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Experiments on fall of open tori

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Abstract

Fall of open tori horizontally dropped in quiescent water is experimentally investigated. The fall is controlled by the Galileo number, the torus aspect ratio and the solid to fluid density ratio. The dependence of the descent velocity and of the drag coefficient on the control parameters is first analyzed. As the control parameters are varied, vertical translation, zigzagging and spiraling descent are observed. These different descent modes are characterized and a phase diagram of the falling tori are presented as function of the three control parameters.

Keywords: Fluid-solid interaction, Falling torus, Fluttering mode, Path instability

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1. Introduction

At the end of the 17th century, Newton performed experiments to investigate the air resistance and reported in *The Principia* that bodies "did not always fall directly down, but sometimes fluttered a little in the air and waved to and fro as they were descending". Almost two centuries later, Maxwell [1] observed that "when a slip of paper falls through the air, its motion, though undecided and wavering at first, sometimes becomes regular". This variety of motions of bodies descending through a fluid, that everybody can experience by observing fall of snowflakes, hailstones or leaves, has excited the curiosity of researchers for a long time.

Studies have mainly concerned bodies with symmetries as spheres, disks or cylinders, for which a review can be found in [2]. For example, successive experimental works of Willmarth et al. [3], Stringham et al. [4], Field et al. [5] and Zhong et al. [6] have provided a comprehensive phase diagram for falling thin disks which reveals, depending on the physical parameters (fluid viscosity and density, disk diameter and density), different motions as steady vertical descent, tumbling, chaotic motion and periodic flutter. Flutter can be spiraling, planar zigzagging or a zigzag motion in a plane

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precessing around a vertical axis. In these studies, the disk thickness is implicitly assumed to be small enough to have no effect, but numerical simulations conducted by Auguste et al. [7] have revealed differences, when compared with zero **thickness** disks, even for diameter-to-thickness ratios as large as 50.

The various descent modes are mainly driven by fluid **loads** induced by the vorticity produced at the body surface and shed in its wake [2]. The knowledge of the flow around the body fixed in a flow can therefore provide insight at least on the transition from vertical translation to nonstraight paths. For this reason, the studies devoted to the flow around open tori, and more generally rings, fixed in a uniform flow are reviewed, then, the works devoted to falling open rings are presented. Unless otherwise stated, the cited studies concern rings held normal to a uniform flow or horizontally released in an initially quiescent fluid.

The flow behind a torus held in a uniform flow is controlled by the torus aspect ratio, defined as the ratio of the torus diameter to that of its cross-section, and by the Reynolds number Re based on the cross-section diameter. Experimental investigations of Roshko [8] for Re up to 500 or Inoue et al. [9] at $Re = 1500$ with tori and Bearman and Takamoto [10] with rings of trapezoidal cross-section for Re between 10^4 and 7×10^4 have evidenced a distinction between high and low aspect ratios, that has been confirmed by the numerical simulations of Sheard et al. [11, 12] or Lazeroms et al. [13]. The limit is found at an aspect ratio around 4 or 5.

For high aspect ratios, the previously cited studies and also the experimental works of Leweke and Provansal [14] reported counter-rotating annular vortices parallel to the torus alternately shed from inside and outside of the torus. This axisymmetric periodic wake is analogous to the well known von Kármán vortex street behind a straight circular cylinder but appears at slightly higher critical values of Re and has lower frequencies by **a** few percent, both quantities asymptotically tending to the values for the cylinder as the aspect ratio is increased. Lower frequencies are due to the self-induced velocity of the vortex rings which results in a slow-down of the vortex convection in the torus wake [14]. Shedding of pair of helical vortices, analogous to the oblique vortices shed in the wake of a cylinder, has also been observed but their appearance required a "violent perturbation" of the axisymmetric wake [14].

Unsteady regimes of low aspect ratio tori wake appear similar to what is observed for a sphere or a disk. They present a planar symmetry and consist of hairpin vortices periodically and alternately shed from opposite sides of the outer diameter of the torus [8, 12, 15].

For all aspect ratios, unsteady regimes described above are preceded by steady attached flow up to Re **of order unity** then steady separated flows up to Reynolds numbers of order 100, both critical Re values depending on the aspect ratio [11, 12, 13, 16]. The second regime is characterized

by recirculation zones in the wake with a special feature at aspect ratios around 2 where a detached recirculation bubble forms on the symmetry axis [11, 15, 16]. The creeping flow, at very low Re , has been the subject of various analytical works as [17, 18].

Fall of tori, and more generally of rings, in a fluid has received little attention. Amarakoon et al. [19] and Roger and Hussey [20] dropped tori and holed disks, respectively, in very viscous liquids to deduce the drag they experience in the Stokes limit. For this low Re range, never reached during the present study, the rings underwent a steady vertical translation. The same motion is reported in the numerical work of Yu et al. [21] for a torus of aspect ratio of 2 falling at $Re = 25$ or 50, and a detached axisymmetric recirculation bubble forms in the wake. This study also showed that when the torus is released with an angle of 30° it returns to the horizontal (0°) after a few damped oscillations. Monson [22] conducted fall experiments using various tori and fluids with the aim to investigate the different regimes of the wake flow but not the torus motions. By means of flow visualizations he reported vortex patterns similar to those observed behind fixed tori described above. That is for aspect ratios higher than about 4 or 5, pair of vortex rings, and more rarely pair of helical vortices, were seen, while for lower aspect ratios, the torus wake behaves like those of spheres or disks with shedding of vortex loops. More recently, Vincent et al. [23] and Bi et al. [24] experimentally studied the effect of a central hole on the fall of disks. Vincent et al. [23] showed how the phase diagram is modified by the hole and Bi et al. [24] reported a new descent mode, that appears for holed disks of low moment of inertia around a diameter, where the ring fall vertically with periodic velocity fluctuations as annular vortices are shed in its wake.

To the author knowledge, the present study is the first to investigate the dynamics of falling tori. Experimental results on tori horizontally released in water at rest are presented and, in particular, the effects of the dimensionless parameters that control the system are considered. The experimental setup is presented in section 2. The results are provided in section 3 and discussed in the last section 4.

2. Experimental details and parameters

Experiments were performed in a tank with transparent walls, of $0.4 \times 0.4 \text{ m}^2$ horizontal cross-section and filled with a height of water of 1 m (see Fig. 1). An open torus is geometrically defined by its cross-sectional diameter d and mean diameter $D > d$, and is hereafter referred to as $d \times D$ where the two diameters are expressed in millimeters.

The release mechanism of the tori is shown in Fig. 2. It consisted of three vertical rods, two were fixed and one could be translated horizontally by means of a lead screw/nut system. The lead screw was mounted on a support plate with two holes for the passage of the fixed rods and an oblong

Figure 1: Schematic of the experimental setup with the coordinate system and torus dimensions.

hole for the moveable rod. The support plate was held partly immersed at the water surface to avoid free surface effects above the release site. The torus was maintained horizontally at the end of the rods, placed inside the torus hole, through a tension exerted by the moveable rod. It was released applying a small displacement of this latter towards the torus center.

The torus fall was recorded on a white background in grayscale images with a camera operating at a frame rate up to 50 s^{-1} . And since we are interested here in the fully developed fall regimes, the field of view was of about 0.7 m high from 0.2 m below the release site to 0.1 m above the tank bottom. Two perpendicular views (x, z) and (y, z) were simultaneously captured using a vertical mirror carefully positioned at 45° as shown in Fig. 1.

Figure 2: Release mechanism.

Consecutive frames of the video recordings were then processed to deduce the coordinates of the torus center $x(t)$, $y(t)$ and $z(t)$ as function of time. The processing consisted in inverting and applying a threshold to the images to obtain a binarized images in which the two torus projections appear white on a black background. Then ellipses were fitted to these projections. With this method, the coordinates of the torus center, whose projections correspond to the ellipse centers, were determined to within 1 pixel that is to ± 0.2 mm. The calibrations for the conversions from pixel to length unit were determined with images of vertical and horizontal scales fixed inside the tank facing the two projection planes. The estimated resolution for the vertical velocities is of about 10^{-2} m.s $^{-1}$.

The tori were released without initial velocity and initial angle (i.e. with their axis of revolution aligned with the vertical axis z) in water at rest. To be more precise, the initial angle was measured a posteriori on the video recordings and was found to be less than 5 degrees, but it has been verified (see section 3.2) that such a small initial angle did not affect the final state of the torus descent. Moreover, the successive experiments were conducted leaving tori at the bottom of the tank to avoid introducing further perturbations and with a time delay of at least ten minutes allowing the water to return to a quiescent state. After a series of experiments, the tori were recovered and the tank was left at rest for many hours. The height of fall is supposed to be high enough to reach a fully developed fall regime. This is supported by the results reported in section 3 showing that the tori have reached a limit vertical velocity (see section 3.1) and that the amplitudes of periodic flutters are constant (see Fig. 6) suggesting saturated states. Moreover the tank cross-section is assumed to be large enough for wall effect to be negligible that is the measured limit velocity in the tank is the same as the the velocity in infinite medium. This point is critical because most of the quantitative results presented in the following are related to this velocity. The wall effect has been the object of numerous studies dating back to Newton in *The Principia*, most of them have dealt with the fall of spheres as e.g. Fidleris and Whitmore [25], Chhabra et al. [26] or Arsenijević [27]. The wall effect depends on and decreases with increasing Reynolds and decreasing blockage ratio, which is the ratio of the projected area of the body to the tank cross-section. Fidleris and Whitmore [25] found that the wall effect for blockage ratios of 25×10^{-4} and 10^{-2} is less than 1% as soon as Re exceeds 5 and 30 respectively. Here the blockage ratio is between 9×10^{-4} and 2.2×10^{-2} with much higher Re values exceeding 130 and 1900 respectively (section 3.1). For this reason small wall effect can be anticipated, at least smaller than the data dispersion obtained as the experiments were repeated which is between 3 and 8%. The height of fall is supposed to be high enough to reach a fully developed fall regime and, because the Reynolds number of this regime is always more than 100, the tank cross-section is assumed to be large enough for confinement and With these conditions, the fall of a torus is defined by

six physical parameters, namely the mean D and cross-sectional d diameters of the torus, and its material density ρ , the density $\rho_w < \rho$ and kinematic viscosity ν_w of water, and the gravitational acceleration g . The problem is then governed by three dimensionless parameters, we used here the torus aspect ratio D/d , the density ratio ρ/ρ_w and the Galileo number

$$Ga = \sqrt{(\rho/\rho_w - 1)gd^3}/\nu_w \quad (1)$$

that measures the ratio of net gravitational to viscous forces.

During the present tests, the water temperature was of 16°C so that its density was $\rho_w = 1000 \text{ kg.m}^{-3}$ and its kinematic viscosity $\nu_w = 1.1 \times 10^{-6} \text{ m}^2.\text{s}^{-1}$. Temporal variation of the water temperature during the experiments was lower than 1°C and its spatial variation from the top to the bottom lower than 0.5°C. **Thirty-two** tori of different sizes were tested, 17 were made of nitrile rubber (O-ring) of density $\rho = 1250 \text{ kg.m}^{-3}$ and 15 of stainless steel of density $\rho = 7720 \text{ kg.m}^{-3}$. **The homogeneity of the torus material was not tested but tori with visible defects were eliminated and for most of the tori, experiments were repeated with two or more samples with similar results.** Each torus was released at least twenty times.

Influences of the three dimensionless parameters were investigated by considering tori of cross-sectional diameter $2 < d < 10 \text{ mm}$, mean diameter $14 < D < 110 \text{ mm}$. The parameter values were in the ranges $3.5 \leq D/d \leq 16$ and $120 < Ga < 1430$ for the rubber tori $\rho/\rho_w = 1.25$, and in the ranges $6 \leq D/d \leq 16$ and $1200 < Ga < 7400$ for the steel tori $\rho/\rho_w = 7.72$.

3. Results

3.1. Descent velocity and drag coefficient

When a torus is released with no velocity in quiescent water, it first undergoes a phase of acceleration then reaches a limit mean vertical velocity U_L hereafter referred to as descent velocity. The measured descent velocities, derived from the temporal evolution of the vertical position of the torus center $z(t)$, are found to range from 0.07 to 0.23 m.s^{-1} for the rubber tori $\rho/\rho_w = 1.25$ and from 0.59 to 1.05 m.s^{-1} for the steel tori $\rho/\rho_w = 7.72$. They are displayed in Fig. 3 under the non dimensional form of a Reynolds number based on the cross-sectional diameter d , $Re = U_L d / \nu_w$, as function of the Galileo number Ga . This Reynolds number is useful for comparisons with fixed body studies for which it is a control parameter. It varies between 130 and 2080 for $\rho/\rho_w = 1.25$ and between 1610 and 9520 for $\rho/\rho_w = 7.72$. The figure shows the monotonic increase of the descent Reynolds number Re with the Galileo number Ga and the continuity of the two curves of equal aspect ratio $D/d = 6$ and 11 as ρ/ρ_w is changed suggests that the parameter ρ/ρ_w alone has no effect on Re . Moreover, the figure, and particularly the

Figure 3: Reynolds number Re as function of Galileo number Ga , circles are for $\rho/\rho_w = 1.25$ and triangles for $\rho/\rho_w = 7.72$, open and closed symbols refer to flutter and translational fall, respectively (see next section). Insets: close-up around $Ga = 360$ and 1867.

zooms around the two values of Ga for which tori of different aspect ratio are tested, shows that Re is a decreasing function of D/d .

The descent velocity results from the balance between net gravitational forces $((\rho - \rho_w)gd^2D\pi^2/4)$ and drag forces $(\rho_w C_D d D U_L^2 \pi/2)$ in such a way that the drag coefficient C_D is deduced from the measured descent velocities with

$$C_D = \frac{\pi}{2} \left(\frac{\rho}{\rho_w} - 1 \right) \frac{dg}{U_L^2}. \quad (2)$$

The experimental drag coefficients C_D are plotted in Fig. 4 as a function of the Reynolds number Re together with the drag coefficient for a fixed circular cylinder (i.e. a torus of infinite aspect ratio) in a uniform flow. The empirical expression derived in [28] is used for the drag coefficient of circular cylinders.

Drag coefficient C_D for the falling tori is found to evolve between 0.7 and 1.4 for $10^2 < Re < 10^4$, first decreasing down to $Re \approx 2000$ then increasing, following the same tendency as the drag coefficient of a circular cylinder. But C_D values of falling tori appear smaller than those of the cylinder and approach them as the aspect ratio D/d is increased as already noted by Sheard et al. [29] for fixed tori. Moreover, results of this latter numerical

Figure 4: Drag coefficient C_D as function of the Reynolds number Re , symbols as in Fig. 3.

study for $D/d = 5$ and 10 , also displayed in Fig. 4, are coherent with the present experimental values for $D/d = 6$ and 11 in the same Re range, along with the experiments of Yan et al. [30] for a fixed torus of aspect ratio $D/d = 3$ at larger Re . As the experiments are repeated, the results present a dispersion represented by error bars in Fig. 4. The width of these dispersion intervals is 8% the mean value at the most.

3.2. Descent modes

When tori of different sizes or densities are horizontally released in water, a variety of descent modes is observed. The observation of the different modes for the 32 tori tested here are reported in the three-dimensional phase diagram $(Ga, D/d, \rho/\rho_w)$ in Fig. 5. Two main categories of motion are distinguishable, namely vertical translation and periodic flutter, flutter can be of zigzagging or spiraling type. The different descent modes are illustrated in Fig. 6 with stroboscopic visualizations of the torus fall along two perpendicular views and paths of the torus center. Periodic behaviors are characterized by their frequency f expressed under the dimensionless form of a Strouhal number $St = fd/U_L$ (Fig. 7). Frequential analysis was only performed for the rubber tori because the maximum frame rate of 50 fps and transit times in the view field of less than or order 1 s for the steel tori give a maximum frequency and a frequency resolution not compatible with the characteristic frequencies of the steel torus fall. Indeed, the frequencies estimated by

Figure 5: Phase diagram: triangles are for $\rho/\rho_w = 1.25$, circles for $\rho/\rho_w = 7.72$, closed symbols refer to translational fall, thin open symbols to zigzagging flutter and thick open circle to spiraling flutter.

extrapolation of the results obtained with the rubber tori, and presented in Fig. 7, would be of a few hertz for zigzagging steel tori and a few tens hertz for steel tori in vertical translation.

For both tested values of the density ratio, vertical translation, corresponding to the closed symbols in the phase diagram of Fig. 5, is observed for the largest aspect ratio values. In this mode the torus **descends** without **any** rotation, its rotational symmetry axis remains vertical (Fig. 6(a)), and its center does not show significant horizontal displacement (Fig. 6(d)). However, the vertical velocity of the torus center presents low amplitude periodic oscillations whose nondimensionalized frequencies St are displayed, as function of Re , with closed circles in Fig. 7, they are found to increase from 0.18 to 0.22 with Re . Dimensionless frequency of the parallel vortices shed in the wake of a fixed circular cylinder is also reported on the graph (thick line in Fig. 7) using the analytical expression introduced in [31]. Frequencies of the vertical velocity fluctuations of the falling tori appear lower than the vortex shedding frequency behind a cylinder and they approach this value as the aspect ratio is increased. **During vertical translation the tori are constantly normal to the relative flow exactly as a torus normally fixed in an incoming flow, making relevant a comparison between both configurations. And the frequency of the annular vortices shed behind fixed tori has been found to follow the same trend as the frequency of the velocity oscillations**

of the translating tori, namely it is lower by a few percent to the frequency of the parallel vortex shed behind a cylinder and approaches it as the aspect ratio is increased [8, 11, 14]. The comparison suggests that the velocity oscillations observed during the vertical translation mode are driven by the shedding of the vortex rings in the wake of the tori. A behavior that has been confirmed for thin holed disks for aspect ratios higher than about 9, where the aspect ratio is defined here as the ratio of the difference between the outer radius and the inner radius to the mean diameter, by means of simultaneous torus and flow visualizations [24]. It should be pointed out that no velocity fluctuations were detected for the two heaviest rubber tori, this could be due to their high translational inertia that should result in too small oscillation amplitudes for the velocity resolution.

Small aspect ratio tori fall following flutter motions (open symbols in Fig. 5). A fluttering torus undergoes periodic oscillations while descending and its center follows an elliptical helical path. Except for the 4×14 rubber torus (thick open circle in Fig. 5) which is the tested torus of the lowest aspect ratio, helical paths of the center of the fluttering tori have a high ellipticity, as seen on the projection into the horizontal plane (x, y) in Fig. 6(e). Therefore this motion appears quasiplanar and is referred to as zigzagging flutter in contrast to the spiraling flutter of the 4×14 rubber torus for which the helical path of the center is almost circular (Fig. 6(f)). Note that the orientation of the vertical plane of oscillation for the zigzagging flutter has been seen to be arbitrary as the experiments were repeated. Dimensionless characteristic frequencies of both these types of flutter are reported in Fig. 7 where it can be seen that zigzagging and spiraling flutters differ also by their characteristic frequency, $St \approx 0.024$ for the former and of about 0.043 for the latter. The inset in Fig. 7 shows the frequency of zigzagging tori normalized using the outer diameter $(D + d)$ and the velocity scale $((1 - \rho_w/\rho)g(D + d))^{1/2}$, namely $St^* = f(D + d)/((1 - \rho_w/\rho)g(D + d))^{1/2}$, rather than d and U_L used to define St . This dimensionless frequency St^* has a value of about 0.1 almost independent of the Reynolds number. This is similar to the experimental results reported by Fernandes et al. [32] for zigzagging rising disks where $(D + d)$ must be replaced by the disk diameter in the definition of St^* .

In Fig. 6(d-f) the origin $(x = 0, y = 0)$ coincides with the horizontal position of the torus center at the top of the field of view i.e. at $z \approx 0.2$ m and can differ from its horizontal position at the release site $z = 0$. Actually, during experiments a small drift and/or damped oscillations are sometimes observed during the first stage of the fall that are suspected to be due to a small initial angle between the vertical and the rotational symmetry axis of the torus which is controlled to a few degrees in the present study. Some

Figure 6: Descent modes of tori in water: Two perpendicular stroboscopic visualisations (top) with a time interval of $1/10$ s and corresponding paths of the torus center (bottom, where the scale in the horizontal x - and y -direction is stretched with respect to the vertical z -direction) for the vertical translation of the 4×44 rubber torus (a,d), the zigzagging 6×36 (b,e) and spiraling 4×14 (c,f) rubber tori.

Figure 7: Characteristic dimensionless frequencies of the torus motion as function of the descent Reynolds number, symbols as in Fig. 5.

experiments are performed for investigating the effect of such an initial angle showing that the final descent mode is not affected as long as the initial angle is not "too large". A similar behavior has been observed in experimental studies of falling thin disks [3, 33], thick disks [34] and also in numerical simulations on a falling torus released with an initial angle of 30° [21]. Effect of an initial angle is illustrated in Fig. 8(a) where the field of view is extended up to the release site. The stroboscopic visualization of the fall of a 4×34 rubber torus released with an angle of about 35° exhibits horizontal drift and damped oscillations before reaching the vertical translation descent mode. However, for still larger initial angle, as in Fig. 8(b) for the 4×24 rubber torus with an initial angle of about 70° , the torus falls vertically.

4. Concluding discussion

A phase diagram for the fall of tori in a viscous fluid has been experimentally established (Fig. 5). In the explored region of the control parameters, the diagram reveals a distinction, as previously reported for the wake flow of tori held in a uniform flow, in the behavior of tori of low and high aspect ratios with a limit which depends on the Galileo number Ga . The limit is found at $8.5 < D/d < 11$ for $Ga \approx 1870$, $6 < D/d < 8.5$ for $Ga \approx 360$ and lower than $D/d = 6$ at $Ga \approx 230$. For D/d above this limit the tori fall vertically

Figure 8: Effect of an initial angle: Stroboscopic visualisations of the 4×34 rubber torus released with an angle of about 35° (time interval of $1/30$ s) (a) and of the 4×24 with an angle of about 70° (time interval of $1/15$ s) (b).

straight with periodic velocity oscillations due to the shedding of annular vortices, analogous to the straight vortices behind a cylinder. In contrast, low aspect ratio tori flutter periodically during their descent describing either a quasi-planar zigzagging motion or a spiraling three-dimensional motion. Most of the studies on falling bodies use a dimensionless moment of inertia and the descent Reynolds number as parameters, although this latter is not a true control parameter **since the descent velocity is not known a priori**. To compare, the phase diagram is presented in Fig. 9 as function of ρ/ρ_w , $Re = U_L d / \nu_w$ and the dimensionless moment of inertia $I^* = I / (\rho_w (D + d)^5)$ where I is the moment of inertia of a torus around a mean diameter $I = (\rho \pi^2 d^2 D / 32) (D^2 + 5d^2 / 4)$. Here I^* lies between 10^{-3} and 3×10^{-2} , it ranges from 5×10^{-4} to 3×10^{-3} and from 8×10^{-3} to 2×10^{-1} in the holed disks experiments of Bi et al. [24] and Vincent et al. [23], respectively; in these studies the Reynolds number is twice Re because it is based on the length scale defined by the outer minus inner diameter.

Although the torus differ of the holed thin disk by a finite thickness d and by rounded corners, the comparison of the phase diagram of Fig. 9 with those in [23, 24] and even with the one for the whole disk in [6] reveals similarities. First, comparison shows that the periodic vertical translation is never observed for falling disks, coherently with the fact that disks does

Figure 9: Phase diagram in the $(Re, I^*, \rho/\rho_w)$ space, symbols as in Fig. 5.

not present unsteady axisymmetric wake, and is only seen for holed disks of small moment of inertia $I^* \lesssim 10^{-3}$ [24]. This is in line with our own observations where it maintains up to much higher Reynolds number of order 10^5 . Secondly, zigzagging and spiraling flutter modes experienced by tori have also been reported for both whole [6, 35] and holed disks [23, 24]. Further investigations are required to complete the phase diagram and to check the existence of others modes as those observed for falling disks with or without central hole as steady vertical translation, tumbling and chaotic descent. The experiments of Amarakoon et al. [19] have shown that tori descending at very low Reynolds numbers, for which the flow does not separate, fall vertically and steadily. This descent mode could be also expected for tori falling in the Re range where the wake consists in steady recirculation regions which has been found to extend up to $Re \approx 90$ at $D/d = 3.5$ and up to $Re \approx 50$ at $D/d = 20$ for fixed tori [11] well below the values measured during the present work for corresponding aspect ratios. **The investigation of the region of the low Reynolds number or equivalently, according to Fig. 3, of the low Galileo number,** required to work either with tori of smaller cross-sectional diameter d and/or made of a lighter material or using more dense and/or more viscous fluid. Finally, the works of Vincent et al. [23] and Bi et al. [24] suggest that chaotic and tumbling descents could be obtained for tori of larger moment of inertia I^* than considered in the present study.

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