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The present study explores engineering students' mathematical problem posing competencies in relation to integral calculus, and their attitudes towards mathematical problem posing. The sample comprised of 135 undergraduate engineering students from a public university in Iran. Students' problem posing competencies were explored using a test including eight problem posing tasks related to the fundamental theorem of calculus and integral-area relationships. Furthermore, students completed a questionnaire that explored their attitudes towards mathematical problem posing. Nine students also participated in a semi-structured interview. The findings show that many students could improve their problem posing abilities further, and around 60 percent of students had positive attitudes towards mathematical problem posing activities.

Keywords: calculus, mathematics for engineers, mathematical problem posing, attitudes towards problem posing, integral calculus.

INTRODUCTION

Engineering relies heavily on using mathematics for building and designing projects for the use of society, however, attracting and retaining students in engineering degrees are sometimes problematic because of the role of mathematics in engineering (Flegg, Mallet, & Lupton, 2012). Mathematical problem posing can be considered as one of the central activities in mathematics and is a useful tool in mathematical teaching and learning (NCTM, 2000). Problem posing referred to "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (Stoyanova & Ellerton, 1996, p. 1). To pose mathematical problems, several skills are required such as the abilities to formulate mathematical situations, recognizing relationships between different mathematics concepts, and choosing an appropriate approach for each situation (Abu-Elwan, 1999). Problem posing has potential benefits for improving the quality of teaching and learning of mathematics. For instance, it could help students to develop their mathematical understanding (e.g., Cai & Hwang, 2002) and problem-solving skills (Cai & Hwang, 2002). Also, it can help teachers to identify students' mathematical misconceptions and difficulties (Chen, Van Dooren, & Verschaffel, 2015).

Calculus is an important topic in advanced mathematics, and has various applications in other disciplines such as physics and engineering (Jones, 2015). It is essential for students to fully understand calculus concepts and be able to apply them in different situations (Mahir, 2009). Integral calculus is a valuable topic in calculus, and is a prerequisite for further coursework (Sealey & Oehrtman, 2005; Thompson &

Silverman, 2008). It consists of important concepts such as the fundamental theorem of calculus and integral-area relationships.

Many studies have explored students' attitudes towards mathematics and mathematical problem solving (e.g., Lim & Chapman, 2013; OECD, 2013). Most of these studies have shown a close relationship between various domains of attitudes towards mathematics and mathematics achievement (Lavy & Bershadsky, 2003; OECD, 2013; Samuelsson & Granstrom, 2007). However, a few studies have explored students' attitudes towards mathematical problem posing (e.g., Chen et al., 2015). Considering the potential value of problem posing activities in the teaching and learning of mathematics, this study explores undergraduate engineering students' problem posing competencies in relation to integral calculus, and their attitudes towards mathematical problem posing. Therefore, the research questions of this study are: What are undergraduate engineering students' competencies in posing problems related to integral? And what are their attitudes towards mathematical problem posing activities?

LITERATURE REVIEW

The literature review section reviews the previous studies related to problem posing, integral calculus, and attitudes towards mathematics.

Problem posing

Problem posing activities offer potential benefits to develop students' mathematical understanding. Problem posing activities could have a positive influence on students' creativity (e.g., Bonotto & Dal Santo, 2015), attitudes toward mathematics (e.g., Chen et al., 2015), and critical thinking skills (Nixon-Ponder, 1995). Furthermore, several studies have reported that there is a close relationship between students' problem posing and problem-solving competencies (Cai & Hwang, 2002; Silver & Cai, 1996; Xie & Masingila, 2017). For instance, Silver and Cai (1998) analysed middle school students' responses to problem posing and problem solving tasks. They found that problem solving and problem posing performance are closely related, and successful problem solvers can pose more complex mathematical problems compared to unsuccessful problem solvers. Several frameworks have been proposed to design problem posing tasks (e.g., Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman, 2005; Stoyanova & Ellerton, 1996). For instance, Christou et al. (2005) have designed a taxonomy for designing problem posing tasks that has four categories: *Editing quantitative information*- posing problems without restriction, *selecting quantitative information*- posing problems based on a given answer, *comprehending quantitative information*-posing problems based on a given calculation/equation, and *translating quantitative information*- posing problems based on a given graph, diagram or table (Christou et al. 2005). In relation to analysing students' posed problem, different frameworks have been proposed (e.g., Leung, 2013). Recently, Cankoy and Özder (2017) have proposed a framework that can be used to analyse students' posed problems across five dimensions: solvability; reasonability; mathematical structure; context; and language.

Integral calculus

Many studies have reported that students have various misconceptions in learning integral calculus (Jones, 2013; Kouropatov & Dreyfus, 2013; Radmehr & Drake, 2017, 2019; Sealey, 2014). Integral calculus includes important topics such as the Fundamental Theorem of Calculus (FTC) and integral-area relationship. FTC links definite and indefinite integrals and is often used to solve definite integral problems (Radmehr & Drake, 2017). Several studies have highlighted that many students rely on learning routine procedures and integral techniques, and do not develop a conceptual understanding of integral calculus (e.g., Radmehr & Drake, 2019). Sealey (2014) explored students' understanding of the definite integral, and suggested a framework to characterize students' understanding of Riemann sums and the definite integral. The results indicated that "conceptualizing the product of $f(x)$ and Δx proves to be the most complex part" (p. 230) for students. Radmehr and Drake (2017) have explored university students' mathematical performance, and metacognitive experiences and skills in relation to FTC. The results showed that several students had difficulties in solving problems related to the FTC. For example, in relation to $F(x) = \int f(x)dx$, many students did not understand that $f(x)$ is the rate of change of the accumulated area function $F(x)$.

Attitude

Attitude could be defined as "a predisposition to respond to a certain object either in a positive or in a negative way" (Zan & DiMartino, 2007, p. 28), consequently, students' attitudes towards mathematics underlie their tendency to engage in mathematical activities. Students' attitudes towards mathematics can impact directly on students' mathematical learning, problem solving, and achievement (Ngurah & Lynch, 2013; Sarouphim & Chartouny, 2017). Positive attitudes towards mathematics can encourage students to engage more in mathematical learning activities (Singh Granville, & Dika, 2002) while negative attitudes towards mathematics can increase students' mathematics anxiety (Trujillo & Hadfield, 1999). Several studies have reported that there is a strong relationship between different attitude domains (e.g., enjoyment of mathematics; motivation to do mathematics) and mathematics achievement (e.g., Ngurah & Lynch, 2013; OECD, 2013; Sarouphim & Chartouny, 2017). Though, a literature search exposed only one study which explores students' attitudes toward problem posing (Chen et al., 2015). Chen et al. (2015) investigated students' problem posing and problem solving competencies, as well as their attitudes towards mathematical problem posing and problem solving. Their findings showed that problem posing activities had a positive impact on students' problem-solving abilities, and attitudes towards problem posing and problem solving also improved.

RESEARCH METHODS

The present study takes a sequential explanatory mixed method approach. To form a comprehensive understanding of students' problem posing abilities and their attitudes towards problem posing in mathematics, first, students participated in a problem posing test, and completed a questionnaire about their attitudes towards problem posing. Then, nine students were invited to participate in semi-structured interviews. The sample comprised of 135 undergraduate students from different engineering majors of a public university in Iran. For the problem posing test, eight problem posing tasks were designed based on Christou's problem posing taxonomy (2005) related to two topics in integral calculus: the FTC and integral-area relationships. The attitude questionnaire consisted of twelve items on a five-point Likert-style scale and two open-ended questions. To illustrate, two items of the questionnaire were "I get a great deal of satisfaction from posing a mathematical problem" and "By practicing mathematical problem posing, I become a better mathematical problem solver". The problem posing test and the attitude questionnaire were piloted with nine students from an engineering calculus 1 course. After piloting and refining, 135 students participated in the problem posing test and completed the attitude questionnaire. Students' problem posing abilities were analysed using an adapted version of Cankoy and Özder's (2017) rubric. Using purposeful sampling, nine students with different levels of performance on the problem posing test were selected to participate in a semi-structured interview. To explore the validity of the attitude questionnaire, two senior lecturers in mathematics education examined the readability of the questionnaire items, and factor analysis was also conducted to examine the relationships among the questionnaire items. To explore the reliability of the questionnaire, Cronbach's alpha was calculated, the value 0.89, indicates that the questionnaire items had good internal consistency. To explore the validity of the problem posing test, two senior lecturers in mathematics examined the problem posing tasks and then it was piloted with nine students.

RESULTS

This section comprises the results of analysing responses to the problem posing tasks, the attitude questionnaire, and students' responses to the interview questions. The results of two tasks are described in this paper because of the page limits, one related to the integral-area relationship (Figure 1) and one related to the FTC (Figure 3).

Students' responses to the problem posing tasks

Task 1 is classified as *translating quantitative information* based on Christou's (2005) problem posing taxonomy because students are asked to pose a problem based on the given graph. Ninety-eight out of 135 (72.6%) students posed a problem for this task, however, the remaining 37 (27.4%) did not pose any problem. Students' posed problems were classified into three categories (Table 1).

Task 1. Please write a problem based on the given graph which its solution would require using area under curves (The red graph is $y = (x-1)^3 + 1$ and the green is $y = x$).

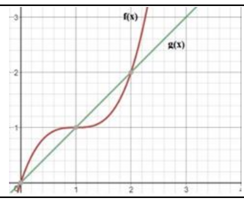

Figure 1. Task 1

Categories	A sample response	N
Finding the enclosed area between two curves	Find the area between $f(x) = (x-1)^3 + 1$ and $g(x) = x$ in [0,2].	77 (79%)
Calculating integral	Calculate the following integral: $$\int_0^2 ((x-1)^3 + 1 - x)dx$$	11 (11%)
Real-world context	Two runners are in a running competition. The first runner runs with the speed of $v(t) = (t-1)^3 + 1$. The speed equation of the second runner is $v(t) = t$. Calculate the displacements of these two runners after one minute?	10 (10%)

Table 1. Posed problems for the integral area-relationship task (N=98)

Furthermore, the results showed that 90 out of 98 (92%) problems were solvable and only 8 (8 %) problems were unsolvable. Ten (10%) problems were based on real-world context while 88 (90%) problems were 'bare tasks without contexts' (Vos, 2020). Ninety out of 98 (92%) posed problems had clear language and only 8 (8%) problems were not clear. Furthermore, analysing students' posed problems showed that many students had several difficulties when posing problems. The interviewed students were asked to pose a new problem for each task during the interviews, and also solved their posed problems. During this process, also some difficulties were identified. Students' difficulties in relation to posing a problem for Task 1 are summarized in Table 2. For example, 30 students in the problem posing test and three interviewed students did not understand that the enclosed area between curves should be always positive as some of them calculated the enclosed area between the two curves zero or negative (Figure 2).

$$\int_0^2 ((x-1)^3 + 1 - x)\,dx = \int_0^2 (x-1)^3\,dx + \int_0^2 dx - \int_0^2 x\,dx = \left[\frac{(x-1)^4}{4} + x - \frac{x^2}{2}\right]_0^2$$
$$= \frac{1}{4} + 2 \to -\frac{1}{4} = 0$$

Figure 2. A sample response to Task 1

Types of difficulty	Test	Interview
Difficulties with the concept of signed areas	30 (22.2%)	3 (33%)
Not checking whether the upper and lower functions change within the enclosed area	15 (11.1%)	7 (78%)
Difficulty with identifying applications of enclosed area between curves in the real world	-	6 (67%)

Table 2. Students' difficulties in Task1

Task 2. “Please can you write a problem based on the following graph whose solution would require using the FTC?” (Radmehr & Drake, 2017, p. 1052).

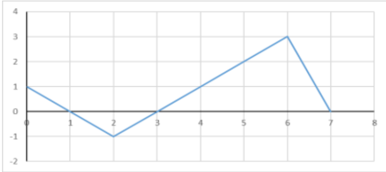

Figure 3. Task 2

Task 2 is also classified as *translating quantitative information* based on Christou’s (2005) problem posing taxonomy as students are required to pose a problem based on the given graph. Forty-two (31.1%) students posed a problem for this task, however, the remaining 93 (68.8%) did not. The posed problems were classified into two categories (Table 3). Forty out of 42 (95.2%) problems were solvable and two (4.8%) were unsolvable. Twenty (47.6%) problems were based on a real-world context while 22 (52.3%) problems were bare tasks without contexts. Forty-one (97.6%) posed problems had clear language, and only one problem was not clear.

Categories	A sample response	N
Find the area under curves	Find the enclosed area between x-axis and the given graph in [0,7].	22 (52%)
Real-world context	The given graph shows the speed of a car between t=0 and t=7 minutes. Calculate the distance travelled by the car.	20 (48%)

Table 3. Posed problems for to the FTC task (N=42)

Students also had several difficulties when posing problems for Task 2 in the test and during the interviews (Table 4). The results showed that ten students in the problem posing test and six out of nine interviewed students had difficulties in identifying how FTC could be used in the real-world. For instance, a posed problem was “the given graph shows the distance a man ran in a running competition. Calculate the acceleration of the man between t=0 and t=5”. In this problem, it seems the student incorrectly thought integrating the displacement equation, that could be obtained from the graph, results acceleration function. However, integrating an acceleration equation results the velocity function, and integrating the velocity equation results the displacement function.

Types of difficulty	Test	Interview
Difficulties in understanding the role of $F(x)$ in the FTC	13 (30%)	7 (78%)
Difficulties in understanding the applications of the FTC in the real-world	10 (23.8)	6 (67%)
Difficulties in calculating antiderivatives	5 (11.9%)	4 (44%)

Table 4. students’ difficulties in Task 2

Students’ attitudes towards mathematical problem posing

Students’ responses to the attitude questionnaire showed that over 50 percent of students enjoyed the problem posing activities, and more than 60% of students believed that problem posing and problem solving are closely related. Students’ responses to open-ended questions showed that they believed engaging in problem posing activities help them to develop their mathematical understanding. For instance, one student said “practicing problem posing activities might increase our creativity in mathematics and also helps us to solve more complicated problems which need more creativity”. The results of the interviews showed that eight out of nine students believed problem posing tasks are enjoyable activities and could be included in the teaching of mathematics. An examples was: “After I posed problems, I finally understood the applications of the mathematics we learned in the school and university. In fact, problem posing activities make mathematics more practical and bring it to our real life”. These eight students also expressed that problem posing tasks could be used in the mathematical exams.

DISCUSSION

The present study explored undergraduate engineering students’ competencies and attitudes towards mathematical problem posing in integral calculus. The findings showed that many engineering students could develop their problem posing skills. Of the 1080 problems that potentially could have been posed for the eight tasks, only 501 (46%) problems were posed. Of these 501 problems, 411 (81%) were solvable which was consistent with previous studies which have reported most of the students’ posed problems are solvable (Bonotto & Dal Santo, 2015). One possible reason for the high percentage of solvability in the present study is that many of the posed problems were bare tasks without contexts. Moreover, only 157 problems (31.3 %) were based on the applications of integrals in the real world and 344 problems (68.6 %) were bare tasks without contexts which could be an indication of students’ lack of knowledge about the applications of integrals in the real world. The language used in the 440 of the posed problems (88%) was clear and understandable which might indicate that students at university level could pose clear and understandable problems. Furthermore, the study findings suggest that problem posing tasks could be used by teachers and lecturers to explore students’ mathematical understanding. In this study, using problem posing tasks, several students’ difficulties in relation to integral calculus were identified. The difficulties that have been identified are in line with previous studies that have explored students’ understanding of integral calculus (e.g., Mahir, 2009; Radmehr & Drake, 2017, 2019). In relation to students’ attitudes towards mathematical problem posing, the findings showed that more than 50% of the engineering students believed problem posing is an enjoyable activity. This is consistent with previous studies (Arikan & Ünal, 2015) which have reported that students enjoyed practicing problem posing tasks. Students also expressed that problem posing tasks could improve their mathematical learning, and they brought several reasons for their responses. For examples, they mentioned problem posing activities help them to foster their creativity, and identify their mathematical misunderstandings. To conclude, this study suggests that problem posing activities could be used to improve the teaching and learning of integral calculus in engineering mathematical courses as the problem posing tasks could identify students' difficulties in integral calculus and motivate them to improve their understanding of applications of integral calculus in real life. Moreover, since many students believed problem posing activities are enjoyable and help them to improve their mathematical learning, using such tasks could encourage students to be more active in mathematical classrooms, and might motivate them to learn mathematical concepts meaningfully.

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