Modeling turbulent stream flows over rough permeable beds

G. Rousseau, I. Pascal & C. Ancey

Laboratory of Environmental Hydraulics, Department of Civil Engineering, Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland

ABSTRACT: In steep streams, turbulent flows run over coarse sediments that are similar in size to flow depth. Under these conditions, a significant part of the flow may seep through the permeable bed. Based on experimental data, this paper presents a model able to reproduce the vertical structure of flows over rough permeable beds in low relative submergence conditions. Experiments were performed on open-channel flows passing over tilted coarse-grained beds with slopes within the 0.5% – 4% range. Fluid velocities were measured by Particle Image Velocimetry (PIV) and a technique called Refractive Index-Matched Scanning (RIMS), allowing the interior of the bed to be examined. By applying the *double averaging methodology*, porosity and mean velocity, as well as turbulent and dispersive stresses profiles were collected from the subsurface to the free surface. A turbulent boundary layer over the rough bed was observed while experiments were run at intermediate Reynolds numbers, i.e. $Re = O(1000)$. Under these flow conditions, viscosity played a non-negligible role through the van Driest damping effect. Based on the Prandtl mixing length theory, we propose a model for the turbulent stress that takes into account the continuous porosity profile, damping and dispersive effects. Finally, we show a good agreement between the model and classic flow resistance laws employed for river studies. Our model contrasts with existing boundary-layer models which generally assume a discontinuous porosity profile at bed interface, whether the bed is permeable or impermeable.

Keywords: Turbulent boundary layers, low relative submergence, steep streams, permeable beds

1 INTRODUCTION

Steep streams exhibit distinctive features unshared by lowland rivers: (i) their flow-depth and roughness scales are similar; and (ii) their bed permeability is much higher. Both features explain why predicting flow resistance or the threshold of incipient motion is more complicated for mountain streams than for lowland rivers.

These flows may advantageously be analyzed through the prism of the *double averaging* framework. Considering a flow volume parallel to the bed (as shown in Figure 1), the *double averaging* method consists of averaging over the temporal scale (this is the classic *Reynolds averaging* step), and then averaging is performed over the spatial scale of a thin layer parallel to the bed (Nikora et al. (2007)). The stream-wise length scale L on which spatial averagings are made to obtain quasi-uniform conditions defines the mesoscopic scale.

As observed in Figure 1, applying *spatial averaging* to naturally shaped beds results in smooth porosity profiles, with porosities increasing from bed porosity ϵ_b , to a porosity equal to 1 in the surface layer above the roughness crest z_{rc} . When bed roughness is similar in size to flow depth, i.e in case of low relative submergence, a non negligible part of the flow seeps trough the roughness and subsurface layers. Additionnaly, the flow depth definition ($h_f = z_{surf} - z_{bed}$) is not univocal and an arbitrary origin must be chosen for z_{bed} between z_{rc} and the troughs of the roughness elements z_t . As highlighted by Pokrajac et al. (2006), this definition problem is essential in low relative submergence conditions. It affects values taken by quantities such as the shear velocity $u^* = \sqrt{gh_f i}$ or the Shields number $Sh = h_f i / ((\rho_s / \rho_f - 1) d_p)$, where g denotes gravity acceleration, d_p is the grain diameter, i is bed slope, and ρ_f and ρ_s are the fluid and grain

Figure 2. Sketch of the experimental set-up

coarse-grained bed. The median diameter was $d_p = d_{50} = 8$ mm. A bimodal size distribution has been used because otherwise the beads arrange itself in structured parallel layers causing undesirable bias in the averaged porosity and velocity profiles (as observed by Ni & Capart (2015)). Before each run, the upper layer was flattened out to form a uniform bed of height $h_s = 5$ cm.

The Refractive Index-Matched (RIM) technique has been employed for locating beads and measuring interstitial velocities. This involves matching the fluid refractive index with that of the glass beads, which makes it possible to access the information about beads location and interstitial flow velocities. The iso-index fluid was prepared by mixing volumic concentrations of 60% benzyl alcohol and 40% ethanol (BAE). Fluid and sediment characteristics were measured (see Table 1).

Table 1. Fluid and sediment characteristics measured at 20°C

i (%)	n_f^*	d_p (mm)	h_s (cm)	ρ_s (kg m ⁻³)	ρ_f (kg m ⁻³)	ν (mP s)
1.0 – 4.0	1.472	8.0	5.0	2200	950	3.0

*fluid refractive index

Compared to other materials used in RIM techniques, combining borosilicate, ethanol and benzyl alcohol leads to mixtures whose relative density between solid and fluid is close to that found in rivers. Meeting similarity criteria such as the Shields and Froude numbers was necessary to obtain flow conditions that mimicked those encountered in real-world scenarios. If we had followed Ni & Capart (2015) and used a sediment of polymethyl methacrylate (PMMA), with a density of $\rho_s = 1190$ kg·m⁻³, it would have been impossible to conduct experiments on steep slopes without observing sediment transport. The same problem would have occur if we had employed a popular RIM fluid mixture made of NaI (e.g. as in Voermans et al. (2017)) since, in this case, the fluid has an unexpectedly high density: $\rho_{f,NaI} = 1770$ kg·m⁻³. In the present context, we needed a stable bed resisting to the stream's erosive action for supercritical flows.

2.2 Velocimetry and scanning

Using PIV, we seeded the flow with micrometric tracers lit up by a laser. In contrast with earlier PIV measurements using a fixed laser sheet, we scanned the flow across its width by moving laser sheet. This methodology is similar to Ni & Capart (2015) and has been called PIV-RIMS (the S means *Scanning*). With this procedure we collected double-averaged profiles such as the porosity, the streamwise velocity U_x as well as the turbulent τ_t and dispersive τ_d stresses (see Section 3 for

Figure 3. Three-dimensional visualization of the horizontal velocity. (a) Side view of a slab of the velocity field at $Y = 25$ mm from the sidewall. (b) Front view of the flow, sliced along y . This view enables us to appreciate the sidewall's influence. Y is the distance from the sidewall. The origins of Z and X are arbitrary.

definitions). The PIV algorithm deducing the local displacement of features is available in open source from the public GitHub repository: <https://github.com/groussea/opyflow> (see the supplementary information in Rousseau (2019) – Annex C). The laser sheet's linear movement perpendicular to the flow enabled us to scan a volume whose dimensions were $73.8 \times 30 \times 34.5$ mm. Thus, the mesoscopic scale L over which spatial averaging was performed was about 10 bead diameters. Importantly, measurements were performed between 1 cm to 4 cm away from the sidewall to neglect the side boundary layer effects as observed in Figure 3. Uniformity and repeatability were examined in Rousseau (2019) and in Rousseau & Ancy (2020), which provide further information on the experimental procedure and findings.

2.3 The vertical origin definition: the crucial role of bed arrangement

To compare velocity and porosity profiles, we first needed to set the origin of the vertical axis. There is no standard procedure for doing this with rough beds. We examined various possibilities. The roughness crest – generally estimated by $z_{rc} \sim z_{\epsilon=0.99}$, as suggested by Pokrajak et al. (2006) – was unsuitable because it created significant scatter between profiles when bed was rearranged. Indeed, $z_{\epsilon=0.99}$ was highly influenced by individual grains that were slightly higher than the average bed level. The origin had to be fixed at a bed height where the scatter between the porosity profiles was minimal. We found $z_{\epsilon=0.8}$ (see Figure 4 and Figure 4.12 in Rousseau (2019)), that is, at approximately $0.3d_p$ below the roughness crest, which was the shift that Voermans et al. (2017) obtained using the porosity inflection method, and which was close to what Nezu (1993) prescribed. The Reynolds numbers based on this choice were slightly different from those estimated from the height of the roughness crest, as the flow depth h_f was computed from $z_{\epsilon=0.8}$, which was above $z_{\epsilon=0.99}$.

3 MODELING

3.1 Double-averaged momentum equation

Double-averaging the Navier–Stokes equations produces the double-averaged momentum equations, whose projection on the streamwise x -direction for steady uniform flows is:

$$0 = \epsilon \rho_f g i + \frac{d\tau_d}{dz} + \frac{d\tau_t}{dz} + \frac{d\tau_v}{dz} + f_{p,x} + f_{v,x}, \quad (1)$$

where $\tau_t = -\rho_f \epsilon \langle \overline{u'_x u'_x} \rangle$ and $\tau_d = -\rho_f \epsilon \langle \tilde{u}_x \tilde{u}_x \rangle$ are the turbulent and dispersive stresses respectively. The symbol ' $\overline{\bullet}$ ' denotes the time averaged variable, the brackets ' $\langle \bullet \rangle$ ' indicate the intrinsic space averaged variable (i.e. over the fluid phase only). Equation 1 is obtained by employing the decomposition of the local instantaneous velocity $u_i = U_i + \tilde{u}_i + u'_i$ (in the i -direction) in term of temporal fluctuation u'_i , the spatial disturbance $\tilde{u}_i$ and the double-averaged velocity $U_i = \langle \overline{u_i} \rangle$ (see Nikora et al. (2007) for further details). U_x is the averaged streamwise velocity and $U_y \sim 0$

to verify quasi-uniformity at the mesoscopic scale. $f_{p,x}$ and $f_{v,x}$ are called the *pressure drag* and *viscous drag* on the solid element surfaces. $\tau_v = \nu d^2 U_x / dz^2$ is the *viscous stress*.

3.2 Closure choices

Following Prandtl's mixing-length approach, we express the turbulent stress by:

$$\tau_t = \rho_f \epsilon l_m^2 \left(\frac{dU_x}{dz} \right)^2 \quad (2)$$

The *damping effect*, introduced by van Driest (1956) and documented by Nezu & Rodi (1986) in open channel flows, is expected to play an active role for our experimental conditions. Indeed, for intermediate Reynolds numbers, i.e. $Re = O(10 - 1000)$, the damping effect reduces the vertical momentum transfer by turbulence in near-bed regions. Unlike van Driest (1956) who stated that roughness is expected to make the damping effect negligible, our results strongly suggest that this effect remains important for rough beds as soon as Reynolds numbers are sufficiently low. Thus, based on the original van Driest damping formulation (van Driest 1956), we express l_m by:

$$l_m = \kappa Z \left(1 - \exp \left(- \frac{\sqrt{Z_{vD} U_x(z) / \nu}}{Re_{vD}} \right) \right) + C_{vD} l_p \quad (3)$$

where κ denotes the van Kármán “constant” that is fixed at $\kappa = 0.41$. Re_{vD} and C_{vD} are two constants that must be provided. l_p is the pore length scale inside the bed given by the ratio of the volume of voids V_v to their surface area A_v , $l_p = V_v / A_v = \epsilon_b d_p / (6 - 6\epsilon_b)$. Within the van Driest approach, the value of the *damping factor* $1 - \exp(-\sqrt{Z U_x(z) / \nu} / Re_{vD})$ is given by relating the local Reynolds number $Z U_x(z) / \nu$ to Re_{vD} with Z the distance from the bed.

The definition of Z must be clarified for rough bed and the integrating procedure is suggested:

$$Z = \int_{-\infty}^z \sqrt{\frac{\epsilon(z) - \epsilon_b}{1 - \epsilon_b}} dz \quad (4)$$

The integration approach has the principal advantage to discard the origin definition ambiguity from the model. For the drag forces exerted on the porous bed, i.e. for $\epsilon(z) < 1$ below the roughness crest, we employed the Ergun (1952) relation which consider pressure and viscous energy losses:

$$f_{v,x} = -180 \frac{(1 - \epsilon)^2 \nu \rho_f}{\epsilon d_p^2} U_x, \quad f_{p,x} = -1.75 \frac{(1 - \epsilon)}{d_p} \rho_f U_x^2 \quad (5)$$

Note that $f_{v,x}$ is the Kozeny-Carman term that is dominant for low velocities in porous medium. From mechanistic arguments, the following expression for the dispersive stress is suggested:

$$\tau_d = 1/2 \rho_f \epsilon \lambda_+ \left(\frac{1 - \epsilon(z)}{1 - \epsilon_b} \right)^{3/2} U_x^2(z) \quad (6)$$

where λ_+ is the fraction of the fluid volume at a vertical coordinate where the local cross product $-\tilde{u}_x \tilde{u}_z$ is positive. For conciseness, we only summarized our choice here. Chapter 3 in Rousseau (2019) is devoted to this closure problem where terms for equations (2 - 6) are discussed.

4 RESULTS

4.1 Hydraulic conditions and vertical structure

In this section we report the results regarding two experimental runs with the same flow discharge per unit width ($q_f = 0.30 \pm 0.015 \text{ m}^2/\text{s}$). Hydraulic characteristics are detailed in Table 2 and double-averaged profiles are shown in Figure 4. Fully turbulent flows were observed when experiments were performed at intermediate Reynolds numbers, i.e., $Re = O(1000)$. Thus, turbulent stress τ_t is the dominant mechanisms for transferring momentum in the surface layer and follows the trend of the gravity force integrated between z_{surf} and z , i.e. defined by : $G = - \int_z^{z_{surf}} \epsilon(z) \rho_f g dz$ which is equal to $(z - z_{surf}) \rho_f g$ above z_{rc} . τ_t has its maximum at the roughness crest and is damped inside the roughness layer.

Figure 4. Vertical profiles for A1 ($i = 0.5\%$) and A3 ($i = 2\%$) runs with a constant flow discharge. (a) Velocity profiles: small velocities are shown with a logarithmic scale on the abscissa. The thin horizontal error bars represent uncertainties at free surface level. (b) Porosity profile. (c) Stress compared with G . The vertical coordinate is normalized by the mass-median-diameter d_p and defined by $z'_{\epsilon=0.8} = z - z_{\epsilon=0.8}$. (d) Plot of measured $U_{x,SSL}$ values that represents subsurface velocities below z_t for 17 trials with various slopes. [Thin line] Ergun prediction, i.e. by solving $f_{v,x}(U_x) + f_{p,x}(U_x) = \epsilon_b \rho g i$ with $f_{v,x}(U_x)$ and $f_{p,x}(U_x)$ given by equation (5). [Thick line] Kozeny-Carman prediction, i.e. by solving $f_{v,x}(U_x) = \epsilon_b \rho g i$.

Table 2. Hydraulic parameters of two runs

Run	$i(\%)$	h_f [mm]	U_b [m/s]	u_* [mm/s]	Re_b	Fr^*	Re_K^{**}	Sh
A1	0.5	13	0.26	26	1147	0.92	2.2	0.007
A3	2.0	10	0.32	45	1095	1.29	3.8	0.019

* $Fr = U_{surf} / \sqrt{gh_f}$ is the Froude number

** $Re_K = \frac{\sqrt{K_{KC}} u_*}{\nu}$ is the permeability Reynolds number as defined by Voermans et al. (2017) with

$K_{KC} = \frac{\epsilon_b^3}{180(1-\epsilon_b)^2} d_p^2$ the Kozeny-Carman equivalent permeability

The dispersive stress τ_d has its maximum at the vertical coordinate $z_{\epsilon=0.8}$ with a maximum being 5% of τ_t maximum. These vertical structures are in agreement with Voermans et al. (2018) observations although our experiments were performed at higher slopes and lower submergence values. Owing to slope variability, subsurface velocities are affected. Influence of gravity on the fluid in the subsurface layer corresponds to Ergun law predictions (see Figure 4 – (d)).

4.2 Measured and modeled profiles

Equation (1) with closures (2 – 6) constitute the 1D model describing the vertical flow structure. We solve this system with a finite difference technique by employing the numerical scheme presented in Rousseau (2019) – Annex B. Simulated profiles were computed with hydraulic conditions given in Tables 1 and 2 for A3 run. In the model, the porosity profile $\epsilon(z)$ is required. These profiles can be deduced from experimental work or estimated with a synthetic $\epsilon(z)$. The first solution has been chosen as we collected $\epsilon(z)$ from the PIV-RIMS procedure. For predicting flows in real river, assuming a synthetic profiles may be required (we use this solution in the next section). Figure 5 shows a comparisons between experimental profiles and the simulated profiles. Three parameters have been adjusted $\lambda_+ = 0.3$, $Re_{vD} = 70$ and $C_{vD} = 0.5$. Note that this parametrization has been tested with slopes ranging from 0.5 to 8% in Rousseau (2019).

Figure 5. Simulated and measured double-averaged profiles for A3 run. Vertical profiles of (a) velocity, (b) porosity, (c) turbulent τ_t , dispersive τ_d and viscous τ_v stresses and (d) pressure and viscous drag forces. Experimental and simulated l_m is plotted with reference axis at the top. Experimental l_m points are deduced by reversing equation (2) with the knowledge of $\tau_{t,exp}$ and $U_{x,exp}$.

Figure 6. [Left] Velocity profiles W-38 and W-42 computed with the 1D model and real-world stream conditions - W-31 ($d_p = 0.1$ m, $i = 1$ %, $h_f = 0.5$ m) and W-34 ($d_p = 0.1$ m, $i = 5$ %, $h_f = 0.1$ m). [Right] Computation of 48 combinations in terms of the non-dimensional Chézy coefficient $C = U_b / \sqrt{gh_f i}$ versus relative submergence. Combinations W-31 and W-34 have been positioned on the plot. Symbols size depends on grain-size, and color is related to bed slope. The equations selected for the comparisons are: the Chézy-Eytelwein equation $C = 16$, Strickler equation $C = 24(h_f/d_p)^{1/6}/g^{1/2}$, Keulegan equation $C = 1/0.4 \ln(12 * h/d_p)$, and the Rickenman & Recking's (2011) relation $C = 4.41 (h_f/d_p)^{1.9} (1 + (h_f/d_p/1.3)^{1.6})^{-1.08}$ which is calibrated on 2890 selected data from a large data set of 3942 field measurements (tiny blue stars).

4.3 Depth averaged simulated profiles and flow resistance

The above mentioned simulations and experiments were performed at $Re = O(1000)$. These values are two orders of magnitude lower than typical Reynolds numbers for gravel bed rivers. Establishing if such modeling assumptions are transferable to real-world streams is important. As a first step, the comparison can be done by employing the following procedure: (i) simulating profiles with the 1D model for higher Re , i.e. with water, real hydraulic conditions and with the above

calibrated parameters C_{vD} , Re_{vD} and λ_+ , (ii) averaging profiles over depth to deduce U_b and the non-dimensional Chézy coefficient $C = U_b/\sqrt{gh_f i}$, (iii) comparing C with well-known flow resistance laws used in steep rivers. Simulations were performed with $d_p = [0.002, 0.01, 0.1, 0.5]$ m, $h_f = [0.001, 0.01, 0.1, 0.5, 3]$ m, and $i = [0.5, 1, 5]$ ‰. All possible combinations result in 60 simulations producing different relative submergence conditions. As explained above, with low relative submergence, flow depth definition is crucial. We used two definitions: (i) as above, the flow depth defined by $h_f = z_{surf} - z_{\epsilon=0.8}$, and (ii) a flow depth estimated from the total flow discharge from mass conservation $h_f^* = q_f/U_b$. This alternative definitions is common in field studies since flow discharge is more accurately monitored than flow depth in mountain rivers (Recking, pers. comm.). The non-dimensional Chézy coefficient C is plotted against the relative submergence h_f/d_p in Figure 6. Interestingly, the resistance values computed with the present model and the classic equations provide results with the same order of magnitude and similar trends. It confirms that under various hydraulic conditions, our model is consistent with extant empirical equations. For high relative submergence flows ($h_f/d_p > 10$), computed points do not show substantial sensibility to the flow depth definitions. This is a consequence that d_p is negligible with respect to h_f . For low relative submergence conditions ($h_f/d_p < 10$), variability is more important, reflecting the importance of the flow depth definition. It comes as no surprise to see that the curve given by Rickenman & Recking (2011) is located between the h_f and q_f/U_b points. Indeed, this equation was fitted to field data where various definitions for flow depth are applied.

5 CONCLUSION

This proceeding presented a model able to reproduce the vertical structure of turbulent flows over rough permeable beds in low relative submergence conditions. This model was successfully compared to experimental profiles collected at intermediate Reynolds numbers ($Re = O(1000)$). In this condition, we revealed the role played by the damping effect on mixing length. While laboratory experiments were performed at Reynolds numbers two orders of magnitude lower than those of regular streams, computed flow resistance deduced from vertical profiles were successfully compared to flow resistance equation employed in river studies. A next step would be to identify the negligible terms in the 1D model in order to derive a flow resistance law for rivers.

REFERENCES

- van Driest, E. R. 1956. On turbulent flow near a wall. *Journal of the Aeronautical Sciences*. 23 (11), 1007–1011.
- Ergun, S., 1952. Fluid flow through packed columns. *Chem. Eng. Prog.* 48, 89–94.
- Mignot, E., Barthelemy, E., & Hurther, D. 2009. Double-averaging analysis and local flow characterization of near-bed turbulence in gravel-bed channel flows. *Journal of Fluid Mechanics* 618: 279-303.
- Nezu, I. & Rodi, W. 1986. Open-channel flow measurements with a laser doppler anemometer. *Journal of Hydraulic Engineering*. 112 (5), 335–355.
- Nezu, I. & Nakagawa H., 1993. *Turbulence in open-channel flows*. IAHR-monograph.
- Ni, W.J. & Capart, H., 2015. Cross-sectional imaging of refractive-index-matched liquid-granular flows. *Experiments in Fluids*, 56(8), p.163.
- Nikora, V., McEwan, I., McLean, S., Coleman, S., Pokrajac, D. & Walters, R., 2007. Double-averaging concept for rough-bed open-channel and overland flows: Theoretical background. *Journal of Hydraulic Engineering*, 133(8), pp.873-883.
- Pokrajac D., Finnigan J.J., Manes C., McEwan I. & Nikora V., 2006. On the definition of the shear velocity in rough bed open channel flows. *Proc. of the International Conference on Fluvial Hydraulics, "RiverFlow 2006"*, Vol. 1: pp. 89-98.
- Rickenmann, D. & Recking, A., 2011. Evaluation of flow resistance in gravel-bed rivers through a large field data set. *Water Resources Research*, 47(7).
- Rousseau G., 2019. *Turbulent flows over rough permeable beds in mountain rivers: Experimental insights and modeling*. PhD thesis, École Polytechnique Fédérale de Lausanne.
- Rousseau G. & Ancey C., 2020. Scanning PIV of turbulent flows over rough permeable beds using refractive index matching. In revision in *Experiments in Fluids*.
- Voermans, J.J., Ghisalberti, M. & Ivey, G.N., 2017. The variation of flow and turbulence across the sediment-water interface. *Journal of Fluid Mechanics*, 824, p.413.