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A probabilistic approach for automated lane identification based on sensor information

1st Jennie Lioris
ENPC, Paris, France
jennie.lioris@enpc.fr

2nd Neila BHOURI
COSYS-GRETTIA, Université Gustave Eiffel-IFSTTAR
77454 Marne-la-Vallée, France
neila.bhourri@univ-eiffel.fr

Abstract—The level lane location problem of sensor equipped vehicles circulating within arbitrary highway infrastructures is addressed. A first approach of a flexible probabilistic decision-making policy is developed utilizing sensor signals. Unmanned vehicles independently of the automation degree are related to challenging executive schemes such as adaptive cruise control systems, real time routing models involving lane changing options and speed control, platoon formation operations etc. An adaptive, closed loop methodology is presented localizing *suitable* detections while involving *uncertainty within data*, sensor vagueness and trust. The whole scheme is associated with *low computational complexity* where no additional investment on external devices is required. The outlined framework pronounces a significantly progressed study regarding a previously presented elementary pattern. The new model focuses in the case of invalid sensor detections due to traffic context, various environmental disturbances and failures for which no response was previously available. The effectiveness of the suggested scheme is measured when applied to detailed simulation scenarios fed by ground truth data. Different complex spatiotemporal contexts elicit varying driving profiles and pragmatic behavior-change interventions inaccessible from direct recordings provided by professional drivers. The proposed methodology is compared with a non-probabilistic model. Analysis illustrates noteworthy accuracy, precision and frequency on the resulting responses.

Index Terms—Highly Automated Driving Functions (HADF), lane identification, real time path and target tracking control, data mining, road navigation, autopilot perception, environment perception, adaptive policy for smart sensor utilization

I. INTRODUCTION

During the last years, a significant increase in the number of the driving abilities of automated vehicles is observed which is also accompanied by an increase in the complexity of the involved functional components. Perception modules destined for “planning and control” such as *navigation*, *guidance*, *stabilization* are associated with complicated, pertinent tasks which at least must remain manageable if not optimized. Since the DARPA challenge [1] deep knowledge was obtained implying that accurate and reliable vehicle positioning is a key feature for automated vehicles. Experiences have shown that navigation based on a global positioning system incorporated with an Inertial Measurement Unit (IMU) and digital cartography implies a satisfactory solution for the positioning of autonomous vehicles [2], [3]. Decisions on lane change possibilities taking into consideration perception uncertainties are examined in [4] by means of a dynamic Bayesian network and an unscented variance transform. Moreover, other

transportation fields involving autonomous cars also demand accurate environmental detection. Thus, the mesoscopic and macroscopic modeling of traffic flow comprised of vehicles moving according to intelligent cruise control (ICC) and co-operative driving is studied in [5]. A methodology identifying the lane position of the GPS-instrumented vehicles when located in a queue of a signalized intersection associated with multiple lanes is presented in [6]. It is shown that the Optimal Bayes Rule employing probability density functions estimated using bivariate statistical mixture models was found to be effective in identifying the lanes. The work confirms that the lane identification is an important step required prior to the queue length estimation. Considering that density distribution may be highly heterogeneous among the different lanes of a highway, real-time lane assignment strategies may have significant advantages in traffic management. Lane policies and lane advice may be achieved if real-time traffic state information per lane is available [7].

The current study proposes a closed loop methodology allowing dynamic level lane positioning of a sensor equipped car (*ego vehicle*) where the whole scheme is associated with low computational complexity. It merits to specify that the suggested approach does not require any additional devices, it exploits basic measurements provided by the required technology of existing automotive vehicle procedures. Similar works suggest procedures simultaneously resolving multiple problems: definition of the total number of lanes within the road segment, road markers detection and finally determination of the object lane [8], [9], [10]. Due to the increased complexity of the dealing problem, a decomposition into subproblems of different nature is here considered. Since sensor observations are often noisy, the model presented in [11] was considered elementary as it deals only with valid detections for which a unique solution is provided by the associated algorithm. A probabilistic aspect was necessary to be conceived, a first approach of which is here presented aiming at accurate performance of the proposed scheme even under invalid data.

The remainder of the paper is organized as follows. §II formalises the dealing problem. §III presents a first performance appraisal of the suggested scheme where it evaluates the probabilistic versus the deterministic approach. §IV summarizes the goal of this work and introduces the next shortcoming findings.

II. STUDY SCOPE

A. Towards a stochastic approach

With the aim of defining a flexible level lane estimation approach a probabilistic model is going to be constructed. Under the assumption that a laser equipped car (*ego-vehicle*) moves within an arbitrary but non-isolated road infrastructure, a methodology estimating the associated lane positioning will be determined based on *noisy* detections as provided by the vehicle equipments, a laser sensor and a smart camera.

The new scheme is comprised of two main criteria categories:

- 1) The first category is composed of algorithms detecting mobile objects, moving on the same direction as the ego vehicle, located within the boundary lanes on the related road segment.
- 2) The second type uses *recently collected* past data and potentially previous estimations to determine the new vehicle behavior during a period where no valid information is available.

Each lane estimation alternative is employed within a specific context, hence a well-defined strategy is developed able to deal with temporary noisy data.

At any detection time t after a specific treatment multiple metrics related to current sensor observations are available such as the object velocity, Cartesian coordinates associated with the associated object regarding the ego vehicle position, other measurements qualifying each detection etc. Moreover, the two distances between the ego vehicle and the right and left lane marks are also available by the smart camera. When the current sensor information is judged *valid* a real time database set is constructed comprised of *well selected* metrics of interest. Figure 1 illustrates how the vehicle environment is defined at a detection instant. Based on reliable information provided by the database set, the ego vehicle lane position can be defined by computations allocating observations one the extreme lanes (under regular or dense traffic flows). However, invalid metrics maybe returned due to a sensible sensor functioning (saturated traffic density, light brightness, failures, meteorological conditions, etc.) within a frequency depending upon the current the vehicle context. Whenever this occurs the previous approach can not be utilized for defining the ego vehicle position. With the purpose of identifying the vehicle location at a regular decision frequency similar to the one regarding the sensor operation, a probabilistic scheme is going to be determined, examining *whether and how* previous estimations can be combined and at which *reliability* level.

B. Building a level lane decision scheme - probabilistic model

At a given moment t , laser scanners detect multiple mobile or immobile objects within a multi-plane and long range vision angle. For each tracked element, the relative lateral and longitudinal distance coordinates are computed with respect to the vehicle, the reference point of which is considered to be located at the ordered pair $(0, 0)$ (see also Figure 1).

Fig. 1. System architecture

Hence, if $N(t)$ describes the total number of points detected at time t by the *selected* vehicle sensors, a set of reference points $\mathcal{P}$ can be constructed at time t potentially involving any captured object. Thus,

$$\mathcal{P}(t) = \{p_i(t) = (x_i(t), y_i(t)), \text{ such that: } v_i(t) > 0, i = 1, \dots, N(t)\}. \quad (1)$$

with

$$\text{card}(\mathcal{P}(t)) = \begin{cases} > 0, & \text{if } N(t) > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Due to sensor temporal failures (visibility limitations etc.) or currently saturated traffic state, no appropriate object may be detected at the considered time instant. Thus, set $\mathcal{P}$ may become empty at t . That is what Equation 2 expresses.

Remark 1: In the present study detections obtained but a laser sensor are only employed. However and since autonomous cars are often equipped with different sensor types one could decide of which technology is preferable to utilize at the decision instant.

Taking into account that the reference ego vehicle is located at a particular candidate lane, a dynamical coordinate system is progressively created. For such a system, the y coordinate of each lane boundary at time t can be computed.

Set $\mathcal{B}_j(t) = \{(y_{i,j}^l(t), y_{i,j}^r(t)), i = 1, \dots, m\}$ can be defined, where

- m is the total number of lanes in the reference vehicle direction (available information by a GPS cartography)
- j is the considered candidate lane where the system origin is placed, $j \in [1, m]$

- $y_{i,j}^l(t)$ is the y coordinate of the left boundary of lane i regarding lane j at time t
- $y_{i,j}^r(t)$ is the y coordinate of the right boundary of lane i regarding lane j at time t .

Let w denote the lane width and d_r, d_l the right and left distances of the ego vehicle from the right and left lane boundaries respectively then the right and left boundaries of lane i , $y_{i,j}^l$ and $y_{i,j}^r$ regarding the candidate lane j at time t can be determined as follows:

$$y_{i,j}^l(t) = \begin{cases} d_l(t) + (j - i) \times w, & \text{if } i < j \\ -(d_r(t) + (i - j - 1) \times w), & \text{otherwise.} \end{cases} \quad (3)$$

$$y_{i,j}^r(t) = \begin{cases} d_l(t) + (j - i - 1) \times w, & \text{if } i < j \\ -(d_r(t) + (i - j) \times w), & \text{otherwise.} \end{cases} \quad (4)$$

Remark 2: In order to simplify notations, the same lane width is taken everywhere in the network, at the current stage of the study. However, this value may vary according to the road infrastructure (when counting narrow lanes or road shoulders frequently employed for decreasing the required right-of-way).

Likewise, each element of set $\mathcal{P}(t)$ can be potentially identified by taking the orthogonal projection of the y coordinate onto the Y axis. Combination of fine criteria intelligently selecting collected sensor observations and a favorable current vehicle environmental context will allow a *well constructed* set $\mathcal{P}(t)$ comprised of objects located at a *close* neighborhood $\mathcal{V}(t)$ of the reference vehicle. Consequently, in that advantageous case a unique candidate lane should verify that all points of set $\mathcal{P}(t)$ belong to the considered region $\mathcal{V}(t)$. Nevertheless, multiple solutions may arise from a roughly constructed set $\mathcal{P}(t)$. Hence, advanced decision schemes should be determined permitting to define an optimized vehicle neighborhood $\mathcal{V}(t)$ and consequently an efficacious observation set $\mathcal{P}(t)$.

Thus, if ϵ defines a desired positive number, then, at time t , set $\mathcal{V}_\epsilon$ should define constraints determining the region within which the desired observations should be located, that is

$$\mathcal{V}_\epsilon(t) = \{p(t) \in \mathcal{P}(t), d(p(t), p') \leq \epsilon, p' = (0, 0)\}. \quad (5)$$

where distance d and parameter ϵ should explicitly be determined. In order to ensure that the selected observation set will be non-empty, parameter ϵ can be increased progressively until a desirable set $\mathcal{V}_\epsilon$ is reached. Moreover, if Q denotes the number of the considered quantitative constraints, then the constraint set $\mathcal{C}$ is comprised of all the quantitative constraints that an observation should satisfy

$$\mathcal{C}_q = \{c_q, q = 1, \dots, Q\}. \quad (6)$$

where each constraint c_q should be also explicitly defined.

Consequently, the set of the selected observation points $\mathcal{P}_\epsilon$ will be comprised of observations in the vehicle neighborhood $\mathcal{V}_\epsilon$ satisfying all constraints in set $\mathcal{C}$, that is

$$\begin{aligned} \mathcal{P}_\epsilon(t) &= \{p(t) \in \mathcal{V}_\epsilon(t), \text{ s.t} \\ c_{q'}(p(t)) &\text{ satisfied } \forall c_{q'} \in \mathcal{C}_q \\ &\text{with } c_{q'} \text{ concerning point } p\}. \end{aligned} \quad (7)$$

Considering a candidate lane j , set $\mathcal{O}$ can be constructed defining two reference points of set $\mathcal{P}_\epsilon(t)$ belonging to the first and last lane respectively.

$$\begin{aligned} \mathcal{O}(t) &= \{p^k(t) = (x^k(t), y^k(t)), k = 1, 2 : \\ p^k(t) &\in \mathcal{P}_\epsilon(t), k = 1, 2, \\ y^1(t) &\in [y_{1,j}^l(t), y_{1,j}^r(t)], \\ y^2(t) &\in [y_{m,j}^l(t), y_{m,j}^r(t)]\}. \end{aligned} \quad (8)$$

Depending upon the definition of the observation set $\mathcal{P}_\epsilon(t)$ some reference points may be shifted and Equation 8 can be satisfied by multiple observations which do not necessarily belong in the considered candidate lanes. Since multiple responses may be implied by this decision criteria additional or complementary schemes should be employed in order to restrict acceptable responses. Furthermore, due to multiple causes set $\mathcal{O}$ may become empty at any time t . For instance, this would be the case if at least one constraint of Equation 6 is not satisfied and thus set $\mathcal{P}_\epsilon$ will be empty for any vehicle neighbourhood $\mathcal{V}_\epsilon$. Such outcomes may also occur whenever the laser detection region is limited e.g. large objects located close to the ego vehicle. In order to deal with similar situations and hence be able to provide a reliable estimation regarding the level lane of the ego vehicle at the sensor functioning frequency a confidence indicator is going to be involved. More precisely, whenever validated sensor data are associated with a unique response, through a defined probability low, a probability value will be determined regarding the available solution. This value will vary according to multiple parameters. For example, for a vehicle of a relatively constant or increasing speed superior to a given threshold, the confidence value will decrease with the time. Thus, only recent estimations will be considered in such a context.

Let's consider function θ taking strictly positive values when a unique lane estimation is available that is, $\theta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with

$$\theta(t) = \begin{cases} > 0, & \text{if lane identification is achieved at time } t \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Moreover, the following notations are considered:

- t_0 : time at which the first sensor detection is associated with valid data
- t_k : the last time with sensor detection for which the lane id was determined (computed or estimated value)
- t_c : the current instant with $t_0 < t_k < t_c$.

Set $\mathcal{D}$ can be determined at any time $t < t_k$ comprised of all the time instants where the ego vehicle level lane was defined,

$$\mathcal{D}(t) = \{t \in [t_0, t_k] \text{ s.t } \theta(t) > 0\}. \quad (10)$$

Let $\rho : \mathcal{D} \rightarrow [0, 1]$ be the probability function defined on set $\mathcal{D}$ with $\rho(t)$ measuring the reliability of the response $\theta(t)$.

For each element of set $\mathcal{D}$ a reliability measure can be associated at time t and thus set $\mathcal{R}$ can be constructed at time t such that

$$\mathcal{R}(t) = \{(t, \rho(t)) : t \in \mathcal{D}(t)\}. \quad (11)$$

Remark 3: Since set $\mathcal{R}$ will be updated at the sensor operation frequency, at each decision time it would be wiser to consider an *appropriate* subset of interval $[t_0, t_k]$ for which the vehicle level lane was identified.

If at time t_c invalid data are available or a non unique response is computed a *relatively* recent solution can be searched as an element of set $\mathcal{R}(t_k)$. If such solution is feasible a reliability measure will be also allocated the value of which will depend upon the considered probability law. Thus, for this first probabilistic approach of this study a quite restrictive probability distribution is defined allowing reduced confidence indicators to estimations provided when invalid data are involved.

Let's define parameters ϵ_{r_1} and ϵ_{r_2} corresponding to strictly positive numbers such that

- ϵ_{r_1} will be associated with the acceptable age of the past estimations
- ϵ_{r_2} will involve the reliability indicator for trusting a previous response
- ϵ_{r_3} will be associated with the acceptable elapsed time between the last estimation and the current time t_c .

These two parameters may be defined as constants (e.g. mean values) or they can be dynamically defined so as to adapt to the current vehicle context and thus providing a more realistic value.

Consider that at time t_c no lane estimation is possible. Let $n(t)$ denote the number of lanes at the related road segment where the ego vehicle moves at time t_c , $\tilde{c}$ is a constant value related to the road infrastructure. If equation

$$|t_c - t_k| \leq \epsilon_{r_3} \quad (12)$$

is satisfied, suppose that exists $\tilde{t} \in \mathcal{D}$ such that:

$$|\tilde{t} - t_k| < \epsilon_{r_1} \quad (13)$$

$$n(t) = \tilde{c}, \forall t \in [\tilde{t}, t_c] \quad (14)$$

$$\rho(\tilde{t}) > \epsilon_{r_2}. \quad (15)$$

Then the ego vehicle level lane at t_c could be the one at time $\tilde{t}$, that is $\theta(t_c) = \theta(\tilde{t})$. An appropriate reliability indicator $\rho(t_c)$ should be associated to $\theta(t_c)$ and time t_c , $(t_c, \rho(t_c))$ should be elements of sets $\mathcal{D}(t_c)$ and $\mathcal{R}(t_c)$ respectively.

When a constant value is associated with parameters ϵ_{r_1} and ϵ_{r_3} a rough approach would be to consider the max necessary time required by the vehicle to realize a lane change. Taking into consideration both the driver behavior and current vehicle context (traffic flow, mobile objects in the vehicle neighborhood, etc.) could provide a more precise estimation for ϵ_{r_1} and ϵ_{r_3} .

Similarly a strict value for parameter ϵ_{r_2} would be to consider a $\tilde{t} \in \mathcal{D}$, for which all constraints 13, 12, 14, 15 are satisfied and such that there exists $t' \in \mathcal{D}$ with $t' \geq \tilde{t}$ and for which $\rho(\tilde{t}) > \rho(t'') \forall t'' \in \mathcal{D}$ with $t'' \geq t'$. In other words one could select the most reliable and recent (within the acceptable time limits) lane identification value. However involving strict values for these three parameters could limit the flexibility associated with the proposed algorithm.

Remark 4: Equation 12 ensures that only recent past predictions can be employed. One may think that constraint expressed by Equation 14 may disrespect the previously made assumption considering recent estimations. However, constraint 15 eliminates this possibility. As previously mentioned whenever Equation 12 is satisfied values $\rho(t)$ of set $\mathcal{R}(t)$ are updated for any $t \in [\tilde{t}, t_k]$. The reliability value of an estimation is a decreasing function regarding time. The older an estimation becomes the smaller reliability is allocated.

III. FIRST ASSESSMENT

The benefits of the new lane identification decision approach are going to be evaluated when the probabilistic scheme is applied to a series of complex scenes. Thus, simulation scenarios supplemented by real recordings are employed modeling multiple traffic contexts under various designs of road infrastructure. The road environment and the related constraining effects imply confounding situations imposing varying driver profiles across time and space. The latter element is very important since under professional drivers recordings are biased due to a non realistic driving behavior. Hence critical elements are absent from such recordings compromising the outcomes of the study. The impacts of behavior-change interventions as provided by detailed simulation data form a precious source of inspiration for an optimal tuning of the suggested probabilistic methodology. The considered scenario is of one hour and half (5,400 secs) where the sensor frequency is of 25Hz. The performance of the suggested methodology is compared to the one resulting within the same scenario when a non probabilistic approach is utilized. The first evaluations noticeably accentuate the importance of the new stochastic scheme where a non deterministic behavior occurs everywhere in the model (traffic and road context, sensor functioning, driver behavior).

A. Measuring Accuracy

The real value of the followed lane id is illustrated in Figure 2 through the grey thick line as a function of time. In the same figure the estimated value of the vehicle lane id is also depicted by a thinner pink line. As one may observe the two plots coincide for the majority of the trip duration. However, an incompatibility is observed as shows the pink line in Figure 2 plotted outside the grey line localizing the vehicle at lane two where the real vehicle position at this moment is lane three. Additionally, on the same figure the total number of lanes of the road segment associated with each detection time is also represented by the red line. Despite the observed incompatibility the estimated lane never exceeds the

total number of lanes at the corresponding highway stretch. A micro statistical analysis will explain this observation. For any time within interval $[620, 679]$ the real vehicle position is lane three. However, the estimated value as provided by the level lane decision algorithm positions the ego vehicle at lane two during a period of 10 secs occurring within the time interval $[645, 655]$. A similar result was observed within period $[680, 695]$. Dense traffic is observed during these time periods creating temporary congestions. The laser detector placed in the front part of the ego vehicle has a limited detection area as it is closely surrounded by large cars almost immobile. As soon as these vehicles are able to move the laser detection area clears and moving objects in all lanes of the associated road segment are detected. It is of worth to accentuate that despite the congested flows the sensor provide valid data. Thus, the probabilistic approach is not utilized in this case. Employment of additional decision schemes detecting lane changes would prohibit this error to occur. Such schemes are not developed in this study and consequently not applied. Alternatively, taking into consideration the variation of the vehicle speed (low values) one could avoid this erroneous estimation.

Fig. 2. Vehicle lane trajectory

B. Probabilistic versus Deterministic Approach-First Comparison

The response frequency is measured when separately the probabilistic and deterministic approaches are employed under the same scenario describing the vehicle trajectory.

As Figure 3 represents the deterministic algorithm provides no response during the 20.3% of the total decision time. This is due to various reasons such as invalid data, multiple feasible solutions etc. When the new introduced probabilistic approach is employed, the percentage of no response falls down to 9.3%. Figure 4 illustrates this case. At this stage of the study and as

the probabilistic approach is recently developed a rather strict model is considered when defining the probability distributions associated with each response at a given time instant t . As previously discussed, these numbers concern the current data.

Remark 1: Under more flexible rules the percentage of no response is even more significantly reduced as the outcomes of the ongoing study show. However, this work is not developed here where a first approach of the probabilistic model is introduced.

Although the considered trip scenario is rather complex and relatively long, many more data are required and further improvements should be considered in the proposed structure in order to provide definite conclusions. Regarding the current scenario, the traffic flow is relatively fluid on average. Traffic congestions are not very frequent and when they occur a short duration is observed. Under a different traffic context, sensors will present a different behavior involving a higher density of invalid data. The response frequency related to the deterministic scheme will certainly be reduced. This subject is part of a research in progress and will not be profoundly developed at present. Additional work is required for providing solid conclusions.

Fig. 3. Response quantification - non probabilistic scheme

C. Quantifying Related Error

The related error associated with the response ability of the proposed scheme is now considered. During the 5,400 seconds of the associated ego vehicle trip, 135,000 decisions should be taken concerning the vehicle lane position. As we previously saw, the considered methodology responds during the 90.7% of the total decision time, that is a response is provided for 122,445 decisions. Moreover, for 25 seconds in total an erroneous solution is provided. Figure 5 represents the error quantification when the proposed decision scheme is applied to the considered data. Hence for the 99,5% of the total decision instants for which the suggested algorithm provided a solution a correct lane identification was returned. However, at this stage of the study this number represents only the error of the considered scenario. Under different entry

Fig. 4. Response quantification - probabilistic scheme

patterns a different number could be implied. That is one reason justifying the importance of realistic simulation data where rich environmental conditions can be easily created at no risk and reduced costs. The considered methodology can be then profoundly studied. Analysis of the system behavior will lead to improvements covering many particular cases requiring explicit treatment. Hence, performance guarantees will be provided which is the ultimate goal of this work.

Fig. 5. Error quantification

IV. DISCUSSION-FUTURE WORK

A stochastic approach detecting the level lane position of a vehicle equipped with a laser detector and a smart camera is introduced. The proposed scheme is mainly based on noisy sensor detections and for which a previously developed work was unable to respond. Due to technology sensibility, traffic context and failures observations frequently are judged invalid. Nevertheless, under reliable sensor outputs the proposed decision making scheme can provide accurate response regarding the

vehicle position, crucial information for the effective control of automotive structures. A probabilistic approach was judged necessary to be developed, where the main algorithm localises the ego vehicle lane by allocating appropriate detections on the extreme lanes of the road segment within the vehicle direction. Whenever sensor outputs are judged unreliable, probability laws are employed quantifying suitable confidence indicators associated with recent past estimations. The effectiveness of the proposed methodology was examined in carefully designed simulation experiments for a real highway stretch and real traffic scenarios. Thus many realistic complex contexts were now involved which would be impossible to study and analyze under recordings provided by professional drivers which mainly maintain a rather good driver behavior. A first evaluation of the suggested approach highlights the benefits and necessity of the probabilistic profile. The low computational complexity associated with the advantages of the decision making scheme encourages for a real-world application involving:

- the use of the closed loop decision model which now has a probabilistic and thus realistic aspect
- the extensive use of limited and available sensor data, devices with which autonomous cars are equipped for multiple purposes and hence no extra investment is required
- the use of low cost calculations.

It is of worth to be emphasized that despite the encouraging results, further improvements are required in order to generalize the efficiency of the lane estimation policy and thus provide robust guarantees regarding the response frequency and accuracy.

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