An Hybrid Wigner-Ville Approach to Estimate the Ultrasound Attenuation in Soft Biological Tissue

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Abstract— In this paper, we compare several techniques to evaluate the frequency shift related to the attenuation phenomenon. More precisely, we compare the short time Fourier analysis and the short time autoregressive analysis to new techniques based upon the Wigner-Ville transform with parametric and non parametric approaches. In the parametric methods, we propose a new frequency estimator called the mean resonating frequency. All of these methods are evaluated in terms of relative error and standard deviation of the attenuation estimation.

Keywords— Ultrasound, Tissue Characterization, Attenuation estimation, time-frequency analysis, parametric, non-parametric.

I. INTRODUCTION

IN the field of tissue characterization, ultrasound attenuation has proved to be a discriminant quantitative parameter which is complementary to the qualitative information obtained from echographic images. The attenuation phenomenon shifts the instantaneous spectrum to lower frequencies as a pulse wave propagates into the biological tissue. The spectral analysis of nonstationary signals requires a signal processing tool that presents both good time and frequency resolutions. It is well-known that the most commonly used short-time Fourier transform is limited by the intrinsic trade-off between time and frequency resolutions.

To overcome this resolution problem three approaches are possible : (i) the Wigner-Ville transform which allows a better compromise, (ii) the parametric methods and (iii) an hybrid method which is based on the Wigner-Ville transform (non parametric) computed by a parametric technique.

In the parametric methods, we propose a new frequency estimator : the mean resonating frequency. It is compared to the centroid and the maximum energy frequencies [1].

In this paper, we quantify the performances of these methods compared to those of the short time Fourier analysis.

II. ATTENUATION MODELING AND SIMULATED DATA

In order to compare the proposed methods, we have built artificial signals that simulate the Radio-Frequency (RF) lines reflected by a soft biological tissue. This modeling

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takes into account the pulse response of the measurement system, the absorption and the scattering of the medium. The diffraction phenomenon which biases the attenuation measurement could be compensated by a filter [2], so we did not consider it in our model [3]. It is usually accepted that attenuation α , in most biological tissues, is a linear function of the frequency: $\alpha = \beta f$ where β is the slope of the attenuation.

It is also considered that the spectral density of the emitted signal has a gaussian form.

The simulated echographic lines are obtained with a sampling frequency of 400 MHz, an emitted central frequency of 45 MHz and a bandwidth of 50%. The homogeneous medium is simulated for different attenuation coefficient values ranging from 1 to 5 dB/cm/MHz. For each attenuation coefficient value, a set of 100 RF lines are computed. With a wave velocity of 1530 m/s, the duration of the backscattered signal corresponds to a depth of 2 mm.

III. THE TIME-FREQUENCY ANALYSIS

The attenuation phenomenon is usually observed in the frequency domain through the frequency shift observed as the ultrasound pulse propagates in the soft medium. This time-frequency analysis is generally performed by a Fast Fourier Transform (FFT) [4].

A. The short time Fourier analysis

Owing to the nonstationary character of the radio-frequency signal, a Hamming window is shifted with a constant step along the signal and in each window, the Power Spectral Density (PSD) is evaluated via the computation of the FFT. This method implies that we have to make a choice for the length and the overlap of the windows. Indeed, the larger the windows, the better is the frequency resolution but the less the stationarity hypothesis is verified. The choice is obtained by a trade-off between the frequency and time resolutions. In order to have a better compromise, we propose the use of the Wigner-Ville transform.

B. The Wigner-Ville (WV) transform

The Wigner-Ville distribution is a well adapted tool to study nonstationary phenomena. It seems to have a good behavior for reflected signals [3].

The Wigner-Ville distribution is defined by :

$$W(t, f) = \int_{-\infty}^{\infty} \zeta(t + \tau/2) \zeta^*(t - \tau/2) e^{-2\pi j f \tau} d\tau$$

where, in our study, $\zeta(t)$ is the analytical signal of the RF signal and $\zeta(t + \tau/2)\zeta^*(t - \tau/2)$ is the kernel. We used the numerical implementation of the windowed discrete Wigner-Ville distribution [5]. This distribution is defined by :

$$W^g(n, f) = \sum_{m=-(M-1)/2}^{m=(M-1)/2} g(m)\zeta(n+m)\zeta^*(n-m)e^{-4\pi jfm/M}$$

where $g(m)$ is the Hamming window. Another solution is to analyze a modeled signal : this is, for example, the case in autoregressive modeling.

C. Autoregressive (AR) modeling

The echographic signal is modeled as the output of linear filter driven by a complex white Gaussian noise $u(n)$ with zero mean variance σ_u^2 [6] :

$$\zeta(n) = -\sum_{i=1}^p a_i \zeta(n-i) + u(n)$$

where a_i are the complex autoregressive parameters and p is the order of the AR model. $\zeta(n)$ is the analytical windowed signal. The optimal AR order is obtained (see figure 1) when both the standard deviation and relative error of the attenuation coefficient are the lowest. There are many algorithms to compute AR parameters required for the poles and spectrum estimation. We will focus on the most popular one : the least-square algorithm [6], [1]. Finally, from known AR parameters the whole spectrum can be evaluated.

D. The autoregressive Wigner-Ville (AR VW) method

This method is a mix between the Wigner-Ville method and the autoregressive method. In order to compute the Fourier transform of the kernel as in the conventional Wigner-Ville approach, we implement the computation of the square-root of the AR kernel spectrum. This is not restrictive, others solutions can be proposed.

IV. FREQUENCY ESTIMATORS

A. Parametric frequency estimators

The whole spectrum can be evaluated using the AR parameters and the corresponding poles:

$$H(z) = \frac{1}{1 + \sum_{i=1}^p a_i z^{-i}}$$

$$S(f) = \sigma_u^2 |H(z)|_{z=\exp(-2\pi jf)}^2$$

$$S(f) = \frac{\sigma_u^2}{|\prod_{i=1}^p (z - z_i)|_{z=\exp(-2\pi jf)}^2}$$

where f is the normalized frequency $0 \leq f \leq 0.5$ and σ_u^2 is the white noise energy. The centroid frequency [8] and the maximum energy frequency [7] can be evaluated directly on the spectrum. The centroid is given by :

$$f_c = \frac{\int_{-\infty}^{+\infty} f S(f) df}{\int_{-\infty}^{+\infty} S(f) df}$$

Fig. 1.

Relative Error in % (a and c) and Standard Deviation in dB/cm/MHz (b and d) of the attenuation coefficient for 2 and 4 dB/cm/MHz versus the Autoregressive Order computed via the mean resonating frequency. Dark plot (with *) corresponds to the autoregressive method ; Light plot (with o) corresponds to the autoregressive Wigner-Ville method.

The resonating frequencies (see Figure 2) that correspond to the pole p_i are evaluated by:

$$fp_i = fe \frac{\arg(z_i)}{2\pi}$$

with $\arg(z_i) \in [0, \pi]$ and where fe is the sampling frequency. In this case the mean resonating frequency is given by:

$$f_{mr} = \frac{fe}{p2\pi} \sum_{i=1}^p \arg(z_i)$$

B. Non parametric frequency estimators

In the non parametric case, only the centroid frequency is authorized. Indeed, as variance of the frequency estimation evaluated by the maximum energy frequency is too high (due to the random character of the spectrum), it can not be used.

Figure 3 illustrates the evolution of f_c in the non-parametric case and f_{mr} in parametric case. These curves are averaged with 100 RF lines. These curves are almost parallel to those of theoretical frequencies. A significant bias appears for non-parametric approaches, but it does not disturb attenuation estimation because its slope constitutes the relevant information.

Fig. 2.

Autoregressive Spectrum with a 6 order AR model.

V. RESULTS

First, we compare the presented parametric methods in terms of relative error and standard deviation of the attenuation estimate (see Table I). This is performed for a theoretical attenuation coefficient of 3dB/cm/MHz , with 100 simulated RF lines, by means of the three presented frequency estimators. These results are obtained with a 4 order autoregressive model. Indeed, Figure 1 indicates that a 4 order autoregressive model is well adapted to our application. Similar results are obtained with other attenuation coefficients (from 1 to 5dB/cm/MHz).

TABLE I

Frequency estimators	Methods			
	Autoregressive		AR Wigner-Ville	
	RE	SD	RE	SD
Centroid	0.56	0.55	0.3	0.45
Maximum Energy	≤ 0.1	0.91	3.0	0.82
Mean resonating	0.8	0.22	0.1	0.30

Relative Error (RE in %) and Standard Deviation (SD in dB/cm/MHz) for a theoretical attenuation coefficient of 3dB/cm/MHz with a 4 order autoregressive method.

Thus, table I points out that in the case of the classical centroid and maximum energy frequencies, the best method, in term of standard deviation, is the hybrid Wigner-Ville. This is due to the combination of properties of the conventional Wigner-Ville and parametric methods.

Let us now consider the new frequency estimator : the mean resonating frequency. Table I highlights that the best attenuation estimation is given by the autoregressive

Fig. 3.

The time frequency tracking with the four methods for a theoretical attenuation coefficient of 3dB/cm/MHz .

method via the computation of the mean resonating frequency. We can stipulate that this good behavior is due to the fact that we have performed both statistical (100 RF lines) and frequency averages (corresponding to the pole arguments). These results suggest the joint use of mean resonating frequency estimator and autoregressive modeling.

The results of a comparison between parametric and non parametric methods is summarized in Tables II and III.

TABLE II

Theoretical attenuation in dB/cm/MHz	Methods			
	Non Parametric		Parametric	
	FFT	WV	AR	AR WV
1	1.20	3.60	3.40	4.40
2	2.45	0.90	0.10	0.35
3	0.17	0.53	0.87	0.10
4	0.30	0.75	0.93	1.58
5	0.32	0.44	0.42	1.10

Relative Error (in %) of the attenuation coefficient estimate in the case of parametric and non-parametric methods

The non-parametric Wigner-Ville approach improves the standard deviation of the attenuation estimate, as compared to the Fourier approach, by a factor of 14%. This factor increases to around 45% for the hybrid Wigner-Ville approach and to around 58% for the AR approach.

TABLE III

Theoretical attenuation in dB/cm/MHz	Methods			
	Non Parametric		Parametric	
	FFT	WV	AR	AR WV
1	0.543	0.479	0.226	0.296
2	0.549	0.489	0.249	0.295
3	0.547	0.457	0.222	0.302
4	0.512	0.445	0.199	0.263
5	0.521	0.428	0.231	0.301

Standard Deviation (in dB/cm/MHz) of the attenuation coefficient estimate in the case of parametric and non-parametric methods

VI. CONCLUSIONS

In this paper, we have compared the performances of time-frequency methods such as the Fourier, the Wigner-Ville and the autoregressive approaches as well as a new autoregressive Wigner-Ville method for attenuation coefficient estimation. We have also presented a new way to estimate the frequency of parametric power spectral density and compared it with classical frequency estimators. These comparisons were performed with simulated RF lines in biological tissues with different attenuation coefficient values ranging from 1 to 5dB/cm/MHz.

In the field of parametric analysis, we suggest the joint use of autoregressive models and the mean resonating frequency estimator. For non parametric analysis, the Wigner-Ville distribution leads to a better attenuation estimation. Indeed, the standard deviation of these methods is lower. Consequently the estimation of the attenuation coefficient should require a lower number of RF lines than for classical methods.

Our future work will be focused on the adaptation of the autoregressive order for each attenuation value as well as on the application of these methods to real echographic lines.

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