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Schwarz algorithms for ocean-atmosphere coupled problems including turbulent boundary layer parameterizations

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DD26, December 9th 2020

PhD under the supervision of Eric Blayo and Florian Lemarié
Application of ocean-atmosphere coupling

Various physical phenomena are governed by the ocean-atmosphere coupling: long term predictions to short term predictions.

climate modeling  seasonal forecasts  short term predictions

Improve the representation of ocean-atmosphere interactions
Complexity of ocean-atmosphere fluxes

- Turbulent Boundary layer $\rightarrow$ complex parametrization.

- Near the interface: fluxes estimated by (complicated) formulas depending on the jump of the solution.
The ocean-atmosphere coupling algorithms

Two current approaches, both mathematically unsatisfactory:

- Synchronous coupling at the time step (local in time)
- Asynchronous coupling by time windows (global in time)

- a lot of communication ⇒ inefficient implementations
- physical validity and numerical stability issues Lemarié & al. (2015), Beljaars & al. (2017)
- balance of the average flows over each time window
- synchronization problem
Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

Objectives : improve the mathematical coupling methods
Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

Objectives: improve the mathematical coupling methods

A numerical method that would solve these problems
⇒ Schwarz algorithms

French COCOA ANR Project: Study of an iterative process on ocean-atmosphere coupling. In particular

- Implementation of Schwarz algorithms in realistic climate models
- Theoretical work on these algorithms in this context.
A 1D Simplified coupled ocean-atmosphere model

Hypotheses:
- Focus on the dynamical part
- Physical restriction (1D)
- Taking into account turbulent parametrisations

A reasonably realistic model on $\mathbf{U} = (u, v)^T$ horizontal ocean/atmosphere currents.

Interface is a buffer zone with its own parameterization.
Non-linear coupled ocean-atmosphere model

\[ \partial_t U + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} U - \partial_z (\nu_{atm}(u^*, z) \partial_z U) = F \]

**ocean atmosphere specificities:**
- Coriolis effect
- non constants viscosities
- non linear equation
- non linear interface condition

**SBL** \( \| U(\delta_{atm}) - U(\delta_{oce}) \| \) \( \rightarrow U^* \)

\[ \rho_a \nu_{atm}(u^*, \delta_{atm}) \partial_z U(\delta_{atm}) = \rho_a (u^*)^2 e_\tau \]
\[ = \rho_o \nu_{oce}(u^*, \delta_{oce}) \partial_z U(\delta_{oce}) \]
Linear coupled ocean-atmosphere model

\[ \begin{align*}
\frac{\partial t}{\partial t} \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{atm}(z) \partial_z \mathbf{U}) &= \mathcal{F} \\
\rho_a \nu_{atm}(0^+) \partial_z \mathbf{U}(0^+) &= \rho_o \nu_{oce}(0^-) \partial_z \mathbf{U}(0^-) \\
\mathbf{U}(0^-) &= \mathbf{U}(0^+) \\
\frac{\partial t}{\partial t} \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{oce}(z) \partial_z \mathbf{U}) &= \mathcal{F}
\end{align*} \]

\[ \begin{align*}
\Rightarrow \text{Ekman Problem} \\
\text{widely used by physicists:} \\
\text{Ekman (1905), Madsen (1977), Grisogono (1995), Lewis & Belcher (2004)}
\end{align*} \]

Linear problem specificities:
- Coriolis effect
- non constants viscosities
1 Ocean atmosphere coupling
2 Schwarz algorithm and OA specificities
3 Impact of Coriolis effect
4 Impact of non-constants viscosities
5 Particular case OA coupling
6 Current and future work
Schwarz algorithm

\[ \mathcal{L}_2(u_2) = 0 \text{ in } \Omega_2 \]
\[ B_2(u_2) = 0 \text{ on } \partial \Omega_2 \setminus \partial \Omega_1 \]
\[ C_2(u_2|_\Gamma, u_1|_\Gamma) = 0 \]
\[ \mathcal{L}_1(u_1) = 0 \text{ in } \Omega_1 \]
\[ B_1(u_1) = 0 \text{ on } \partial \Omega_1 \setminus \partial \Omega_2 \]
\[ C_1(u_1|_\Gamma, u_2|_\Gamma) = 0 \]

first guess \( u_2^0 \) then

\[
\begin{aligned}
\mathcal{L}_1 u_1^n &= \mathcal{F}_1 & \text{on } \Omega_1 \times ]0, T[ \\
B_1 u_1^n &= \mathcal{G}_1 & \text{on } \partial \Omega_1^\text{ext} \times ]0, T[ \\
u_1^n(t = 0) &= u_0 & \text{on } \Omega_1 \\
C_{1,1} u_1^n &= C_{1,2} u_2^{n-1} & \text{on } \Gamma \\
\end{aligned}
\]

\[
\begin{aligned}
\mathcal{L}_2 u_2^n &= \mathcal{F}_2 & \text{on } \Omega_2 \times ]0, T[ \\
B_2 u_2^n &= \mathcal{G}_2 & \text{on } \partial \Omega_2^\text{ext} \times ]0, T[ \\
u_2^n(t = 0) &= u_0 & \text{on } \Omega_2 \\
C_{2,2} u_2^n &= C_{2,1} u_1^n & \text{on } \Gamma \\
\end{aligned}
\]
Convergence factor for linear problems

\[ \rho^{obs} = \frac{\| e_j^n(z = 0) \|_2}{\| e_j^{n-1}(z = 0) \|_2} \quad e_j^n = u_j^n - u^{exact} \]

- 1D Stationary case: solve the equation for each iteration
- 1D Non-stationary case: use Fourier transform in time

\[ \Rightarrow \rho(\omega) = \frac{\| \hat{e}_j^n(\omega, 0) \|}{\| \hat{e}_j^{n-1}(\omega, 0) \|} \]
Convergence factor for linear problems

\[ \rho^{obs} = \frac{||e_j^n(z = 0)||_2}{||e_j^{n-1}(z = 0)||_2} \quad e_j^n = u_j^n - u^{exact} \]

- **1D Stationary case**: solve the equation for each iteration
- **1D Non-stationary case**: use Fourier transform in time

\[ \Rightarrow \rho(\omega) = \frac{||\hat{e}_j^n(\omega, 0)||}{||\hat{e}_j^{n-1}(\omega, 0)||} \]

\[ \min_{\omega_{\text{min}} \leq |\omega| \leq \omega_{\text{max}}} \rho(\omega) \leq \rho^{obs} \leq \max_{\omega_{\text{min}} \leq |\omega| \leq \omega_{\text{max}}} \rho(\omega) \]

Discretized-in-time algorithm: \( \omega_{\text{max}} = \frac{\pi}{\Delta t} \) and \( \omega_{\text{min}} = \frac{\pi}{T} \)
Coupled Ekman problems

\[ \partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{atm}(z) \partial_z \mathbf{U}) = \mathcal{F} \]

- Dirichlet conditions

\[ \rho_a \nu_{atm}(0^+) \partial_z \mathbf{U}(0^+) = \rho_o \nu_{oce}(0^-) \partial_z \mathbf{U}(0^-) \]

\[ \mathbf{U}(0^-) = \mathbf{U}(0^+) \]

\[ \partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{oce}(z) \partial_z \mathbf{U}) = \mathcal{F} \]

⇒ Specificities:
- Coriolis effect (components are coupled)
- Non constant diffusion coefficients with interface discontinuity
## State of the art

<table>
<thead>
<tr>
<th></th>
<th>Constante diffusion</th>
<th>Variable in space diffusion</th>
</tr>
</thead>
</table>
| **Stationary** | adv-diff 2D Japhet et al., 2001  
*eq. Helmholtz* Dubois, 2007 Magoulès et al., 2004 | *eq. de diffusion* Lions, 1990 |
| **Nonstationary** | heat equation Gander and Halpern, 2003  
*reaction-reaction-diff 2D* Bennequin et al., 2016 and Gander et al., 2007 | diffusion 1D eq. Lemarié et al., 2013 |
| without Coriolis | 2D shallow water Martin, 2003 *primitives eq. 3D* Audusse et al., 2009 | |
| with Coriolis | | diffusion 1D eq. + Coriolis Thery et al., 2020 |
Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components

\[
\partial_t \mathbf{U}_j + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U}_j - \nu_j \partial_z^2 \mathbf{U}_j = \mathbf{F}_j
\]

+ Dirichlet external conditions
+ Initial conditions
\[
\mathbf{U}_1(0^-) = \mathbf{U}_2(0^+),
\nu_1(0^-) \partial_z \mathbf{U}_1 = \nu_2(0^+) \partial_z \mathbf{U}_2
\]
Impact of the Coriolis effect: example with constant viscosities

- Coriolis effect → coupling \( \mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix} \) components
- Study of the convergence with change of variable \( \phi = u + iv \).

\[
\partial_t \phi_j + i f \phi_j - \nu_j \partial_z^2 \phi_j = F_{\phi_j}
\]

+ Dirichlet external conditions
+ Initial conditions
\[
\begin{align*}
\phi_1(0^-) &= \phi_2(0^+) \\
\nu_1(0^-) \partial_z \phi_1 &= \nu_2(0^+) \partial_z \phi_2
\end{align*}
\]
Impact of the Coriolis effect: example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error $e_j$

\[
\begin{aligned}
\partial_t e_j + i f e_j - \nu_j \partial_z^2 e_j &= 0 \\
+ \text{Dirichlet external conditions} \\
+ \text{Null initial condition} \\
e_1(0^-) &= e_2(0^+) \\
\nu_1 \partial_z e_1 &= \nu_2 \partial_z e_2
\end{aligned}
\]
Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error $e_j$
- Fourier Transform $\Rightarrow$ frequencies shifted by $f$ :

$$i(\omega + f)\hat{e}_j - \nu_j \partial_z^2 \hat{e}_j = 0$$

+ Dirichlet external conditions

$$\hat{e}_1(0^-) = \hat{e}_2(0^+)$$

$$\nu_1 \partial_z \hat{e}_1 = \nu_2 \partial_z \hat{e}_2$$
With Dirichlet-Neumann interface conditions

\[
\begin{align*}
\hat{e}_1^n(\omega, 0) &= \hat{e}_2^{n-1}(\omega, 0) \\
\nu_2 \partial_z \hat{e}_2^n(\omega, 0) &= \nu_1 \partial_z \hat{e}_1^n(\omega, 0)
\end{align*}
\]

Convergence factor

- Infinite domains:

\[
\rho_{\text{cst}}^{DN}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}}
\]

Independent of time frequency

- Finite domains:

\[
\rho_{\text{cst}}^{DN}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}} \left| \frac{\tanh \left( z_2^\infty \sqrt{i \frac{\omega + f}{\nu_2}} \right)}{\tanh \left( z_1^\infty \sqrt{i \frac{\omega + f}{\nu_1}} \right)} \right|
\]
Case 1: \(|z_2^\infty \sqrt{\nu_1}| > |z_1^\infty \sqrt{\nu_2}|\)

- Convergence factor behavior: \(|\rho(\omega)| < \frac{z_2^\infty}{z_1^\infty} \frac{\nu_1}{\nu_2}\) and \(\rho(\omega) \xrightarrow{|\omega| \to \infty} \sqrt{\frac{\nu_1}{\nu_2}}\)
- Impact of Coriolis: shifts the local maximum and non symmetric graph
Case 2: $|z_2^\infty \sqrt{\nu_1}| < |z_1^\infty \sqrt{\nu_2}|$

- Convergence factor behavior: $|\rho(\omega)| < \sqrt{\frac{\nu_1}{\nu_2} \frac{Q(x_1)}{Q(x_2)}}$ and $\rho(\omega) \underset{\omega \to \infty}{\to} \sqrt{\frac{\nu_1}{\nu_2}}$
  
  with $Q(x) = |\tanh((1 + i)x)|$ and $x_1, x_2$ solution of the transcendental equation

- Impact of Coriolis: shifts the local minimum and non symmetric graph
The effect of turbulence

Parametrisation of turbulence $\Rightarrow$ non constant viscosity

In ocean-atmosphere context:
- KPP viscosity (O’Brien, 1970)
  - affine profile close to the surface
  - parabolic or cubic profile in the turbulent zone
  - constant profile in free zone
$\Rightarrow$ convergence for variable viscosity profile
Convergence factor with non-constants viscosities

\[ i(f + \omega)\hat{e}_j(z, t) - \partial_z(\nu_j(z)\partial_z\hat{e}_j(z, t)) = 0 \]

Mathematical tools to calculate converge:

- with \( \nu_j(z) = a_jz + b_j \) \( \rightarrow \) Bessel’s functions
- with \( \nu_j(z) = a_jz^2 + bjz + cj \) \( \rightarrow \) Legendre polynomials.

\( \Rightarrow \) The convergence factor depends on the global viscosity profile
**Convergence factor with non-constants viscosities**

\[ i(f + \omega)\hat{e}_j(z, t) - \partial_z(\nu_j(z)\partial_z\hat{e}_j(z, t)) = 0 \]

Mathematical tools to calculate converge:
- with \( \nu_j(z) = a_jz + b_j \) → Bessel’s functions
- with \( \nu_j(z) = a_jz^2 + b_jz + c_j \) → Legendre polynomials.

⇒ The convergence factor depends on the global viscosity profile

**Dirichlet-Neumann interface conditions**

\[
\rho_{DN}(\omega) \xrightarrow{|\omega| \to \infty} \sqrt{\frac{\nu_1(0)}{\nu_2(0)}}
\]

for all viscosities profiles

\[
\rho_{DN}^{aff}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{\mu_1 \ln(1 + 1/\mu_1)}
\]

\[
\rho_{DN}^{par}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{4\mu_1 \arccos(\sqrt{1 + 4\mu_1})}
\]

with \( \mu_1 = \left| \frac{\nu_1(0)}{\partial_z\nu_1(0)z_1^\infty} \right| \)
The particular case ocean-atmosphere coupling

Stationary case for any interface condition:

- without Coriolis → free zones have a huge influence
- with Coriolis → turbulent zones have a bigger influence
The particular case ocean-atmosphere coupling

Non-Stationary case for any interface condition:

- $|\omega + f| < 10^{-11} \rightarrow$ influenced by free zone
- $|\omega + f| > 10^{-5} \rightarrow$ influenced by turbulent zone

Example: convergence for all frequencies except frequencies close to $-f$
Conclusion for linear problems

- Impact of Coriolis effect: shift of the graph
  ⇒ perturbation of the algorithm’s behavior
- Convergence factor depends of the global viscosities profile
  ⇒ mathematical tools to calculate the convergence factor (Bessel and Legendre functions)
  ⇒ simplification must be made with caution

⇒ Results are shown in: S. Thery, C. Pelletier, F. Lemarié and Blayo E., 2020: Coupling two Ekman layers with a Schwarz algorithm. under review
Conclusion for linear problems

- Impact of Coriolis effect: shift of the graph
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Ocean-atmosphere coupling

The Coriolis effect and the turbulence zones have a big impact on the convergence
The non-linear model

\[ \partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{atm}(u^*, z) \partial_z \mathbf{U}) = \mathcal{F} \]

\[ \nu_{atm}(u^*) \partial_z \mathbf{U} = (u^*)^2 e_\tau \]

\[ u^* \propto (\| \mathbf{U}(\delta_{atm}) - \mathbf{U}(\delta_{oce}) \|) \]

\[ \rho_{oce} \nu_{oce}(u^*) \partial_z \mathbf{U} = -\rho_{atm}(u^*)^2 e_\tau \]

\[ \partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{oce}(u^*, z) \partial_z \mathbf{U}) = \mathcal{F} \]

Application of Schwarz algorithms on non-linear model
- Study of the well-posedness of the problem
- Study of convergence of the algorithm

Current and future work
To a realistic model

Sophie THERY (UGA, LJK)
Schwarz algorithms for OA coupling
DD26, December 9th 2020 23 / 26
Current work on the non-linear model

**Stationary case:** using tools from linear problem

- unique solution consistent with the physical constraints
- without Coriolis effect: fast divergence & free zones have a huge influence
- with Coriolis effect: fast convergence & turbulent zones have a bigger influence
Current work on the non-linear model

Stationary case: using tools from linear problem

- unique solution consistent with the physical constraints
- without Coriolis effect: fast divergence & free zones have a huge influence
- with Coriolis effect: fast convergence & turbulent zones have a bigger influence

Non stationary case:

No theoretical method for solve this problem
⇒ experimental results ⇒ similarities with linear problem behavior:

- unique solution consistent with the physical constraints
- if \( \omega_{\text{min}} \leq |f| \leq \omega_{\text{max}} \) → divergence
- if \( |f| \leq \omega_{\text{min}} \) or \( \omega_{\text{max}} \leq |f| \) → convergence
**Test on a real model**

### IPSL-CM (3D)
- **test on a climate model (3D)**
- **Convergence in two iterations for 90% points**

Thank you