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*Space-Time-Block-Coded Transmissions for
Downlink MC-CDMA Transmissions Over
Frequency-Selective Fading Channels*

S. JAMALLULIL, J. P. CANCES and V. MEGHDADI

Groupe d'Etude des Systèmes de Télécommunications
Ecole Nationale Supérieure d'Ingénieurs de Limoges
Parc ESTER
B.P 6804, 87068 LIMOGES CEDEX
(cances@ensil.unilim.fr)

Abstract

Mitigation of multipath fading effects and suppression of multiuser interference (MUI) constitute major challenges in the design of the third generation wireless mobile systems. Space-time (ST) coding offers a attractive solution to cope with multipath fading, but most existing ST coding schemes assume flat fading channels that may not be realistic for wide-band communications. Single-user ST coded orthogonal frequency-division multiplexing transmissions over frequency-selective channels suffer from deep fades. Especially multiuser ST block-coded transmissions through multipath fading channels present unique challenge in suppressing not only MUI but also intersymbol/chip interference. In this paper, we design ST multiuser transceivers for MC-CDMA quasi-synchronous systems, capable to reliably transmit over frequency-selective multipath channels (downlink or uplink). For quasi-synchronous MC-CDMA systems relying on symbol blocking and a single receive antenna (downlink situation), ST block codes are derived and MUI is eliminated without destroying the orthogonality of ST block codes. The proposed system is able to provide transmit diversity and to guarantee symbol recovery in multiuser environments, regardless of unknown multipath. Unlike existing approaches, the mobile does not need to know the channel of other users. In addition to decoding simplicity, computer simulations show the performance merits of the proposed transceiver.

Index Terms: Fading channels, space-time codes, diversity methods, multiuser detection.

I. INTRODUCTION

The major driver for broadband wireless communications has been reliable high-data-rate services (e.g., real-time multimedia services). This, together with the scarcity of bandwidth resources, motivate research towards developing efficient coding and modulation schemes that improve the quality and bandwidth efficiency of wireless systems. In wireless links, multipath fading causes performance degradation and constitutes the bottleneck for increasing data rates. Moreover, in the case of multiuser transmission, the other major impairment comes from multiuser interference (MUI).

Traditionally, the most popular technique to combat fading has been the exploitation of diversity. Space-time (ST) coding has been proved effective in combat fading, and enhancing data rates [1-4] and references therein. Exploiting the presence of spatial diversity offered by multiple transmit and/or receive antennas, ST coding relies on simultaneous coding across space and time to achieve diversity gain without necessarily sacrificing precious bandwidth. Two typical examples of ST codes are ST trellis codes [5] and ST block codes [2] [6].

ST codes were originally designed for single-user systems and investigated for slow flat fading channels [7-9]. However this assumption does not hold any more in wide-band communications, which motivated recent extensions of ST coding to frequency-selective channels for orthogonal frequency-division multiplexing (OFDM) [10-11] and generalized OFDM (GOFDM) systems [12-13]. With transmit diversity, ST coded (G)OFDM can improve (but not eliminate) fading effects caused by channel nulls (fades). However, when transmit antennas are not well separated, the multiple channels become correlated, and symbol recovery is not assured because nulls common to the multiple channels are possible to occur. Different from [10-11], the so-called "root-rotating" variant of [12-13] avoids persistent deep fading caused by common channel nulls but symbol recovery is still not guaranteed.

ST coding for multiuser systems is challenging due to the presence of severe MUI. For an M -user system where each user is equipped with N_t transmit antennas, MUI is caused by $(M-1)N_t$ instead of $M-1$ interfering signals. Assuming flat fading channels, [14] exploited the structure of ST block codes to suppress MUI in space using a beamforming approach that relies on multiple receive antennas. However, in order to cancel MUI, a minimum of M receive antennas is required, which limits its application to base station transceivers. Furthermore, the

per-user performance of [14], when compared to the single user system in [2] and [6], may be degraded after the beamforming-based suppression of MUI.

In this paper, we develop novel ST multiuser transceivers for MC-CDMA multiple access through frequency-selective channels. In our proposed system, each user is equipped with multiple transmit antennas, but only one receive antenna will be sufficient.

We study the context of quasi-synchronous MC-CDMA systems where the relative delays between each transmitted user is a small fraction of the period symbol and may be captured by the FIR transmission channel model.

In the context of quasi-synchronous MC-CDMA, we develop three key points:

- 1) We adopt and extend the the generalized multicarrier code-division (GMC-CDMA) design of [15-16] to our ST context.
- 2) We eliminate MUI deterministically regardless of finite-impulse response (FIR) channels encountered by judicious design of user's spreading codes.
- 3) Having suppressed MUI, we then rely on symbol blocking and develop novel ST block coders to achieve transmit diversity gain. Redundant transmit precoding is used to guarantee symbol recovery regardless of channel nulls.

Unlike [14] where channel knowledge of all users is needed and multiple antennas are required at the receiver, we only need to know the channel of the desired user. Thus, our receiver is particularly suitable for downlink transmissions.

The paper is organized as follows. In Section II, we describe the system model. In Section III the construction of users' spreading codes for deterministic MUI elimination is presented. Then Section IV deals with ST coding and symbol recovery designs. Simulation results are given in Section V. Finally Section VI concludes the paper resuming the main results.

II. SYSTEM MODEL

Fig. 1 depicts the discrete-time equivalent baseband model of a chip-sampled quasi-synchronous multiuser communication system as in [16-17] with N_t transmit antennas and a single receive antenna for each user. Each of the M users transmits the information sequence $s_m(n)$ in blocks of size $K \times 1$: $s_m(n) := [s_m(n.K), \dots, s_m(n.K + K - 1)]^T$. The filterbank described by the $Q \times K$ matrix Θ_m , with complex entries, constitutes our redundant ($Q > K$) precoder that will facilitate symbol recovery. The $Q \times 1$ precoded blocks

$$\tilde{s}_m(n) := [\tilde{s}_m(nQ), \dots, \tilde{s}_m(nQ + Q - 1)]^T = \Theta_m s_m(n) \quad (1)$$

are fed into the ST encoder $\mathcal{E}(\cdot)$. The ST encoder takes N_s consecutively precoded symbols $\tilde{s}_m(n)$, which we denote by $\tilde{s}_m(i) := [\tilde{s}_m^T(iN_s), \dots, \tilde{s}_m^T(iN_s + N_s - 1)]^T$, to output the following $N_d \times Q \times N_t$ matrix of ST encoded blocks :

$$\bar{S}_m(i) = \mathcal{E}(\tilde{s}_m(i)) = \begin{bmatrix} \bar{s}_{m1}(iN_d) & \cdots & \bar{s}_{mN_t}(iN_d) \\ \vdots & \ddots & \vdots \\ \bar{s}_{m1}(iN_d + N_d - 1) & \cdots & \bar{s}_{mN_t}(iN_d + N_d - 1) \end{bmatrix} \quad (2)$$

whose rows are generated in successive time intervals with each of the N_t blocks in a given row forwarded to one of the N_t transmit antennas. We denote :

$\bar{s}_{mn_i}(n) := [\bar{s}_{mn_i}(nQ), \dots, \bar{s}_{mn_i}(nQ + Q - 1)]^T$ the $Q \times 1$ block transmitted through the n_i transmit-antenna at time n . The link between the ST encoded output block $\bar{s}_{mn_i}(n)$ and the precoded input block $\tilde{s}_m(n)$ will offer transmit diversity gain as it will be demonstrated in Section IV. Before transmission, each user's spreading code described by the $P \times Q$ matrix $\mathbf{C}_m$ is applied to $\bar{s}_{mn_i}(n)$ to produce the block $\mathbf{u}_{mn_i}(n) := \mathbf{C}_m \bar{s}_{mn_i}(n)$ of length $P > Q$. Each user's spreading code matrix $\mathbf{C}_m$ will be designed to eliminate the MUI in section III.

Fig. 1. Discrete-time equivalent baseband system for the m th user

After parallel-to-serial (P/S) conversion, the P -long block $\mathbf{u}_{mn_i}(n)$ propagates through the L th-order discrete-time equivalent FIR channel $\{h_{mn_i}(l)\}_{l=0}^L$. As we said before, the channel includes transmit-receive pulse-shaping filters, multipath and quasi-synchronism among users. We underscore that except for an upper bound L on the maximum channel order that is assumed available, we do not need to know the coefficients $\{h_{mn_i}(l)\}_{n_i=0}^{N_t}$ at the transmitter. In order to eliminate the interblock interference (IBI) caused by the FIR channels, we pad L zero symbols at the end of $\mathbf{u}_{mn_i}(n)$, the $J \times P$ matrix $\mathbf{T}_{zp} := [\mathbf{I}_P^T \mathbf{0}_{L \times P}^T]^T$ and $J := L + P$. Padding L guard zeros reduces bandwidth efficiency, but this reduction can be made arbitrarily small by increasing the block size K . Zero-padding (ZP) is an alternative to the cyclic prefix (CP) that is introduced at OFDM transmitters and is discarded at OFDM receivers to remove IBI [18]. The FIR channel operation on the transmitted data is described by the $J \times J$ Toeplitz matrix $\bar{\mathbf{H}}_{mn_i}$ with first row $[h_{mn_i}(0) \mathbf{0}_{1 \times (J-1)}]$ and first column $[h_{mn_i}(0) \dots h_{mn_i}(L) \mathbf{0}_{1 \times (J-L-1)}]^T$. After serial-to-parallel (S/P) conversion (not shown on Fig. 1) the n th received IBI-free block: $\mathbf{y}(n) := [y(nJ), \dots, y(nJ + J - 1)]^T$ from all M users is given by

$$\mathbf{y}(n) = \mathbf{x}_m(n) + \sum_{i=0, i \neq m}^{M-1} \mathbf{x}_i(n) + \mathbf{w}(n) \quad (3)$$

where $\mathbf{w}(n) := [w(nJ), \dots, w(nJ + J - 1)]^T$ denotes additive white Gaussian noise (AWGN). The quantity $\mathbf{x}_m(n) := [x_m(nJ), \dots, x_m(nJ + J - 1)]^T$ is the noise-free received block from the m th user that is expressed as

$$\mathbf{x}_m(n) = A_m \cdot \sum_{n_t=1}^{N_t} \mathbf{H}_{mn_t} \mathbf{C}_m \bar{\mathbf{s}}_{mn_t}(n) \quad (4)$$

where $\mathbf{H}_{mn_t} := \bar{\mathbf{H}}_{mn_t} \mathbf{T}_{zp}$ is a Toeplitz channel matrix of size $J \times P$ and A_m is the m th user's amplitude. The vector $\mathbf{y}(n)$ in (3) consists of three terms: the first term is useful information from the desired (here the m th) user, the second term is MUI, and the third term is additive receiver noise.

Given the received block $\mathbf{y}(n)$, the recovery of $\mathbf{s}_m(n)$ will be sought in three steps. First $\mathbf{y}(n)$ will be filtered by the user-separating filter described by the $Q \times J$ matrix $\mathbf{G}_m$ to eliminate MUI. Second, the MUI-free $\mathbf{y}_m(n)$ will be processed by the ST decoder $\bar{\Xi}(\cdot)$ to collect the embedded transmit diversity built by the ST encoder Ξ (see Fig. 1). Finally, a low-complexity block linear equalizer described by the $K \times J$ matrix Γ_m will be used to eliminate ISI and recovers $\mathbf{s}_m(n)$. Thus, the goal of our system design consists in three steps

- 1- *MUI Elimination Stage*: It consists of designing the users' spreading code matrices $\mathbf{C}_m$ and their separating filters $\mathbf{G}_m$, and selecting their dimensions to guarantee deterministic MUI elimination regardless of the underlying FIR channels, the chosen constellations, and with minimum redundancy.
- 2- *ST Encoding/Decoding Stage*: It consists in designing the ST encoder $\Xi(\cdot)$ and decoder $\bar{\Xi}(\cdot)$, and selecting the parameters N_s, N_d, N_t to achieve transmit diversity gain regardless of the underlying FIR channel filters, the chosen constellations, and with minimum redundancy.
- 3- *Symbol recovery Stage*: It consists in designing the precoder Θ_m and the equalizer Γ_m , and selecting the block sizes K and Q to guarantee symbol recovery of the regardless of the FIR channel nulls, the signal constellation, and with minimum redundancy.

III. USER CODES FOR DETERMINISTIC MUI ELIMINATION

Due to the multi-antenna transmissions, the MUI in (3) consists of $(M - 1) \cdot N_t$ interfering signals instead of $M - 1$ as in [15-16]. Furthermore, one can observe that the same spreading code is chosen for the N_t transmit-antenna branches of each user. Accordingly, the $(M - 1) \cdot N_t$ interfering signals in (3) can be partitioned into $M - 1$ groups with respect to their spreading codes. The basic idea to cancel MUI is to adopt the user-separating capability of the single transmit- and receive-antenna GMC-CDMA design [15-16] and generalize it to our particular

ST context. As in [15-16], we will design jointly the user's spreading codes $\{C_\mu\}_{\mu=0}^{M-1}$ and their separating filters $\{G_\mu\}_{\mu=0}^{M-1}$ to eliminate the MUI deterministically.

We first choose $P = M \cdot Q$ and define the set $\mathcal{P} := \{\rho_0, \dots, \rho_{P-1}\}$ where ρ_p s are *distinct points* on the complex plane. The set $\mathcal{P}$ will be named the signature set and its members (ρ_p s) are the signature points. With a total of P signature points, the μ th user is allocated Q signature points $\{\rho_{\mu,q}\}_{q=1}^{Q-1}$ (M corresponds to the spreading factor) and different signature set are allocated to different users. Let the set $I := \{1, 2, \dots, P\}$ be indexing the collection of all signature points in $\mathcal{P}$. Mathematically, the allocation of signature points can be represented by partitioning the set I into M nonintersecting subsets I_μ .

$$\bigcup_{\mu=0}^{M-1} I_\mu = I \quad I_\mu \cap I_m = \emptyset \quad \forall \mu \neq m \quad (5)$$

where $\emptyset$ denotes the empty set and the subset I_μ collects the indices of $\{\rho_{\mu,q}\}_{q=1}^{Q-1}$ in the set $\mathcal{P}$. To link I_μ with the operation of signature point allocation, we further define for the μ th user the signature selector matrix $\Phi_\mu = \mathbf{I}_P(:, I_\mu)$, where $\mathbf{I}_P(:, I_\mu)$ is a $P \times Q$ permutation matrix built from the I_μ columns of the $P \times P$ identity matrix $\mathbf{I}_P$. Because permutation matrices have orthogonal columns, it follows that the signature selector matrices are orthogonal to each other, i.e.,

$$\Phi_\mu^T \Phi_m = \delta(\mu - m) \mathbf{I}_Q \quad (6)$$

where $\delta(\cdot)$ denotes Kronecker's symbol. The orthogonality (6) will be exploited for MUI elimination. Before, we process MUI, we have to define two Vandermonde matrices from the set $\mathcal{P}$.

$$\mathcal{V}_1 := \begin{bmatrix} 1 & \rho_0^{-1} & \dots & \rho_0^{-(P-1)} \\ 1 & \rho_1^{-1} & \dots & \rho_1^{-(P-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \rho_P^{-1} & \dots & \rho_P^{-(P-1)} \end{bmatrix}_{P \times P} \quad \mathcal{V}_2 := \begin{bmatrix} 1 & \rho_0^{-1} & \dots & \rho_0^{-(J-1)} \\ 1 & \rho_1^{-1} & \dots & \rho_1^{-(J-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \rho_P^{-1} & \dots & \rho_P^{-(J-1)} \end{bmatrix}_{P \times J} \quad (7)$$

Recalling that $\{\rho_p\}_{p=0}^{P-1}$ were chosen to be distinct, we deduce that the matrix $\mathcal{V}_1$ is full rank, i.e., its inverse $\mathcal{V}_1^{-1}$ always exists. The μ th user's spreading code matrix C_μ and corresponding separating filter G_μ are designed as :

$$C_\mu = \mathcal{V}_1^{-1} \Phi_\mu, \quad G_\mu = \Phi_\mu^T \mathcal{V}_2 \quad \mu = 0, 1, \dots, M-1 \quad (8)$$

If ρ_s in (7) are chosen equispaced on the unit circle (FFT grid), then $(\mathcal{V}_1^{-1})\mathcal{V}_2$ corresponds to (I)FFT matrices and the selector in (8) allocate distinct user-specific subcarriers. Substituting (8) into (4), we obtain from (3) the receiver filter output $y_m(n) := \mathbf{G}_m \cdot y(n)$ as follows :

$$\begin{aligned} y_m(n) &:= A_m \cdot \sum_{n_t=1}^{N_t} \Phi_m^T \mathcal{V}_2 \mathbf{H}_{mn_t} \mathcal{V}_1^{-1} \Phi_m \bar{s}_{mn_t}(n) \\ &+ \sum_{\mu=0, \mu \neq m}^{M-1} A_\mu \sum_{n_t=1}^{N_t} \Phi_\mu^T \mathcal{V}_2 \mathbf{H}_{\mu n_t} \mathcal{V}_1^{-1} \Phi_\mu \bar{s}_{\mu n_t}(n) + \Phi_m^T \mathcal{V}_2 \mathbf{w}(n) \end{aligned} \quad (9)$$

To eliminate the MUI [second term in the right sight of (9)], we use the following properties :

-1 The toeplitz channel matrix $\mathbf{H}_{\mu n_t}$ is diagonalized by performing $\mathcal{V}_2 \mathbf{H}_{\mu n_t} \mathcal{V}_1^{-1}$ that means

$$\mathcal{V}_2 \mathbf{H}_{\mu n_t} \mathcal{V}_1^{-1} = \mathbf{D}_{H_{\mu n_t}} := \text{diag}[H_{\mu n_t}(\rho_0), \dots, H_{\mu n_t}(\rho_{P-1})]$$

where $H_{\mu n_t}(\rho_p) := \sum_{l=0}^L h_{\mu n_t}(l) \rho_p^{-l}$ is the Z-transform of the channel $h_{\mu n_t}(l)$ evaluated at the signature point ρ_p . For circular channel matrices that are created when CP is used instead of ZP, this property amounts to their diagonalization via (I)FFT matrices [16].

-2 Denoting $\mathcal{D}_{\mu n_t} := \Phi_\mu^T \mathbf{D}_{H_{\mu n_t}} \Phi_\mu = \text{diag}[H_{\mu n_t}(\rho_\mu, 0), \dots, H_{\mu n_t}(\rho_\mu, Q-1)]$, we have

$$\mathbf{D}_{H_{\mu n_t}} \Phi_\mu = \Phi_\mu \mathcal{D}_{\mu n_t}$$

This simply accounts for the fact that, with appropriate changes in dimensionalities, diagonal matrices commute with permutation matrices. Exploiting properties 1 and 2 and the orthogonality of (6), the user separating filter output $y_m(n)$ in (9) can be rewritten as

$$\begin{aligned} y_m(n) &= A_m \cdot \sum_{n_t=1}^{N_t} \Phi_m^T \mathcal{V}_2 \mathbf{H}_{mn_t} \mathcal{V}_1^{-1} \Phi_m \bar{s}_{mn_t}(n) + \Phi_m^T \mathcal{V}_2 \mathbf{w}(n) \\ &= A_m \cdot \sum_{n_t=1}^{N_t} \mathcal{D}_{mn_t} \bar{s}_{mn_t}(n) + \eta_m(n) \end{aligned} \quad (10)$$

where $\eta_m(n) := \Phi_m^T \mathcal{V}_2 \mathbf{w}(n)$ denotes the residual noise. It is clear from (10) that the MUI has been eliminated deterministically through the mutually orthogonal designs of $\mathbf{C}_\mu$ and $\mathbf{G}_\mu$ in (8). The MUI elimination here is achieved without requiring channel state information (CSI) at the receiver. Observe also that the equivalent channel matrix $\mathcal{D}_{mn_t}$ is diagonal instead of the Toeplitz convolution matrix $\mathbf{H}_{mn_t}$ that corresponds to our frequency-selective channels in (4). In fact, our MUI elimination has converted each frequency-selective channel into a set of frequency-flat subchannels. The diagonal structure of $\mathcal{D}_{mn_t}$ will be exploited to develop ST block codes' design.

The deterministic MUI elimination procedure can be summarized as follows:

a- Choose $P = M \cdot Q$ and $J = P + L$.

- b- Select P distinct complex points $\{\rho_p\}_{p=0}^{P-1}$ and assign them to disjoint subsets $\{\mathcal{I}_\mu\}_{\mu=0}^{M-1}$.
 - c- Set up the user's spreading codes $\mathbf{C}_\mu$ and their separating filters $\mathbf{G}_\mu$ according to (8).
- The next step, after MAI canceling, consists in the design of the ST block encoders/decoders that collect transmit diversity gains.

IV. SPACE-TIME BLOCK CODING AND SYMBOL RECOVERY

One advantage of the MUI-free transceivers is that multiple users can be separated by design and each user can be viewed as transmitting through his/her own multiple-antenna channels. Thus, ST coding design for a multiuser system has been converted into an equivalent set of single-user systems, which greatly reduces the receiver complexity per user. Without loss of generality, we focus on the m th user and we use the MUI-free model already obtained in (10). We wish to design the ST encoder $\Xi(\cdot)$ and decoder $\Xi^*(\cdot)$ to recover $s_m(n)$ from $y_m(n)$ with transmit diversity. At this moment, we suppose that $\{\mathcal{D}_{m, l_i}\}_{l_i=1}^{N_t}$ (or equivalently the channel coefficients $h_{m, l_i}(l)$) are known at the receiver. Its important to remind that the m th user's receiver does not need to know the channels from all interfering users and only has to estimate his own channels i.e., $h_{m, l_i}(l)$. The issue of channel estimation in our system will be discussed later.

The design of ST block codes was cast as a generalized complex orthogonal design (GCOD) problem in [6], for single-user symbol-by-symbol transmissions over flat fading channels. The orthogonal structure of ST block codes leads to low complexity ML decoding. However, as indicated in (3), we deal in our system with block-by-block transmissions through frequency-selective channels where code orthogonality is broken. However, recall from (10) that MUI elimination has converted each frequency-selective channel to a set of flat subchannels. Our basic objective is to achieve transmit diversity through these flat subchannels by developing a new counterpart of GCOD which is suitable for block transmissions. Note that the generalized real orthogonal designs of [6] are not applicable here because the equivalent channels $\mathcal{D}_{m, l_i}$ could be complex valued even when real constellations are used.

A. GCOD

Details for GCOD can be found in [6]. We only briefly review the main results here. We start by the following definition.

Definition : Define the $L_s \times 1$ vector $\mathbf{s} := [s_0, \dots, s_{L_s-1}]^T$ and let $\mathcal{G}(\mathbf{s})$ be an $L_d \times N_t$ matrix with entries: $0, -s_0, +s_0, -s_0^*, +s_0^*, \dots, -s_{L_s-1}, +s_{L_s-1}, -s_{L_s-1}^*, +s_{L_s-1}^*$, or their linear combinations. If

$\mathcal{G}(\mathbf{s})\mathcal{G}^H = \alpha \left(|s_0|^2 + |s_1|^2 + \dots + |s_{L_s-1}|^2 \right) \mathbf{I}_{N_t}$ for some positive constant α , then $\mathcal{G}(\mathbf{s})$ is termed a

GCOD in variables $s_0, \dots, s_{L_s-1}$ of size $L_d \times N_t$ and rate $R = L_s/L_d$. A GCOD of size $L_d \times N_t$ is designed to achieve transmit diversity gain of order N_t for a system with N_t transmit antennas [6]. For a fixed number of transmit antennas, one is interested in GCOD's with maximum rate R that are spectrally more efficient. On the existence of GCOD's, we know from [6] the following results:

- a- for $N_t = 2$, a GCOD with maximum rate $R = 1$ exists;
- b- for $N_t = 3, 4$, GCOD's exist but with rate $R = 3/4$;
- c- for $N_t > 4$, GCOD's exist with rate $R = 0.5$.

The most famous example is given for $N_t = 2$ and is known in the open literature as Alamouti's scheme.

$$\mathcal{G}_2 = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0 \end{bmatrix} \quad (11)$$

We will adopt here the GCOD's of [6] and adapt them to the system described in Section II.

B. Block ST Encoder Design

ST encoder design in our system depends on the selection of the parameters N_t , N_s and N_d . The choice of N_t is mainly dictated by complexity, size, and cost considerations at the transmitter. For a given N_t , choosing N_s and N_d starts with looking for an available GCOD, call it $\mathcal{G}_{N_t}$, of size $L_d \times N_t$ with maximum rate R , if possible. Then N_d and N_s are chosen as $N_d = L_d$ and $N_s = L_s = R.L_d$. Based on GCOD's of the previous subsection, the selection of (N_s, N_d) is listed as follows:

$$(N_s, N_d) = \begin{cases} (2, 2) & \text{if } N_t = 2 \\ (3, 4) & \text{if } N_t = 3, 4 \\ (N_t, 2N_t) & \text{if } N_t > 4 \end{cases} \quad (12)$$

Suppose that the selected $\mathcal{G}_{N_t}$ is an $N_d \times N_t$ GCOD in variables $s_0, \dots, s_{N_s-1}$. We proceed to design out ST encoder $\mathcal{E}(\cdot)$ by first representing $\mathcal{G}_{N_t}$ as in [6].

$$\mathcal{G}_{N_t} = \sum_{n_s=0}^{N_s-1} (\mathbf{A}_{n_s} s_{n_s} + \mathbf{B}_{n_s} s_{n_s}^*) \quad (13)$$

where $\mathbf{A}_{n_s}$ and $\mathbf{B}_{n_s}$ are $N_d \times N_t$ real matrices which are uniquely determined by $\mathcal{G}_{N_t}$. Our novel ST encoder $\mathcal{E}(\cdot)$ is constructed as the *block* counterpart of $\mathcal{G}_{N_t}$ in (13). Specifically, using the matrix pairs $\{\mathbf{A}_{n_s}, \mathbf{B}_{n_s}\}_{n_s=0}^{N_s-1}$, the matrix $\bar{\mathbf{S}}_m(i)$ in (2) that defines our block GCOD is specified as

$$\bar{\mathbf{S}}_m(i) = \sum_{n_s=0}^{N_s-1} [\mathbf{A}_{n_s} \otimes \tilde{\mathbf{s}}_m(iN_s + n_s) + \mathbf{B}_{n_s} \otimes \tilde{\mathbf{s}}_m^*(iN_s + n_s)] \quad (14)$$

where $\otimes$ denotes the Kronecker product. Clearly, without any blocking ($K = 1$), our ST block codes of (14) reduce to those in [2] and [6]. For example, when $N_t = 2$, we obtain from (11) that :

$$\begin{aligned} \mathbf{A}_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & \mathbf{A}_1 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ \mathbf{B}_0 &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} & \mathbf{B}_1 &= \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \end{aligned} \quad (15)$$

and it follows from (14) that

$$\bar{\mathbf{S}}_m(i) = \begin{bmatrix} \bar{s}_{m1}(2i) & \bar{s}_{m2}(2i) \\ \bar{s}_{m1}(2i+1) & \bar{s}_{m2}(2i+1) \end{bmatrix} = \begin{bmatrix} \tilde{s}_m(2i) & \tilde{s}_m(2i+1) \\ -\tilde{s}_m^*(2i+1) & \tilde{s}_m^*(2i) \end{bmatrix} \quad (16)$$

which is nothing but the block counterpart of [6]. Our ST encoder design is summarized as follows:

- a- given N_t , select N_s, N_d according to (12) ;
- b- find $\mathcal{G}_{N_t}$ of size $N_d \times N_t$ with rate $R = N_s/N_d$;
- c- use (13) to obtain $\{\mathbf{A}_{n_s}, \mathbf{B}_{n_s}\}_{n_s=0}^{N_s-1}$;
- d- build the ST block codes $\bar{\mathbf{S}}_m(i)$ according to (14).

Given the MUI-free received blocks $\mathbf{y}_m(n)$ of (10), we are now able to exploit the available transmit diversity embedded in (2) to recover $\mathbf{s}_m(n)$ with diversity gain. The channel matrices $\{\mathcal{D}_{mn_t}\}_{n_t=1}^{N_t}$ are supposed available and will be assumed time-invariant for at least N_d blocks.

C. Block ST Decoder Design

Defining the $Q \times Q$ diagonal matrix $\mathcal{D}_{\bar{s}_{mn_t}}(n) := \text{diag}[\bar{s}_{mn_t}(nQ), \dots, \bar{s}_{mn_t}(nQ + Q - 1)]$ and the $Q \times 1$ channel vector $\mathbf{h}_{mn_t} := [H_{mn_t}(\rho_m, 0), \dots, H_{mn_t}(\rho_m, Q - 1)]^T$, we use the identity :

$$\mathcal{D}_{mn_t} \bar{s}_{mn_t}(n) = \mathcal{D}_{\bar{s}_{mn_t}}(n) \mathbf{h}_{mn_t}$$

to rewrite (10) as :

$$\mathbf{y}_m(n) = \mathbf{A}_m \cdot \sum_{n_t=1}^{N_t} \mathcal{D}_{\bar{s}_{mn_t}}(n) \mathbf{h}_{mn_t} + \boldsymbol{\eta}_m(n) \quad (17)$$

Let us collect N_d consecutive blocks $\mathbf{y}_m(n)$ into a superblock $\tilde{\mathbf{y}}_m(i) := [\mathbf{y}_m^T(iN_d), \dots, \mathbf{y}_m^T(iN_d + N_d - 1)]^T$ and use (17) to obtain

$$\tilde{\mathbf{y}}_m(i) = A_m \times \underbrace{\begin{bmatrix} \mathcal{D}_{\bar{s}_{m1}}(iN_d) & \cdots & \mathcal{D}_{\bar{s}_{mN_t}}(iN_d) \\ \vdots & \ddots & \vdots \\ \mathcal{D}_{\bar{s}_{m1}}(iN_d + N_d - 1) & \cdots & \mathcal{D}_{\bar{s}_{mN_t}}(iN_d + N_d - 1) \end{bmatrix}}_{:= \mathbf{D}_{\bar{s}_m}(i)} \times \begin{bmatrix} \mathbf{h}_{m1} \\ \vdots \\ \mathbf{h}_{mN_t} \end{bmatrix} + \tilde{\eta}_m(i) \quad (18)$$

where $\tilde{\eta}_m(i)$ is defined similar to $\tilde{\mathbf{y}}_m(i)$. By comparing $\mathbf{D}_{\bar{s}_m}(i)$ to $\bar{\mathbf{S}}_m(i)$ of (2), we infer from (14) that

$$\mathbf{D}_{\bar{s}_m}(i) = \sum_{n_s=0}^{N_s-1} [\mathbf{A}_{n_s} \otimes \mathcal{D}_{\bar{s}_m}(iN_s + n_s) + \mathbf{B}_{n_s} \otimes \mathcal{D}_{\bar{s}_m}^*(iN_s + n_s)] \quad (19)$$

where $\mathcal{D}_{\bar{s}_m}(n) := \text{diag}[\tilde{s}_m(nQ), \dots, \tilde{s}_m(nQ + Q - 1)]^T$. Plugging (19) into (18) and using the identity $\mathcal{D}_{\bar{s}_m}(n)\mathbf{h}_{mn_t} = \mathcal{D}_{mn_t}\tilde{s}_m(n)$, we obtain

$$\tilde{\mathbf{y}}_m(i) = A_m \sum_{n_s=0}^{N_s-1} \sum_{n_t=1}^{N_t} \left\{ [\mathbf{A}_{n_s}(:, n_t) \otimes \mathcal{D}_{mn_t}] \tilde{s}_m(iN_s + n_s) + [\mathbf{B}_{n_s}(:, n_t) \otimes \mathcal{D}_{mn_t}] \tilde{s}_m^*(iN_s + n_s) \right\} + \tilde{\eta}_m(i) \quad (20)$$

Our ST decoder $\bar{\Xi}(\cdot)$ consists of N_s branches of detectors each described by the $Q \times 2N_dQ$ partitioned matrix $\mathcal{L}_{m,n_s}$ as follows

$$\mathcal{L}_{m,n_s} := \begin{bmatrix} \sum_{n_t=1}^{N_t} \mathbf{A}_{n_s}^T(:, n_t) \otimes \mathcal{D}_{mn_t}^* & \sum_{n_t=1}^{N_t} \mathbf{B}_{n_s}^T(:, n_t) \otimes \mathcal{D}_{mn_t} \end{bmatrix} \quad (21)$$

To decode $\tilde{s}_m(iN_s + n_s)$ with diversity gain, we concatenate $\tilde{\mathbf{y}}_m(i)$ and its conjugate and premultiply the result by $\mathcal{L}_{m,n_s}$, to output the $Q \times 1$ block $\mathbf{z}_m(iN_s + n_s)$ as

$$\mathbf{z}_m(iN_s + n_s) = \mathcal{L}_{m,n_s} \begin{bmatrix} \tilde{\mathbf{y}}_m(i) \\ \tilde{\mathbf{y}}_m^*(i) \end{bmatrix} \quad (22)$$

Substituting (21) into (22), we obtain :

$$\begin{aligned} \mathbf{z}_m(iN_s + n_s) &= \alpha A_m \left(\sum_{n_t=1}^{N_t} \mathcal{D}_{mn_t}^* \mathcal{D}_{mn_t} \right) \tilde{s}_m(iN_s + n_s) + \tilde{\eta}_m(iN_s + n_s) \\ &= \alpha A_m \mathcal{D}_m \ominus_m \mathbf{s}_m(iN_s + n_s) + \tilde{\eta}_m(iN_s + n_s) \end{aligned} \quad (23)$$

where for the second equality we used (1) and the definitions $\mathcal{D}_m := \sum_{n_t=1}^{N_t} \mathcal{D}_{mn_t}^* \mathcal{D}_{mn_t} = \text{diag} \left[\sum_{n_t=1}^{N_t} |H_{mn_t}(\rho_m, 0)|^2, \dots, \sum_{n_t=1}^{N_t} |H_{mn_t}(\rho_m, Q-1)|^2 \right]$ and $\tilde{\eta}_m(iN_s + n_s) := \mathcal{L}_{m,n_s} [\tilde{\eta}_m^T(i) \tilde{\eta}_m^{*T}(i)]^T$. Equation (23) shows how N_t -fold transmit diversity has been gained through $\mathcal{D}_m$. For example, when $N_t = 2$ ($N_s = N_d = 2$), the two branches of our ST decoder $\bar{\Xi}(\cdot)$ are given by [cf (15 and (21)]

$$\begin{bmatrix} \mathcal{L}_{m,0} \\ \mathcal{L}_{m,1} \end{bmatrix} = \begin{bmatrix} \mathcal{D}_{m1}^* & \mathbf{0}_{Q \times Q} & \mathbf{0}_{Q \times Q} & \mathcal{D}_{m2} \\ \mathcal{D}_{m2}^* & \mathbf{0}_{Q \times Q} & \mathbf{0}_{Q \times Q} & -\mathcal{D}_{m1} \end{bmatrix} \quad (24)$$

Substituting (24) into (22), the two output blocks $z_m(iN_s)$ and $z_m(iN_s + 1)$ can be formed as

$$\begin{bmatrix} z_m(2i) \\ z_m(2i+1) \end{bmatrix} = \begin{bmatrix} \mathcal{D}_{m1}^* & \mathcal{D}_{m2} \\ \mathcal{D}_{m2}^* & -\mathcal{D}_{m1} \end{bmatrix} \begin{bmatrix} y_m(2i) \\ y_m^*(2i+1) \end{bmatrix} \quad (25)$$

Having decoded our block ST transmissions, our next subsection will illuminate the importance of redundant precoding that we introduced in Θ_m .

D. Precoder Design for Guaranteed Symbol Recovery

Following a similar approach as in [15-16], this subsection will rely on $z_m(iN_s + n_s)$ to recover the information symbols $s_m(iN_s + n_s)$. It is deduced from (23) that the necessary and sufficient condition to retrieve $s_m(iN_s + n_s)$ with a linear equalizer *regardless* of the information symbol constellation is to have :

$$\text{rank}(\mathcal{D}_m \Theta_m) \geq K \quad (26)$$

Constellation-independent linear equalizers are desirable from a computational perspective and useful for initializing constant-modulus or decision-feedback equalization algorithms. Furthermore, linear constellation-irrespective equalizers obviate error propagation effects.

Since $h_{mn_t}(l)$ has order $\leq L$, at most L of $\{H_{mn_t}(\rho_{m,q})\}_{q=0}^{Q-1}$ can be zero. However, it is very unlikely to have $\sum_{n_t=1}^{N_t} |H_{mn_t}(\rho_{m,q})|^2 = 0$, when each user's channel are uncorrelated. If we ignore this chance, we just choose $Q = K$ and $\Theta_m = \mathbf{I}$ in (1). But even channels are correlated and $\mathcal{D}_m$ loses rank due to common channels nulls, we can still guarantee symbol recovery using the redundant precoders of [15-16]. Finding that in the worst case, $\text{rank}(\mathcal{D}_m) = Q - L$, we design our redundant precoder Θ_m to satisfy : $Q = K - L$ and any K rows of Θ_m to be linearly independent. One simple choice of Θ_m is

$$\Theta_m = \begin{bmatrix} 1 & \rho_{m,0}^{-1} & \cdots & \rho_{m,0}^{-(K-1)} \\ 1 & \rho_{m,1}^{-1} & \cdots & \rho_{m,1}^{-(K-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \rho_{m,Q-1}^{-1} & \cdots & \rho_{m,Q-1}^{-(K-1)} \end{bmatrix} \quad (27)$$

where the m th user's signature points $\{\rho_{m,q}\}_{q=0}^{Q-1}$ are distinct so that any K rows of the Vandermonde matrix in (27) are linearly independent. Redundancy and judicious choice of the precoder satisfies condition (26) for any channel matrix $\mathcal{D}_m$ which assures symbol recovery no matter what FIR channel is encountered. In a way, redundant precoding “shapes the ISI channels” to ensure an invertible one-to-one input-output relationship. Furthermore, Θ_m introduces extra-diversity (over the complex field) in addition to the ST diversity gained through the GCOD.

Depending on complexity versus performance tradeoffs, *any single user linear equalizer* can be applied to retrieve $s_m(n)$ from (23). For example, we may choose the zero-forcing (ZF) equalizer (see Fig. 1) which is given by

$$\Gamma_m = (\mathcal{D}_m \Theta_m)^\dagger \quad (28)$$

where $\dagger$ stands for the pseudo-inverse matrix operator. After the survey of the overall system design, some remarks have to be made here.

Remark 1: The proposed system of Fig. 1 is applicable to both the uplink and the downlink. However, since only one antenna is required at the receiver, our proposed system is especially attractive for downlink applications. For the downlink, transmissions for all M users originate at the base station and they share the same multiple transmit antennas. In this case, a total of N_t instead of MN_t antennas are deployed at the base station. With $M = 1$ and skipping the MUI elimination stage, our multiple-access design is also applicable to the single-user setting that is dealt with in [11-13]. It is in fact that only ST block-coded single user system that guarantees symbol recovery regardless of frequency-selective propagation.

Remark 2: One advantage of our design (inherited from [15-16]) is that MUI is eliminated without requiring CSI at the receiver. Thus, the number of channels to be estimated for each user's symbol recovery is N_t instead of MN_t . Blind estimation of multiple channels with a single receive antenna is challenging. For a single user with two transmit antennas, blind channel estimation has been reported in [12-13] relying on a deterministic constant modulus algorithm, and in [19] using a subspace method with time-varying (TV) redundant precoding. Without TV precoding, constellation irrespective blind channel estimation in our multiuser scenario is also possible using an “antenna switching” method, where for estimating $h_{m\bar{n}_t}(l)$ only the $\bar{n}_t$ th antenna is switched on, while all $n_t \neq \bar{n}_t$ antennas are off. Because only a single channel has to be estimated at the receiver, any single channel estimator is applicable. However, by using “antenna switching” for channel estimation, the system becomes “uncoded” when acquiring CSI and no diversity gain is achieved. The simulations in Section VI will indicate that our system outperforms OFDM even with uncoded transmissions through multipath channels. To maintain an overall blind and simple receiver, it is also possible to use

the blind channel estimator based on the filterbank approach of [20]. Such a blind channel estimator capitalizes on input redundancy which is also present in our transmitter design in the form of L guard zeros. The simulations will illustrate its reliable performances.

Remark 3: We infer from (23) that the normalized equivalent channel gain for $\tilde{s}_m(nQ + q)$ is

$G_m(q) := (1/N_t) \sum_{n_t=1}^{N_t} |H_{mn_t}(\rho_{m,q})|^2$. Since $G_m(q)$ is the sum of squares of $H_{mn_t}(\rho_{m,q})$, the equivalent channel model in our system can be zero or weak only if $H_{mn_t}(\rho_{m,q})$'s for all n_t are zero or weak. In [14], however, the equivalent channel gain after MUI suppression is the sum of squares of linear (beamformed) combinations of multiple-antenna channels. Hence, the equivalent channel could be zero or weak even when the multiantenna channels themselves are strong. Thus, the performance of each user may suffer considerably from the beamforming-based MUI suppression of [14], which is not the case for our system. Observe that $G_m(q)$ depends also on the choice of signature points. If CSI is available at the transmitter, e.g., via a feedback channel, we have flexibility to optimize the allocation of signature points $\rho_{m,q}$. If the channels are unknown to the transmitter, then similar to [21-22], hopping the signature points may be helpful to avoid persistent fading.

V. DESIGN CONSIDERATION

So far we have designed a general ST coded multicarrier transceiver that guarantees symbol recovery with transmit diversity gain regardless of the underlying frequency-selective channels. In this section, we further consider the choice of design parameters taking into account practical issues such as complexity, efficiency, and performance.

A. Overall Bandwidth Efficiency

Redundancy has been exploited so far in three places, correspondingly for three purposes : i) IBI elimination (through L guard zeros); ii) guaranteed symbol recovery (through redundant precoding); and iii) ST coding for $N_t > 2$ transmit antennas. Because redundancy certainly affects the transmission rate, it is of interest to check the bandwidth efficiency of our designs. As described in Sections III and IV, the transmitted block has length $J = P + L = M(K + L) + L$ including the L guard zeros ($P = MQ$ and $Q = K + L$). From the viewpoint of bandwidth utilization, each of the M users sends N_s information blocks of length K for every N_d transmitted blocks of length J (see also Section II). Therefore, the overall bandwidth efficiency of our ST coded multiuser system is

$$\xi = \frac{M \cdot N_s \cdot K}{N_d \cdot [M(K + L) + L]}$$

Depending of the number of transmit antennas, we have then :

$$\xi = \begin{cases} \frac{MK}{M(K+L)+L} & \text{if } N_t = 2 \\ \frac{3.MK}{4M(K+L)+4L} & \text{if } N_t = 3, 4 \\ \frac{MK}{2M(K+L)+2L} & \text{if } N_t > 4 \end{cases} \quad (29)$$

where in deriving the second equality we used (12). Clearly, the overall bandwidth efficiency ξ depends on the ratio K/L . Recall that we consider in this paper a QS multiuser system (see Section II) where quasi-synchronism among all users ensures that the channel order L is relatively small. Hence, for a sufficiently (but reasonably) large $K \gg L$, we will have $\xi \cong 1$ when $N_t = 2$; however, $\xi \cong 0.75$ when $N_t = 3, 4$ and $\xi \cong 0.5$ when $N_t > 4$. Hence, similar to [6], the bandwidth efficiency is sacrificed only when $N_t > 2$ transmit antennas are deployed. On the other hand, (23) implies that higher diversity gains can be achieved with more than two transmit antennas. By regaining bandwidth efficiency through larger constellation sizes, we will see in Section IV that the overall performance improves as the number of transmit antennas increases.

B. Signature Point Selection and Allocation

At the end of Section III, step b- does not specify how to select the signature points ρ_p and decide their allocation I_μ . The optimal design of ρ_p and I_μ may rely on CSI knowledge at the transmitter which may not be practical for wireless access. We consider here a particular choice of ρ_p s which leads to low-complexity designs. Specifically, similar to [15-16], we recommend selecting ρ_p s regularly around unit circle on the FFT grid, i.e., we choose as signature points

$$\rho_p = e^{j(2\pi/P)p}, \forall p \in [0, P-1] \quad (30)$$

Plugging (30) into (7) leads to $\mathcal{V}_1 = \mathbf{F}$ and $\mathcal{V}_2 = \mathbf{F} \cdot \mathbf{R}_{\text{oa}}$, where $\mathbf{F}$ is the $P \times P$ FFT matrix and $\mathbf{R}_{\text{oa}} = [\mathbf{I}_P \mathbf{I}_{\text{oa}}]$ is a $P \times (P + L)$ matrix implementing the overlap-add (oa) operation. Accordingly, the user spreading code $\mathbf{C}_\mu$ and the user separating filter $\mathbf{G}_\mu$ become $\mathbf{C}_\mu = \mathbf{F}^H \Phi_\mu$ and $\mathbf{G}_\mu = \Phi_\mu^T \mathbf{F} \cdot \mathbf{R}_{\text{oa}}$, which suggests that the matrix multiplication by $\mathbf{C}_\mu$ and $\mathbf{G}_\mu$ can be implemented by low-complexity FFTs. Note also that the entries of the matrices Φ_μ and $\mathbf{R}_{\text{oa}}$ are either 0 or 1 and thus their operation involves simple addition. As a result, our design has low complexity if the choice of ρ_p s in (30) is adopted.

Selecting ρ_p s to be complex exponentials as in (30) corresponds to an OFDM-like multiuser system with a total of P subcarriers allocated to M users. The subcarrier allocation is specified through the choice of the index subsets in I_μ in (6). Two possible choices for I_μ 's are, $\forall \mu \in [0, M-1]$

$$I_\mu = \{v.M + \mu + 1, v = 0, \dots, Q-1\} \quad (31.a)$$

$$I_\mu = \{\mu.Q + v + 1, v = 0, \dots, Q-1\} \quad (31.b)$$

The choice (31.a) assigns P subcarriers periodically interleaved to M users as in the generalized OFDMA (G-OFDMA) system of [15-16], while (31.b) corresponds to a digital FDMA assignment where each user is allocated Q contiguous subcarriers. Because Q signature points designate Q subchannels [see the definition of $\mathcal{D}_m$ in (23)] for every user, the allocation of (31.a) renders these subchannels more uncorrelated than the subchannels corresponding to (31.b), and thus increases diversity. Consequently, the chance that the subchannels corresponding to (31.a) suffer from deep fades simultaneously is lower than that corresponding to (31.b). The comparison between (31.a) and (31.b) will be investigated also through simulations in Section VI.

C. Performance Versus Complexity Tradeoffs

The MUI-elimination design in Section III converts an M -user system into M single-user systems and greatly reduces not only the receiver complexity but also the burden of multichannel estimation, as well as timing acquisition and synchronization. However, the price paid is a slight performance loss by coloring the noise $\eta_m(n)$ in (10). Let us consider the special choice of ρ_p in (30). It follows from the definition of $\eta_m(n)$ that its correlation matrix can be written as

$$\Lambda_{\eta_m} = N_0 \cdot \Phi_m^T \mathcal{V}_2 \mathcal{V}_2^{\mathcal{H}} \Phi_m = N_0 \cdot \Phi_m^T \cdot (P \cdot \mathbf{I}_P + \mathbf{F} \cdot \mathbf{I}_{\text{oa}} \cdot \mathbf{I}_{\text{oa}}^{\mathcal{H}} \cdot \mathbf{F}^{\mathcal{H}}) \cdot \Phi_m \quad (32)$$

where N_0 is the power spectral density of $w(n)$. It is easy to verify that the maximum entry of the matrix $\mathbf{F} \cdot \mathbf{I}_{\text{oa}} \cdot \mathbf{I}_{\text{oa}}^{\mathcal{H}} \cdot \mathbf{F}^{\mathcal{H}}$ is L . If we choose $K \gg L$ (and thus $P \gg L$), the matrix Λ_{η_m} can be well approximated as: $\Lambda_{\eta_m} \approx N_0 \cdot P \cdot \mathbf{I}_Q$. For sufficiently large K , the residual noise $\eta_m(n)$ will thus be approximately white in which case the loss introduced by MUI elimination will be negligible. Note also that with $\eta_m(n)$ being white, the noise terms $\tilde{\eta}_m(iN_s + n_s)$ in (23) for different n_s become uncorrelated. Hence, separating the detection of $\tilde{s}_m(iN_s + n_s)$ from that of $\tilde{s}_m^T(iN_s + n'_s)$, $\forall n'_s \neq n_s$ will not incur any performance loss. Thus, as far as performance and bandwidth efficiency are concerned, choosing a large K is preferable. But, as K increases, the detection delay increases as well, since the receiver has to wait for $N_d[M(K+L)+L]$ received symbols to perform ST decoding. Furthermore, as K increases, the receiver complexity grows accordingly. Recall that our St design in Section IV requires that channels remain invariant for at least N_d blocks of length $J = M(K+L)+L$. Thus, increasing K also requires longer channel coherence time. In practice, how large K should be depends on complexity versus performance tradeoffs and the application environment.

VI. PERFORMANCE ANALYSIS

Assuming $w(n)$ in (3) is AWGN with spectral density N_0 , theoretical BER evaluation is possible for a given constellation when we adopt the ZF equalizer of (28). Starting with (23), the decision vector $\hat{s}_m(iN_s + n_s) := \Gamma_m \mathbf{z}_m(iN_s + n_s)$ for $s_m(iN_s + n_s)$ is given by

$$\tilde{\mathbf{z}}_m(iN_s + n_s) = \alpha \cdot A_m \cdot s_m(iN_s + n_s) + \bar{\eta}_m(iN_s + n_s) \quad (33)$$

where $\bar{\eta}_m(iN_s + n_s) := (\mathcal{D}_m \Theta_m)^\dagger \tilde{\eta}_m(iN_s + n_s)$. We deduce from the definition of $\tilde{\eta}_m(iN_s + n_s)$ and straightforward matrix manipulations that

$$\tilde{\eta}_m(iN_s + n_s) = \mathcal{L}_{m,n_s} \tilde{\mathbf{G}}_m \tilde{\mathbf{w}}(i) \quad (34)$$

where the $2.N_d.Q \times 2.N_d.J$ matrix $\tilde{\mathbf{G}}_m$ and the $2.N_d.J \times 1$ vector $\tilde{\mathbf{w}}(i)$ are given by

$$\tilde{\mathbf{G}}_m := \begin{bmatrix} \mathbf{I}_{N_d} \otimes \mathbf{G}_m & 0_{N_d.Q \times N_d.J} \\ 0_{N_d.Q \times N_d.J} & \mathbf{I}_{N_d} \otimes \mathbf{G}_m^* \end{bmatrix} \quad \text{and} \quad \tilde{\mathbf{w}}(i) := \begin{pmatrix} w(iN_d) \\ \vdots \\ w(iN_d + N_d - 1) \\ w^*(iN_d) \\ \vdots \\ w^*(iN_d + N_d - 1) \end{pmatrix} \quad (35)$$

Let us define the $K \times 2.N_d.J$ matrix $\Phi_m(n_s) := (\mathcal{D}_m \Theta_m)^\dagger \mathcal{L}_{m,n_s} \tilde{\mathbf{G}}_m$, whose k th row is denoted by $\phi_{m,k}^T(n_s)$, and write out (33) componentwise, $\forall k \in [0, K-1]$

$$\hat{s}_m((iN_s + n_s)K + k) = \alpha \cdot A_m \cdot s_m((iN_s + n_s)K + k) + \phi_{m,k}^{\mathcal{H}}(n_s) \tilde{\mathbf{w}}_m(i) \quad (36)$$

To proceed further with our performance analysis, we assume quadrature phase-shift keying (QPSK) modulated transmissions. Our figure of merit is the average BER, defined as

$\bar{P}_e = (N_s \cdot K)^{-1} \sum_{n_s=0}^{N_s-1} \sum_{k=0}^{K-1} P_{n_s,k}$, where $P_{n_s,k}$ denotes the BER for $s_m((iN_s + n_s)K + k)$ whose symbol energy is denoted by E_s . It follows from (36) that

$$\bar{P}_e = (N_s \cdot K)^{-1} \sum_{n_s=0}^{N_s-1} \sum_{k=0}^{K-1} Q \left(\sqrt{\frac{\alpha^2 \cdot A_m^2 \cdot E_s}{N_0 \cdot \phi_{m,k}^{\mathcal{H}}(n_s) \cdot \phi_{m,k}(n_s)}} \right) \quad (37)$$

Apart from the closed-form BER expression in (37), we will test the performance and reveal additional features of our system with simulated examples.

In all simulations, signature points are chosen as in (30) and their allocation is described by (31.a) unless specified otherwise. Channels are assumed to be zero-mean Rayleigh faded with unit variance. The simulated BER is averaged over 200 channel and noise realizations for each E_b/N_0 point.

Example 1 (Single-User Performance and Comparison with OFDM): We test our system on a single-user ($M = 1$) setup with $N_t = 1, 2, 3, 4$ antennas and compare it with a single-antenna OFDM. The channels are assumed flat ($L = 0$). For $N_t = 1$ (uncoded) and $N_t = 2$ transmit antennas, 8-PSK modulation is used. But for fairness, 16-QAM modulation is adopted for $N_t = 3, 4$ transmit antennas so that bandwidth efficiency for different number of transmit antennas is identical. In OFDM, 32 subcarriers are used and 8-PSK modulation is employed. We choose block length parameters as: $Q = K = 32$. From Fig. 2, it is seen that our uncoded ($N_t = 1$) system's performance coincides with that of OFDM over the flat fading channel. However, with multiple transmit antennas our system outperforms OFDM. For example, our system with $N_t = 2, 3, 4$ transmit antennas gives, respectively, about one, two, and three orders of magnitude better performance than OFDM at SNR = 20 dB. For frequency-selective channels of order $L = 3$ and block length parameters: $Q = 64$ and $K = 61$, the simulation is performed and the results are depicted in Fig. 3. Interestingly, even without ST coding, our uncoded ($N_t = 1$) system still outperforms OFDM. To explain the improvement, note that even without ST coding, each user in our system transmits over $K + L$ subcarriers instead of K used in OFDM. Consequently, our system does not suffer from deep channel fades as discussed in Section III. With multiple transmit antennas, our system outperforms OFDM and the uncoded system significantly.

Example 2 (Multiuser Performance): We implement our system with multiple ($M = 16$) users with different N_t 's and block length parameters $Q = 64$ and $K = 61$. Again, we employ 8-PSK modulation for $N_t = 1, 2$ and 16-QAM for $N_t = 3, 4$. Channels are assumed to be frequency-selective of order $L = 3$. Fig. 4 confirms that ST coding with multiple transmit antennas improves performance considerably in multiuser environments.

Example 3 (The Role of Redundant Precoding): Suppose $L = 2$, $M = 1$, $Q = 8$, $N_t = 2$, and QPSK modulation is employed. Two discrete-time equivalent channels are chosen to be respectively $H_{m1}(z) = (1 + j.z^{-1}).(1 + 0.5.z^{-1})$ and $H_{m2}(z) = (1 + j.z^{-1}).(1 + 0.8.z^{-1})$. As shown in Fig. 5, two channels share a common zero at $\rho = -j$. The BERs with redundant precoder Θ_m and without Θ_m are shown in Fig. 6. It is seen that the system with Θ_m improves performance considerably at least for this pair of channels. With Θ_m as in (27) and under the same system set-up, we average BER over 100 independent channel realizations and compare our system to ST coded GOFDM in [12-13] where eight subcarriers are used. The results in Fig. 7 confirm that our system with Θ_m outperforms [12-13]. It is important to note that channel fading may be slow and different channels may share common nulls for a long time in some scenarios. Hence, the use of the precoder Θ_m is recommended in our system.

Example 4 (Importance of Signature Point Allocation): Our system designs in Section III and IV provide flexibility to allocate signature points to different users through the design of

I_μ . To investigate the effects of signature allocation on BER performance, we compare the two allocation schemes specified in (31.a : G-OFDMA) and (31.b : FDMA), respectively. We choose parameters $M = 8$, $K = 61$, $L = 3$, $N_t = 2$, and QPSK modulation. The result depicted in Fig. 8 illustrates that the allocation scheme of (31.a) is better than that of (31.b), especially at high SNR values. Although symbol recovery in our system does not depend on the allocation of signature points, this example implies that judicious signature point allocation is still possible without CSI knowledge at the transmitter.

Example 5 (Estimation of Unknown Channels): To simulate performance of the antenna switching channel estimation approach given in *Remark 2*, we choose $N_t = 2$, $L = 1$, $M = 4$, $Q = 16$ and $K = 15$. During CSI acquisition, only one antenna is switched on each time so that multichannel estimation reduces to single-channel estimation at the receiver. For the single-user channel estimation, the blind approach of [20] is implemented with 32 received blocks used for single-channel estimation (64 blocks for two channels). Once CSI has been acquired, the system reverts to its normal mode described in this paper. BER performance with estimated channels is shown against that with perfect CSI. The average BER is depicted on Fig. 9 where we observe that the blind method entails only a small penalty in the overall system performance.

Fig. 2: Single user with flat fading channels

Fig. 3: Single user with frequency-selective channels

Fig.4: Sixteen users with frequency-selective channels

Fig. 5: Channel nulls

Fig. 6: Precoding for a specific frequency-selective channel

Fig. 7: Comparison with ST GOFDM [12-13]

VII. CONCLUSION

We designed in this paper a new ST coded multiuser system suitable for frequency-selective multipath fading channels. Relying on symbol blocking and ST redundancy, MUI was canceled first deterministically, and novel ST block codes were designed next to achieve diversity gain. The system is able to provide transmit diversity and to guarantee symbol recovery in multiuser environments regardless of the underlying FIR multipath. In addition to simplicity, simulations demonstrated the system's flexibility and superior performance.

This paper's ST coded multiuser system supports single-rate services and relies on a single receive antenna. The extensions to multirate services and multiple receive antennas are currently under investigation.

APPENDIX: PROOF OF (23)

According to the defining property of GCOD, namely, $\mathcal{G}_{N_t}^{\mathcal{H}} \cdot \mathcal{G}_{N_t} = \alpha \cdot \left(\sum_{n_s=0}^{N_s-1} |s_{n_s}|^2 \right) \mathbf{I}_{N_t}$, we first deduce from (13) that

$$\mathbf{A}_{n_s}^T \mathbf{A}_{n'_s} + \mathbf{B}_{n_s}^T \mathbf{B}_{n'_s} = \begin{cases} \alpha \cdot \mathbf{I}_{N_t} & \text{if } n_s = n'_s \\ \mathbf{0}_{N_t \times N_t} & \text{if } n_s \neq n'_s \end{cases}$$

$$\mathbf{A}_{n_s}^T \mathbf{B}_{n'_s} = \mathbf{0}_{N_t \times N_t} \quad \forall n_s, n'_s \quad (38)$$

Neglecting the noise term from $\tilde{\eta}_m(i)$ from (20) and plugging (21) and (20) into (22), we obtain

$$\begin{aligned} z_m(iN_s + n_s) = & A_m \cdot \sum_{n'_s=0}^{N_s-1} \sum_{n_t=1}^{N_t} \sum_{n'_t=1}^{N_t} \left\{ \left[\mathbf{A}_{n_s}^T(:, n'_t) \otimes \mathcal{D}_{mn'_t}^* \right] \left[\mathbf{A}_{n'_s}(:, n_t) \otimes \mathcal{D}_{mn_t} \right] \times \tilde{\mathbf{s}}_m(iN_s + n_s) \right. \\ & + \left[\mathbf{A}_{n_s}^T(:, n'_t) \otimes \mathcal{D}_{mn'_t}^* \right] \left[\mathbf{B}_{n'_s}(:, n_t) \otimes \mathcal{D}_{mn_t} \right] \times \tilde{\mathbf{s}}_m^*(iN_s + n'_s) \\ & + \left[\mathbf{B}_{n_s}^T(:, n'_t) \otimes \mathcal{D}_{mn'_t} \right] \left[\mathbf{A}_{n'_s}(:, n_t) \otimes \mathcal{D}_{mn_t}^* \right] \times \tilde{\mathbf{s}}_m^*(iN_s + n'_s) \\ & \left. + \left[\mathbf{B}_{n_s}^T(:, n'_t) \otimes \mathcal{D}_{mn'_t} \right] \left[\mathbf{B}_{n'_s}(:, n_t) \otimes \mathcal{D}_{mn_t}^* \right] \times \tilde{\mathbf{s}}_m(iN_s + n'_s) \right\} \end{aligned} \quad (39)$$

Let us now recall the distributive property of Kronecker products of matrices : $(\mathbf{F}_1 \otimes \mathbf{F}_2)(\mathbf{F}_3 \otimes \mathbf{F}_4) = (\mathbf{F}_1 \mathbf{F}_3) \otimes (\mathbf{F}_2 \mathbf{F}_4)$. Applying this identity to (39) and using (38), it follows that

$$z_m(iN_s + n_s) = \alpha A_m \cdot \left(\sum_{n_t=1}^{N_t} \mathcal{D}_{mn_t}^* \mathcal{D}_{mn_t} \right) \tilde{\mathbf{s}}_m(iN_s + n_s) + \tilde{\eta}_m(iN_s + n_s)$$

which is the proof of (23).

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