An Optimal Control Problem of Unmanned Aerial Vehicle
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I. Introduction

An unmanned aerial vehicle (UAV) has grown rapidly the last years. They are being engaged in many types of missions, ranging from military to agriculture passing from entertainment and rescue or even delivery. This work consists to ensure the convergence of positions and yaw angle, to their desired trajectories while maintaining stability of roll and pitch angle. For this, formulate this problem to an optimal control problem which minimizes the distance between state and desired state in free final time. Our contribution is to use the Bocop software to solve this problem. And to implement Shooting method with Matlab software.

II. Quadrotor Flight Model

Let

\[ X = [\phi, \theta, \psi, x, y, z, \dot{x}, \dot{y}, \dot{z}]^T \]

(1)

The model dynamic of the quad-rotor is given under the state space as \( \dot{x} = f(x) + g(x, u) \) considering \( X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}, x_{11}, x_{12}]^T \), and \( u = [u_1, u_2, u_3, u_4]^T \) be the state, the control vector of the system respectively is given as follows:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8 \\
\dot{x}_9 \\
\dot{x}_{10} \\
\dot{x}_{11} \\
\dot{x}_{12}
\end{bmatrix}
= \begin{bmatrix}
-I_a \xi_1 \\
-I_a \xi_2 \\
-I_a \xi_3 \\
-I_a \xi_4 \\
-I_a \xi_5 \\
-I_a \xi_6 \\
-I_a \xi_7 \\
-I_a \xi_8 \\
-I_a \xi_9 \\
-I_a \xi_{10} \\
-I_a \xi_{11} \\
-I_a \xi_{12}
\end{bmatrix} u_4 - g,
\]

where

\[ g(m/s^2) \] : gravity acceleration; \( I_x, I_y, I_z(kg/m^2) \) : roll, pitch and yaw inertia moments respectively; \( J_r(kg/m^2) \) : the rotor inertia; \( m(kg) \) : mass; \( x, y, z(m) \) : longitudinal, lateral and vertical motions respectively; \( \phi, \theta, \psi(rad) \) : roll, pitch and yaw angles, respectively; \( w_k(rad/s) \) : rotor angular velocity, where, \( k \) equal to 1, 2, 3 and 4; \( 4; d(m) \) : the distance between the quadrotor center of mass and the propeller rotation axis; \( u_1, u_2, u_3(N, m) \) : aerodynamical roll, pitch and yaw moments respectively; \( u_4(N) \) : lift force.

With \( \Omega = \omega_1 - w_2 + w_3 - w_4, a_1 = \frac{\omega_1}{I_z}, a_2 = \frac{w_1 - \omega_1}{I_z}, a_3 = \frac{\omega_1 - \omega_2}{I_z}, a_4 = \frac{\omega_1 - \omega_3}{I_z}, b_1 = \frac{\omega_1 - \omega_2}{I_z}, b_2 = \frac{\omega_3 - \omega_4}{I_z}, b_3 = \frac{\omega_4 - \omega_1}{I_z} \),

The one of the main objectives of this paper is to find control which ensure the convergence of positions \( \{x, y, z\} \) and yaw angle \( \psi \), to their desired trajectories respectively \( \{x_d, y_d, z_d\} \), and yaw angle \( \psi_d \) while maintaining stability of roll and pitch angle \( \{\phi, \theta\} \) in free final time.
Then, the criterion is formulated as follows:

\[ J = \int_0^t \frac{1}{\rho}(x_5 - \psi_d)^2 + (x_7 - x_d)^2 + (x_9 - y_d)^2 + (x_{11} - z_d)^2 \, dt \]

where \( t_f(\text{second}) \) : free final time ; \( \{x_d, y_d, z_d\} \) : desired trajectory of \( \{x, y, z\} \); \( \psi_d \) : desired trajectory of \( \psi \).

### III. Shooting Indirect method

The shooting indirect method is used to obtain the value of \( p(0) \) necessary to the solution of the problem characterized by the Pontryagin principle. If it is possible, from the condition of minimization of the Hamiltonian to express the control extremal function of \( (x(t), p(t)) \) then the extremal system is a differential system of the form \( \dot{v}(t) = G(t, v(t)) \) where \( v(t) = (x(t), p(t)) \).

The Hamiltonian of the system 2 is given by:

\[
H = p_1 x_2 + p_2 (a_1 x_1 x_6 + a_2 x_1 \dot{\Omega} + b_1 u_1) + p_3 (x_4) + p_4 (a_3 x_2 x_6 + a_4 x_2 \dot{\Omega} + b_2 u_2) + p_5 (x_6) + p_6 (a_5 x_4 + b_3 u_3) + p_7 (x_8) + \frac{p_8}{m} (\cos x_1 \sin x_2 \cos x_5 + \sin x_1 \cos x_5) + p_9 (x_{10}) + \frac{p_{10}}{m} (\sin x_1 \sin x_2 \sin x_5 - \sin x_1 \cos x_5) + p_{11} (x_{12}) + \frac{p_{12}}{m} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5) + \frac{p_{13}}{m} (\sin x_1 \sin x_2 \sin x_5 - \sin x_1 \cos x_5)
\]

Let us define a control law:

\[
\begin{aligned}
\frac{\partial H}{\partial p_1} &= p_2 b_1 + \frac{1}{\rho} u_1, \\
\frac{\partial H}{\partial p_2} &= p_4 b_2 + \frac{1}{\rho} u_2, \\
\frac{\partial H}{\partial p_3} &= p_6 b_3 + \frac{1}{\rho} u_3.
\end{aligned}
\]

Then, the control is:

\[
\begin{aligned}
u_1 &= \frac{p_2 b_1}{\rho} u_1, \\
u_2 &= \frac{p_4 b_2}{\rho} u_2, \\
u_3 &= \frac{p_6 b_3}{\rho} u_3, \\
u_4 &= \frac{p_{11}}{m} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5) + \frac{p_{12}}{m} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5),
\end{aligned}
\]

The transversality condition of the Hamiltonian is defined as follows:

\[
H(t_f, x_i, p_i, i = 1, 12, j = 1, 4) = 0
\]

To implement shooting method, consider \( |u_i| \leq 1, \ j = 1, 4 \).

Then, we have to solve the following system:

\[
\begin{align*}
\dot{v}_1 &= v_2, \\
v_2 &= a_1 v_2 v_6 + a_2 v_4 \dot{\Omega} + b_1 u_1, \\
v_3 &= v_4, \\
v_4 &= a_3 v_2 v_6 + a_4 v_2 \dot{\Omega} + b_2 u_2, \\
v_5 &= v_6, \\
v_6 &= a_5 v_2 v_4 + b_3 u_3, \\
v_7 &= v_8, \\
v_8 &= \frac{m}{a_1} (\cos x_1 \sin x_2 \cos x_5 + \sin x_1 \sin x_5), \\
v_9 &= v_{10}, \\
v_{10} &= \frac{m}{a_4} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5), \\
v_{11} &= v_{12}, \\
v_{12} &= \frac{m}{a_2} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5), \\
v_{13} &= -v_{12}, \\
v_{14} &= -v_{13}, \\
v_{15} &= \frac{m}{a_3} (\cos x_1 \sin x_2 \cos x_5 + \sin x_1 \cos x_5), \\
v_{16} &= \frac{m}{a_4} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5), \\
v_{17} &= \frac{m}{a_2} (\cos x_1 \sin x_1 \sin x_5 - \sin x_1 \cos x_5), \\
v_{18} &= \frac{m}{a_1} (\cos x_1 \sin x_2 \cos x_5 + \sin x_1 \cos x_5) - \frac{1}{\rho} u_4, \\
v_{19} &= -2 (v_7 - x_d), \\
v_{20} &= -v_1, \\
v_{21} &= -2 (v_9 - y_d), \\
v_{22} &= -v_2, \\
v_{23} &= -2 (v_11 - z_d), \\
v_{24} &= -v_3, \\
v_1(0) &\in \mathbb{R}, \ i = 1, 24.
\end{align*}
\]

Let \( v(t, x_i, u_i, i = 1, 12) \) be the solution of the previous system at time \( t \) with the initial condition \( v_1(0), i = 1, 24 \).

We construct a shooting function given by:
In the solution, we used the Newton’s method. The solution of \( \varphi(p(0)) = 0 \) is to find \( p(0) \) such that \( \varphi(p(0)) \) gives the desired value of \( x(t_f) = (\psi_d, x_d, y_d, z_d) \), which in our case uses the method quasi-newton (implemented in ‘fsolve’ of Matlab).

IV. SIMULATION AND DISCUSSION

\[
\varphi(p) = \begin{pmatrix}
v_0(t_f, x_i, p_i, i = 1, 12) \\
v_7(t_f, x_i, p_i, i = 1, 12) \\
v_9(t_f, x_i, p_i, i = 1, 12) \\
-2v_{11}(t_f, x_i, p_i, i = 1, 12)
\end{pmatrix}
\]

The results given by Bocop software are presented in figures 1-8. This figures shows that the distance between the
Fig. 6. Trajectory state of $x_{11}$ and $x_{12}$ respectively.

Fig. 7. Control $u_1$ and $u_2$ respectively.

Fig. 8. Control of $u_3$ and $u_4$ respectively.

Fig. 9. Trajectory state of $x_1$ and $x_2$ respectively.

Fig. 10. Trajectory state of $x_3$ and $x_4$ respectively.

Fig. 11. Trajectory state of $x_5$ and $x_6$ respectively.
Fig. 12. Trajectory state of $x_7$ and $x_8$ respectively.

Fig. 13. Trajectory state of $x_9$ and $x_{10}$ respectively.

Fig. 14. Trajectory state of $x_{11}$ and $x_{12}$ respectively.

Fig. 15. Control $u_1$ and $u_2$ respectively.

Fig. 16. Control of $u_3$ and $u_4$ respectively.

V. CONCLUSION

In this work, we have solved an optimal control problem of unmanned aerial vehicle to minimize the distance between the state and the desired state in free final time. The results are adequate for our purpose in the computational time is 9.63s in 141 iterations with Bocop software. The convergence is fast and the computational time is small. But, with Shooting method, the sensibility of the initial condition of the adjoint state, go slower time with a lot of iterations to find an optimal solution.

state $\{x, y, z, \psi\}$ and their desired trajectories $\{0, 0, 0, 2\}$ is ensured in 141 iterations with 9.63 second, and final time $t_f = 0.77036$ second. And the results which are given by Shooting method are presented in figures 9-14. Final time with Shooting method is 0.7889 second, in 2 iterations with 15.869417 seconds. To ensure the convergence of Newton method which is the method’s used to founded the zero of the Shooting function, we uses a result of the book of Ortega and Rheinboldt [27]; indeed if the discretization step $h_{ij}$ are small and tend to zero.