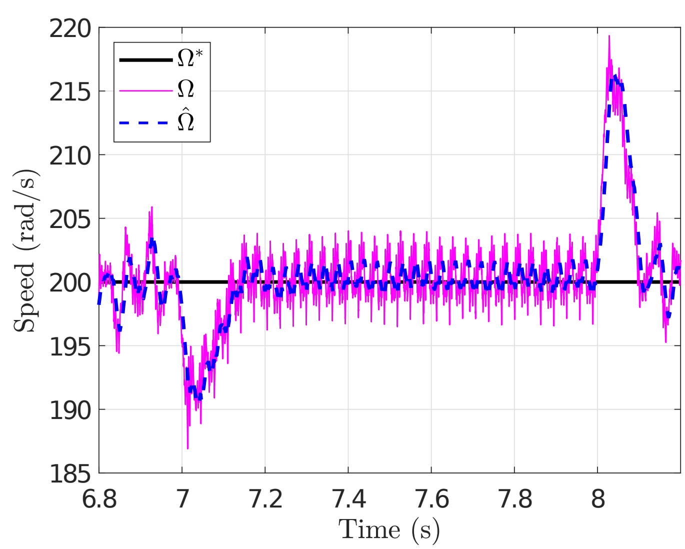


Fig. 6: Speed under load torque

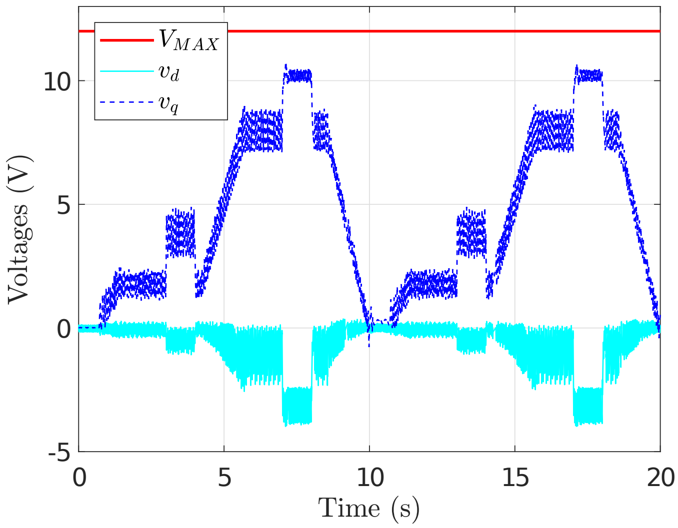


Fig. 7: d-q voltages

The phase voltages v_d and v_q are represented in Fig. 7. In the d-q coordinates one has

$$\left\| \begin{bmatrix} v_d & v_q \end{bmatrix} \right\| = \sqrt{v_d^2 + v_q^2} < V_{MAX} = 12\ V$$

The applied torque at high speed reaches these limits. It explains the fact that overshoots are present at high speed but not at low speed. Fig. 5a also shows the accuracy of the speed observation. Note that the motor is a low cost motor that has a high cogging torque and low cost encoder. It results in period the oscillations on Fig. 6, indeed, the oscillation period around $0.031s$ corresponds to the motor speed at $200rad/s$. In addition, to the good tracking performances, one can see that the observed speed is filtered compared to the measured speed (Fig. 6).

All the figures presented in this section demonstrate the efficiency of the proposed control strategy with the use of four tuning gains only.

VI. Conclusion

In this article, a new method to tune sliding mode controllers and observers based on STA has been presented. This method relies on the high-gain theory and allows to easily tune the gains of the algorithm by using one gain only. It is essential to mention that the novelty of this article is based on the gain tuning method and not on the control design of the STA. Furthermore, a stability analysis, based on a Lyapunov approach, which guarantees the finite-time convergence towards zero of the proposed algorithm has been presented, where sufficient conditions have been obtained. The performances of the method have been evaluated experimentally on a PMSM and shows very good performances. This new method will make the tuning of controllers much easier for the control engineers because it reduces the number of tuning gains and the tuning only consists in increasing each gain till achieving the desired convergence time. Future developments will target the extension to higher order systems as well as the reduction of the conservatism of the stability condition.

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