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On the fluidization/sedimentation velocity of a homogeneous suspension in a low-inertia fluid

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Abstract

The modeling of the fluidization or sedimentation velocity of a suspension of solid particles is revisited by examining experiments conducted in either a liquid or a gas. A general expression is found in the case of negligible fluid inertia, i.e. at low Reynolds or Archimedes number. It is built as the product of the velocity of an isolated particle by three non-dimensional corrections that each takes into account a specific physical mechanism. The first correction reflects the variation of the buoyancy with the particle concentration. The second correction describes how the drag force increases with the concentration in case of negligible particle inertia. The third one accounts for the further increase of the drag when the particle inertia is increased. Remarkably, each correction only relies on a single of the three independent non-dimensional groups that control the problem: (1) the particle volume fraction Φ_s ; (2) the ratio Φ_s/Φ_{pack} where Φ_{pack} is the bed packing concentration; (3) the Stokes number St_0 , which characterizes the inertia of the particles and controls their agitation. Moreover, the onset of the instability that separates the homogeneous regime from the heterogeneous one is found to be controlled similarly by the Stokes number. Empirical expressions of the corrections are given, which provide a reliable tool to predict fluidization and sedimentation velocities for all values of the three non-dimensional numbers. The present results emphasize the crucial role of particle inertia, which is often disregarded in previous modeling approaches, such as that of Richardson and Zaki.

Keywords: Fluidization velocity, Sedimentation velocity, Liquid-solid fluidized beds, Gas-solid fluidized beds, Particulate suspensions.

1. Introduction

This article revisits two closely related fundamental problems: the fluidization of solid particles by an upward flow of fluid and the sedimentation of population of solid particles in a fluid at rest. The physics of these configurations is complex because of the intricate interplay between the continuous fluid phase and the dispersed solid phase. Especially, the interactions between the particles may involve hydrodynamic forces, shocks between particles, solid friction, short-range adhesion forces... In this work, we focus on the case where hydrodynamic forces are predominant, shocks possibly play a significant role, whereas the other effects are negligible. This situation is achieved when the continuous phase is liquid, provided that the particles are not too small to avoid colloidal or Brownian effects, and that their concentration is not too close to packing to avoid solid friction and jamming. It can also be obtained in a gas that is heated in order to prevent capillary forces resulting from moisture. In this context, there exists a range of solid volume fractions in which the flow is homogeneous. Apart from a narrow region close to the wall, the average particle velocity, fluid velocity and volume fraction Φ_s are spatially uniform. The sedimentation and fluidization processes thus only differ by a Galilean change of reference frame, so that the fluidization velocity U_f and the sedimentation velocity U_{sed} are equal, and will be referred as U in the following of this paper. This range is limited by an upper boundary Φ_{up} and a lower boundary Φ_{low} . Let us consider a fluidization experiment in which the fluidization velocity is regularly increased while the concentration of the mixture decreases. Concentration Φ_{up} is reached when the solid friction between the particles ceases to play a significant role and the influence of the walls becomes negligible. It is close to the concentration of the bed at the minimum fluidization velocity and corresponds to the end of the jamming state. Concentration Φ_{low} is reached when strong large-scale fluctuations

of the particle concentration develop, giving birth to the well-known bubbling regime in gas-solid fluidized beds. Within the homogeneous regime, the dynamics is controlled by four main forces: (1) the effective weight of the particles, which depends on the density difference between the two phases; (2) the viscous stresses within the fluid, which control the dissipation of mechanical energy; (3) the fluid inertial stresses, which influence the flow between the particles; (4) the particle inertia, which determines the intensity of particle-velocity fluctuations relative to those of the fluid. The relative magnitude of these forces depends on the physical parameters that characterize the two phases, which makes it difficult to provide a unified description, valid for a wide range of situations, and to find a universal law capable of describing the relation between U and Φ_s .

Historically, two different ways have been opened to address the problem. The first, initiated by Richardson and Zaki [1, 2] considers the dilute state as a starting point, whereas the second, developed by Abrahamsen and Geldart [3], starts from the packing state.

Let's begin with the Richardson-and-Zaki's approach. The average velocity U is modeled as a correction to the velocity U_i of an isolated settling particle in the corresponding flow regime,

$$U = U_i (1 - \Phi_s)^n , \quad (1)$$

where n is an exponent that is expected to depend on the particle Reynolds number Re , being equal to 4.65 in the limit of vanishing Re . This approach has become very popular and many studies have shown that eq. 6 describes experimental results well, provided that U_i and n are adequately chosen (see [4] and references therein). Then, several works have investigated how this law can be extended to account for more complex effects, such as suspensions of binary particles [5], adhesion forces depending on temperature [6], magnetic field [7], multisized irregular particles [8]... However, even considering the simplest case of a monodisperse homogeneous suspension of spheres in the absence of adhesion forces, this approach has limitations. First, eq. 6 predicts that the sedimentation velocity U becomes null at $\Phi = 1$ although it actually vanishes

when the jamming occurs, at a concentration which is less than unity. As a consequence, such a law necessarily ceases to be valid when approaching the packing state. On the other hand, the homogeneous regime is destabilized at a concentration Φ_{low} , which is not necessarily small. There is hence no reason to expect that a law that is relevant for $\Phi > \Phi_{low}$ would still be valid in the limit of vanishing concentration, and the value of U_i involved in eq. 6 does not represent the terminal velocity of an isolated settling particle, as it will be shown later. In addition, by following the original dimensional analysis of Richardson and Zaki [2], the proposed expressions for n and U_i [4] generally do not account for the particle inertia, which questions their validity when the particle-to-fluid density ratio can significantly vary from one case to another.

The alternative approach is more specific to the fluidized-bed configuration. The fluidization velocity is modeled as a correction to the minimum velocity U_{mf} required to fluidize the bed [3],

$$U = U_{mf} + \frac{g(\rho_s - \rho_f)d^2}{210\mu_f} \left[\frac{(1 - \Phi_s)^3}{\Phi_s} - \frac{(1 - \Phi_{pack})^3}{\Phi_{pack}} \right], \quad (2)$$

where g is the gravitational acceleration, ρ_s the density of the particle, ρ_f the density of the fluid, d the particle diameter, μ_f the fluid dynamic viscosity and Φ_{pack} the particle volume fraction of the bed just before expansion. This approach is based on the description of the flow through a porous medium of porosity $\epsilon = 1 - \Phi_s$, and can be seen as an extension of the original work of Ergun [9]. Being based on the properties of the bed at U_{mf} , it is expected to be relevant at concentrations close to Φ_{up} . But it can hardly constitute a universal law, since its evolution away from Φ_{pack} does not involve parameters that may account for the variations, between the many possible different fluid/particle systems, of the magnitude of the four main forces listed above. Nevertheless, this approach emphasizes that a model that intends to describe the entire homogeneous regime should probably involve Φ_{pack} .

Suspensions are often described as an equivalent continuous medium of effective viscosity μ_m . Since the pioneering work of Einstein in 1905, which dealt with the dilute limit, and that of Krieger and Dougherty [10], which proposed

a way to continuously connect the dilute and the concentrated regime, many authors have been attempted to model the effective rheology of a suspension (see [11] for a recent review). The definition of an effective viscosity requires to consider a volume that contains enough particles so that the average particle
90 volume concentration can be considered as a relevant parameter at this scale [12]. Consequently, if the effective viscosity is suitable to predict the sedimentation velocity of a large body falling in a suspension of small particles, it is questionable to relate it to the settling velocity of the small particles that make up the suspension. For that reason, although both problems are closely related,
95 the literature on sedimentation/fluidization velocity is disconnected from that on suspension rheology, except in rare cases as [13]. Considering an effective viscosity determined from the mean sedimentation velocity of the particles, which differs from that measured in large-scale sheared suspensions, is however not without interest. This important issue has been recently addressed by two of
100 the authors [14], who defined the effective mixture viscosity μ_m by balancing the buoyancy force acting on a spherical particle, $g(\rho_s - \rho_f)\frac{\pi}{6}d^3$, to the Stokes drag, $3\pi\mu_m d(1 - \Phi_s)U$, acting on a particle immersed in a fluid of viscosity μ_m that moves at velocity $\frac{U}{1 - \Phi_s}$ relative to the fluid. This definition turns out to be the only way to gather the experimental values of U , measured for three
105 different types of small particles fluidized by a gas, on a master curve of the form

$$\frac{\mu_m}{\mu_f} = \frac{U_0(1 - \Phi_s)}{U} = F\left(\frac{\Phi_s}{\Phi_{pack}}\right), \quad (3)$$

where

$$U_0 = \frac{g(\rho_s - \rho_f)d^2}{18\mu_f} \quad (4)$$

is the Stokes velocity for an isolated settling particle, and F is an empirical function that is independent of the nature of the particles. This equation describes
110 non-cohesive fine particles suspended by a gas all over the homogeneous regime. In particular, this law is expected to be valid for fluid catalytic cracking (FCC) or volcanic ash which are both able to generate highly expanded suspensions at high temperature [15, 16]. However, the question of its generalization to lower

particle-to-fluid density ratios, as those corresponding to particles in liquids,
115 remains open and represents the principal objective of this present paper.

In the present work, a fluidization column is used to measure the fluidization
and the sedimentation velocities of a suspension of solid particles in a liquid. A
series of experiments were carried out for various particle sizes, particle densities
and liquid viscosities. Combined with experimental results of our previous work
120 [14] which were performed by using a gas, this amounts to exploring a wide
range of the control parameters. From the analysis of these data, we propose an
expression able to accurately predict the value of U for any systems involving
a suspension of particles all over the homogeneous regime ranging from Φ_{low} to
 Φ_{up} , provided that the inertia of the fluid is negligible. This law relies on the key
125 physical parameters that describe the particle/fluid system, namely the Stokes
velocity of an isolated particle, the particle volume fraction, a specific packing
concentration, and a Stokes number. In addition, the values of Φ_{up} and Φ_{low} are
also determined and found to be simple functions of the Stokes number, which
provides a prediction of the achievable expansion in a fluidization column.

130 This paper is organized as follows. Section 2 reviews the involved physical
parameters and introduces the relevant dimensionless groups. Section 3 de-
scribes the experimental setup and presents the investigated systems. Section 4
analyzes the results by means of Richardson-Zaki's and Abrahamsen-Geldart's
approaches. Section 5 describes our approach and introduces a new sedimen-
135 tation/fluidization law. Section 6 examines the boundaries of the homogeneous
range. Finally, concluding remarks are given in Section 7.

2. Dimensional analysis

We consider the fluidization and the sedimentation of a homogeneous sus-
pension of non-cohesive particles in a fluid. Each particle is characterized by its
140 density ρ_p , its equivalent diameter $d = (6\vartheta_p/\pi)^{1/3}$, and its shape. In the case
where all particles do not have the same size, we consider that d is either the
mean or median of the diameter distribution. The solid particles are made of a

homogeneous material of density ρ_s , so that $\rho_p = \rho_s$.

The concentration of the suspension is characterized by the particle (or solid)
 145 volume fraction, Φ_s , or equivalently by the fluid volume fraction (or bed porosity), $\epsilon = 1 - \Phi_s$. In order to be able to describe large concentration cases, it is useful to introduce a packing concentration, Φ_{pack} , which is defined as the particle volume fraction of a fluidized bed just below the minimum fluidization velocity or that of a settling suspension at the point where the sedimentation
 150 velocity vanishes. Note that, in a fluidized bed, Φ_{pack}/Φ_s corresponds to the bed expansion, which can be measured from the ratio between the fluidized bed height and the initial bed height, without knowing the value of Φ_{pack} . It is also worth mentioning that, when considering a population of particles with non-uniform distributions of size and shape, Φ_{pack} embeds important information
 155 about these distributions, which can be enough to determine the sedimentation/fluidization velocity U in certain cases, such as those of heated volcanic ash in a gas [14].

Then, we consider a Newtonian fluid which is characterized by its density, ρ_f and dynamic viscosity, μ_f . Finally, we account for a uniform gravity field of
 160 acceleration g . Since gravity is only involved in weight and buoyancy, it is better to consider the effective weight $g(\rho_p - \rho_f)$. Also, since the particles are moving relatively to a fluid, it is better to consider their effective inertia, $\rho_p + C_M\rho_f$, where $C_M\rho_f$ represents for the mass of fluid entrained by a particle while C_M is the added-mass coefficient, which is equal to $1/2$ for a sphere.

The problem is thus controlled by seven parameters, two non-dimensional
 165 ones (Φ_s , Φ_{pack}) and five dimensional ones (ρ_f , μ_f , d , $\rho_p + 1/2\rho_f$, $g(\rho_p - \rho_f)$), and involves three physical dimensions (length, mass, time). The problem is thus fully characterized by $7 - 3 = 4$ independent non-dimensional groups. We thus need to build two non-dimensional groups in addition to Φ_s and Φ_{pack} .
 170 Since numerous practical configurations involve fine particles, viscous forces are often dominant. It is thus relevant to introduce non-dimensional numbers that compare inertial forces to viscous ones.

Let us begin by considering the inertia of the fluid. The Reynolds num-

ber, $Re = \rho_f dU / \mu_f$, is the non-dimensional group that compares fluid inertial stresses to viscous ones. However, U being not an initial parameter, we need to replace it by a velocity scale that is built on the control parameters. Considering the settling velocity U_0 of a single particle in the Stokes regime (Eq. 4), we get the Archimedes number, $Ar = \frac{\rho_f(\rho_p - \rho_f)gd^3}{18\mu_f^2}$.

Evaluating the importance of the particle inertia force is more delicate. Indeed, this force does not play any role when a particle moves at a constant velocity and is therefore associated with the velocity fluctuations that take place in a settling or fluidized suspension. We thus introduce the Stokes number, $St = \tau_p / t_f$, which compares the particle response time, τ_p , to the time scale of the fluctuations of the fluid velocity, t_f . By balancing particle inertia, $\frac{(\rho_p + 1/2\rho_f)\pi d^3 U}{6\tau_p}$ and Stokes drag, $3\pi\mu_f dU$, τ_p is found to scale with $\frac{(\rho_p + 1/2\rho_p)d^2}{\mu_f}$. Then, t_f can be estimated as d/U , which leads to $St = \frac{(\rho_p + 1/2\rho_f)dU}{\mu_f}$. Finally, by replacing U by U_0 , we obtain a Stokes number that depends only on the initial control parameters: $St_0 = \frac{(\rho_p + 1/2\rho_f)(\rho_p - \rho_f)gd^3}{18\mu_f^2}$.

The problem is then fully characterized by Φ_s , Φ_{pack} , Ar and St_0 . Note that Ar is the only non-dimensional group that can be constructed from the three physical parameters when $\rho_p + 1/2\rho_f$ is removed, while St_0 is the only one when ρ_f is disregarded. Therefore, in cases where Ar (or Re) is small, the inertia of the fluid can be neglected, whereas in cases where St_0 (or St) is small, that is the inertia of the particle that is negligible.

3. Experimental setup, procedures and regime characterization

We report fluidization and sedimentation experiments that were carried out in the experimental setup schematized in figure 1. It includes a transparent vertical column of height $H = 0.7$ m which has a rectangular cross-section of sides $x_0 = 0.2$ m and $w_0 = 0.3$ m. A liquid, supplied by a centrifugal pump, can be injected at the bottom of the column through an array of straws discharging in a stack of large glass pebbles and smaller lead beads, all covered by a mesh filter. This injection system ensures a uniform liquid flow and prevents the

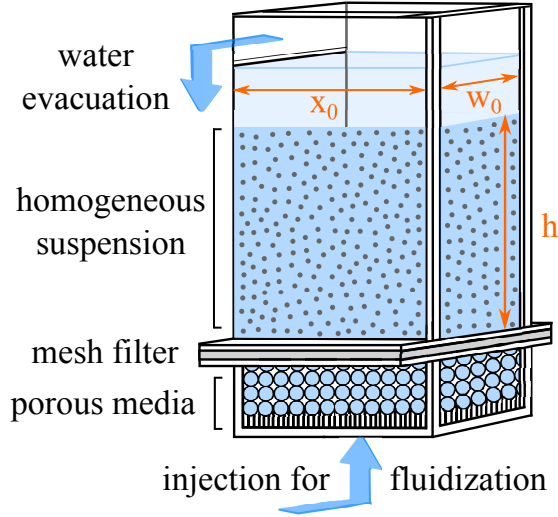


Figure 1: Scheme of the fluidization column.

solid particles from leaving the column. At the top, an evacuation system is connected to an external tank which allows a closed-loop flow.

205 The experimental procedure can be described as follows. The column is filled with a mixture of solid particles and a liquid. The particles being denser than the liquid, they form a bed of initial height h_0 . The bed consists of a loose random packing at a concentration Φ_{pack} . The total volume of solid ϑ_s having been preliminarily measured, the initial concentration is determined as

210 $\Phi_{pack} = \frac{\vartheta_s}{x_0 w_0 h_0}$. Then, the liquid is injected from the bottom at a given flow rate Q , corresponding to a fluidization velocity $U_f = \frac{Q}{x_0 w_0}$, determined with an accuracy of $\pm 2\%$. Provided U_f is larger than the minimum fluidization velocity, the bed expands, reaching a height $h > h_0$ and a concentration Φ_s , which correspond to an expansion $E = \frac{h}{h_0}$, determined with an accuracy of

215 $\pm 2\%$. The normalized concentration is thus obtained from $\frac{\Phi_s}{\Phi_{pack}} = \frac{1}{E}$ while the concentration is given by $\Phi_s = \frac{\Phi_{pack}}{E}$, with an accuracy of $\pm 4\%$. When the liquid injection is stopped, the sedimentation velocity is measured from the duration Δt taken by the bed to settle: $U_{sed} = \frac{h-h_0}{\Delta t}$, with an accuracy of $\pm 4\%$. A preliminary fluidization-sedimentation cycle is performed before the collect of

Experimental parameters	<i>PMMA</i>	<i>GB</i> ¹	<i>GB</i> ²	<i>GB</i> ³	<i>Sand</i>	<i>Ash</i> ¹	<i>Ash</i> ²	<i>FCC</i>
Particle density ρ_p [kg m ⁻³]	1200	2500	2500	2500	2650	1600	1490	1420
Mean particle diameter d [μ m]	210	160	240	335	310	80	65	70
Fluid density ρ_f [kg m ⁻³]	998	998	998	998	998	0.79	0.79	0.79
Fluid viscosity μ_f [Pas]	10 ⁻³	10 ⁻³	10 ⁻³	10 ⁻³	10 ⁻³	2.45 10 ⁻⁵	2.45 10 ⁻⁵	2.45 10 ⁻⁵
Limit of fluidization $\frac{\phi_{up}}{\phi_{pack}}$	0.987	0.955	0.966	0.972	0.967	0.94	0.95	0.95
Limit of stability $\frac{\phi_{low}}{\phi_{pack}}$	0.06	0.16	0.38	0.62	0.47	0.70	0.71	0.82
Range of homogeneous regime $\frac{\phi_{up}}{\phi_{low}}$	16.45	5.97	2.54	1.80	1.57	1.34	1.34	1.16
$Ar = \frac{\rho_f(\rho_p - \rho_f)gd^3}{18\mu_f^2}$	1	3	11	31	28	0.6	0.3	0.4
$St_0 = \frac{(\rho_p - \rho_f)(\rho_s + \frac{1}{2}\rho_f)gd^3}{18\mu_f^2}$	1.6	10	34	94	88	1176	545	655

Table 1: Experimental parameters for both liquid-solid suspensions (*PMMA*; *GB*¹; *GB*²; *GB*³; *Sand*) at 20°C and gas-solid suspensions (*Ash*¹; *Ash*²; *FCC*) at 170°C.

data, so that the initial packing state Φ_{pack} is the result of particle sedimentation and not to an arbitrary configuration following the filling of the column. Then, a series of cycles are carried out for different liquid flow rates in order to measure how U_f and U_{sed} evolve with the particle concentration. For each considered system of a fluid and particles, the boundaries of the homogeneous regime are determined: Φ_{up} , is associated with the minimum flow-rate for which a visible bed expansion is achieved; Φ_{low} corresponds to the limit of stability of the bed beyond which visible fluctuations of concentration develop and its surface begins to be agitated. Note that this transition is quite abrupt, which makes possible its determination with a good accuracy.

Various systems are investigated (see Table 1 for a summary of their properties). Five sets of particles made of three different solid materials have been studied, the density of which has been measured by means of a pycnometer: one set of light PMMA beads, three sets of glass beads of different sizes (*GB*¹, *GB*² and *GB*³), and one set of sand grains. Ten samples of each set of particles have been analyzed by using a laser granulometer. Figure 2 shows the distributions of the particle equivalent diameters ($d = (6\vartheta_p/\pi)^{1/3}$). Beside, the shape of the

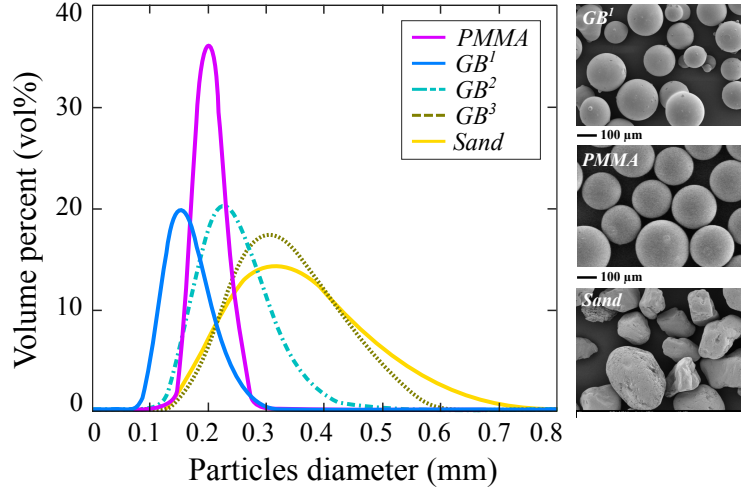


Figure 2: Particle size distribution and microscopic pictures of the materials used in the experiments.

particles was observed using a microscopic image of each sample, also shown in fig. 2. PMMA beads are almost spherical with a narrow size distribution. Glass beads are also almost spherical with a size distribution that is broader for the sets of larger particle sizes. Sand particles are less spherical and have a rather broad size distribution. In what follows, the particle size of each set will be characterized by the median diameter d_{50} of the distribution, we will thus be assimilated to d . In any case, the liquid is water but two different operating temperatures are used in order to vary the viscosity: $\mu_l=1.00\times 10^{-3}$ Pa s at 20°C or $\mu_l=0.72\times 10^{-3}$ Pa s at 35°C.

In addition to these new experiments, the present analysis will also consider results obtained in a recent study [14]. This previous work investigated two sets of non-spherical ash particles (Ash¹ and Ash²) and one set of almost spherical FCC particles, which were fluidized in air at 170°C [15]. The physical parameters of these experiments are also reported in Table 1. Even though two different setups are used, the experimental procedures of the previous and present experimental campaigns are similar and their results can thus be compared without limitations. Combining results obtained in either a liquid or a gas allows us to

explore a very broad range of the Stokes number and thus to reveal the role of
the particle inertia.

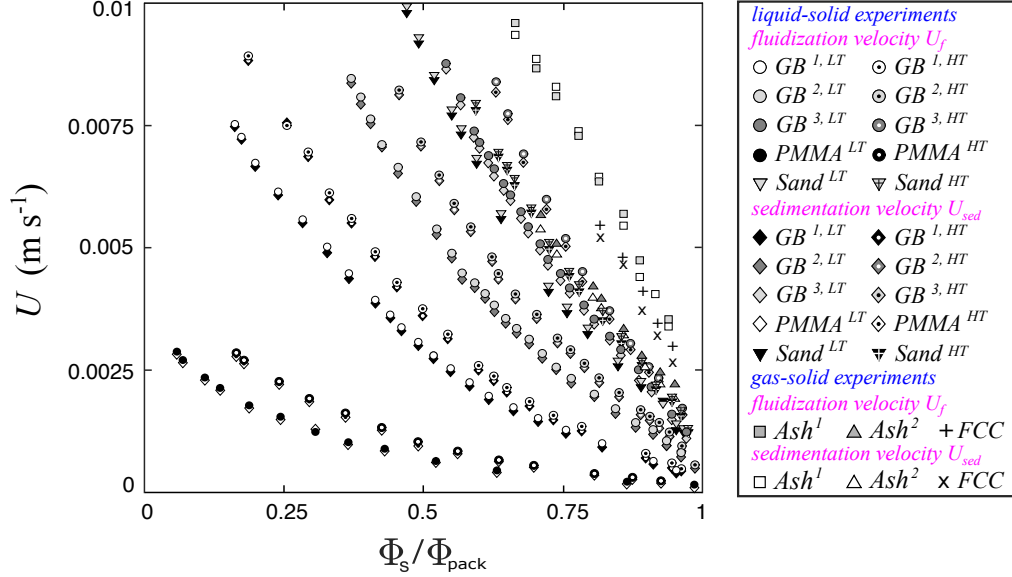


Figure 3: Fluidization velocity U_f and sedimentation velocity U_{sed} represented as a function of the normalized solid particle volume fraction $\frac{\Phi_s}{\Phi_{pack}}$ for both liquid-solid and gas-solid suspensions.

Figure 3 shows the measured fluidization velocity U_f and sedimentation velocity U_{sed} as a function of the normalized concentration $\frac{\Phi_s}{\Phi_{pack}}$ over the whole range of the stable homogeneous regime between $\frac{\Phi_{low}}{\Phi_{pack}}$ and $\frac{\Phi_{up}}{\Phi_{pack}}$ for all the systems under investigation. In all cases, U_f and U_{sed} are equal within the measurement accuracy, which confirms that wall effects are negligible. In what follows,
we will no longer distinguish them and consider a single velocity U , the value of which is set equal to the sedimentation velocity U_{sed} . Velocity U decreases with the concentration, but depending on the system under consideration, its values differ greatly. In particular, light PMMA particles in water feature the lowest values, the weakest decrease and the broader homogeneous range, whereas the reverse is true for particles in gas. Considering non-dimensional quantities is therefore necessary to interpret the results.

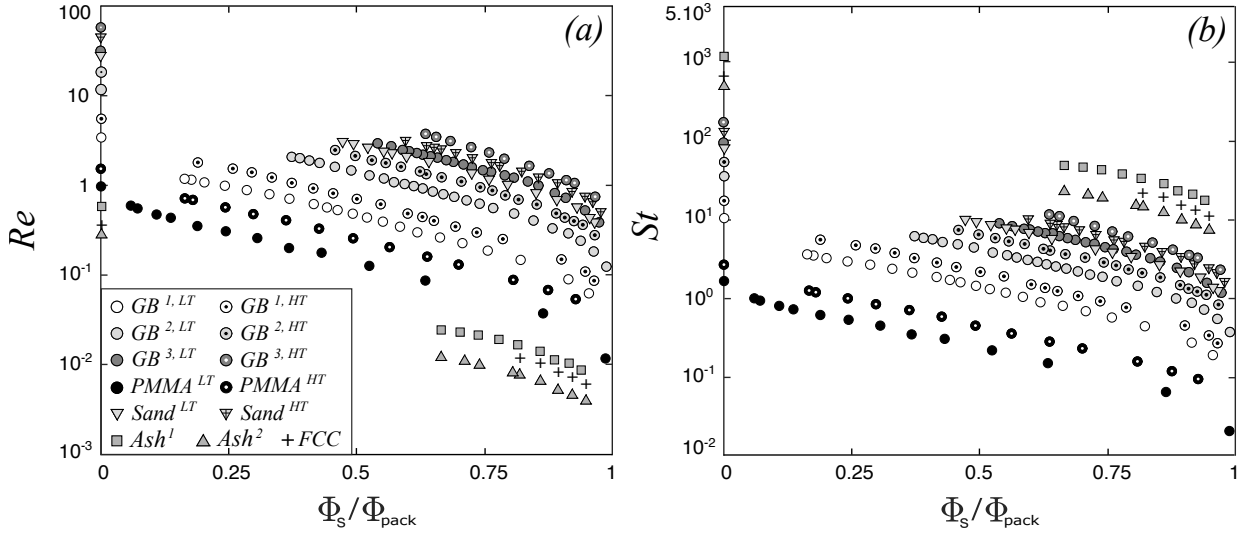


Figure 4: (a) Particles Reynolds number Re and (b) Stokes number St as a function of Φ_s/Φ_{pack} .

Table 1 gives the non-dimensional control parameters \mathcal{Ar} and St_0 for all sets of solid particles in water at 20°C and in air at 170°C. The values of \mathcal{Ar} remain moderate in any case and are the smaller for PMMA particles in water and solid particles in air ($\mathcal{Ar} \leq 1$). On the other hand, differences in St_0 are considerable, especially between PMMA in water ($St_0 < 2$) and particles in air ($St_0 > 500$). Disparities between the values of U of the different fluid-and-particle systems are thus rather associated with variations of particulate inertia than to fluid one. However, since \mathcal{Ar} and St_0 are based on the Stokes velocity U_0 of an isolated settling particle, it is not straightforward to determine the flow regime within a concentrated suspension from their values. This can be better done by examining Reynolds number Re and the Stokes number St which are based on the actual velocity U corresponding to each concentration. Figure 4 shows the evolution of Re and St with the normalized concentration. For comparison purposes, the value of \mathcal{Ar} (respectively St) corresponding to each fluid-and-particle system are reported at $\frac{\Phi_s}{\Phi_{pack}} = 0$ in fig. 4a (respectively in fig. 4b). The values of Re are in general significantly lower than those of \mathcal{Ar} . Their maximum

value, which is reached at Φ_{low} for the larger glass beads in water at 35°C, is
 285 about 5, while all values are less than unity close to Φ_{up} . The values of St are
 lower than those of St_0 too, but their variation range remains very wide: from
 2.0×10^{-2} to 9.8×10^{-1} for PMMA particles in 20°C-water, and from 7.5 to 50
 for Ash^1 particles in 170°C-air.

In these experiments, it is therefore reasonable to conclude that fluid inertia
 290 plays a minor role. We will thus disregard the Archimedes number and only
 examine the data by considering the other non-dimensional groups: St_0 , Φ_s and
 Φ_{pack} . This choice will be proved to be relevant since all data can be modeled
 by accounting for these only three parameters.

4. Discussion of existing laws

295 In this section, we confront the two classical approaches with our results.
 We begin with that of Abrahamsen and Geldart. Fig. 5 compares the predic-
 tions of eq. 2 with experimental results. Eq. 2 involves a free parameter that
 is the minimum fluidization velocity U_{mf} for which we used the value of U_f
 measured at Φ_{up} , so that experiments and predictions necessarily match at this
 300 concentration. For particles in gas, Eq. 2 follows rather the evolution of U at
 high concentrations. However, it fails at low concentrations and is clearly not
 suitable for particles in water. Therefore, we do not think that this approach is
 relevant to gather the data, obtained from configurations of contrasted proper-
 ties, into a unique description.

305 Now we examine the popular approach of Richardson and Zaki. Eq. 6 in-
 volves two parameters: velocity U_i and exponent n . As it is usually done, the
 experimental results have been represented in Fig. 6a under the form $\log_{10}(\frac{U}{U_i})$
 as a function of $\log_{10}(1 - \Phi_s)$. The value of U_i corresponding to each experimen-
 tal system has been determined such that the extrapolation of the best fit of the
 data by Eq. 6 intercepts $\frac{U}{U_i} = 1$ at $\Phi_s = 0$. Fig. 6a shows that the data of all
 310 systems gather on a straight line of slope $n = 3.75$ within values of $\log_{10}(1 - \Phi_s)$
 ranged from approximately -0.3 to -0.05. This exponent is expected to depend

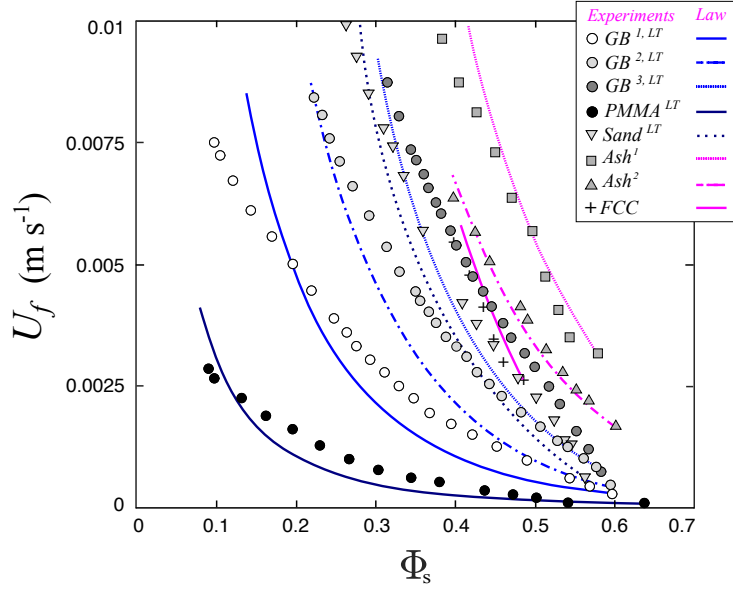


Figure 5: Fluidization velocity U_f as a function of the particle volume fraction Φ_s . Plain lines represent the Abrahamsen-Geldart's [3] (Eq. 2) prediction for both the liquid-solid and the gas-solid suspensions.

on the Reynolds number. Since the exponent turns out to be the same for all considered systems, which include some cases at very low Re , the value found
315 here should correspond to the low- Re limit. However, the present value $n = 3.75$ is significantly lower than the low- Re value $n = 4.65$ proposed by Richardson and Zaki. This is nevertheless not so surprising since, as pointed by [4], various exponents have been reported in the literature. Fig. 6b shows the values of
320 U_i , normalized by the Stokes velocity U_0 of an isolated settling particle, as a function of St_0 . They are observed to vary from $0.3 U_0$ to $0.6 U_0$ depending on the system under consideration. According to Richardson and Zaki, U_i can be affected by the column dimension or the Reynolds number. Here the ratio x_0/d between the minimum column side and the particle diameter is between 600 and 3000, which is enough to ensure that the results are independent of this parameter.
325 By the way, it is worth mentioning that the existence of a dependence of U on the column dimension is incompatible with the homogeneity of the flow

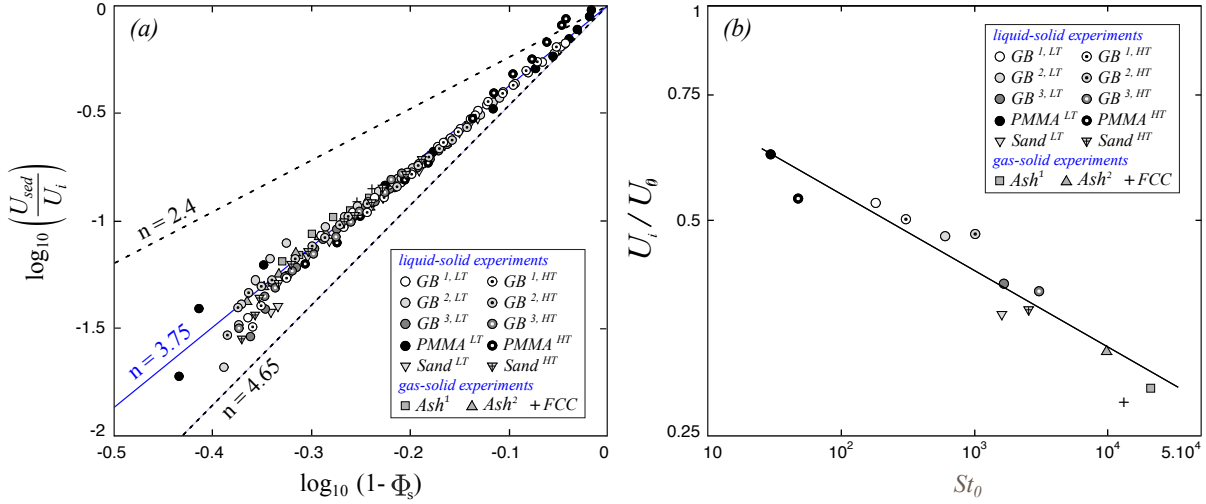


Figure 6: Presentation of the experimental results according to the Richardson and Zaki approach [2] (Eq. 1). (a) Decimal logarithm of the sedimentation velocity, $\log_{10}\left(\frac{U_{sed}}{U_i}\right)$, as a function of that of the bed porosity $\log_{10}(1 - \Phi_s)$. The dashed lines correspond to exponents $n = 4.65$ of the low-Re regime and $n = 2.4$ of the high-Re regime. (b) Velocity U_i normalized by the Stokes velocity U_0 as function of the Stokes number St_0 .

in the transverse direction. Regarding the Reynolds number, we note that the gas cases show the largest deviations to the Stokes velocity although they correspond to the lowest Re , which is much less than unity. On the other hand, $\frac{U_i}{U_0}$ can be described as a regular monotonous function of St_0 , which indicates that, in agreement with the flow regime characterization presented in the previous section, particle inertia is the main cause of discrepancy between the various systems. Yet, the inertia of a particle does not affect its motion when moving at a constant speed. It is thus clear that the value of U_i that allows the results to gather do not correspond to the velocity of an isolated particle. To conclude, the Richardson-Zaki approach, with an appropriate value of n and a value of U_i which depends on St_0 , allows us to model the present results, provided that we renounce to describe the evolution of U at too small or large concentrations.

5. A more universal approach

Our objective is to gather the results obtained for all systems of materials over the entire homogeneous regime. We reconsider the idea presented in our previous study [14] that dealt with solid particles in a gas and thus was limited to high Stokes numbers. Let us consider that the mixture of fluid and particles can be considered as a homogeneous fluid of density ρ_m and viscosity $\mu_m = \mu^* \mu_f$. The force balance on a test spherical particle settling at velocity U under the action of gravity within in this fluid is, in the regime of negligible fluid inertia,

$$\frac{\pi d^3}{6}(\rho_p - \rho_m)g = 3\pi\mu^*\mu_f dU. \quad (5)$$

Knowing that the mixture density is of $\rho_m = \Phi_s \rho_p + (1 - \Phi_s)\rho_f$ and introducing the Stokes velocity U_0 of an isolated particle (Eq. 4), leads to

$$\frac{U_0(1 - \Phi_s)}{U} = \mu^* \left(\mathcal{St}_0, \Phi_s, \frac{\Phi_s}{\Phi_{pack}} \right). \quad (6)$$

This non-dimensional number a priori depends on the three non-dimensional control parameters. It describes the excess of viscous friction acting on the test particle due to the presence of the other particles. It is therefore greater than unity. It tends towards one in the dilute limit and diverges towards infinity when Φ_s tends toward Φ_{pack} . It is also expected to increase with \mathcal{St}_0 from what we saw in Fig. 6b.

Fig. 7 shows experimental values of the non-dimensional viscosity $\mu^* = \frac{U_0(1 - \Phi_s)}{U}$, and its inverse the non-dimensional velocity $U^* = \frac{U}{U_0(1 - \Phi_s)}$, as a function of $\frac{\Phi_s}{\Phi_{pack}}$. Even if they contain the same information, these two representations are complementary. The evolution of μ^* emphasizes the differences between the various cases at large concentrations, while that of U^* highlights the differences at low concentration. Each system of materials is characterized by a specific value of the Stokes number \mathcal{St}_0 . We remark that the evolution of μ^* or U^* against $\frac{\Phi_s}{\Phi_{pack}}$ looks similar at all \mathcal{St}_0 . The only differences lie in their overall magnitude and the limit of stability of the homogeneous regime. The values of μ^* corresponding to solid particles in a gas are above the others and

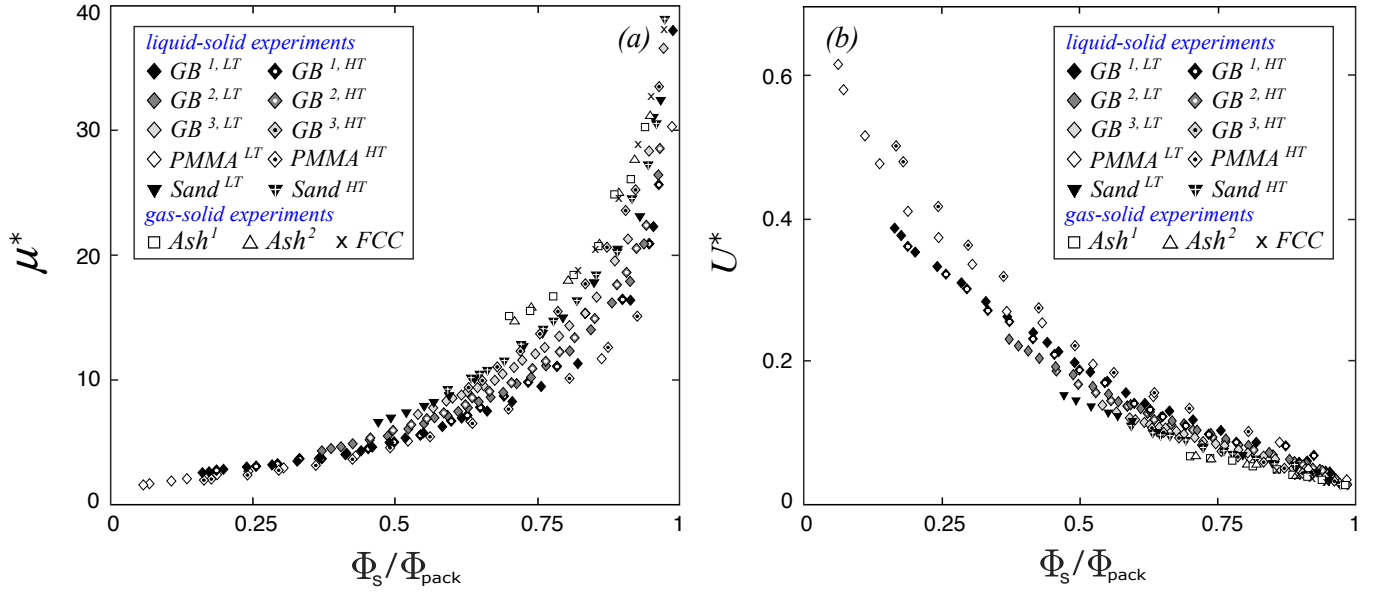


Figure 7: Experimental results on the non-dimensional form $\mu^* = \frac{U_0}{U(1-\Phi_s)}$ or $U^* = \frac{U(1-\Phi_s)}{U_0}$ as a function of $\frac{\Phi_s}{\Phi_{\text{pack}}}$.

defined on a short range, whereas as those corresponding to PMMA particles in water are below and defined on a very broad range. Comparing all systems, we can claim that the smaller $\mathcal{S}t_0$, the smaller μ^* and Φ_{low} . A closer look reveals that, over their definition range, the curves are proportional and only differ by a factor, which depends only on $\mathcal{S}t_0$. It means that there exist a function $\mathcal{K}(\mathcal{S}t_0)$ so that the evolutions of $\frac{\mu^*}{\mathcal{K}}$ can be described by a unique function of $\frac{\Phi_s}{\Phi_{pack}}$. The expression of μ^* thus simplifies into

$$\mu^*(\mathcal{S}t_0, \Phi_s, \Phi_{pack}) = \mathcal{K}(\mathcal{S}t_0) \mathcal{F}\left(\frac{\Phi_s}{\Phi_{pack}}\right), \quad (7)$$

where the Stokes number and the concentration are now involved in two separate functions, and where Φ_s and Φ_{pack} only appear through their ratio. The experimental values of \mathcal{K} are easily determined from the data. Since it does not depend on the concentration, we can chose any given value Φ^* of $\frac{\Phi_s}{\Phi_{pack}}$ to calculate them. The i^{th} value is given by

$$\mathcal{K}(\mathcal{S}t_0^i) = c \frac{\mu^*(\mathcal{S}t_0^i, \Phi^*)}{\mu^*(\mathcal{S}t_0^1, \Phi^*)}, \quad (8)$$

where c is a constant that can be arbitrarily included in \mathcal{K} or \mathcal{F} without changing their product, and thus without changing the value of μ^* according to Eq. 7. We choose the value of c so that \mathcal{F} tends towards unity when $\left(\frac{\Phi_s}{\Phi_{pack}}\right)$ tends toward zero. Fig. 8 represents the experimental data in the form of $\mathcal{F}\left(\frac{\Phi_s}{\Phi_{pack}}\right)$. The corresponding values of $\mathcal{K}(\mathcal{S}t_0)$, obtained by using Eq. 8, are shown in Fig. 9.

The excellent collapse of all data into a unique master curve seen in Fig. 8 proves the validity of the simple model expressed by Eq. 7. In the absence of a theory predicting the function \mathcal{F} , it is interesting to search for an empirical expression. It turns out that it cannot be accurately described by either a power law or an exponential function. In fact, the low concentration range shows an exponential increase, whereas the divergence at high concentration is well described by a power law. These considerations lead us to propose an expression of the form

$$\mathcal{F}\left(\frac{\Phi_s}{\Phi_{pack}}\right) = C_0 \left(\exp \left[-C_1 \left(1 - \frac{\Phi_s}{\Phi_{pack}} \right) \right] + C_2 \left(1 - \frac{\Phi_s}{\Phi_{pack}} \right)^{-C_3} \right), \quad (9)$$

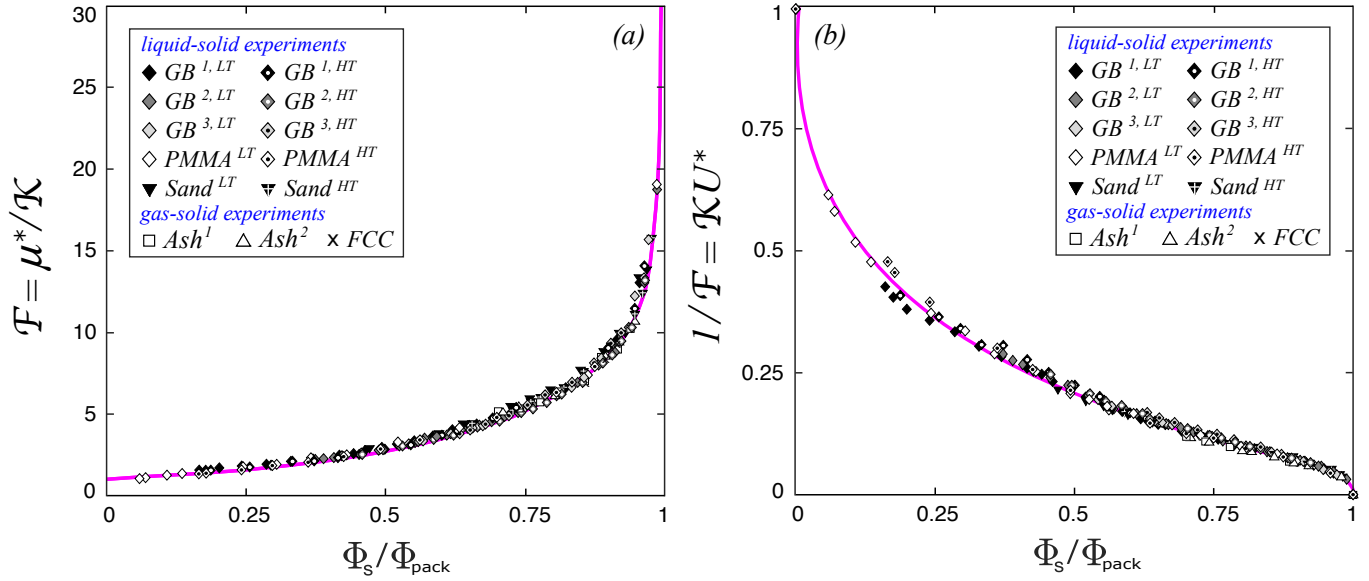


Figure 8: Non-dimensional function $\mathcal{F}\left(\frac{\Phi_s}{\Phi_{pack}}\right) = \frac{\mu^*}{\mathcal{K}(\mathcal{S}t_0)}$, which reflects the dependence of U on the concentration due to the enhanced viscous dissipation in the presence of many particles. Symbols represent experiments, the pink curve shows empirical law 9.

where the constants C_i are positive numbers and $C_0 = \frac{1}{\exp(-C_1+C_2)}$. This expression satisfies the two conditions $\mathcal{F}(0) = 1$ and $\mathcal{F}(1) = \infty$. The best fit is obtained with $C_1 = 3$, $C_2 = 0.08$ and $C_3 = 2/3$. It is represented by the pink curve in Fig. 8 and accurately describes the experimental results.

The evolution of \mathcal{K} against $\mathcal{S}t_0$, plotted in Fig. 9, reveals the effect of the particle inertia on the fluidization or sedimentation velocity. The experimental results show that $\mathcal{K}(\mathcal{S}t_0)$ is an increasing function, with a slope that is large at low $\mathcal{S}t_0$ but then decreases continuously as $\mathcal{S}t_0$ increases, finally reaching a plateau at high $\mathcal{S}t_0$. Such a behavior can be described by means of a simple saturation function of the form

$$\mathcal{K}(\mathcal{S}t_0) = (K_\infty - K_0)g(\mathcal{S}t_0) + K_0, \quad (10)$$

where K_0 and K_∞ are respectively the limits of $\mathcal{K}(\mathcal{S}t_0)$ at zero and infinity, and

$$g(x) = \frac{x}{x+1}. \quad (11)$$

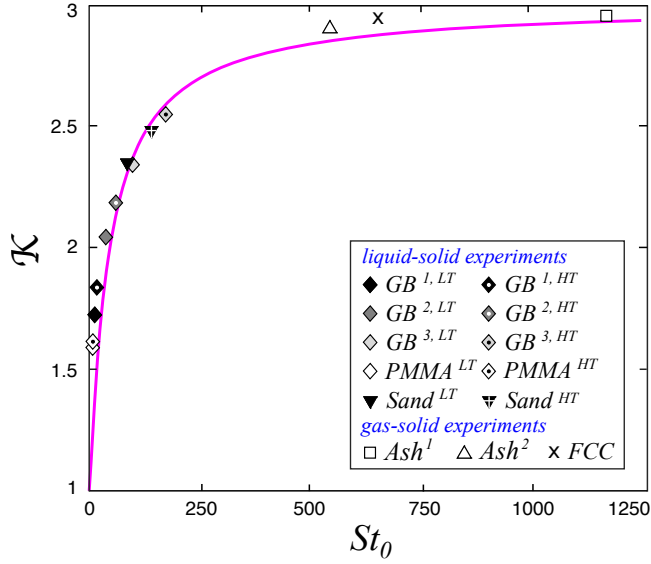


Figure 9: Non-dimensional function $K(St_0)$, which reflects the increase of viscous dissipation due to the difference between instantaneous motions between inertial particles and the carrier fluid. Symbols represent experiments, the pink curve shows empirical law 10.

Now recall that the smaller St_0 , the smaller Φ_{low} . With light particles, dilutions strong enough for U to approach U_0 can be achieved while remaining in the homogeneous regime. Thus, if we stand that Φ_{low} tends towards zero when St_0 tends towards zero, we obtain from Eqs. 6 and 7

$$\mu^* \left(St_0 = 0, \frac{\Phi_{low}}{\Phi_{pack}} \right) = \mu^* (0, 0) = K_0 \mathcal{F}(0) = K_0 = \frac{U_0(1-0)}{U_0} = 1. \quad (12)$$

Given that $K_0 = 1$, K_∞ is the only free parameter in Eq.10. A good fit of the experimental results, shown by the pink curve in Fig. 9, is obtained by setting $K_\infty = 3$.

We finally end up with a general model of the sedimentation/fluidization velocity U of a suspension that is valid all over the homogeneous range and for all systems of fluid and particles, provided that the inertia of the fluid can be neglected compared to viscous forces. From Eqs. 6 and 7, U can be written as

a product of four terms

$$U = \underbrace{\frac{g(\rho_p - \rho_f)d^2}{18\mu_f}}_{\text{1. Isolated particle, } U_0} \underbrace{(1 - \phi_s)}_{\text{2. mixture density}} \underbrace{\frac{1}{\mathcal{F}\left(\frac{\phi_s}{\phi_{pack}}\right)}}_{\text{3. bed porosity, Eq. 9}} \underbrace{\frac{1}{\mathcal{K}(\mathcal{S}t_0)}}_{\text{4. particle agitation, Eq. 10}}, \quad (13)$$

which can be interpreted as follows:

- 415 1. The first term is the theoretical Stokes terminal velocity U_0 of an isolated sphere of density ρ_p settling under the action of gravity in a fluid of density ρ_f and viscosity μ_f . It is the value reached by U when both $\Phi_s = 0$ and $\mathcal{S}t_0 = 0$. The three other terms are non-dimensional corrections to U_0 accounting for the various effects associated with the presence of many
420 particles.
2. The second term simply accounts for the evolution of the density ρ_m of the mixture which affects the buoyancy force acting on each solid particle in Eq. 5. As noticed in [14], Eq. 5 can also be obtained by considering a test particle moving at velocity $\frac{U}{1-\Phi_s}$ relative to the fluid of density ρ_f instead
425 of moving at velocity U relative to the mixture of particles and fluid at density ρ_m . Thus, the second term can also be interpreted as reflecting the increase of the relative velocity between the particle and the fluid when the concentration increases. It is worth mentioning that this term is not associated with interactions between particles nor with an increase of the
430 drag force acting on the particles.
3. As well as in a porous medium, the fluid follows complex paths within the interstices between the particles. The friction on each particle and the corresponding viscous dissipation are therefore increased compared to the case of an isolated particle. This causes the increase of the average
435 drag force on the particle and the decrease of U . The third term of Eq. 13, modeled by Eq. 9, quantifies this effect in the case of particles of negligible inertia, i.e. $\mathcal{S}t_0 = 0$. Function $\frac{1}{\mathcal{F}}$ decreases from unity to zero as $\frac{\phi_s}{\phi_{pack}}$ goes from zero (isolated particle) to one (packed state).

4. In fluidized beds or sedimenting suspensions, both fluid and particle velocities undergo fluctuations, even at low Reynolds number [17, 18]. These fluctuations contribute to the dissipation of mechanical energy and thus affect the average velocity U . In particular, the fluctuations of the relative velocity between the fluid and the particles are intimately related to how U depends on the concentration [19]. However, these fluctuations do not only depend on the concentration but also on the Stokes number. Particles of negligible inertia ($St_0 \ll 1$) instantaneously follow any local fluctuations of the fluid that surrounds them, while particles of significant inertia follow trajectories that differ from those of the fluid. For particles of negligible inertia, the effect of the fluctuations is already embedded in $\frac{1}{\mathcal{F}}$ and turns out to vanish at strong dilution. The fourth term of Eq. 13, modeled by Eq. 10, describes the evolution of this effect with the Stokes number. It turns out that $\frac{1}{\mathcal{K}}$ is a decreasing function of St_0 which reaches a minimum value of about 1/3 at large St_0 . The larger St_0 , the smaller U , which is at most three times smaller for high-inertia particles compared to low-inertia ones.

6. Boundaries of the homogeneous range

Eq. 13 provides a general relation between U and all the control parameters of the problem. To achieve a complete description, we need henceforth to determine the boundaries of the homogeneous range in which this law is valid.

Fig. 10a shows the upper boundary, $\frac{\Phi_{up}}{\Phi_{pack}}$, as a function of St_0 . Note that Φ_{up} is defined as the maximum concentration below which the bed of particles is fully fluidized, i.e. $U_f = U_{sed}$. In principle, this point may differ from the concentration Φ_{mf} corresponding to the minimum fluidization velocity, where the presence of the wall can still play a significant role and cause U_f to be larger than U_{sed} . However, it turned out that U_f was approximately equal to U_{sed} at the first point where measurable expansion was detected. We can thus make no distinction between Φ_{up} and Φ_{mf} in the present experiments. This is not

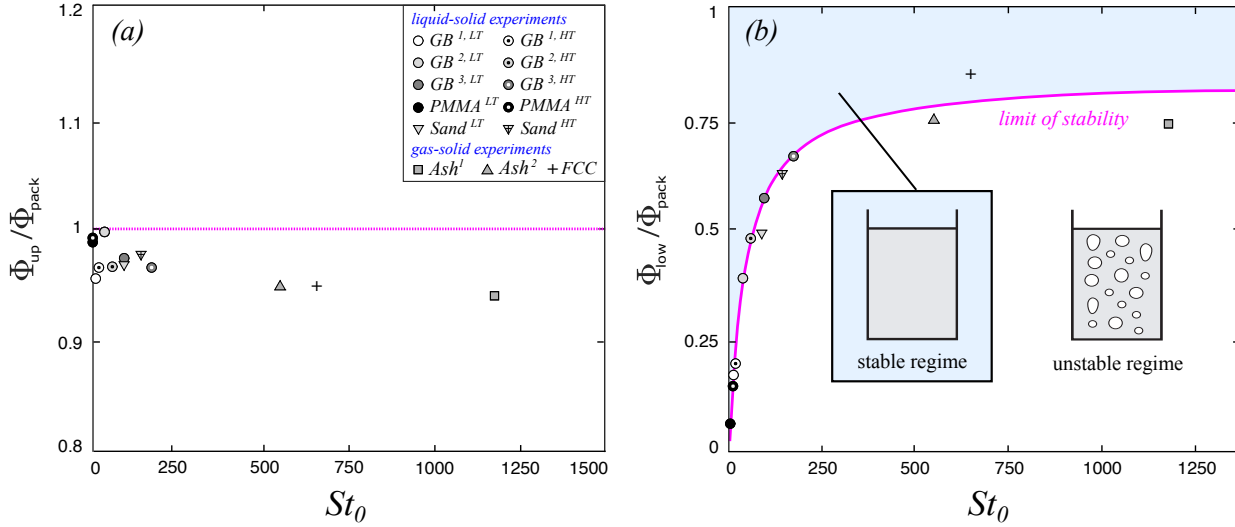


Figure 10: Boundaries of the homogeneous range at a function of St_0 . (a) Upper limit $\left(\frac{\phi_{up}}{\phi_{pack}}\right)$ determined from the minimum fluidization velocity; (b) lower limit $\left(\frac{\phi_{low}}{\phi_{pack}}\right)$ corresponding to the onset of the bed instability.

surprising since the ratio between Φ_{up} and Φ_{pack} remains in any case ranged from 0.94 and 1, without showing any clear correlation with St_0 .

470 The study of the lower boundary Φ_{low} is much more interesting, since it characterizes the limit of stability of the homogeneous regime when the concentration is decreased as well as the bed expansion. Above Φ_{low} , the bed shows no noticeable fluctuations. Below Φ_{low} the surface of the bed becomes strongly agitated and visible fluctuations of concentration at large scales compared to d are visible. Fig. 10b shows $\frac{\Phi_{low}}{\Phi_{pack}}$ as a function of St_0 . It is immediately noticeable that this behavior is similar to that of $\mathcal{K}(St_0)$ presented in Fig. 9 and can thus be described by an expression similar to Eq. 10,

$$\frac{\Phi_{low}}{\Phi_{pack}}(St_0) = \left(\left[\frac{\Phi_{low}}{\Phi_{pack}} \right]_{\infty} - \left[\frac{\Phi_{low}}{\Phi_{pack}} \right]_0 \right) g(St_0) + \left[\frac{\Phi_{low}}{\Phi_{pack}} \right]_0, \quad (14)$$

where function g is still given by Eq. 11. The best fit of the experimental results, given by $\left[\frac{\Phi_{low}}{\Phi_{pack}} \right]_0 = 0$ and $\left[\frac{\Phi_{low}}{\Phi_{pack}} \right]_{\infty} = 0.8$, is represented by the pink curve in Fig. 10b. Considered together, the evolutions of $\mathcal{K}(St_0)$ and $\frac{\Phi_{low}}{\Phi_{pack}}(St_0)$ draw an interesting picture. First, it is worth recalling that $\left[\frac{\Phi_{low}}{\Phi_{pack}} \right]_0 = 0$ means that

a bed of particles of negligible inertia should be expanded without limit while remaining homogeneous. At $St_0 = 0$, there is no discontinuity between the case of an isolated particle ($\Phi_s = 0$) and the onset of the transition towards jamming
485 ($\Phi_s = \Phi_{pack}$), U being determined by the three first terms of Eq. 13. At $St_0 > 0$, the velocity U is divided by a factor $\mathcal{K}(St_0) > 1$ and, concurrently, the homogeneous regime is restricted to concentrations larger than $\Phi_{low}(St_0) > 0$. In this case, the existence of a bifurcation at Φ_{low} limits the evolution of U that cannot be extrapolated towards $\Phi_s = 0$ to recover the value U_0 corresponding
490 to an isolated particle. Increasing St_0 leads to the increase of \mathcal{K} and $\frac{\Phi_{low}}{\Phi_{pack}}$ by following a similar law, which indicates that the same mechanism related to the inertia of the particles is responsible for both the instability of the suspension and the decrease of U .

7. Conclusion

495 From a thorough examination of experimental results conducted with systems of contrasted properties, we have established a general law (Eq. 13) for the sedimentation/fluidization velocity U of a non-cohesive suspension of particles in a low-inertia fluid, either gaseous or liquid. This law takes into account all the physical parameters that control the flow. It is valid over the whole range
500 of particle volume fractions where a fully fluidized homogeneous suspension is stable, between the lower limit Φ_{low} below which large-scale fluctuations of concentration develop, and the upper limit Φ_{up} above which solid friction starts to play a significant role. A major finding of this work is that the expression of U can be decomposed into the product of four terms, each of them accounting for
505 a different physical mechanism: (1) the settling velocity of an isolated particle, (2) the effective particle weight, (3) the effect of the concentration of particles of negligible inertia (4) the effect of particle inertia. Remarkably, the function $\mathcal{F}\left(\frac{\Phi_s}{\Phi_{pack}}\right)$ modeling term (3) is independent of function $\mathcal{K}(St_0)$ modeling term (4). Also, the flow instability beyond a given concentration is directly related
510 to the inertia of the particles. A bed of inertialess particles ($St_0 = 0$) can be

indefinitely expanded while remaining stable. Increasing St_0 , $\frac{\Phi_{low}}{\Phi_{pack}}$ and $\mathcal{K}(St_0)$ both increase by following the same law. Thus, the larger the Stokes number, the lower U and the less the maximum stable bed expansion.

Combined to general expression 13, empirical laws 9, 10 and 14 provide
515 a reliable tool for engineers needing to predict the behavior of a fluidized bed. Now, let us discuss two possible limitations of the present result. We can wonder whether there could be an effect of the width of the size distribution. Although the particle sets investigated in a liquid (Fig. 2) as well as those investigated in a gas (Fig. 1 of [16], [14]) have various size distributions, our model is able to
520 describe them by only accounting for the average diameter of the particles. We thus think that the particle distribution can be disregarded, provided that its width is narrow enough to prevent size segregation from occurring [20]. Note that the mixture was made with the same material and also prevents the density segregation. Moreover, in experiments made in gas, one set of roughly spherical
525 FCC particles and two sets of non-spherical randomly shaped volcanic ash were studied (see pictures in Fig. 1 of [16], [14]). In experiments made in liquid, glass beads and PMMA particles are spherical, whereas sand grains are not (see pictures in Fig. 2). Since the present model is found to work with all the considered materials, it seems that Φ_{pack} embeds the most important information about
530 the particle shape, at least for shapes that are moderately anisotropic.

Lastly, it is important to recall that the present model described situations where the effect of the fluid inertia can be neglected, i.e. small Reynolds numbers. To deal with all possible cases, a model must provide the dependence of U with all four non-dimensional parameters: Φ_s and Φ_{pack} , St_0 and Ar . Is it
535 possible to extend the present model to include the Archimedes number ? A more general expression for the velocity of the isolated particle (term 1) could be considered in order to account for a finite-Reynolds-number drag. Also, the effect of the concentration on the mixture density (term 2) should remain unchanged. However, it is more difficult to anticipate about the two other terms.
540 In particular, it would be interesting to know whether a separation of variables as that expressed by Eq. 7 is still relevant when the role of Ar is considered.

A Future work based on further experimental investigations will address these issues.

References

- 545 [1] J. F. Richardson, W. N. Zaki, The sedimentation of a suspension of uniform spheres under conditions of viscous flow, *Chemical Engineering Science* 3 (2) (1954) 65–73.
- [2] J. F. Richardson, W. N. Z. P. D, Sedimentation and fluidisation: Part I, *Trans. Inst. Chem. Eng.* 32 (1954) S82–S100.
- 550 [3] A. R. Abrahamsen, D. Geldart, Behaviour of gas-fluidized beds of fine powders part I. Homogeneous expansion, *Powder Technology* 26 (1) (1980) 35–46.
- [4] O. J. I. Kramer, P. J. de Moel, E. T. Baars, W. H. van Vugt, J. T. Padding, J. P. van der Hoek, Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics, *Powder Technology* 343 (2019) 465–478.
- 555 [5] N. Funamizu, T. Takakuwa, An improved Richardson-Zaki formula for computing mixed layer composition in binary solid-liquid fluidized beds, *Chemical Engineering Science* 50 (19) (1995) 3025–3032.
- 560 [6] P. Lettieri, D. Newton, J. G. Yates, Homogeneous bed expansion of FCC catalysts, influence of temperature on the parameters of the Richardson–Zaki equation, *Powder Technology* 123 (2-3) (2002) 221–231.
- [7] J. M. Valverde, A. Castellanos, A modified Richardson–Zaki equation for fluidization of Geldart B magnetic particles, *Powder Technology* 181 (3) (2008) 347–350.
- 565 [8] M. Bargiel, E. M. Tory, Extension of the Richardson–Zaki equation to suspensions of multisized irregular particles, *International Journal of Mineral Processing* 120 (2013) 22–25.

- [9] S. Ergun, Fluid flow through packed colmuns, Chemical Engineering Progress 48 (2) (1952) 89–94.
- [10] I. M. Krieger, T. J. Dougherty, A mechanism for non-Newtonian flow in suspensions of rigid spheres, Transactions of the Society of Rheology 3 (1) (1959) 137–152.
- [11] É. Guazzelli, O. Pouliquen, Rheology of dense granular suspensions, Journal of Fluid Mechanics 852 (2018) 35–73.
- [12] E. J. Hinch, An averaged-equation approach to particle interactions in a fluid suspension, Journal of Fluid Mechanics 83 (4) (1977) 695–720.
- [13] S. Koo, Sedimentation velocity of bidisperse suspensions, Journal of Industrial and Engineering Chemistry 14 (5) (2008) 679–686.
- [14] L. Girolami, F. Risso, Sedimentation of gas-fluidized particles with random shape and size, Physical Review Fluids 4 (7) (2019) 074301.
- [15] L. Girolami, Dynamique et sédimentation des écoulements pyroclastiques reproduits en laboratoire, Ph.D. thesis, Université de Clermont II (2008).
- [16] L. Girolami, T. H. Druitt, O. Roche, Towards a quantitative understanding of pyroclastic flows: Effects of expansion on the dynamics of laboratory fluidized granular flows, Journal of Volcanology and Geothermal Research (296) (2015) 31–39.
- [17] N.-Q. Nguyen, A. J. C. Ladd, Sedimentation of hard-sphere suspensions at low Reynolds number, Journal of Fluid Mechanics 525 (2005) 73–104.
- [18] M. Abbas, E. Climent, O. Simonin, M. R. Maxey, Dynamics of bidisperse suspensions under Stokes flows: Linear shear flow and sedimentation, Physics Of Fluids 18 (12) (2006) 121504.
- [19] E. Alméras, O. Masbernat, F. Risso, R. O. Fox, Fluctuations in inertial dense homogeneous suspensions, Physical Review Fluids 4 (10) (2019) 102301.

- [20] J. L. P. Chen, D. L. Keairns, Particle segregation in a fluidized bed, The Canadian Journal of Chemical Engineering 53 (4) (1975) 395–402.