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Cross-relaxation and ion clustering in $\text{Tm}^{3+}:\text{CaF}_2$ crystals

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ABSTRACT

We systematically study cross-relaxation (CR) and ion clustering in $\text{Tm}^{3+}:\text{CaF}_2$ crystals using a spectroscopic approach. For this, the luminescence from the $^3\text{H}_4$ and $^3\text{F}_4$ states was monitored for a broad range of Tm^{3+} doping concentrations, from 0.01 at.% to 7 at.%. The decay curves were fitted using a model of two ions classes, namely isolated ions showing no energy-transfer processes and ions with neighbors exhibiting both CR and energy-transfer upconversion (ETU), and accounting for energy-migration. The fraction of ions with neighbors and the microscopic concentration-independent CR and ETU parameters are deduced. The critical Tm^{3+} doping level for which at least half of the active ions are clustered is only 0.7 at.%. The obtained results are relevant for achieving efficient laser operation of $\text{Tm}^{3+}:\text{CaF}_2$ crystals at the $^3\text{F}_4 \rightarrow ^3\text{H}_6$ (at $\sim 1.9 \mu\text{m}$) and the $^3\text{H}_4 \rightarrow ^3\text{H}_5$ (at $\sim 2.3 \mu\text{m}$) transitions.

Keywords: calcium fluoride, thulium ions, cross-relaxation, ion clustering, luminescence.

1. INTRODUCTION

Cubic (fluorite-type) calcium fluoride (CaF_2) crystals are well-known for fabrication of optical elements because of their good thermo-mechanical properties (thermal conductivity: $\kappa \sim 9.7 \text{ W}/(\text{mK})$ for an undoped crystal), broad transparency range (0.13–10 μm) and low refractive index ($n = 1.424$ at $\sim 2 \mu\text{m}$). CaF_2 crystals are also suitable for doping with trivalent rare-earth ions (RE^{3+}) for laser applications [1-3]. They feature a strong RE^{3+} ion clustering even at low doping concentrations about 0.1–1 at.% [4,5] resulting in broad and smooth spectral bands both in absorption and emission (the so-called glassy-like spectroscopic behavior). It is of interest for broadly tunable [6] and mode-locked [7,8] lasers. The good thermal properties of the host matrix allow for power scaling.

Thulium (Tm^{3+}) doped CaF_2 crystals exhibit a broadband emission at $\sim 1.9 \mu\text{m}$ due to the $^3\text{F}_4 \rightarrow ^3\text{H}_6$ electronic transition. Efficient $\text{Tm}^{3+}:\text{CaF}_2$ lasers were reported [2,9]. Camy *et al.* reported on a laser-pumped $\text{Tm}:\text{CaF}_2$ laser delivering $\sim 135 \text{ mW}$ at 1887 nm with a slope efficiency of 41% and a laser threshold of only 68 mW [2]. Liu *et al.* reported on power scaling of a $\text{Tm}:\text{CaF}_2$ laser under diode-pumping yielding 2.71 W with a higher slope efficiency of 70.1% [9]. The same group of authors demonstrated continuous wavelength tunability for this laser between 1.85 and 2.04 μm (189 nm broad) and mode-locked operation employing a SESAM. In the latter case, 15 ps-long pulses were obtained at a central wavelength of 1886.8 nm with a repetition rate of 96.35 MHz [9]. Thin films of $\text{Tm}:\text{CaF}_2$ were grown [10].

It was proven that the ion clustering determines the emission properties of Tm^{3+} ions in CaF_2 [5]. For low doping levels (0.01-0.1 at.%), the Tm^{3+} ions are mostly isolated and they accommodate in sites with different symmetry depending on the charge compensation mechanism involving interstitial F^- anions. For higher doping levels, ion clusters are formed. To prevent the clustering of active ions, the CaF_2 crystals were codoped by Tm^{3+} ions and optically passive “buffer” ions Ln^{3+} such as Gd^{3+} , Lu^{3+} , La^{3+} or Y^{3+} [11,12]. Laser operation with such codoped crystals was also demonstrated. Zhang *et al.* reported on a $\text{Tm},\text{La}:\text{CaF}_2$ laser delivering 4.27 W at 1921 nm with a slope efficiency of 67.8% [12].

A key spectroscopic process which determines the laser efficiency of $\sim 1.9 \mu\text{m}$ Tm lasers is the cross-relaxation (CR) for adjacent Tm^{3+} ions, $^3\text{H}_4(\text{Tm}_1) + ^3\text{H}_6(\text{Tm}_2) \rightarrow ^3\text{F}_4(\text{Tm}_1) + ^3\text{F}_4(\text{Tm}_2)$, potentially leading to a pump quantum efficiency of 2 [13]. This means that after an excitation at $\sim 0.8 \mu\text{m}$ (to the $^3\text{H}_4$ pump level), there can be up to 2 excited ions in the upper laser level ($^3\text{F}_4$). CR in Tm^{3+} -doped laser materials increases the laser slope efficiency and reduces the heat dissipation. Up to now, no detailed study of CR for $\text{Tm}:\text{CaF}_2$ crystals was performed.

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The aim of the present work is to study the cross-relaxation (and other energy-transfer processes, such as energy-transfer upconversion (ETU)) in Tm:CaF₂ crystals accounting for the ion clustering. For this, we involve the spectroscopic model of distinct ion classes which was recently developed [14].

2. EXPERIMENTAL RESULTS

For the spectroscopic studies, we grew a series of Tm³⁺:CaF₂ crystals with the doping level ranging from 0.01 at.% to 7 at.%. The Tm:CaF₂ crystals were grown using a conventional Bridgman technique with a homemade furnace. A mixture of high purity (4N) CaF₂ and TmF₃ powders was introduced in a graphite crucible. A good vacuum (<10⁻⁵ mbar) was realized before introducing Ar and CF₄ gases to avoid oxygen pollution. The furnace temperature was set to about 50 K higher than the melting point of CaF₂. After soaking, the charge was thoroughly melted and the crystal growth was carried out with a rate of 2-4 mm/h. When the growth was finished, the crystal was cooled down to room temperature (RT, 293 K) within 48 hours.

2.1 Absorption and luminescence

The RT absorption spectra of Tm³⁺:CaF₂ crystals containing predominantly isolated ions (0.05 at.% Tm) and ion clusters (1.5 at.% and 4.5 at.% Tm) are shown in Fig. 1. It is known that in Tm³⁺-doped CaF₂ crystals with very low doping levels, the active ions are distributed over three types of sites with the symmetry C_{3v} (trigonal), C_{4v} (tetragonal) or O_h (cubic). The particular symmetry depends on the charge compensation mechanism (as trivalent Tm³⁺ ions are replacing the divalent Ca²⁺ ones). The latter is provided by an interstitial fluorine anion (F_i⁻). Here, we do not consider the possible charge compensation by oxygen (O²⁻) or univalent alkali metal cations (Na⁺, K⁺) leading to other types of sites. As a result, the absorption spectra of the 0.05 at.% Tm³⁺:CaF₂ crystal represent a superposition of sharp absorption lines of Tm³⁺ ions in all three sites. For high doping levels, the Tm³⁺ ions are mostly arranged in clusters leading to smooth and broad spectral bands. There is no significant change in the absorption spectra of 1.5 at.% Tm and 4.5 at.% Tm-doped crystals, except of the ³H₆ → ³H₅ transition. The small changes may reflect the transition from simple clusters (dimers, trimers) to large agglomerates of active ions.

Figure 1. RT absorption cross-section, σ_{abs} , spectra of Tm³⁺:CaF₂ crystals with different Tm doping concentrations (0.05 at.%, 1.5 at.% and 4.5 at.%): transitions from the ground-state (³H₆) to the excited-states: (a) ³F₄, (b) ³H₅, (c) ³H₄ and ³F_{2,3}, and (d) ¹G₄ and ¹D₂.

For the ³H₆ → ³H₄ pump transition, the maximum absorption cross-section σ_{abs} is $0.46 \times 10^{-20} \text{ cm}^2$ at 766.2 nm and the full width at half maximum (FWHM) of the absorption band is 13.2 nm (for the 4.5 at.% Tm doping). For the 0.05 at.% Tm doping, the peak σ_{abs} value is much lower, $0.17 \times 10^{-20} \text{ cm}^2$ at ~768 nm.

The evolution of RT luminescence spectra for the $^3\text{H}_4 \rightarrow ^3\text{H}_6$ (at $\sim 0.8 \mu\text{m}$) and $^3\text{F}_4 \rightarrow ^3\text{H}_6$ (at $\sim 1.9 \mu\text{m}$) transitions in emission was studied for a broad range of Tm doping concentrations (0.01–7 at.%). This comparative study revealed a critical Tm concentration (0.5–0.7 at.%) for which a significant change in the shape of the emission bands is observed: the spectra become smooth and broad and they resemble to a great extent those for Tm^{3+} -doped fluoride glasses. This is an indication of a change in the predominant ion coordination (a transition from isolated ions to clusters). Figure 2 shows such a comparison for the emission from the pump level ($^3\text{H}_4$) at $\sim 0.8 \mu\text{m}$.

Figure 2. Normalized RT luminescence spectra of $\text{Tm}^{3+}:\text{CaF}_2$ crystals with different doping concentrations (the $^3\text{H}_4 \rightarrow ^3\text{H}_6$ transition), $\lambda_{\text{exc}} = 769 \text{ nm}$ (non-selective excitation).

2.2 Luminescence decay

We monitored the decay of Tm^{3+} luminescence from the $^3\text{H}_4$ and $^3\text{F}_4$ multiplets for a broad range of Tm doping levels (0.01–7 at.%) in CaF_2 . Figure 3 shows the measured luminescence decay curves for the $^3\text{H}_4 \rightarrow ^3\text{H}_6$ transition in emission (at $\sim 0.8 \mu\text{m}$). For very low doping level (0.01 at.% Tm), the luminescence decay curve is close to a single-exponential one with a characteristic lifetime τ_{lum} of 3.55 ms. Despite the existence of several species contributing to this emission (namely, isolated Tm^{3+} ions in various sites), the intrinsic lifetimes of the $^3\text{H}_4$ state for the C_{3v} and C_{4v} coordinated Tm^{3+} ions in CaF_2 are rather close: 3.03 ms and 3.59 ms, respectively (as determined in site-selective excitation experiments). With increasing Tm doping, the decay from the $^3\text{H}_4$ state becomes faster and the luminescence decay curves notably deviate from the single-exponential law.

Figure 3. Luminescence decay curves from the $^3\text{H}_4$ Tm^{3+} state in $\text{Tm}^{3+}:\text{CaF}_2$: *black* – experimental data, *red* – their fitting with the rate-equation model of distinct ion classes, $\lambda_{\text{exc}} = 765 \text{ nm}$, $\lambda_{\text{lum}} = 820 \text{ nm}$. Measured under ns pulse excitation.

To explain this behavior, we used the model of spectroscopically distinct ion classes [14]. A simplified scheme of Tm^{3+} energy-levels was considered, Fig. 4. The populations of the Tm^{3+} ground-state ($^3\text{H}_6$) and two relevant excited-states ($^3\text{F}_4$ and $^3\text{H}_4$) were monitored; the population of the intermediate $^3\text{H}_5$ excited-state was assumed to be almost zero as this state is depopulated by very efficient multi-phonon non-radiative (NR) relaxation. The states above $^3\text{H}_4$ were not taken into account. The following spectroscopic processes were considered: (i) radiative (R) decay from the $^3\text{F}_4$ and $^3\text{H}_4$ states, (ii) cross-relaxation (CR) $^3\text{H}_4(\text{Tm}_1) + ^3\text{H}_6(\text{Tm}_2) \rightarrow ^3\text{F}_4(\text{Tm}_1) + ^3\text{F}_4(\text{Tm}_2)$, (iii) energy-transfer upconversion, $^3\text{F}_4(\text{Tm}_1) +$

$^3F_4(Tm_2) \rightarrow ^3H_6(Tm_1) + ^3H_4(Tm_2)$ (ETU₁) and $^3F_4(Tm_1) + ^3F_4(Tm_2) \rightarrow ^3H_6(Tm_1) + ^3H_5(Tm_2)$ (ETU₂), and (iv) energy-migration (EM) to impurities leading to de-excitations from the 3F_4 and 3H_4 multiplets. We assumed the existence of two distinct ion classes: (a) isolated ions showing no energy-transfer processes (CR and ETU) and (b) ions with neighbors exhibiting both CR and ETU. The fraction of ions with neighbors f , the macroscopic rates of CR (W_{CR}) and ETU (W_{ETU1} and W_{ETU2}) and the 3H_4 level lifetime τ_3 accounting for EM were used as free parameters of the fitting (for luminescence decay curves shown in Fig. 3). The model gives a satisfactory agreement with the experiment.

Figure 4. The scheme of energy-levels of Tm^{3+} ions (for a free-ion) showing relevant spectroscopic processes associated with the conventional pumping scheme (to the 3H_4 state): R – radiative relaxation, NR – multiphonon non-radiative relaxation, CR – cross-relaxation, ETU – energy-transfer upconversion, EM – energy-migration.

3. DISCUSSION

The results on the fraction of Tm^{3+} ions with neighbors f are shown in Fig. 5. With increasing the Tm doping level, the fraction of isolated ions ($1 - f$) gradually decreases. The critical Tm doping level for which at least half of the active ions are clustered is only 0.7 ± 0.1 at.%. This agrees well with our conclusion drawn from the measured luminescence spectra at $\sim 0.8 \mu m$. The dependence of f on the doping concentration was fitted using a statistical approach [14,15]:

$$P_{m,n} = \frac{n!}{m!(n-m)!} p^m (1-p)^{n-m}. \quad (1)$$

Figure 5. Ion clustering in $Tm^{3+}:CaF_2$: fraction of ions with neighbors f , *symbols* – data extracted from the modeling of the experimental luminescence decay curves using the rate-equation model of distinct ion classes, *curve* – their fit using a statistical approach.

Here, $P_{m,n}$ is the probability for occurrence of neighbor Tm^{3+} ions in the first coordination sphere of the considered Tm^{3+} ion, n is the coordination number (C.N.) by nearest-neighbor rare-earth sites, m is the actual number of Tm^{3+} ions in the first coordination sphere ($m = 0$ corresponds to a single ion and $m \geq 1$ - to an ion pair or cluster), and $p = N_{Tm}/N_{max}$ represents the relative doping concentration (N_{max} is the maximum possible concentration). The expression for the fraction of ions with neighbors ($m \geq 1$, not specifying the exact m number) is:

$$f = 1 - [1 - (N_{\text{Tm}} / N_{\text{max}})]^{\text{C.N.}} \quad (2)$$

For calculation, we assume $N_{\text{max}} = 10$ at.% and C.N. = 10 which gives a good agreement with the data obtained from the measured luminescence curves.

The model also yields the macroscopic rates W_{CR} , W_{ETU1} and W_{ETU2} [in s^{-1}], as shown in Fig. 6. The CR in $\text{Tm}^{3+}:\text{CaF}_2$ is much stronger than the ETU. W_{CR} and $W_{\text{ETU1(2)}}$ show a quadratic dependence on the Tm doping concentration (N_{Tm}):

$$W_{\text{CR}} = C_{\text{CR}} N_{\text{Tm}}^2, \quad (3a)$$

$$W_{\text{ETU1(2)}} = C_{\text{ETU1(2)}} N_{\text{Tm}}^2, \quad (3b)$$

where C_{CR} and $C_{\text{ETU1(2)}}$ are the concentration-independent microscopic energy-transfer parameters. CR in $\text{Tm}^{3+}:\text{CaF}_2$ is about one order of magnitude stronger than that in other well-known fluoride crystal, $\text{Tm}^{3+}:\text{LiYF}_4$ [16].

Figure 6. Energy-transfer processes in $\text{Tm}^{3+}:\text{CaF}_2$: macroscopic rates of cross-relaxation (CR) W_{CR} and energy-transfer upconversion (ETU) $W_{\text{ETU1(2)}}$, symbols – data extracted from the modeling of the experimental luminescence decay curves using the rate-equation model of distinct ion classes, lines – their quadratic fits.

4. CONCLUSIONS

To conclude, we have quantified ion clustering (in terms of fraction of ions with neighbors) and CR and ETU energy-transfer processes (in terms of macroscopic rates and microscopic concentration-independent parameters) in Tm^{3+} -doped CaF_2 crystals, for the first time, to the best of our knowledge. This information is of key importance for the design of $\text{Tm}:\text{CaF}_2$ lasers operating on the $^3\text{F}_4 \rightarrow ^3\text{H}_6$ ($\sim 1.9 \mu\text{m}$) and $^3\text{H}_4 \rightarrow ^3\text{H}_5$ ($\sim 2.3 \mu\text{m}$ [17]) transitions. Indeed, for the $\sim 1.9 \mu\text{m}$ lasers, quantification of CR is important to determine the possibility to reach high pump quantum efficiency of 2 for the upper laser level ($^3\text{F}_4$) [13]. For the $\sim 2.3 \mu\text{m}$ lasers which have not been demonstrated yet in $\text{Tm}:\text{CaF}_2$, in contrast, CR is a detrimental effect as it quenches the lifetime of the upper laser level ($^3\text{H}_4$). However, ETU1 in such lasers can play a key role in refilling the $^3\text{H}_4$ state leading to increased laser slope efficiency [16,18]. The proposed spectroscopic model can be extended to describe the ion clustering in $\text{Tm}^{3+}, \text{Ln}^{3+}:\text{CaF}_2$ crystals codoped with “buffer” ions.

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