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Direction Blanking with Controlable Width in MIMO Radar via Phase Code Optimization

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Abstract—In this article, we consider the problem of creating null-energy directions of controlable width with a MIMO radar, for surveillance applications where the energy radiated should be uniformly spread over the remaining angular directions. In a MIMO radar, the antenna beam pattern can be modified by a judicious choice of the transmitted waveforms. We restrict our attention here to phase code waveforms. We first propose an optimization program that enables to minimize the energy in a given narrow direction while uniformly spreading the energy over the other directions. Then we propose to extend this solution to create larger null angular intervals. Interestingly the proposed solution involves the well-known Discrete Prolate Spheroidal Sequences [1]. We show on simulations the efficiency of the proposed approach.

I. INTRODUCTION

In a classical phased array radar, the same waveform is transmitted by all the antenna elements. The phased array beam is focused in a particular direction thanks to a judicious choice of the elementary phases of each antenna [2]. Since the early 2000's however, a new approach has been proposed in the literature, and has gained considerable attention since then: the MIMO radar [3], [4], [5]. In a MIMO radar, each transmitting antenna transmits its own waveform. The MIMO radar framework can be used in two distinct frameworks: the non coherent MIMO radar, also known as Statistical MIMO, corresponds to a multistatic system with widely separated antennas [6], [7]. And the coherent colocated MIMO radar corresponds to a classic colocated phased array antenna geometry with close antennas that can be processed coherently.

In this article, we will consider only the coherent colocated MIMO radar. Clearly this kind of MIMO radar encompasses the particular case of the phased array, where simply the signals transmitted by the different antennas are chosen to be identical. However the MIMO radar provides much more flexibility thanks to the possibility to choose different waveforms for the different transmitting antennas. It can be easily shown that the MIMO radar transmit beam pattern is then directly linked to the autocorrelation matrix of the transmitted waveforms. A very common choice is the use of orthogonal waveforms at transmit. Such a choice naturally leads to spread the transmitted energy uniformly over the entire angular domain. However many other choices may be of interest, in order to adapt the waveforms to specific problematics such as target detection, improved clutter rejection or better multipath coping [8]. A huge literature exists on the subject of optimizing signal

waveforms for MIMO radar [9], [8], [10], [11], [12], [13], [14], [15] with different problematics at hand. In particular, a specific optimization program was proposed in [14] to create antenna patterns presenting multiple peaks and nulls.

In this article, we consider the more specific problem of optimizing transmitted waveforms so as to create a transmit beam pattern presenting a null in a specific direction, with the possibility to enlarge this null to a full blank angular interval, while ensuring a flat uniform pattern in the other directions. Due to the complexity of the problem at hand and the huge variety of possible waveforms, we will moreover restrict ourselves here to the specific case of unimodular phase codes. The problem to solve is thus to minimize the energy transmitted in a given direction over a non convex set of waveforms. We show that ensuring a uniform beam pattern in the rest of the angular domain can be written in terms of additional constraints to the original optimization program. In order to solve this resulting constrained optimization program, we propose to relax it in the form of a penalized cost function with no constraints. Both programs with initial cost function and penalized cost function remain non convex. Many different solutions have been proposed to deal with such difficult problems, for instance simulated annealing algorithms [9] or genetic algorithms [14]. However these solutions tend to be extremely costly, especially when dealing with long signal waveforms and/or many transmitting antennas. We will preferably resort here to a gradient descent algorithm that has proved recently to perform very well in the context of phase codes optimization [16], [17], [18], [15], [19].

We also propose in this article to modify the cost function to optimize in order to enlarge the null over an extended angular interval. This can be done by minimizing the energy integrated over the interval to blank. We show that this new cost function is obtained as the trace of the product between the waveform autocorrelation matrix and a specific matrix defining the angular interval under consideration. Interestingly, in the specific case of a linear array with antennas separated by half the wavelength, it turns out that this matrix eigenvectors correspond to the well-known Discrete Prolate Spheroidal Sequences (DPSS) [1] that were defined in order to solve the spectral concentration problem. These DPSS have since been extensively used in the signal processing community for many different applications [20], [21], [22]. The author has regularly encountered them when trying to deal with subspace rejection

Fig. 1. Classical linear MIMO array.

[23] or robust detectors and robust mismatched filters for off-grid targets [24], [25].

This article is organised as follows: in section II, we will present the MIMO radar and main principles that will be useful for the remainder of the paper. In section III, we will propose a solution to create a beam pattern presenting a null direction and a uniform energy spreading in the remaining angular domain. In section IV, we will extend this solution to cope with an extended angular domain. Finally we will present some simulation results in section V.

II. MIMO RADAR TRANSMIT BEAM PATTERN

In this section, we will review the main principle and properties of the coherent colocated MIMO radar. We will not consider any reception array here since this will not have any influence on the radiated beam pattern.

A. Transmitted signal and transmitted beam pattern

Let us consider a MIMO antenna array composed of N_t transmitting antennas, assumed to be colocated and coherent. For simplicity, we will consider in the following a linear array and thus only one angular direction. The m^{th} antenna is located at a position denoted by vector $\mathbf{x}_m$, and transmits its own waveform $s_m(t)$ where $s_m(t)$ may differ from $s_{m'}(t)$ when $m \neq m'$. The overall signal transmitted by the MIMO array in a given direction θ is provided by

$$s_\theta(t) = \sum_{m=0}^{N_t-1} e^{j\mathbf{x}_m^T \mathbf{k}(\theta)} s_m(t),$$

where $\mathbf{k}(\theta)$ represents the wave vector in direction θ . Clearly the signal transmitted in a given direction θ_1 will differ from the signal transmitted in another direction θ_2 : $s_{\theta_1}(t) \neq s_{\theta_2}(t)$.

Denoting by $\mathbf{V}(\theta)$ and $\mathbf{s}(t)$ the vectors whose m^{th} entries are $\mathbf{V}_m(\theta) = e^{-j\mathbf{x}_m^T \mathbf{k}(\theta)}$ and $\mathbf{s}_m(t) = s_m(t)$, the transmitting signal can be simply written

$$s_\theta(t) = \mathbf{V}^H(\theta) \mathbf{s}(t).$$

The energy transmitted by the MIMO array in a given direction θ over the pulse duration is provided by

$$\begin{aligned} E(\theta) &= \int_0^{T_p} |s_\theta(t)|^2 dt \\ &= \int_0^{T_p} \mathbf{V}^H(\theta) \mathbf{s}(t) \mathbf{s}^H(t) \mathbf{V}(\theta) dt \\ &= \mathbf{V}^H(\theta) \mathbf{\Lambda}_s \mathbf{V}(\theta), \end{aligned}$$

where the (m, m') entry of the correlation matrix $\mathbf{\Lambda}(\mathbf{s})$ is defined by

$$\Lambda_{(m, m')}(\mathbf{s}) = \int_0^{T_p} s_m(t) s_{m'}^*(t) dt.$$

Since $\mathbf{V}(\theta)$ depends only on the antenna array geometry that we will assumed fixed here, designing a desired antenna array diagram will thus resort to optimizing $\mathbf{\Lambda}(\mathbf{s})$.

B. Case of orthogonal and identical waveforms

For orthogonal waveforms verifying the following "weak" orthogonal property (weak in the sense that the orthogonality is verified only for synchronous transmitted waveform)

$$\int_0^{T_p} s_m(t) s_{m'}^*(t) dt = \begin{cases} 1 & \text{si } m = m', \\ 0 & \text{sinon,} \end{cases}$$

it comes that $\mathbf{\Lambda}_s$ is the identity matrix, so that the transmitted energy is

$$E(\theta) = \mathbf{V}^H(\theta) \mathbf{V}(\theta) = N_t,$$

i.e. it is the same in all directions and equal to N_t . Recall however that this property is valid only when considering the overall pulse duration. At a given instant during the pulse duration, the instantaneous radiated beam pattern may be completely different, and will never be uniform. However it will permanently change during the pulse duration, so that the energy integrated over the pulse duration will finally be uniform.

The important implication of this property is that if one wants to design a non-uniform antenna pattern, the corresponding waveforms should NOT be orthogonal to each other, since orthogonality of the waveforms implies a uniform energy diagram.

For identical waveforms, $s_m(t) = s(t), \forall m$, the transmitted energy is given by

$$E(\theta) = \mathbf{V}^H(\theta) \mathbf{U} \mathbf{V}(\theta) = N_t,$$

where entries of matrix $\mathbf{U}$ are all equal to 1, i.e. $\mathbf{U}_{(m, m')} = 1, \forall (m, m')$. Then for $\theta = 0$, $E(0) = N_t^2$, and the energy is fully focused in the direction orthogonal to the array.

Let us finally consider for the remaining of this article the simpler case of a uniformly-spaced linear array with antennas separated by half the wavelength. It is well-known that in this specific but very classical case, the steering vector in a given direction θ is expressed as

$$\mathbf{V}(\theta) = \left[1, e^{j\pi \sin(\theta)}, e^{j2\pi \sin(\theta)}, \dots, e^{j\pi(N_t-1) \sin(\theta)} \right]^T. \quad (1)$$

III. SINGLE DIRECTION BLANKING VIA PHASE CODE OPTIMIZATION

In this section, the objective will be to look for MIMO radar signal waveforms that provide a non-uniformly spread energy in the angular domain. More precisely we will try to design waveforms that minimize the energy in a specific angular direction θ_0 while presenting a flat diagram in the remaining angular domain.

For simplicity, we will reduce our waveform domain to the specific case of unimodular phase codes of length N , i.e. signals that are defined by a set of N phases so that the m^{th} transmitted signal can be written as

$$\mathbf{s}_m = [e^{j\varphi_{m,1}}, e^{j\varphi_{m,2}}, \dots, e^{j\varphi_{m,N}}]^T,$$

where $\varphi_{m,n}$ is the n^{th} entry of phase code $\mathbf{s}_m$. Besides the N_t transmitted codes will be gathered in matrix $\mathbf{S} = [\mathbf{s}_0, \dots, \mathbf{s}_{N_t-1}]^T$.

Note also that it is possible without loss of generality to choose $\theta_0 = 0$ as the null direction. Indeed, multiplying the optimized family code for that direction by the steering vector corresponding to another direction θ'_0 would naturally translate the null direction to this new direction.

A. Optimization problem for direction blanking

The natural way to search for codes that provide the minimum possible energy in direction θ_0 consists in solving the following optimization problem [14]:

$$\mathbf{S}_{\text{opt}} = \arg \min_{\mathbf{S} \in \mathcal{U}} E(\theta_0) = \arg \min_{\mathbf{S} \in \mathcal{U}} \mathbf{V}^H(\theta_0) \mathbf{\Lambda}_s \mathbf{V}(\theta_0),$$

where $\mathcal{U}$ represents the set of unimodular phase codes.

Due to the nature of the set of unimodular phase codes, this optimization problem is difficult to solve. In particular, the constant modulus constraint inherent to the set $\mathcal{U}$ leads to a non convex search space when expressed in this way. In order to facilitate this optimization, some articles in the recent literature [16], [18], [15], [19] have proposed to directly perform the optimization on the phases rather than on the temporal code. Denoting by Φ gathering the phases of the codes defined by $\mathbf{S}$, whose (m, n) entry is thus provided by

$$\Phi_{m,n} = \varphi_{m,n},$$

and denoting abusively by $\mathbf{S} = \exp(j\Phi)$ the matrix whose (m, n) entry is defined by $\Phi_{m,n} = \varphi_{m,n}$, the autocorrelation matrix $\mathbf{\Lambda}_s$ defined in section II becomes then $\mathbf{\Lambda}_s = \exp(j\Phi)^H \exp(j\Phi)$, and the optimization problem can be written in the new form

$$\Phi_{\text{opt}} = \arg \min_{\Phi} \mathbf{V}^H(\theta_0) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_0). \quad (2)$$

Such a reformulation enables to perform the optimization over a convex set (the phase space) but does not transform the optimization problem into a convex one since the objective function is still not convex. Thus searching for the optimal solution remains complex, and it is necessary to resort to sub-optimal solutions.

A simple solution, that actually proved to be very efficient when searching for very good phase codes, consists in applying a gradient-descent-like algorithm. This type of algorithms is very simple to implement and generally quite fast, especially compared to other optimization algorithms when considering large dimension variable spaces. In the problem at hand here, where the number of variables to optimize is $N \times N_t$, it is incomparably less costly than simulated annealing algorithms [9] or genetic algorithms [14]. We will thus resort in this

article to use a gradient descent algorithm to solve either the optimization problem (2) or the other problems that will be presented in the following.

Note that since unimodular phase codes are considered, the overall energy transmitted by the MIMO array is set to N_t^2 , thus ensuring that some energy is transmitted in other directions. However the energy spreading over these directions may be non uniform without some additional constraints, and may depend on the initialization of the gradient descent algorithm. It may thus be of interest to add additional constraints to ensure the uniform spreading of the energy in the remaining angular domain.

B. Direction blanking and uniform energy spreading in the angular domain

Considering one single null direction $\theta_0 = 0$, the remaining angular domain may be sampled in $N_t - 1$ orthogonal directions $\theta_k^\perp$. In the specific linear array case with antennas separated by half the wavelength, these $N_t - 1$ orthogonal directions are defined by $\sin \theta_k^\perp = \frac{2k}{N_t}$ pour $k \neq 0 \bmod N_t$.

Note here that whatever the transmitted waveform considered, the overall transmitted energy by the MIMO array in the N_t orthogonal angular directions is always N_t^2 . This is verified when considering orthogonal waveforms since the radiated energy is then N_t in each of the N_t orthogonal sampling the angular domain, and also in the classical phased array case using identical waveforms, where the radiated energy is N_t^2 in the focused direction and 0 in the $N_t - 1$ other directions.

In the specific case of a single null direction that is considered here, minimizing the energy in direction θ_0 and ensuring a uniform energy in the $N_t - 1$ remaining orthogonal directions $\theta_k^\perp$ implies that the energy $E(\theta_k^\perp)$ radiated in any of these $\theta_k^\perp$ shall approximately verify

$$(N_t - 1)E(\theta_k^\perp) = N_t^2 \Rightarrow E(\theta_k^\perp) = \frac{N_t^2}{N_t - 1}.$$

In order to ensure a uniform energy spreading in the $N_t - 1$ orthogonal directions $\theta_k^\perp$, and recalling that this energy can be written $E(\theta_k^\perp) = \mathbf{V}^H(\theta_k^\perp) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_k^\perp)$, we thus propose the following modified optimization problem with $N_t - 1$ constraints

$$\begin{aligned} \min_{\Phi} \quad & \mathbf{V}^H(\theta_0) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_0) \\ \text{s.t.} \quad & \mathbf{V}^H(\theta_k^\perp) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_k^\perp) = \frac{N_t^2}{N_t - 1}, \\ & \forall k \in \{1, N_t - 1\} \end{aligned}$$

However the addition of such equality constraints makes the optimization problem even more difficult to solve. Besides a solution verifying exactly these constraints does not necessarily exist, even more when the uniform energy constraint level is only approximate. It is thus generally more efficient to control the gap between the solution and the constraints, which can be done by reformulating the problem as

$$\begin{aligned} \min_{\Phi} \quad & \mathbf{V}^H(\theta_0) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_0) \\ \text{s.t.} \quad & \left| \mathbf{V}^H(\theta_k^\perp) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_k^\perp) - \frac{N_t^2}{N_t - 1} \right|^2 \leq \varepsilon, \\ & \forall k \in \{1, N_t - 1\} \end{aligned}$$

where ε shall be the smallest possible. Then in turn, solving this constrained problem may be done by considering the following non-constrained penalized problem (that can also be viewed as the lagrangian of the optimization problem):

$$\min_{\Phi} \mathbf{V}^H(\theta_0) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_0) + \mu \sum_k \left| \mathbf{V}^H(\theta_k^\perp) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta_k^\perp) - \frac{N_E^2}{N_E - 1} \right|^2,$$

where the penalization variable μ can be chosen freely depending whether one wants to privilege the energy minimisation in direction θ_0 or the uniform spreading of the energy among the orthogonal directions $\theta_k^\perp$. This last optimization program can be solved again in a non optimal way by a gradient descent algorithm.

Note here that these optimization programs can be easily extended to multiple null directions by simply minimizing the sum of the energy in the desired N_{blank} null directions, and constraining the energy in the remaining orthogonal directions to be approximately equal to $\frac{N_t^2}{N_t - N_{blank}}$.

IV. BLANKING OVER AN EXTENDED ANGULAR INTERVAL

The previously proposed optimization programs enable to minimize the energy in a given direction θ_0 , but will thus tend to create a very narrow null area in the angular domain. We will now focus on an optimization program that enables to create energy blanking over extended angular intervals. This resorts to finding the optimum solution to the following optimization problem:

$$\min_{\Phi} \int_{\Delta\theta} E(\theta) d\theta, \quad (3)$$

where $\Delta\theta$ represents the angular domain to be blanked. Typically this interval may be defined by $\Delta\theta = [\theta_{\min}, \theta_{\max}]$.

Recalling that

$$E(\theta) = \mathbf{V}^H(\theta) \mathbf{\Lambda}_s \mathbf{V}(\theta) = \mathbf{V}^H(\theta) (\exp(j\Phi))^H \exp(j\Phi) \mathbf{V}(\theta),$$

and observing that

$$\mathbf{V}^H(\theta) \mathbf{\Lambda}_s \mathbf{V}(\theta) = \text{Tr}(\mathbf{V}^H(\theta) \mathbf{\Lambda}_s \mathbf{V}(\theta)) = \text{Tr}(\mathbf{\Lambda}_s \mathbf{V}(\theta) \mathbf{V}^H(\theta))$$

by exploiting the commutativity property of the trace operator, it can be written that

$$\begin{aligned} \int_{\Delta\theta} E(\theta) d\theta &= \int_{\Delta\theta} \text{Tr}(\mathbf{\Lambda}_s \mathbf{V}(\theta) \mathbf{V}^H(\theta)) d\theta \\ &= \text{Tr}\left(\mathbf{\Lambda}_s \int_{\Delta\theta} \mathbf{V}(\theta) \mathbf{V}^H(\theta) d\theta\right) \end{aligned}$$

where we have also exploited the linearity of the trace operator.

The optimization problem under consideration can thus be written in terms of the matrix

$$\mathbf{M}(\Delta\theta) = \int_{\Delta\theta} \mathbf{V}(\theta) \mathbf{V}^H(\theta) d\theta.$$

In the case of a linear array with antennas separated by half the wavelength, this integral can be further simplified. Recalling from Eq.(1) that the angular steering vector phase is then

linear in $u = \sin \theta$, let us consider classically for the sake of simplicity the steering vector as a function of u

$$\mathbf{V}(u) = [1, e^{j\pi u}, e^{j\pi 2u}, \dots, e^{j\pi(N_E-1)u}]^T,$$

and the corresponding matrix $\mathbf{M}(\Delta u)$ instead of matrix $\mathbf{M}(\Delta\theta)$.

Thanks to the change of variable, the (k, l) entry of matrix $\mathbf{V}(u) \mathbf{V}^H(u)$ is straightforwardly provided by

$$[\mathbf{V}(u) \mathbf{V}^H(u)]_{kl} = e^{j\pi(k-l)u}.$$

Assuming now that the angular interval for the blanking can be simply written $[-\Delta u/2, \Delta u/2]$, then the (k, l) term of matrix $\mathbf{M}(\Delta u)$ can be computed as

$$\begin{aligned} [\mathbf{M}(\Delta u)]_{kl} &= \int_{-\Delta u/2}^{\Delta u/2} e^{j\pi(k-l)u} du \\ &= \frac{1}{j\pi(k-l)} \left[e^{j\pi(k-l)u} \right]_{-\Delta u/2}^{\Delta u/2} \\ &= \Delta u \text{sinc}(\pi(k-l)\Delta u/2). \end{aligned} \quad (4)$$

This matrix is well known in signal processing: its eigenvectors are the Discrete Prolate Spheroidal Sequences (DPSS), that were defined by Slepian in [1] in order to solve the spectral concentration problem.

Two remarks must be stated here:

- first, the calculation leading to the DPSS here differs from the calculations made in [23], [24], [25], and thus the result is different. In these previous works, the DPSS were not directly involved but were more present via a Hadamard product with another matrix depending on the considered application. On the contrary, here the DPSS arise straightly from the computation.
- The fact that the computation leads to the DPSS here directly comes from the use of a linear array with uniformly spaced antennas. In a more general case, the computation of matrix $\mathbf{M}(\Delta u)$ should be more complicated and should not lead to the DPSS.

Finally, recalling that $\mathbf{\Lambda}_s = \exp(j\Phi)^H \exp(j\Phi)$, the optimization problem (3) for minimizing the energy in the extended angular domain $[-\Delta u/2, \Delta u/2]$ becomes simply

$$\min_{\Phi} \text{Tr}(\exp(j\Phi)^H \exp(j\Phi) \mathbf{M}(\Delta u)),$$

where $\mathbf{M}(\Delta u)$ provided by Eq.(4) is the matrix whose eigenvectors are the DPSS.

Of course, it is possible to insert additional constraints to this optimization problem, for instance in order to ensure a uniform energy spreading in the remaining angular domain.

V. SIMULATION RESULTS

In this section, we will present some simulation results to illustrate the different optimization program proposed thereafter. The transmitting array will be a linear array with uniformly spaced antennas separated by half the wavelength.

We will first consider the creation of a single blank direction with uniform energy spreading in the remaining angular

Fig. 2. Transmit beam pattern of a MIMO radar for different phase codes: random initial phase code, and optimized phase codes obtained for penalization factor $\mu = 0, 1$, and 100 . $N_t = 10$, $N = 200$.

domain. We choose to consider sequences of length $N = 200$ and a linear array composed of $N_t = 10$ antennas. Thus the number of variables to optimize $N_t N = 2000$, which is already a relative large optimization program. The gradient descent algorithm is initialized with N_t phase sequences obtained by randomly drawing the phase matrix Φ in $[0, 2\pi]$. We present in Figure 2 the antenna beam pattern obtained with respect to the angle when $\theta_0 = 0$, for the initial randomly drawn phase codes, and for the phase codes optimized for different values of the penalization factor μ , namely $\mu = 0, 1$ and 100 .

We can observe first that the beam pattern provided by the initial random codes is admirably uniform over the entire angular domain, which indicates that these random codes are approximately orthogonal. On the contrary the optimized waveforms provide the desired narrow null direction. Interestingly even without the constraint on uniform energy (case $\mu = 0$), the resulting beam pattern is already quite uniform over the directions orthogonal to θ_0 . This can again be explained by the random initialization which tends to provide orthogonal sequences. It is very likely that a choice of non orthogonal sequences as initial sequences would have led to a much less uniform beam pattern. Anyway, the use of a non-zero penalization clearly enables an improvement on the overall homogeneity of the energy spreading. This can be seen in Figure 3 that presents a zoom over the flat homogeneous energy level. Note that this improvement comes at the price of a slight deterioration of the rejection level in the null direction. Note finally that since we consider a linear array, the direction $\theta = 0$ corresponding to the direction orthogonal to the array is identical to the direction $\theta = \pi$ and thus we retrieve the same null in this direction.

We will now consider the case of an extended angular rejection domain. We will this time consider the case of a linear array with $N_t = 40$ antennas and phase codes of

Fig. 3. Transmit beam pattern of a MIMO radar for different phase codes: random initial phase code, and optimized phase codes obtained for penalization factor $\mu = 0, 1$ and 100 . $N_t = 10$, $N = 200$. Zoom over the uniform energy area: penalization helps flattening the beam pattern.

length $N = 100$. This increases the number of variables up to $N_t N = 4000$. The phases of the initial random codes are again uniformly drawn in $[0, 2\pi]$. We present in Figures 4 and 5 the antenna beam pattern obtained with respect to the angle when $\theta_0 = 0$, for the initial randomly drawn phase codes, for a phase code optimized using the previous cost function designed for narrow direction blanking, and for phase codes optimised with the cost function based on the DPSS that enables an enlarged angular rejection. Several different widths are considered for the null area, expressed in terms of Δu as defined in section IV: $\Delta u = 1/N_t$, $\Delta u = 2/N_t$ et $\Delta u = 4/N_t$. We effectively observe that the use of the DPSS cost function leads to a larger blanking interval. However it must also be noticed that the remaining energy in the zone tends to increase when the considered interval is larger. It seems thus that it is much difficult to completely blank a given interval when this interval gets larger.

VI. CONCLUSION

In this article, we have proposed to optimize phase codes for MIMO radar in order to create beam pattern presenting a blanking area of controllable width, while spreading the transmitted energy uniformly over the other angular directions. The proposed cost function to optimize is composed of a term representing the energy to minimize in a given direction and of an additional penalization term that enables to control the homogeneity of the energy in the orthogonal directions. For larger blanking area, we have shown that the cost function can be simply expressed in terms of the DPSS. Optimization of the proposed cost functions is performed thanks to a gradient descent algorithm. Simulation results show that the resulting phase codes indeed provide the desired beam pattern. In the case of an extended blanking area, the enlargement of the null is obtained at the price of a slight energy increase. Note

Fig. 4. Transmit beam pattern of a MIMO radar for different phase codes: a random initial phase code, and optimized phase codes with the DPSS-based cost function for different angular interval width $\Delta u = 1/N_t$, $\Delta u = 2/N_t$ and $\Delta u = 4/N_t$. $N_t = 40$, $N = 100$.

Fig. 5. Transmit beam pattern of a MIMO radar for different phase codes: a random initial phase code, and optimized phase codes with the DPSS-based cost function for different angular interval width $\Delta u = 1/N_t$, $\Delta u = 2/N_t$ and $\Delta u = 4/N_t$. $N_t = 40$, $N = 100$. Zoom over the extended interval: the optimization effectively leads to a broadened blanking area at the price of an increased energy level inside this area.

that we did not focus on the sidelobe level in range here, but the use of random codes already ensures a satisfying level, that can anyway be optimized at the reception using a dedicated mismatched filter [26]. Note also that all proposed cost function may be easily extended to multiple direction blanking. Besides these cost functions may be inverted to create single or multiple lobes instead of nulls. In that case, the DPSS may be used to create wider mainlobes.

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