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Abstract—This paper proposes a static decentralized controller design method for an HVDC link embedded in a large-scale AC grid based on the Linear Matrix Inequalities (LMIs) technique. The considered studied system is treated as composed by two overlapping subsystems. The interconnection between the two subsystems is nonlinear and is treated by increasing disturbance rejection and robustness against norm-bounded parametric uncertainties of the closed-loop. Indeed, the overall robustness and tracking ability of the entire closed-loop is significantly improved with the proposed controller. This is both analytically ensured in the synthesis of the gains of the regulators and, next proven by validation simulations. The overall robustness and tracking ability of the entire closed loop system can be significantly improved. It is shown that this control can deal with HVDC link with different lengths, including the short ones for each the coupling between the two converters is strong.

Another advantage of the proposed decoupling control is the *resilience*: as no communication is used between the two stations, in case of failure of one converter or loss of measures as control, the control of the other converter is not affected. Moreover, it guarantees the stability of the overall system. Finally, the efficiency and robustness of the proposed controllers are tested and compared with each others, to illustrate the control synthesis and its effectiveness.

Keywords—VSC-HVDC, H_∞ control, robust control, decentralized control, resilience, LMI.

I. Introduction

WITH the rapid development in scale of power grid, due to the need of state and (or) output information from far-off regions, the solution with classical centralized control becomes increasingly infeasible. In addition, with the enormous diversity in power supplying, the required information is much harder to obtain as it is the case for the massive integration of dispersed renewable energies on distribution systems. On one hand, controllers should be decentralized as there are many generators to be dealt with. As a matter of fact, flexibility of modern power systems should be increased to operate the systems closer to their limits. In contrast, the robust optimal decentralized control meet the need of having less required information in control, and can still keep the same performance as in the former techniques. Simultaneously, with the gradual evolution of decentralized control from 1980s to 2010s [1], [2] and robust optimal control from 1990s to 2010s [3]-[5], it is now the time for decentralized control development in power systems.

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Over long distance, with the increasing in length [6] of modern VSC-HVDC, centralized control become costly and ineffective with introduced delay in feedback signals from remote converters. There are many research works for VSC-HVDC, including decoupling control in converter stations using feedback linearization [7], small signal stability [8] and Linear matrix inequalities (LMIs)-based robust control [9]. Also, multi-terminal HVDC [10], [11] as well as DC grid, with many advantages over AC grid [10]-[28], are rapidly developed and intensively studied in recent years. However, these papers hardly cover the uncertain aspects, and non of them met the need of decentralized techniques.

The main contribution of this paper is a robust H_∞ decentralized control for embedded HVDC system with external bounded disturbances, in order to improve the resilience of the implementation, the dynamic behavior performance under wide range of parameter uncertainties and to minimize the tracking reference error. The sufficient conditions are formulated in the format of Linear matrix inequalities (LMIs). The controllers proposed herein are synthesized to satisfy H_∞ performance for disturbance attenuation. Lyapunov function has been employed to satisfy the stability condition. In this paper, we develop LMI stability conditions for the linear system without parametric uncertainties and also for the linear system with uncertainties. The proposed method has the following advantages:

1) Overall resilience, robustness and tracking ability of the entire closed loop system can be significantly improved. Moreover, it guarantees the stability of the overall system.
2) Improves the dynamic behavior performance under wide range of operating conditions and minimize the tracking reference error.

The rest of the paper is organized as follows. In section II, a state space representation of the HVDC dynamics is given. The proposed robust H_∞ decentralized control structure is developed in section III. Simulation results that illustrates the effectiveness of the proposed strategies are presented in section IV. Finally, the conclusions and future prospects are presented in section V.

II. VSC-HVDC POWER STRUCTURE AND ITS DYNAMICS MODEL

A. *VSC-HVDC nonlinear model*

Consider a VSC-HVDC link, which interconnects two AC zones:

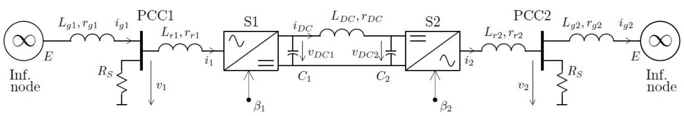

Fig. 1. VSC-HVDC power structure

If average models are taken for converters, he VSC-HVDC nonlinear system (in figure 1) can be modeled by following differential equations [14]-[17]:

$$\left\{\begin{array}{l}
\frac{dx_1}{dt} = \left(-\frac{R_s}{L_{g1}} - \frac{r_{g1}}{L_{g1}}\right) x_1 + \omega x_2 + \frac{R_s}{L_{g1}} x_3 + \frac{E}{L_{g1}} \\
\frac{dx_2}{dt} = \left(-\frac{R_s}{L_{g1}} - \frac{r_{g1}}{L_{g1}}\right) x_2 - \omega x_1 + \frac{R_s}{L_{g1}} x_4 \\
\frac{dx_3}{dt} = \left(-\frac{R_s}{L_{r1}} - \frac{r_{r1}}{L_{r1}}\right) x_3 + \frac{R_s}{L_{r1}} x_1 + \omega x_4 - \frac{x_5}{2L_{r1}} u_1 \\
\frac{dx_4}{dt} = \left(-\frac{R_s}{L_{r1}} - \frac{r_{r1}}{L_{r1}}\right) x_4 + \frac{R_s}{L_{r1}} x_2 - \omega x_3 - \frac{x_5}{2L_{r1}} u_2 \\
\frac{dx_5}{dt} = \frac{3}{2C_1} x_3 u_1 + \frac{3}{2C_1} x_4 u_2 - \frac{2}{C_1} x_6 \\
\frac{dx_6}{dt} = \frac{1}{2L_{DC}} x_5 - \frac{r_{DC}}{L_{DC}} x_6 - \frac{1}{2L_{DC}} x_7 \\
\frac{dx_7}{dt} = \frac{2}{C_2} x_6 + \frac{3}{2C_2} x_{10} u_3 + \frac{3}{2C_2} x_{11} u_4 \\
\frac{dx_8}{dt} = \left(-\frac{R_s}{L_{g2}} - \frac{r_{g2}}{L_{g2}}\right) x_8 + \omega x_9 + \frac{R_s}{L_{g2}} x_{10} + \frac{E}{L_{g2}} \\
\frac{dx_9}{dt} = \left(-\frac{R_s}{L_{g2}} - \frac{r_{g2}}{L_{g2}}\right) x_9 - \omega x_8 + \frac{R_s}{L_{g2}} x_{11} \\
\frac{dx_{10}}{dt} = \left(-\frac{R_s}{L_{r2}} - \frac{r_{r2}}{L_{r2}}\right) x_{10} + \frac{R_s}{L_{r2}} x_8 + \omega x_{11} - \frac{x_7}{2L_{r2}} u_3 \\
\frac{dx_{11}}{dt} = \left(-\frac{R_s}{L_{r2}} - \frac{r_{r2}}{L_{r2}}\right) x_{11} + \frac{R_s}{L_{r2}} x_9 - \omega x_{10} - \frac{x_7}{2L_{r2}} u_4
\end{array}\right. \tag{1}$$

with:

$$\left\{\begin{array}{ll}
x_1 = i_{g1d}, & x_2 = i_{g1q} \\
x_3 = i_{1d}^{\infty}, & x_4 = i_{1q}^{\infty} \\
x_5 = v_{DC1}, & x_6 = i_{DC} \\
x_7 = v_{DC2}, & x_8 = i_{g2d} \\
x_9 = i_{g2q}, & x_{10} = i_{2d}^{\infty} \\
x_{11} = i_{2q}^{\infty} &
\end{array}\right. \quad
\left\{\begin{array}{l}
u_1 = \beta_{1d} \\
u_2 = \beta_{1q} \\
u_3 = \beta_{2d} \\
u_4 = \beta_{2q}
\end{array}\right. \tag{2}$$

The output of the system are:

$$\left\{\begin{array}{l}
y_1 = v_{DC1} = x_5 \\
y_2 = Q_1 = \dfrac{3}{2} R_s x_2 x_3 - \dfrac{3}{2} R_s x_1 x_4 \\
y_3 = P_2 = \dfrac{3}{2} R_s x_8 x_{10} - \dfrac{3}{2} R_s x_{10}^2 + \dfrac{3}{2} R_s x_9 x_{11} \\
\qquad - \dfrac{3}{2} R_s x_{11}^2 \\
y_4 = Q_2 = \dfrac{3}{2} R_s x_9 x_{10} - \dfrac{3}{2} R_s x_8 x_{11}
\end{array}\right. \tag{3}$$

where i_{DC} is the DC line current, v_{DC1} is the DC voltage for VSC on left side, i_{g1d} and i_{g1q} are the grid current on d and q axis for AC filter on the left side, respectively, i_{g2d}, i_{g2q} are the grid current on d and q axis for AC filter on the right side, β_{1d}, β_{1q}, v_{DC1} are variables for VSC on left side, β_{2d}, β_{2q}, v_{DC2} are variables for VSC on right side (β_{ij}, $i \in \{1,2\}$, $j \in \{1,2\}$ are the duty ratios of the converters, i.e., the controls of the VSC system), i_{2d}^{∞}, i_{2q}^{∞}, are AC grid currents on the right side, i_{1d}^{∞}, i_{1q}^{∞}, are AC grid currents on the left side, L_{g1}, L_{g2}, r_{g1}, r_{g2}, L_{r1}, L_{r2}, r_{r1}, r_{r2}, R_s, C_1, C_2, r_{DC}, L_{DC}, E and w are constants, P_2, Q_1,Q_2 are active power and reactive power, repectively. The system parameters are:

S.N	Variable Names	Constants	Values
1	AC grid inductance	$L_{g1} = L_{g2}$	$0.11H$
2	AC grid resistance	$r_{g1} = r_{g2}$	0.01Ω
3	Filter inductance	$L_{r1} = L_{r2}$	$0.025H$
4	Filter resistance	$r_{r1} = r_{r2}$	0.01Ω
5	AC filter resistance	R_s	2Ω
6	DC capacitor filter	$C_1 = C_2$	$220*10^{-6}F$
7	DC line resistance	r_{DC}	1.39Ω
8	DC line inductance	L_{DC}	$1.59*10^{-2}H$
9	Grid voltage	E	$187*10^{3}V$
10	Grid frequency	ω	$314 rad/sec$

B. *Exact Linearization of VSC-HVDC nonlinear model*

The HVDC model (1) is a nonlinear model in the form of:

$$\dot{X} = f(X,U), \qquad Y = g(X) \tag{4}$$

Hence, its linerization around an equilibrium point (X^*, U^*) is:

$$\Delta\dot{X} = A\Delta X + B\Delta U, \qquad \Delta Y = B\Delta X \tag{5}$$

where:

$$\Delta X = X - X^*, \Delta U = U - U^*, A = \left.\frac{\partial f(X,U)}{\partial X}\right|_{(X^*,U^*)}$$

$$B = \left.\frac{\partial f(X,U)}{\partial U}\right|_{(X^*,U^*)}, C = \left.\frac{\partial g(X,U)}{\partial X}\right|_{(X^*,U^*)}$$

$$X^* = \begin{bmatrix} x_1^* & x_2^* & x_3^* & x_4^* & x_5^* & x_6^* & x_7^* & x_8^* & x_9^* & x_{10}^* & x_{11}^* \end{bmatrix}^T$$

$$U^* = \begin{bmatrix} u_1^* & u_2^* & u_3^* & u_4^* \end{bmatrix}^T$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix}, B = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \\ 0 & B_{32} \end{bmatrix}, C = \begin{bmatrix} C_{11} & 0 \\ C_{12} & C_{22} \\ 0 & C_{23} \end{bmatrix}^T$$

with:

$$A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{2}{C_1} \end{bmatrix}^T, A_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{2L_{DC}} \end{bmatrix};$$

$$A_{22} = \begin{bmatrix} -\frac{r_{DC}}{L_{DC}} \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} \frac{-1}{2L_{DC}} & 0 & 0 & 0 & 0 \end{bmatrix}, A_{32} = \begin{bmatrix} \frac{2}{C_2} & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$A_{11} = \begin{bmatrix}
\frac{-R_s}{L_{g1}} - \frac{r_{g1}}{L_{g1}} & \omega & \frac{R_s}{L_{g1}} & 0 & 0 \\
-\omega & \frac{-R_s}{L_{g1}} - \frac{r_{g1}}{L_{g1}} & 0 & \frac{R_s}{L_{g1}} & 0 \\
\frac{R_s}{L_{r1}} & 0 & \frac{-R_s}{L_{r1}} - \frac{r_{r1}}{L_{r1}} & \omega & \frac{-u_1^*}{2L_{r1}} \\
0 & \frac{R_s}{L_{r1}} & -\omega & \frac{-R_s}{L_{r1}} - \frac{r_{r1}}{L_{r1}} & \frac{-u_2^*}{2L_{r1}} \\
0 & 0 & \frac{3u_1^*}{2C_1} & \frac{3u_2^*}{2C_1} & 0
\end{bmatrix}$$

$$A_{33} = \begin{bmatrix}
0 & 0 & 0 & \frac{3u_3^*}{2C_2} & \frac{3u_4^*}{2C_2} \\
0 & \frac{-R_s}{L_{g2}} - \frac{r_{g2}}{L_{g2}} & \omega & \frac{R_s}{L_{g2}} & 0 \\
0 & -\omega & \frac{-R_s}{L_{g2}} - \frac{r_{g2}}{L_{g2}} & 0 & \frac{R_s}{L_{g2}} \\
\frac{-u_3^*}{2L_{r2}} & \frac{R_s}{L_{r2}} & 0 & \frac{-R_s}{L_{r2}} - \frac{r_{r2}}{L_{r2}} & \omega \\
\frac{-u_4^*}{2L_{r2}} & 0 & \frac{R_s}{L_{r2}} & -\omega & \frac{-R_s}{L_{r2}} - \frac{r_{r2}}{L_{r2}}
\end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{x_5^*}{2L_{r1}} & 0 \\ 0 & -\frac{x_5^*}{2L_{r1}} \\ \frac{3x_3^*}{2C_1} & \frac{3x_4^*}{2C_1} \end{bmatrix}; B_{32} = \begin{bmatrix} \frac{3x_{10}^*}{2C_2} & \frac{3x_{11}^*}{2C_2} \\ 0 & 0 \\ 0 & 0 \\ -\frac{x_7^*}{2L_{r2}} & 0 \\ 0 & -\frac{x_7^*}{2L_{r2}} \end{bmatrix}$$

$$B_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix}; B_{22} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$C_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ -\frac{3R_s x_4^*}{2} & \frac{3R_s x_3^*}{2} & \frac{3R_s x_2^*}{2} & -\frac{3R_s x_1^*}{2} & 0 \end{bmatrix}$$

$$C_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; C_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_{23} = \begin{bmatrix} 0 & \frac{3R_s x_{10}^*}{2} & \frac{3R_s x_{11}^*}{2} & \frac{3R_s x_8^*}{2} - 3R_s x_{10}^* & \frac{3R_s x_9^*}{2} - 3R_s x_{11}^* \\ 0 & -\frac{3R_s x_{11}^*}{2} & \frac{3R_s x_{10}^*}{2} & \frac{3R_s x_9^*}{2} & -\frac{3R_s x_9^*}{2} \end{bmatrix}$$

C. Decentralized HVDC linear model

The linearized HVDC system (5) can be rewritten into 2 overlapping subsystems:

$$\begin{aligned} (I) &\begin{cases} \dot{X}_1 = A_1 X_1 + B_1^1 U_1 + B_2^1 W_1 \\ Y_1 = C_1 X_1 \end{cases} \\ (II) &\begin{cases} \dot{X}_2 = A_2 X_2 + B_1^2 U_2 + B_2^2 W_2 \\ Y_2 = C_2 X_2 \end{cases} \end{aligned} \tag{6}$$

where $W_1 = x_7$ for system (I), $W_1 = x_5$ for system (II), x_6 is the common state, and:

$$\begin{aligned} X_1 &= \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T \\ X_2 &= \begin{bmatrix} x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{bmatrix}^T \\ A_1 &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; A_2 = \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix} \\ B_1^1 &= \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}; B_1^2 = \begin{bmatrix} B_{22} \\ B_{32} \end{bmatrix} \\ C_1 &= \begin{bmatrix} C_{11} & C_{12} \end{bmatrix}; C_2 = \begin{bmatrix} C_{22} & C_{23} \end{bmatrix} \\ B_2^1 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{-1}{2L_{DC}} \end{bmatrix}^T \\ B_2^2 &= \begin{bmatrix} \frac{1}{2L_{DC}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \end{aligned}$$

D. Extended System for Reference Tracking

Each subsytem in decentralized linearized HVDC system (equation (6)) is in the form of a linear system with disturbance

$$\dot{X} = AX + B_1 U + B_2 W, \qquad Y = CX \tag{7}$$

with the error between output and the reference

$$e = -CX + Y_{ref}. \tag{8}$$

Consider the extended state and output are:

$$\bar{X} = \begin{bmatrix} \dot{X} \\ e \end{bmatrix}; \bar{Y} = e \tag{9}$$

As a result, the extended system to be stabilized in order to ensure references tracking is:

$$\begin{cases} \dot{\bar{X}} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \bar{X} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \dot{U} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \dot{W} \\ Y = \begin{bmatrix} 0 & I \end{bmatrix} \bar{X} \end{cases} \tag{10}$$

III. Proposed Robust H$_\infty$ Decentralized State Feedback Control for HVDC line

This section presents a systematic proposed robust H$_\infty$ decentralized state feedback ccontrol for HVDC to compensate the effect parameter uncertainties and regulate output variable taking. We will develop first new stability conditions based on LMI [18]-[28] for the uncertainty-free case. After that, we will generalize our results by considering the norm-bounded parametric uncertainties.

Our first objective is the design of the following linear robust H$_\infty$ decentralized state feedback ccontroller using integral action as the following;

$$\dot{U} = \bar{K}\bar{X} \Rightarrow U = \bar{K} \begin{bmatrix} X \\ \int edt \end{bmatrix} \tag{11}$$

A. Synthesis of Uncertainty-Free H$_\infty$ Decentralized Control

Consider the linear system which includes systems as (10):

$$\begin{cases} \dot{X} = AX + B_1 U + B_2 W \\ Y = CX + D_1 U + D_2 W \end{cases} \tag{12}$$

The influence of the disturbance W to the output Y is:

$$Y(s) = G(s) W(s) \tag{13}$$

where:

$$G(s) = (C + D_1 K)(sI - (A + B_1 K))^{-1} B_2 + D_2 \tag{14}$$

By using disturbance rejection, in the case of decentralized control in HVDC link, we successfully minimize the effect of one converter to the other converter's control, i.e., decouple the coupling between these two converters.

$$\|Y\|_2 \le \|G\|_\infty \|W\|_2 \tag{15}$$

Problem 1: For the linear system (12), the H$_\infty$ problem is to design a state feedback control law such that:

$$\sup_{\|W\|_2 \neq 0} \frac{\|Y\|_2}{\|W\|_2} \le \gamma \tag{16}$$

hold for a given positive scalar γ.

In the uncertainty-free case, stability and robustness analysis is summarized in the following Theorem 1.

Theorem 1: The H$_\infty$ problem has a solution if and only if there exist a matrix W_p and a symmetric positive definite matrix X_p , such that:

$$\begin{gathered} \min_{X_P, W_P} \quad \gamma \\ \begin{bmatrix} \Psi & B_2 & (CX_p + D_1 W_p)^T \\ B_2^T & -\gamma I & D_2^T \\ CX_p + D_1 W_p & D_2 & -\gamma I \end{bmatrix} < 0 \\ \Psi = (AX_p + B_1 W_p)^T + AX_p + B_1 W_p \end{gathered} \tag{17}$$

And, the solution of the problem can be given as:

$$K = W_p X_p^{-1} \tag{18}$$

Proof:

It is well-known that with linear system:

$$\left\{ \begin{array}{l} \dot{X} = AX + BU \\ Y = CX + DU \end{array} \right. \tag{19}$$

The H_∞ problem with this system:

$$\left\{ \begin{array}{l} Y = G\left(s\right)U \\ G\left(s\right) = C(sI - A)^{-1}B + D \end{array} \right. \tag{20}$$

is solved by [5], [29] if and only if there exist a symmetric positive definite matrix X_p , such that:

$$\left[\begin{array}{ccc} X_P A^T + AX_P & B & X_P C^T \\ B^T & -\gamma I & D^T \\ CX_P & D & -\gamma I \end{array} \right] < 0 \tag{21}$$

Apply this for the linear system with disturbance (equation (12)), the H_∞ problem for the linear system with disturbance (12), and by defining $W_P = KX_P$, the inequality turns into Theorem 1.

■

B. Synthesis of Uncertainty H_∞ Decentralized Control

In this subsection, we consider the problem of norm-bounded parametric uncertainties. As discussed in the uncertainty-free case, by considering the coupling between the two converters in HVDC link as disturbance, the effect of one converter to the other converter's control is minimized by the mean of disturbance rejection. Here, the uncertainties are taken into account since the HVDC links with uncertainties are popular.

Recall that the norm-bounded parametric uncertainties were assumed the linear system with uncertainty, which includes systems as (10):

$$\left\{ \begin{array}{l} \dot{X} = (A + \Delta A)X + (B_1 + \Delta B_1)U + B_2 W \\ Y = CX + D_1 U + D_2 W \end{array} \right. \tag{22}$$

where:

$$\left\{ \begin{array}{l} \left[\begin{array}{cc} \Delta A & \Delta B_1 \end{array} \right] = HF\left[\begin{array}{cc} E_1 & E_2 \end{array} \right] \\ E_1 = \delta_1 A; |\delta_1| < r_1; r_1 \in R^+ \\ E_2 = \delta_2 B_1; |\delta_2| < r_2; r_2 \in R^+ \\ F^T F \le I \end{array} \right. \tag{23}$$

The influence of the disturbance W to the output Y is determined by:

$$Y\left(s\right) = G\left(s\right)W\left(s\right) \tag{24}$$

where:

$$\begin{aligned} G\left(s\right) =& (C + D_1 K)(sI - ((A + \Delta A) + (B_1 + \Delta B_1)K))^{-1}B_2 \\ &+ D_2 \end{aligned} \tag{25}$$

Problem 2: For the uncertain linear system, the robust H_∞ problem is to design a state feedback control law such that:

$$\sup_{\|W\|_2 \neq 0} \frac{\|Y\|_2}{\|W\|_2} \le \gamma \tag{26}$$

hold for a given positive scalar γ.

The second approach for the design of new LMI stability conditions, for the linear system (22) in the uncertainty case is summarized in the following Theorem 2.

Theorem 2: The robust H_∞ problem has a solution if and only if there exist a positive scalar α, a matrix W_p and a symmetric positive definite matrix X_p , such that:

$$\min_{X_P, W_P} \quad \gamma$$

$$\left[\begin{array}{cccc} \Psi & B_2 & \left(CX_p + D_1 W_p\right)^T & \left(E_1 X_p + E_2 W_p\right)^T \\ B_2^T & -\gamma I & D_2^T & 0 \\ CX_p + D_1 W_p & D_2 & -\gamma I & 0 \\ E_1 X_p + E_2 W_p & 0 & 0 & -\alpha I \end{array} \right] < 0 \tag{27}$$

where, $\Psi = (AX_p + B_1 W_p)^T + AX_p + B_1 W_p + \alpha HH^T$

And, the solution of the problem can be given as:

$$K = W_p X_p^{-1} \tag{28}$$

Proof:

First, we introduce the well-known lemma [5], [29]:

Lemma 1: The following inequality hold:

$$Q + MFN + N^T F^T M^T < 0 \ , \forall F^T F < I \tag{29}$$

if and only if there exist $\alpha > 0$ such that:

$$Q + \alpha MM^T + \alpha^{-1} N^T N < 0 \tag{30}$$

Now, apply theorem 1 with $G\left(s\right)$ in the form of equation (25), we have:

$$\begin{array}{l} \left[\begin{array}{ccc} \langle AX_p + B_1 W_p \rangle_s & B_2 & (CX_p + D_1 W_p)^T \\ B_2^T & -\gamma I & D_2^T \\ CX_p + D_1 W_p & D_2 & -\gamma I \end{array} \right] \\ + \left\langle \left[\begin{array}{c} H \\ 0 \\ 0 \end{array} \right] F \left[\begin{array}{ccc} E_1 X_p + E_2 W_p & 0 & 0 \end{array} \right] \right\rangle_s < 0 \end{array} \tag{31}$$

From this, applying lemma 1 with:

$$\left\{ \begin{array}{l} Q = \left[\begin{array}{ccc} \langle AX_p + B_1 W_p \rangle_s & B_2 & (CX_p + D_1 W_p)^T \\ B_2^T & -\gamma I & D_2^T \\ CX_p + D_1 W_p & D_2 & -\gamma I \end{array} \right] \\ M = \left[\begin{array}{c} H \\ 0 \\ 0 \end{array} \right]; N = \left[\begin{array}{ccc} E_1 X_p + E_2 W_p & 0 & 0 \end{array} \right] \end{array} \right.$$

This inequality can be equivalently written as:

$$\begin{array}{l} \left[\begin{array}{ccc} \Psi & B_2 & (CX_p + D_1 W_p)^T \\ B_2^T & -\gamma I & D_2^T \\ CX_p + D_1 W_p & D_2 & -\gamma I \end{array} \right] \\ + \alpha^{-1} \left[\begin{array}{c} E_1 X_p + E_2 W_p \\ 0 \\ 0 \end{array} \right] \left[\begin{array}{ccc} E_1 X_p + E_2 W_p & 0 & 0 \end{array} \right] < 0 \end{array} \tag{32}$$

Where:

$$\Psi = \langle AX_p + B_1 W_p \rangle_s + \alpha HH^T \tag{33}$$

This inequality, according to Schur complement lemma [5], [29]-[30], is equivalent to the inequality (27) of Theorem 2.

■

IV. Simulations and Results

In this section, our control methodologies are applied to HVDC system with varying line length (110 Km, 750 Km, 1400 Km), in robust (H_{inf}RDC) and non-robust (H_{inf}DC) version, and in case of loss of control in one of the two converter in the HVDC line.

In the first figures (figure 2 to figure 3), the comparison between robust (problem 2) and non-robust (problem 1) version of H_∞ decentralized control is conducted, with 20% uncertainty. The robust version of H_∞ control has much smaller overshoot and shorter transient time. The uncertainty is an increase of 20% of matrices A and B_1 in system (22), i.e. an increase of 20% of $R_s, r_{g1}, r_{r1}, r_{g2}, r_{r2}$, and a decrease of 17.3% of $L_{DC}, L_{r1}, L_{g1}, C_1, L_{r2}, L_{g2}$.

The comparison for the robust HVDC decentralized H_∞ control, in different lengths of line (110 Km, 750 Km, 1400 Km), is presented from figure 4 to figure 5 with uncertainty (robust version). There is an obvious relationship between the line length and the converter's performance: the longer the length is, the better performance the converter's performance is. This can be explained by carefully examining the HVDC equation system, where there is a strong coupling between v_{DC1} and v_{DC2}. By increasing the length, r_{DC} and L_{DC} will be increased, lead to the weaker coupling between these voltages. These couplings are handled here as disturbances and parametric uncertainties in the robust control (problem 2). Improvement is significant for short HVDC lines (110 Km) and oscillations due to interaction are completely damped for longer lines.

In case of loss of control in one converter (input control of this converter is now equal to zero), the other converter still operates properly. This is demonstrated from the figure 6 to figure 7 (loss of control in converter 1, i.e. loss of control in v_{DC1} and Q_1), and from figure 8 to figure 9 (loss of control in converter 2, i.e. loss of control in P_2 and Q_2).

V. Conclusion

In this paper it is proposed a *decentralized control* of HVDC links. It has the advantage to propose two independent controls for each converter. Independence is at all levels: synthesis, implementation and use of measures. This is an advantage over classic vector controls which have separate synthesis but use feedback from the other converter to seperate dynamics.

Our control provides *resilience*: in case of failure on a converter (measure or loss of control), the control of the other converter is not influenced. No communication between the two stations is needed.

The natural coupling between the dynamics of the two converters is treated as disturbance and rejected by advanced control. The gains are synthesized by solving LMIs to analytically guarantee stability of the whole closed-loop (i.e., for the whole HVDC connected to the grid).

Coupling between the converters is quantified by the length of the HVDC link. The H_∞ LMI-baed synthesis mentioned above provides also increased parametric robustness as well as disturbance rejection. As a consequence, the effect of coupling is well counteracted and good results are obtained even for short HVDC links (and thus the strong coupling between the two converters).

In future work we will validate this control methodology on HVDCs inserted in a more detailed AC grid as well as hardware-on-the-loop platform. Also, this decoupling control methodology will be further used for other power systems problems such as, e.g., insertion of renewable sources generators in AC grids with high request of ancillary services.

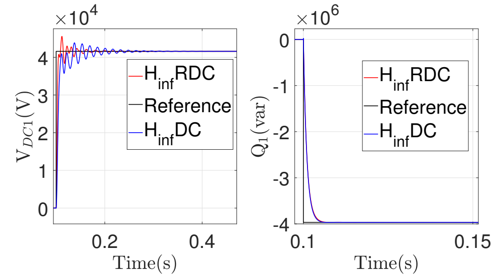

Fig. 2. Voltage v_{DC1} and reactive power Q_1 in 110 Km line (robust and non-robust)

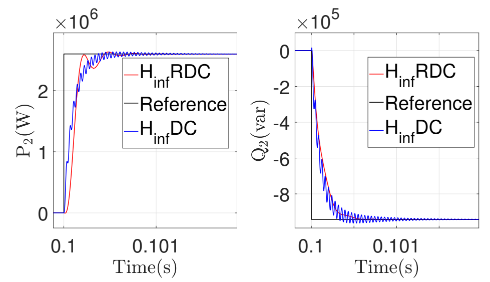

Fig. 3. Active power P_2 and reactive power Q_2 in 110 Km line (robust and non-robust)

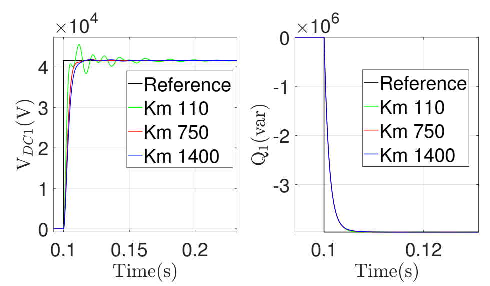

Fig. 4. Voltage v_{DC1} & reactive power Q_1 in 110 , 750, 1400 Km line (20% uncertainty)

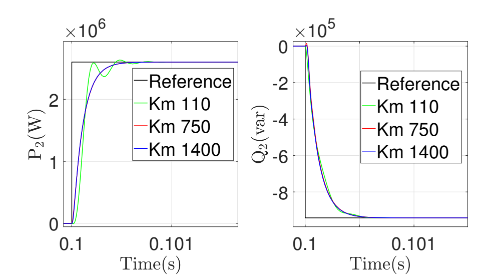

Fig. 5. Active & reactive power P_2, Q_2 in 110 , 750, 1400 Km line (20% uncertainty)

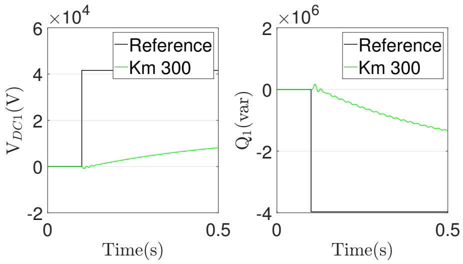

Fig. 6. Voltage & reactive power v_{DC1}, Q_1 (converter 1 control loss)

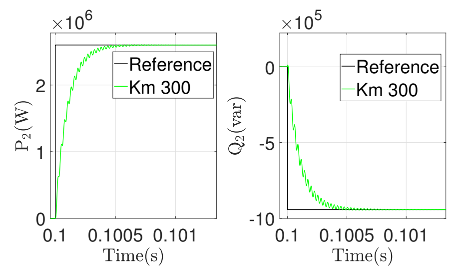

Fig. 7. Active & reactive power P_2, Q_2 (converter 1 control loss)

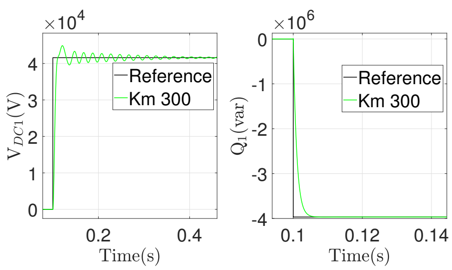

Fig. 8. Voltage & reactive power v_{DC1}, Q_1 (converter 2 control loss)

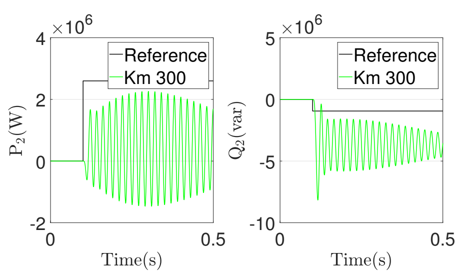

Fig. 9. Active & reactive power P_2, Q_2 (converter 2 control loss)

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