“Moisture Control” of the Precipitation: A Probabilistic Perspective

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ABSTRACT

10 Moisture is a key variable for determining the precipitation, thus the notion
11 of the “moisture control” of the precipitation follows. The present chapter
12 examines this notion from a probabilistic perspective. Here, it is important to
13 distinguish between the probability of precipitation and the expected intensity
14 of precipitation, under a given moisture state. It is suggested that the former
15 increases in a more pronounced manner than the latter with the increasing
16 moisture in the atmosphere. For this demonstration, local rain–gauge station
17 data and a shared sounding over Friuli Venezia Giulia, North-East Italy are
18 analyzed. Soundings provide a column precipitable water, which is used as
19 a measure of the moisture in the atmosphere. Implication of the study is that
20 moisture is more crucial in increasing a chance of precipitation, while it “con-
21 trols” the precipitation intensity only in a lesser extent.

1. Introduction:

Moisture “controls” the precipitation: this is a commonly accepted notion in atmospheric research. The present study is built upon this common notion, and examine it from a probabilistic perspective.

However, a word of caution first: we should keep in mind that the notion of “moisture control” of the precipitation is rather a misnomer. It is like to say “wood builds a wooden house”. Most will agree this is wrong: we need a carpenter to build a house using wood. In the same token, moisture by itself *does not* control the precipitation, although it definitely contributes to the precipitation process. The notion of “control” is loosely borrowed from a control theory originating from Norbert Wiener’s (1961) idea of *cybernetics*. In analogy with controlling a machine with a knob, the moisture is a “knob” in the atmosphere to control the precipitation.

This analogy may further go on to consider the moisture to be a cause of the precipitation. Here, again, we must recognize ambiguity of the notion “cause” in this context (*cf.*, Aristotle’s *Metaphysics*): this analogy takes the moisture as an agent (or more explicitly, like an actor in a drama) actively acting on a process. Of course, this is true only in analogy. Note that the most clear manner of defining the “cause” is to invoke the classical mechanics: any variable, ϕ , may be described by

$$d\phi/dt = f$$

with a tendency defined by f for a zero-dimensional system. Here, f is a cause, and $d\phi/dt$ is an effect. Clearly, moisture does not act on precipitation in this manner.

These moisture-based thinkings are becoming increasingly popular, partially due to recent rapid developments of satellite technologies. A notable example is the Global Navigation Satellite System (GNSS) sensor on board the Global Positioning System (GPS) satellite, which can provide a

retrieval of the column precipitable water (CPW: to be defined by Eq. 1 in Sec. 3) in much higher resolutions both in time and space than hitherto possible (Bevis et al. 1992). Wide availability of this data further stimulates the interests of applying it for short-term prediction of the precipitation (e.g., Champollion, et al. 2004, Brenot et al. 2013, Benevides et al. 2015, et al. 2019, Labbouz et al. 2015, Priego et al. 2016, Yao et al. 2017, Barindelli et al. 2018). These studies establish a well-identifiable correlation between CPW and precipitation, even visible in individual time series. Thus, the results rectify the afore-mentioned notions that moisture “controls and causes” precipitation.

With this background in mind, the purpose of the present chapter is to examine rather ambiguous notions of “control” and “cause” in the context of interplays between moisture and precipitation. For this purpose, we take a probabilistic perspective. Here, various probabilistic statistics (probabilities, expectation values) associated with the precipitation are evaluated under a given CPW (moisture state). By formulating the problem in this manner, we examine the change of these precipitation statistics with a change of CPW. From the obtained tendencies, we infer the question of “control” of moisture to “cause” the precipitation.

Most naively, we expect that both chance and the intensity of precipitation increases with increasing moistures (CPWs) in the atmosphere. Thus, we may infer that higher moisture more likely causes intensive precipitations. Here, we emphasize an importance of distinguishing them, i.e., between the *frequency* (or the chance) and the *intensity* of precipitations: two quantities that are clearly linked together. However, one does not necessarily follow from another. More precisely, the present chapter is going to suggest that, although the precipitation frequency increases with increasing moistures in the atmosphere, an increase of the precipitation intensity does not *necessarily* follow.

For this investigation, combination of sounding-derived data and rain–gauge measurements over Friuli Venezia Giulia (FVG), North–East Italy, is adopted, as described in the next section. The methodologies, as well as theories behind, are introduced in Sec. 3, then followed by a presentation of the results in Sec. 4. The paper is concluded with further discussions in Sec. 5.

2. Data Description:

Studied is the 12–year long data set spanning from 1 January 2006 to 1 January 2018 of soundings and rain gauges available over Friuli Venezia Giulia (FVG), a region of North-East Italy. FVG faces to the north the eastern part of the Alps, and to the south the Grado and Marano lagoon, situating in a north of the Adriatic Sea (Fig. 1). Due to this unique topography, FVG experiences one of the severest precipitation climatology in Central Europe (e.g., Feudale and Manzato 2014, Isotta et al. 2014, Manzato et al. 2016, Poelman et al. 2016, Pavan et al. 2019). The analyses are performed for the full period by default (for full years), but they are also repeated by dividing the data into the convective (May–September) and non–convective (October–April) seasons.

The soundings are launched every 12 hours at 0000 and 1200 UTC from a radio-sounding station (WMO code 16044) located near Udine. To measure the moistness of the atmosphere, the column precipitable waters (CPWs in [mm]), as defined by Eq. (1) in the next section, are calculated from soundings. The obtained CPWs are statistically representative of the atmosphere over the FVG Plain, rather than of a local vertical column immediately above the sounding launch site. In fact, during its ascension a radiosonde typically traverses (moves horizontally) over a distance of the whole FVG Plain, stretching 80 km and 40 km, respectively, in longitude and latitude, taking on average 42 minutes until reaching the tropopause.

A network of mesonet stations (33 in total) organized over FVG provides rain–gauge measurements of the 6–hourly accumulated precipitation amount [mm/6h] ending at 0000, 0600, 1200, and 1800 UTC. In the following, analyses are performed with the accumulated precipitation observed

at both four rain–gauge stations (Udine, Campofornido, Fagagna, and Talmassons) individually, and additionally with the *average* and the *maximum* of all the 33 mesonet rain–gauge stations. The rain–gauge sensibility varies from 0.1 to 0.2 mm/h. By literally measuring precipitation at a single geographical point, a rain gauge provides direct information on local precipitation processes, whileas the use of four stations, close to each other, takes into account of the high spatial variability of precipitation. Furthermore, a greater extent of the precipitation variability over FVG is examined by both the average and maximum of the 33 rain–gauge stations.

By combining these two sets of data, the study examines the statistical characteristics of local precipitation under a given background state of moisture in the atmosphere: e.g., as the moisture increases more as a background state over FVG, will both the “chance of precipitation” and the “expected precipitation intensity” increase at individual points of the surface? From this statistical analysis, we infer an extent that the local precipitation is “controlled” by an atmospheric state over the scale of the FVG Plain.

To decide the timing for pairing 12–hourly sounding–based CPW and six-hourly accumulated precipitations, their lag correlations are first evaluated (Fig. 2). Here, the lag is defined by a lead time of sounding. Well-defined peaks (maximum) in correlations are found with a lag of six hours, *i.e.*, the accumulated precipitation measured six hours after a launch of a sounding, has the highest correlation. Based on this lag–correlation result, in the following, the CPW values computed from the soundings at 0000 and 1200 UTC are paired with accumulated precipitation amount [mm] over 0000–0600 and 1200–1800 UTC periods, respectively. This pairing strategy leads to four pairs of time series consisting of CPW and precipitation from afore–mentioned four rain–gauge stations: 34664 CPW–rain measurement pairs in total. Here, 400 pairs are missing from these four pair time series, with 56 missing CPW and additional 176 missing precipitation measurements. Analyses repeated with the accumulated precipitation with no lag time do not change the results.

3. Analysis methodology and Probability Theory

This section introduces an analysis procedure based on the probability theory, from which we infer “moisture control” of the precipitation.

3.1. Column precipitable water (CPW)

We measure moistness of the atmosphere by the column precipitable water (CPW), I , i.e., a vertical integral of the water-vapour content in a given column atmosphere:

$$I = 1/\rho_w \int_{z_s}^{z_T} \rho q_v dz = 1/\rho_w g \int_{p_T}^{p_s} q_v dp. \quad (1)$$

Here, q_v is the specific humidity, ρ_w and ρ are the liquid water and air densities, respectively. A vertical integral in height, z , is performed from the surface, z_s , to a top, z_T (12 km in the present study), of a sounding measurement. The second expression is obtained from the first with a help of the hydrostatic balance ($dp/dz = -g\rho$), in which the integral is in terms of the pressure, p , with a reversed integral range. Here, g is the acceleration of the gravity.

The CPW is a frequently adopted measure of available moisture for precipitation in the atmosphere: literally speaking, it measures the maximum possible precipitation of water, which may precipitate out from a give atmospheric column, if nothing else (e.g., surface evaporation, moisture advection) happens over a given time span. Of course, this is rather an oversimplified picture, because it is hard to imagine that all CPW would condensate within a given column under any conceivable process. Nevertheless, it would be intuitive enough to interpret CPW as an upper bound for a possible precipitation at a given moment with a given atmospheric column. The analyses are performed over a range of CPW, $(I_1, I_2) = (5, 50)$ mm, and this range is divided into 18 discrete bins. Each bin is characterized by its central value.

3.2. Probability Theory

As stated in the Introduction, we infer the “control” of moisture to “cause” the precipitation by adopting a probabilistic perspective. Here, probabilities (chances) are computed as frequencies of occurrences, and the terms, probability (or chance) and frequency, are used interchangeably in the following. Thus, for example, the probability, $p(I)$, say, of observing CPW, I , is evaluated by dividing a number of occurrences (*i.e.*, frequency) over a bin range $[I - \Delta I/2, I + \Delta I/2]$, $N(I)$, by total number of data, N , *i.e.*, $p(I)\Delta I = N(I)/N$.

For examining the change of precipitation statistics with a change of CPW, I , the most basic quantity is a conditional probability density, $p(R|I)$, of a precipitation rate, R , under a given CPW, I . By further examining the statistics on precipitation, R , *conditioned* by CPW, I , we can infer how the moisture “controls” and “causes” the precipitation. As a result, an average precipitation, $\bar{R}(I)$, under a given CPW, I , *i.e.*, the conditionally-expected precipitation, can be computed by using the conditional probability density, $p(R|I)$, as:

$$\bar{R}(I) = \int_0^{+\infty} p(R|I)RdR. \quad (2)$$

In practice, $\bar{R}(I)$ is obtained by simply averaging over all the precipitations happened over a range, $[I - \Delta I/2, I + \Delta I/2]$.

For defining a probability of precipitation, we first need to define a “precipitation state” (or “precipitation event”). Here, we define it as a state with a precipitation, R , above a pre-fixed threshold, $R_c(> 0)$, *i.e.*, $R > R_c$: a finite-precipitation threshold, R_c , is crucial for two reasons. First, the rain-gauge data records any state below 0.1 mm/6h as zero precipitation, excluding all the drizzlings: we must set $R_c \geq 0.1$ mm/6h to define a precipitation state. More importantly, by analyzing the statistics for intensive precipitations with increasing CPWs, we can infer an extent that a high moisture state “causes” intensive precipitations by increasing the threshold, R_c .

The most basic quantity for inferring the “control” of precipitation by moisture is the probability for precipitation $R > R_c$, with a given CPW, I :

$$P(R > R_c|I) = \int_{R_c}^{+\infty} p(R|I)dR. \quad (3)$$

In practice, it is obtained by counting the number of occurrences, $N(R > R_c, I)$, with $R > R_c$ over a CPW range, $[I - \Delta I/2, I + \Delta I/2]$, and dividing it by $N(I)$, i.e., $P(R > R_c|I) = N(R > R_c, I)/N(I)$.

A likely change of precipitation intensity with increasing CPWs is inferred by computing the expected precipitation intensity with a given CPW, I , under a condition of $R > R_c$:

$$\bar{R}(R > R_c, I) = \int_{R_c}^{+\infty} p(R|R > R_c, I)RdR. \quad (4)$$

In practice, $\bar{R}(R > R_c, I)$ is obtained by simply averaging over all the precipitation measurements with $R > R_c$ over a CPW range, $[I - \Delta I/2, I + \Delta I/2]$. Particularly, “moisture control” of the precipitation is inferred by examining how the average (*or* conditionally-expected) precipitation $\bar{R}(R > R_c, I)$ changes with increasing thresholds, R_c . Here, by increasing the threshold, R_c , we focus our attention more on the most intense precipitations.

Note that a joint probability density, $p(R, R > R_c|I)$, for being R and $R > R_c$ with a given I , can be written in two different manners as

$$\begin{aligned} p(R, R > R_c|I) &= p(R|R > R_c, I)P(R > R_c|I) \\ &= P(R > R_c|R, I)p(R|I). \end{aligned}$$

Furthermore, $P(R > R_c|R, I) = 1$ when $R > R_c$, and otherwise both $p(R, R > R_c|I)$ and $P(R > R_c|R, I)$ are simply zero, making the relation trivial. It immediately follows that

$$p(R|R > R_c, I) = p(R|I)/P(R > R_c|I).$$

Also note that $P(R > R_c|I)$ does not depend on R . Thus, Eq. (4) reduces to

$$\bar{R}(R > R_c, I) = \int_{R_c}^{+\infty} p(R|I)RdR/P(R > R_c|I). \quad (5)$$

177 Note especially when $R_c = 0$, the above expression simply states that the average precipitation, un-
 178 der precipitation ($R > 0$) and with a given CPW, I , is obtained by dividing the average precipitation
 179 with the probability of precipitation ($R > 0$) both under a given CPW:

$$\bar{R}(R > 0, I) = \bar{R}(I) / P(R > 0 | I). \quad (6)$$

181 To see a more general relation than Eq. (6) between the three quantities (2), (3), and (5), we
 182 re-write Eq. (2) as

$$\bar{R}(I) = \int_0^{R_c} p(R|I) R dR + \int_{R_c}^{+\infty} p(R|I) R dR.$$

184 Noting that

$$P(R < R_c | I) = 1 - P(R > R_c | I),$$

186 and recalling the definition of a conditional average, given by Eq. (5), this expression further
 187 reduces to

$$\bar{R}(I) = [1 - P(R > R_c | I)] \bar{R}(R \leq R_c, I) + P(R > R_c | I) \bar{R}(R > R_c, I).$$

189 By re-writing the above expression, we find that an expected precipitation under a precipitation
 190 state with $R > R_c$ (Eq. 4) is alternatively given by:

$$\bar{R}(R > R_c, I) = \bar{R}(I) / P(R > R_c | I) - [1 - P(R > R_c | I)] \bar{R}(R \leq R_c, I) / P(R > R_c | I). \quad (7)$$

192 The exact Eq. (7) may not be easy to interpret. However, note that the second term of the right-
 193 hand side drops out when $R_c = 0$, as already noted above. In general, if Eq. (7) could furthermore
 194 be approximated as

$$\bar{R}(R > R_c, I) \simeq \bar{R}(I) / P(R > R_c | I), \quad (8a)$$

196 then this equation is relatively easy to interpret. Eq. (8a) states that the average precipitation under
 197 a precipitation intensity above R_c and with the CPW, I , may be approximated by the precipitation
 198 averaged over all the cases with a given I divided by the probability of precipitation above R_c

under the given I . The remaining second term in the right hand side may simply be considered a non-trivial correction term, arising from non-zero precipitation threshold, R_c . For this reason, we may consider the first term in Eq. (7) in a stand-alone manner. Eq. (8a) can alternatively be presented as

$$P(R > R_c | I) \simeq \bar{R}(I) / \bar{R}(R > R_c, I), \quad (8b)$$

$$\bar{R}(I) \simeq P(R > R_c | I) \bar{R}(R > R_c, I). \quad (8c)$$

These expressions are exactly only when $R_c = 0$. However, errors with these expressions only increase gradually with the increasing R_c , thus they can be considered to be asymptotic expressions for $R_c \rightarrow 0$.

We furthermore introduce a joint frequency density distribution, $p(I, R)$, of the CPW, I , and the precipitation, R , as a number of occurrences per bin, i.e., $N p(I, R) \Delta I \Delta R$, where N is the total number of data. The distribution is alternatively examined separately for a given CPW as being conditioned by I , thus $p(R|I) = p(I, R) / p(I)$, where the probability density, $p(I)$, of CPW, I , can be evaluated from the joint distribution by an integral:

$$p(I) = \int_0^{+\infty} p(I, R) dR.$$

In practice, this integral is replaced by a sum of the frequencies of precipitation occurrence over the all bins for precipitation, R .

3.3. Normalization

In the following, tendency of a change of the three quantities, (2), (3), and (5), with a change of CPW, I , is analyzed for inferring the “moisture control” of precipitation. The simplest manner of quantifying a tendency is to approximate it by a linear trend. Thus, a quantity of interest, say, $\phi(I)$, is fit into a linear dependence on I : a least-square fit is performed over the maximum range of CPW, (I_{min}, I_{max}) , over which a nonvanishing probability (frequency), $P(R > R_c | I)$, is found for

a given threshold, R_c . The result is then expressed as

$$\phi(I) \simeq \phi_0 [1 + \alpha(I - I_0)/(I_2 - I_1)] \quad (9)$$

in terms of a reference value, ϕ_0 , and a normalized slope, α . Here, $I_0 = (I_1 + I_2)/2$ is a middle point of the range under the consideration (*i.e.*, 5–50 mm) with $I_2 - I_1 = 45$ mm. The reference value, ϕ_0 , is defined as a value, $(\phi(I_1) + \phi(I_2))/2$, at the middle point of a linear least-square fit. The normalized slope, α , measures an increase of the quantity over the range, (I_1, I_2) , relative to the reference value, ϕ_0 . Thus, $\alpha = 1$ means that a normalized value, ϕ/ϕ_0 , varies from 0.5 to 1.5 over the range from $I = I_1$ to I_2 under the linear fit.

4. Results

By following the formulation of the last section, we examine how various precipitation statistics change with a change of CPW, I , in this section.

Fig. 3 shows the average precipitations, $\bar{R}(I)$, *i.e.*, a straight average over all the precipitation measurements (including zero measurements) over a range, $[I - \Delta I/2, I + \Delta I/2]$, with $\Delta I = 2.5$ mm, for the four rain–gauge stations in (a) and for the average (mean) and the maximum of the 33 rain–gauge stations in (b) for full years (green), as well as for convective (May–September: red) and non–convective (October–April: blue) seasons. They increase overall with increasing CPWs, and also overall linearly, as suggested by short–dashed lines for least–square fits (and the chain–dashed lines for the maximum). In performing the least–square fits to straight lines, here and hereinafter, the statistical points are weighted by the numbers of observations used for the evaluation. This weighting properly takes into account the uncertainty associated with small numbers of data over the tails of the CPW distribution.

The result seems to support the popular notion that more moisture “causes” more precipitations. This conclusion does not change regardless of whether we focus on particular seasons, nor of ex-

amining the individual four stations, or the average and maximum of all 33 stations. Nevertheless, we may note that the slope is much steeper for the maximum precipitations of all the 33 stations in the FVG Plain, and also twice steeper for non-convective winter seasons: twice more precipitation is expected with $I > 15$ mm on average during winter than summer.

Recall that the average precipitation, $\bar{R}(I)$, under a given CPW are mainly defined by contributions of the two factors (Eq. 8c): the probability for precipitation, $P(R > R_c|I)$, and the expected intensity of precipitation during the events, $\bar{R}(R > R_c, I)$. We, thus, examine how these two factors contribute to increasing precipitations with increasing CPWs. Though the formula itself is exact only with $R_c = 0$, tendencies with the increasing R_c are of their own interests for inferring the “moisture control” of more intensive precipitations.

Fig. 4(a) shows that the conditional frequency, *or* probability, $P(R > R_c|I)$, of precipitation indeed increases with increasing moistures in the atmosphere, *or* CPWs, with varying thresholds, R_c , over the range 1–20 mm/6h. Here, the four rain-gauge stations are considered for full years, as a standard case. The probability, $P(R > R_c|I)$, overall increases linearly with increasing CPWs, as suggested by short-dashed lines for least-square fits. Furthermore, the precipitation probability decreases with increasing precipitation-event thresholds, R_c , at a constant rate, almost independent of CPW, as suggested by Eq. (8b), which is also overlaid by long-dashed curves.

The expected conditional precipitation intensity, $\bar{R}(R > R_c, I)$, shown in Fig. 4(b), in turn, tends to increase with increasing CPWs, but much less significantly than the average precipitation, $\bar{R}(I)$ (*cf.*, Fig. 3): although more moisture may somehow favor more intensive precipitations, this tendency is much less pronounced than for the precipitation probability. A particularly noticeable feature is that the expected-precipitation intensity is asymptotically extrapolated (say, by taking a linear fit) to a nonvanishing value in the limit $I \rightarrow 0$, although the precipitation probability itself vanished at $I = 0$. In the crudest approximation, $\bar{R}(R > R_c, I)$ may even be considered a constant

270 with CPW, and the average precipitation may even be set

$$\bar{R}(I) \simeq P(R > R_c | I) \bar{R}(R > R_c, I = 0). \quad (10)$$

271 In other words, the increase of the average precipitation with the increasing CPW is overall due
272 to an increase of the precipitation probability, rather than to that of the conditionally-expected
273 precipitation intensity.

274 The finite precipitation intensity, $\bar{R}(R > R_c, I = 0)$, in zero-CPW limit increases with increasing
275 thresholds, R_c . As the threshold, R_c , increases, a relative change of $\bar{R}(R > R_c, I)$ with a change
276 of CPW becomes even weaker compared to a larger limiting value. Overlaid long-dashed curves
277 show that the asymptotic formula (8a) remains a reasonable approximation up to $R_c = 5 \text{ mm/6h}$,
278 although the discrepancy rapidly increases beyond this threshold, and no curve is even plotted for
279 $R_c \geq 15 \text{ mm/6h}$.

280 To see these overall tendencies of $P(R > R_c | I)$ and $\bar{R}(R > R_c, I)$ more clearly, the curves in
281 Fig. 4(a) and (b) are re-plotted in Fig. 4(c) and (d), respectively, under a normalization, $\phi(I)/\phi_0$,
282 defined by Eq. (9), where ϕ is a variable in concern. Strikingly, all these curves superpose each
283 other relatively well after the normalization. Also shown by the short-dashed lines are linear least-
284 square fits. Although the actual curves do fluctuate, the overall tendencies of all these curves are
285 reasonably linear. Keep in mind that larger fluctuations for larger CPWs and larger thresholds, R_c ,
286 are due to statistical uncertainties with lack of sufficient data.

287 We note that these normalized linear slopes in Fig. 4(c) and (d) are much gentler for the expected
288 precipitation intensity, $\bar{R}(R > R_c, I)$, than for the precipitation probability, $P(R > R_c | I)$. It re-
289 iterates the point already suggested by Fig. 4(a) and (b) in a more objective manner: the increase
290 of $P(R > R_c | I)$ with increasing CPWs is more pronounced than that of $\bar{R}(R > R_c, I)$, *i.e.*, a more
291 objective support for a tendency towards Eq. (10).

292 Tendencies of the statistics towards larger thresholds, R_c , are of own interests, because they
 293 help us to infer how the moisture “controls” more intensive precipitations. Here, we notice weak,
 294 but noticeable, changes of the slopes with increasing thresholds, R_c : a gradual steepening and
 295 flattening tendencies, respectively, for $P(R > R_c|I)$ and $\bar{R}(R > R_c, I)$. Thus, as we focus more on
 296 more intensive precipitations, both the tendencies for higher probabilities and for independencies
 297 of the expected precipitation intensity with increasing moistures are more pronounced.

298 Loosely translating the results into a more common terminology: though more moisture clearly
 299 “causes” more frequent precipitations, the resulting precipitation may not always be that strong.

300 Fig. 5 presents further analyses to understand this whole behaviour: Fig. 5(a) shows a joint
 301 frequency distribution, $p(R, I)$, for CPW, I , and the precipitation, R . More precisely, $p(I, R)$, is
 302 plotted in terms of a number of occurrences per bin, *i.e.*, $N p(I, R) \Delta I \Delta R$, where N is the total
 303 number of data, $\Delta I = 2 \text{ mm}$ and $\Delta R = 2 \text{ mm/6h}$ are bin sizes for I and R , respectively. Here, data
 304 with zero precipitation are excluded. The precipitation occurs more frequently over a range of
 305 CPW, $I = 10\text{—}35 \text{ mm}$. Heavy precipitation events, say, above 20 mm/6h are also most frequent
 306 over this range.

307 However, it does not mean that the range, $I = 10\text{—}35 \text{ mm}$, is the most optimal for producing
 308 the intensive precipitations, because a frequent occurrence of precipitation is simply due to a more
 309 frequent occurrence of CPW values in this range. Fig. 5(b) shows this by plotting a number
 310 of occurrences of CPW for every 2 mm bin, both including (blue) and excluding (red) the zero
 311 precipitation measurements. Very small numbers of occurrences makes the statistics uncertain
 312 with the CPW above 40 mm , as already suggested by Fig. 4(d).

313 Fig. 5(c), in turn, shows the conditional frequency distribution, $p(R|I)$, of precipitation [mm/6h]
 314 computed for every CPW bin, separately, using the same bin size as in (a). A conditional frequency
 315 distribution is obtained by dividing the joint distribution in (a) by the frequency of non-zero pre-

precipitation state over a given CPW bin (*i.e.*, red curve in Fig. 5(b) divided by the blue curve):
 $p(R|I) = p(R, I)/p(I)$. The distribution, as shown in the vertical direction, does not change in any
dramatic manner with a change of CPW, as moving in the horizontal direction. Most notably, we
no longer see higher frequencies over the range, $I = 10\text{--}35\text{ mm}$, than other CPW ranges.

This distribution (Fig. 5(c)) is designed to answer the question of whether a chance of intensive
precipitation actually increases with increasing CPWs. Here, we *do* indeed see an indication of this
tendency, with a noticeable more frequent occurrence of some intense precipitations, exceeding
 $20\text{ mm}/6\text{ h}$, above $I = 44\text{ mm}$. However, this tendency is statistically hardly robust due to very few
number of occurrences in the bin boxes as seen in Figs. 5(a) and (b). The highest CPWs could be
associated to specific situations, as for example when there is a continuous advection of moisture.

For substantiating our claim, *i.e.*, more moisture does not “cause” more intense precipitation,
the same is examined by plotting the frequency, $p(R|I)$, as curves in Fig. 5(d): frequencies over a
CPW bin size of 10 mm are shown for a full range of CPW, $I = 0\text{--}50\text{ mm}$. They overlay each other
quite well. It simply suggests that, once it is in a precipitation state (*i.e.*, $R > 0$), the statistical
characteristics of the precipitation are approximately identical independent of the available mois-
ture in the atmosphere. Especially, the expected intensity of precipitation does not change in any
dramatic manner with increasing moistures in the atmosphere, as already suggested by Figs. 4(b)
and (d).

Of course, the match is hardly perfect. The most noticeable deviation is over the lowest CPWs
($0\text{--}10\text{ mm}$: in green), with a faster decrease of the frequency than the other ranges: no precipitation
above $20\text{ mm}/6\text{ h}$ is observed. A large scatter of frequency with the largest CPWs ($40\text{--}50\text{ mm}$:
orange), due to small numbers of occurrences, is also evident: occasional high frequencies of
precipitation are associated with very low frequencies in between. The point would be clearer by
directly examining the distribution in Fig. 5(c): over the CPW range of $(46\text{--}48)\text{ mm}$, for example,

although we find high frequencies for the precipitation ranges of 42–44 mm/6h and 30–32 mm/6h, the chance of precipitation is absolutely zero (if the analysis is trusted) over the range of 32–42 mm/6h, whereas we definitely find a finite chance of precipitation for the same range even below $I = 20$ mm. A smoothed version of distribution, as would be obtained with more number of occurrences, is expected to be closer to distributions with the smaller CPWs, as predicted by the statistical extreme-value theory (Katz 1999, Lucarini et al. 2012, Dutfoy et al. 2014). A further, weak but systematic trend with increasing moistures is just consistent with a gentle increase of the expected precipitation intensity with increasing CPWs already identified in Figs. 4(b) and (d).

A more compact manner of presenting the same results is to plot the normalized slope, α , as introduced in Eq. (9) for the all cases (Fig. 6): for the precipitation probability, $P(R > R_c|I)$ (solid) and for the the expected precipitation intensity, $\bar{R}(R > R_c, I)$ (short dash) with (a) the four rain-gauge stations, (b) the average, and (c) the maximum of the 33 rain-gauge stations. Curves are for full years (green), convective (red), and non-convective (blue) seasons. Note that no value is plotted when no more than three bins are found for a least-square fit, for example, above $R_c = 12$ mm/6h in (a) for $P(R > R_c|I)$ of convective seasons.

As already seen in Fig. 4, an increasing tendency (*i.e.*, slope) of precipitation probability, $P(R > R_c|I)$, with increasing CPWs is enhanced by increasing thresholds, R_c , whereas the same tendency with the expected precipitation intensity, $\bar{R}(R > R_c, I)$, decreases. The latter tendency is similar regardless of periods considered with individual rain-gauge stations: all the normalized slope becomes as small as 0.2 with $R_c = 20$ mm/6h. Average over the 33 rain-gauge stations also behaves in a similar manner. On the other hand, with the maximum precipitation of all the 33 stations, the slope for $\bar{R}(R > R_c, I)$ decreases only to 0.4 with $R_c = 20$ mm/6h for the full years and the convective seasons, and with a consistently larger slope for the non-convective seasons. As a whole, Fig. 6 confirms the earlier conclusion that a main consequence of a more moisture in

the atmosphere is a more chance of intensive precipitation, but without increasing the expected precipitation intensity substantially.

5. Further Discussions

The present study has considered how the moisture “controls” and “causes” the precipitation by examining the precipitation statistics conditioned by the atmospheric moisture state, as measured by the column precipitable water (CPW). An ontologically rather naive premise is taken: change of the statistics with a change of the atmospheric moisture suggests how the latter controls the former. Though this premise is hardly defensible, it leads to some insights on the “moisture control” of the precipitation.

Naively, more moisture in the atmosphere should cause more frequent and intense precipitations. The present study suggests an importance of distinguishing between those two expected tendencies with the increasing moisture in the atmosphere. Unfortunately, the literature is often not careful with this distinction. A probabilistic description of precipitation adopted in our study makes this distinction easier by introducing two separate measures: the probability of rain occurrence and the expected intensity of precipitation under given CPW. Furthermore, dependencies of these statistics on the precipitation–event threshold, R_c , are to be closely examined, because they can infer how moisture “controls” more intensive precipitations. The question has been addressed by analyzing the rain–gauge and the sounding data sets collected over Friuli Venezia Giulia (FVG), North-East Italy.

We have found that the frequency (probability), $P(R > R_c|I)$, of occurrence of precipitation above R_c indeed increases with increasing available moistures in the atmosphere (Fig. 4(a, c)). However, the expected intensity, $\bar{R}(R > R_c, I)$, of precipitation under a condition above, R_c in-

creases only weakly with increasing CPWs (Fig. 4(b, d)). It has been also found that these contrasted tendencies become more pronounced with increasing thresholds, R_c .

The obtained result is rather in odd with a common notion of attributing a high moisture state as a *cause* of extreme precipitation events and floods. Here, however, realize that our analysis is rather subtle: the main analysis is performed by limiting it to a state of precipitation (*i.e.*, $R > R_c$), and the statistics are evaluated under this condition. When the statistics are evaluated by including the non-precipitation state, a different picture emerges: the average precipitation, indeed, increases with increasing available moistures (Fig. 3).

However, the present study points out something more: the average precipitation increases with increasing CPWs mostly because of an increasing chance of precipitation, but not because the intensity of individual precipitation events increases. Thus, the cause of extreme precipitations may better not be attributed to high moisture of the background atmosphere.

Our main conclusion is still to be established in robust manner over an upper tail of CPW, especially, above 44 mm, where not enough data is available in this study. Examination of some specific cases may still be helpful: Table 1 shows the 11 most intense 6h-precipitation observed by a dense network of 104 rain gauges over FVG during the period of February 2006—February 2015 (9 years), adopted from Manzato et al. (2016). Each row lists a station reporting the maximum precipitation over a 6h-period, along with the CPW value as observed by the Udine sounding. We see that in all these cases, for producing precipitation of more than 150 mm/6h, the maximum CPW was only 42 mm, whereas the minimum CPW was 23 mm (followed by 30 mm), and the average CPW was 34 mm. The table shows that a CPW value of just above 30 mm was sufficient to generate an extreme precipitation (up to 270 mm/6h) over mountainous areas of FVG. We may speculate that a minimal moisture must be available to initiate precipitation. In the present analysis, 20 mm of atmospheric moisture appears to be a pre-condition for a substantial chance of precipitation,

say, 15 %, when the precipitation is defined by a threshold of 1 mm/6h (Fig. 4(a)). Above this moisture threshold, the expected precipitation intensity also remains almost constant over a range of 8–10 mm/6h (Fig. 4(b)).

The present study is based on an analysis over a particular region of the globe. For a good demonstration, a choice of the area of study must be reasonably precipitation intensive. By choosing one of the most precipitation intensive areas in Central Europe (Feudale and Manzato 2014, Isotta et al. 2014, Manzato et al. 2016, Poelman et al. 2016, Pavan et al. 2019), the present study easily satisfies this basic data requirement. Nevertheless, some of the details of the results would be specific to the FVG region. For this reason, more extensive observational analyses would still be required to verify a generality of the present result, especially a tendency for constancy of the expected precipitation intensity with increasing available moistures in the atmosphere.

Nevertheless, the present result rather reminds us a basic that the precipitation process is not a simple conversion of moisture into water drops, and then falls of these drops as a precipitation. For a precipitation to form, a certain dynamical mechanism must be in place, for example by a frontogenesis associated with a low-pressure cyclone as seen in weather maps (*cf.*, Shapiro and Grønås 1999), or alternatively, by a local convective instability (Doswell 2001, Houze 2014), or more generally, by a combination of both (Markowski and Richardson 2010), maybe assisted by the presence of mountains, that lift inflow air upwards (Rotunno and Houze 2007). In order to induce an extreme precipitation, a given atmospheric column does not need to contain all the water required to fall, say, in next six hours: there are always atmospheric flows that supply more moisture as required, and the aforementioned dynamical mechanisms can provide means for generating and maintaining such a state by positive feedback processes. Finally, at a smaller scale, cloud microphysics (Pruppacher and Klett 1997, Khain and Pinsky 2018) ultimately defines the precipitation intensity.

434 Only after combining all these different processes together, the available moisture in the atmo-
435 sphere is converted into condensed water, and then precipitates. Ontologically naively speaking,
436 more moisture causes more frequent precipitations, but only with helps of many other processes;
437 more moisture does no necessarily cause more intensive precipitations, but that may happen only
438 with the active contributions of many other processes.

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519 cases happened in the plain, while all the others are in the Prealps or Alps. . . . 27

DATE	PERIOD	STATION	ZONE		MaxRAIN6h [mm/6h]	CPW [mm]
2011/10/26	00-06	Malga Valine	3	Carnic Prealps	271.2	33
2013/09/09	12-18	Cividale	2	NE plain	210.5	30
2012/11/11	12-18	Chievolis	3	Carnic Prealps	175.8	32
2012/11/05	00-06	Resia	3	Julian Prealps	168.8	40
2008/08/15	12-18	Pontebba	4	Julian Alps	160.2	42
2010/11/01	00-06	Piancavallo Mount	3	Carnic Prealps	158.8	31
2010/10/05	00-06	Piancavallo Mount	3	Carnic Prealps	158.6	38
2010/06/13	12-18	Andreis	3	Carnic Prealps	157.2	35
2013/12/26	06-12	Barcis	3	Carnic Prealps	152.0	23
2012/11/11	06-12	Chievolis	3	Carnic Prealps	150.8	32
2006/09/15	12-18	Chievolis	3	Carnic Prealps	150.0	40

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(a) Four Stations

(b) Mean & Max

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