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Applying Massively Parallel Computing to Multi-Scale Ka Dual-Band Transmit-Array Analysis using FETI-2LM

André Barka, Sérgio A. Matos, *Member, IEEE*, Jorge R. Costa, *Senior Member, IEEE*, Carlos A. Fernandes, *Senior Member, IEEE*, Hassan Chreim

Abstract—Transmit-arrays (TAs) are a popular cost-effective solution for high-gain antennas at millimeter waves (mmW). The design of these antennas relies on the fine tuning of the sub-wavelength unit-cells that compose the aperture. The intricacy of the unit-cells increases as new features are implemented, such as dual-band operation and wide-angle beam steering, making this antenna even more computationally challenging. In this work, a high gain (25 dBi @ 20 GHz and 28 dBi @ 30 GHz) multi-scale Ka dual-band TA for beam-steering applications is analyzed using a massively parallel implementation on multicore clusters of the Finite Element Tearing and Interconnecting method, with a two Lagrange Multiplier (FETI-2LM) technique.

The main contribution of this work is the implementation of 3D non-periodic grids with sub-domains of different scales, that is suitable for proper modelling of different regions of the antenna (horn feed, lens cells, air region). An automatic batch file procedure is proposed for the TA meshing, allowing the limited set of constitutive unit-cells of the TA to be meshed separately. Additionally, we are using a block-Krylov strategy to efficiently capture the TA beam-steering capability, without restarting from scratch the interface problem for each feed position. Since other Finite Element Method (FEM) results were not accessible due the size of the problem, the FETI-2LM numerical results provided in this work are compared with other MultiLevel Fast Multipole Method (MLFMM) [1] and Transmission Line Modeling Method (TLM) [2] of CST Microwave Studio solvers, as well as with the measured results of the corresponding Ka dual-band prototype. The computational infrastructure has been used only a fraction of its capacity. Therefore, a much higher gain design (40 dBi) can be assessed, opening a new realm of applicability of TAs in the space segment usually occupied by reflector-based solutions.

Index Terms—Finite Element Tearing and Interconnecting (FETI), Finite Element Method (FEM), Domain Decomposition, Krylov spaces, Ka-band transmit-array, GID pre-processor.

I. INTRODUCTION

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KA-band high-gain antennas providing broadband access services are required for installation on High Throughput Satellites (HTS) and High Altitude Platforms (HAPs). Recent works show that transmit-arrays can also be a viable solution for Satcom On The Move (SOTM) ground terminals in the Ka-band [3], [4]. In [3] a simple mechanical beam-steering antenna concept has been proposed, in which high-gain (28 dBi) is conciliated with wide-angle elevation beam-steering ($-50^\circ, 50^\circ$) by a relative in-plane translation of a thin offset flat lens above a stationary primary feed. The full azimuth coverage is then obtained by a 360° rotation of the TA. In [4], a dual-band version (20 and 30 GHz) of the TA was developed, which required additional complexity of the unit-cell design. The working principle of this antennas is shown in Fig.1. In the single and dual-band cases, the electric size (at 30 GHz) of the aperture is $20 \lambda \times 15 \lambda$ and it comprises several thousand of unit-cells (4524 in [3] and 2352 in [4]) arranged in a non-periodic pattern to collimate and offset the incoming spherical-wave. Although full-wave evaluation of these antennas using commercial solvers is possible, it can take several hours, or even days, to complete, even using high-end state-of-the-art workstations. The optimization of the antenna is quite challenging. Several approximate methods can be applied, based on a mix of analytical modeling and full-wave electromagnetic simulations [5], Physical Optics and Geometrical Optics (PO/GO) techniques [6] and even equivalent dielectric homogenization techniques [7]. However, full-wave simulations of the real structure cannot be removed from the design process, especially when a complex fabrication process is involved. One such example is the sensitive analysis of manufacturing imprecisions, which cannot be undertaken using approximate methods. Moreover, fast and accurate evaluations are required to properly optimize the TA. We should stress that the TA design is intrinsically approximate, since it extrapolates the periodic behavior of the unit-cells to the non-periodic background of the TA. This is usually referred to as the quasi-periodic approximation [8], which does not hold for the regions of the lens where fast phase variations occur. Finally, when higher gains are required, as in the case of Geosynchronous Equatorial Orbit (GEO) satellite terminals (40 dBi) [9], all of the aforementioned problems are exacerbated and the solution is not easily accessible with commercial softwares as CST Microwave Studio and HFSS even on powerful workstations with large memory resources. We should stress that, a 40 dBi gain cannot be achieved by simply scaling-up a

given 30 dBi TA design [10], since aberrations and fast phase variations also scale up. The application of massive parallel computing to TAs is therefore very relevant for this active field of research.

Fig. 1. TA beam-steering antenna concept implemented in [2] for dual-band operation. Focal distance between the horn and the lens $F = 100$ mm, length along the X axis $D_{ax} = 196$ mm, width along the Y axis $D_{ay} = 147$ mm

Fig. 2. Exploded image of the TA domain decomposition (from bottom to top: horn (cyan), empty (purple and green), TA (red))

The objective of this work is to evaluate the potential of the FETI-2LM domain decomposition method [11] [12] for the simulation of large scale transmit-arrays such as those described in Fig. 2 and Fig. 1 on massively parallel clusters using MPI programming. This kind of resources is not farfetched for TA simulations and available High Power Computing resources in France and Europe can be used for this purpose. In France the "Grand Equipement National de Calcul Intensif" (GENCI) consortium provides computing hours to research organizations within the framework of innovative projects [13]. In Europe, the Partnership for Advanced Computing in Europe (PRACE) provides access to world class HPC computing resources [14]. Recent results we obtained within the context of ultra large band microwave materials [15], [16], using the FETI-2LM technique [11] [12], have been completed in this work for antenna gain simulations. TA translation over fixed feed is equivalent to fixed TA with feed in multiple positions. Therefore, a multi-source FETI-2LM formulation [12] is used to address this TA simulation. The iterative

resolution efficiency of the interface problem for assembling the local solutions coming from the overall sub-domains is not favorable due to linearly independent near fields of the different sources (the incident fields on the lens for several feed positions are clearly different), even with standard Krylov recycling techniques. To remedy this problem we used the efficient block-Krylov algorithms initially developed in [12].

The main particularity of our implementation remains in the fact that we do not mesh the initial global domain before the resolution of the electromagnetic problem. As a consequence a mesh splitter tool like METIS [17] is not necessary to decompose the large scale initial mesh into sub-domains, as was done for example for array antennas covered by a radome in [18]. Our method is general and efficient by introducing 3D multi-scale non-periodic grids with sub-domains that are not necessarily of the same size (horn feed, lens unit-cells and empty sub-domains described in Fig. 2 and Fig. 3). Each sub-domain of our decomposition belonging to a limited set of unit-cell meshes, is locally meshed with an automatic procedure of the GID preprocessor [19] based on GID batch files and Tcl scripts [20]. In this way, very complex TA antennas including very fine details (width of loop metallizations comprised between $\frac{\lambda}{200}$ and $\frac{\lambda}{50}$) are efficiently represented with only a few local meshes. The interest of this strategy is that since the method is implemented on parallel machines, each computation core is allocated exclusively to a sub-domain (elementary cell of the TA). Then, with at least 4 GB of memory allocated exclusively to a sub-domain (elementary cell of the TA), relatively fine meshes can be implemented easily in each sub-domain. This methodology makes unnecessary the sometimes very slow adaptive mesh refinement techniques.

The paper comprises the following sections. Section II presents briefly the implementation of a total field FETI-2LM formulation with block-Krylov multi-source algorithms to speed-up the beam-steering simulations of TAs and describes an effective meshing strategy for fundamentally non-periodic applications with sub-domains of different scales. Section III completely describes the geometry of the 30 dBi dual-band TA, as well as cross comparison of FACTOPO (FETI-2LM) simulation results with CST simulation results (TLM and MLFMA) and experimental results. Finally, section IV discusses the efficiency of parallel computation using the new FETI-2LM implementation and gives, by extrapolation, an estimation of the memory and number of computing cores required for a 40 dBi TA simulation, following a recent improvement of the FETI-2LM preprocessing for space radar surveillance [21].

II. FETI-2LM IMPLEMENTATION FOR TRANSMIT-ARRAY SIMULATIONS

The Finite Element Method (FEM) is one of the most successful frequency domain computational methods for electromagnetic simulations [22] of antennas or microwave circuits engineering applications. For computationally challenging TA applications, FEM method requires the solution of problems with billion of unknowns, that are very difficult to solve with

Fig. 3. Three types of sub-domains: lens sub-domain (left side), empty sub-domain (center), horn sub-domain (right side)

a single computational domain using commercial softwares as CST Microwave Studio [23] or HFSS [24]. Such simulation problems can be very difficult to solve, especially when the problems of interest involve multi-scale geometries, with very small details compared to the wavelength, are used in Ka band transmit-array applications. From this perspective, the FETI-2LM method proposed to solve the Maxwell equations [11], [25], [26], is splitting the initial computational domain into non overlapping sub-domains. Finite Element solution of the local problems are obtained first with the Intel PARDISO direct solver [27]. Then, 2 Lagrange Multipliers are introduced to impose tangent field continuity at the interfaces of the sub-domains, and the reduced problem at the interfaces is solved

with an iterative ORTHODIR method [28].

The scanning of the beam is obtained by linear translation of the primary source (here the horn placed under the TA). Consequently, the analysis of a large number of beams , requires efficiently computation of multi-source problems with the FETI-2LM iterative method. Once the interface problem was solved with the Krylov space method for the first source (first position a_0 of the primary feed under the TA as described in Fig. 1), a set of search direction vectors was built and can be stored in memory. These direction vectors obtained for all the successive primary feeds investigated under the TA provide a natural set of vectors to be used for implementing a preconditioner which decreases the number of iterations during ORTHODIR iterations. With this technique, the number of new iterations required for each new feed position decreases dramatically. It is well established that this recycling technique is very efficient for linearly dependent sources (plane wave excitations for example) as discussed for example in [12] where an acceleration of the calculation time by a factor of 115 is obtained. Unfortunately, it is not the case for TA applications for which the translation of the primary feed above the TA implies the sources to be linearly independent resulting in a much less rapid convergence of the recycling algorithm. To fix this problem, we used the block-Krylov recycling [12], whose main consequence is to improve the parallel efficiency of the method. The idea consists in solving the successive problems in blocks, with a few right-hand sides (typically 10-20) solved at the same time. This technique consists in computing several search directions at each iteration of the interface problem and all of the search directions of the block are used for all of the right-hand sides. The efficiency of the block-Krylov recycling technique allowing an acceleration by a factor 11 of the simulation has been demonstrated in section III Tab. X for the calculation of the complete Ka dual-band transmit-array proposed in this work.

Fig. 4. Zoom on lens unit-cells to be meshed (front view)

Fig. 5. Dual-band unit cell (geometry of the 30 lens cells)

The complete mesh of the dual-band TA (Figure 2), which is very tricky to achieve and requires considerable memory resources, is therefore not built into our simulation methodology. The TA being non-periodic, a single domain pre-treatment step would be long and difficult to implement even with an automatic mesh splitter such as METIS [17]. The alternative strategy proposed in this work makes it possible to virtually build the mesh of the complete TA with only 3 types of sub-domain meshes ("lens unit cell", "empty unit-cell" and "horn" described on Fig. 3). As a consequence, our implementation of the FETI-2LM method only requires that the mesh has a reasonable size for each sub-domain. Each unit-cell of the TA has been designed in [4] to provide a phase correction at position (x,y) defined by:

$$\Phi_{lens}(x,y) = k_0 \sqrt{x^2 + y^2 + F^2} - k_0 x \sin \alpha_0 \quad (1)$$

where F is the focal distance and α_0 is the zenithal beam angle when the feed is at the central position ($x=0, y=0$). They

comprise metallized layers of concentric squared patches and strip loops printed on substrate layers as described in Fig. 3 and Fig. 5. The in-plane TA is designed with a set of $N_{elem}=30$ elementary cells populating the dual-band transmit-array. The geometry and mesh is generated automatically via batch files that are executed by the GID software (Fig. 4). The batch files used to create the geometry are interfaced to an Excel table giving the geometric dimensions involved in the $N_{elem}=30$ cells (squared patches and strip loops geometrically described on Fig. 5). Finally, a Fortran script is interfaced with another Excel table (Tab. I) containing the position (x,y,z) and type of each of the $N_{TAcels}=2352$ sub-domains of the complete TA to completely populate the TA by creating N_{elem} GID batch-files. The N_{elem} local meshes are then processed with short *Tcl* [20] script called in the command line of the GID preprocessor that launches the geometry and mesh batch files. In summary, the simulation strategy used is as follows:

TABLE I
STRUCTURE OF THE SHIFT FILE DESCRIBING THE POSITION AND GEOMETRY TYPE (FROM 1 TO 30) OF EACH OF THE 2352 IN-PLANE SUB-DOMAINS

Sub-domain	Type	x	y	z
1	9	-0.097	0.072	0.0
...	...	...	...	...
2352	6	0.096	-0.072	0.0

- Step 1: Design and optimization of the $N_{elem}=30$ unit-cell geometry with CST Microwave Studio [23]
- Step 2: Automatic mesh generation of the $N_{elem}=30$ elementary unit-cells with the GID pre-processor [19]
- Step 3: Assignment of the $N_{elem}=30$ elementary mesh cells populating the lens to the sub-domains and calculation cores through Tab. I
- Step 4: FETI-2LM simulations using massively parallel clusters
- Step 5: Extraction of the radiation patterns of the TA

III. SIMULATION RESULTS OF THE MULTI-SCALE DUAL-BAND TRANSMIT-ARRAY

This transmit-array simulation test case was proposed as part of the ISAE-2018 electromagnetic workshop on radar signatures and the geometry is given through STEP files that can be downloaded from the ISAE website [29]. In order to cross-check the FETI-2LM results, several electromagnetic solvers of the CST Microwave Studio tool have been used: the iterative MultiLevel Fast Multipole Method (MLFMM) [1] and the Transmission Line Modeling Method (TLM) [2]. A FEM solution, to be compared with FETI-2LM solution, could not be obtained with the CST solver, due to the large size of the TA problem (few billion unknowns) when solved with a single domain in its current non parallelized version. The objective is not to compare the intrinsic performances of these methods because the simulations could not be done on the same computer. The objective is only to recall their particularities for these TA calculations and to give some trends on the memory occupation, the computing times as well as their level of parallelization. Remember that the frequency domain FETI-2LM and MLFMM methods can use very effective strategies

for applications with multiple sources ([30], [12]). This is not the case with the TLM method where the whole simulation must be repeated completely for each new position of the primary source. On the other hand, the TLM time domain method is very effective to solve the problem on a broad frequency band.

The dual-band TA designed and detailed in [4] operating at 20 GHz and 30 GHz is composed of 2352 unit-cells. The 30 dual-band constitutive unit-cells (Fig. 5) designed in [4] are properly distributed throughout the aperture to provide beam collimation. These unit-cells are composed of seven metallized layers of concentric squared patches and strip loops printed on six Rogers Duroid 5880 substrate layers (Tab. II). The thickness of each dielectric layer is $s = 1.617$ mm and the in-plane width of the unit-cell is $P = 3.5$ mm. The dimensions of the metallization patterns of each layer (in mm) can be found in Fig. 5 and Tab. II. One can observe that the width W_j of the loop metallization is comprised between $\frac{\lambda}{200}$ and $\frac{\lambda}{50}$ while the width L_j of the patches is comprised between $\frac{\lambda}{25}$ and $\frac{\lambda}{4}$ at 30 GHz (Tab. III). The unit-cell details are then very small compared to the TA aperture at 30 GHz ($20 \lambda \times 15 \lambda$) and the total calculation volume ($18 \lambda \times 20 \lambda \times 15 \lambda$) making the problem very difficult to solve.

TABLE II
DUAL-BAND UNIT-CELL

Rogers Duroid 5880	
Dielectric permittivity	$\epsilon_r=2.2$
Dielectric loss tangent	$\tan \delta =0.0009$
Metal	PEC with zero thickness
Unit cell	
In plane dimensions, (P $\times$ P) (mm ²)	3.5×3.5
Thickness of the dielectric layers, s (mm)	1.617
Total height, T (mm)	9.702
Metallization dimensions	
Patches in-plane dimensions (mm ²)	$L_j \times L_j$ j=1,4
Strip loops widths (mm)	W_j , j=1,4

TABLE III
SIZE OF SUBDOMAINS IN TERM OF WAVELENGTH AT 30 GHz

Lens unit cell	
(PxP) (λ^2)	$0.35\lambda \times 0.35 \lambda$
lens unit cell height, T (λ)	0.97λ
loops width, L_j	$\frac{\lambda}{200} \leq W_j \leq \frac{\lambda}{50}$
patches width, L_j	$\frac{\lambda}{25} \leq L_j \leq \frac{\lambda}{4}$
Air unit cell	
(PxP) (λ^2)	$0.35\lambda \times 0.35\lambda$
Air unit cell height, H_1, H_2 (λ)	$4 \lambda, 4.8 \lambda$
Horn cell	
Aperture ($P_{horn} \times Q_{horn}$) (λ^2)	$3.6 \lambda \times 3.15 \lambda$
Horn height, T_{horn} (λ)	7.5λ

Two 15-dBi standard-gain rectangular horn antennas are

used to illuminate the transmit-array at each band (Flann Microwave N° 20240 15 used for 20 GHz and Flann Microwave N° 22240 15 used for 30 GHz), with their phase centers positioned at a distance of 100 mm from the bottom face of the transmit-array. One horn orientation (y-pol) is used to simulate the response for each lens position (Fig. 1).

The TA is designed to produce a beam offset $\alpha_0 = 40^\circ$ for a focal distance $F = 100$ mm when the horn is aligned with the geometric center of the TA in the xy plane. The top view of the lens is represented in Fig. 1. The aperture size is 196×147 mm² (Tab. IV). One should note that the TA is symmetric relative to the xy plane. Elevation beam-scanning is produced by linear displacement of the TA (Fig. 1). This displacement is controlled by the parametric variable a . For this test, three TA positions are considered: $a = 0$ mm (corresponding to the center of the TA aligned with the feed phase center), $a = -15$ mm (TA moves to the right) and $a = 44$ mm (TA moves to left). Please note that the supporting structure shown in Fig. 1 was not considered in the simulation because it was confirmed that it has no significant effect on the antenna performance.

TABLE IV
DUAL-BAND TA ANTENNA

TA	
Focal distance, F (mm)	100
In-plane dimensions (mm ²)	196×147
Number of unit cells	2352
Beam direction ($^\circ$)	
Without lens displacement, $a = 0$ mm	40°
Right lens displacement, $a = -15$ mm	50°
Left lens displacement, $a = +44$ mm	15°

The FETI-2LM numerical simulation of the dual-band TA is implemented on 9,601 Intel Xeon Broadwell E5-2680v4 processing cores of the Bull high-performance parallel machine "Occigen" of the "Centre Informatique National de l'Enseignement Supérieur"(CINES). The configurations of the calculators used by the software packages CST and FACTOPO are specified in Tab. V and the problem sizes at 20 GHz and 30 GHz are specified in Tab. VI.

TABLE V
CONFIGURATIONS OF MACHINES USED

Software	Method	Machine
CST	MLFMM	2 Xeon Dual (E5-2620 V4 @ 2.1 GHz), 8 cores
	TLM	2 Xeon Dual (E5-2620 V4 @ 2.1 GHz), 8 cores
FACTOPO	FEM-FETI	Intel E5-2690 V4 @ 2.8 GHz, 9,601 cores

The simulations were carried out with MLFMM, TLM and FETI-2LM methods on meshes of different types and of very large sizes up to 6.2×10^{10} tetrahedral cells (CST/TLM) and 2.26×10^9 edges (FETI-2LM) as indicated in Tab. VI. We can first observe that the results of the full-lens problem including the horn source were obtained with both CST methods on computers with 8 Xeon E5-2620V4 cores (MLFMM, TLM).

TABLE VI
TA PROBLEM SIZE AT F = 20 GHz AND F = 30 GHz

Software	Method	Problem size
CST	MLFMM	20 GHz: 6,80x10 ⁵ mesh cells 30 GHz: 1,36x10 ⁶ mesh cells
	TLM	20 GHz: 5,50x10 ¹⁰ mesh cells 30 GHz: 6,20x10 ¹⁰ mesh cells
FACTOPO	FEM-FETI	2,26x10 ⁹ unknowns (edges)

TABLE VII
COMPUTING RESOURCES AT F = 20 GHz AND F = 30 GHz (ONE BEAM)

Software	Method	Elapse	Memory
CST	MLFMM	20 GHz: 7h10m 30 GHz: 19h25m	20 GHz: RAM 79 GB 30 GHz: RAM 126 GB
	TLM	20 GHz: 76h06m 30 GHz: 59h37m	20 GHz: RAM 29 GB, GPU 10 GB 30 GHz: RAM 33 GB, GPU 12 GB
FACTOPO	FEM-FETI	20 GHz: 45m 30 GHz: 45m	RAM peak/core: 2.5 GB

As illustrated in Fig. 2, the FETI-2LM domain decomposition method uses 9,601 subdomains (lens cells, air cells and horn cell) assigned to 9,601 cores Intel E5-2690V4. Indeed, the physical in-plane lens comprises 56×42 unit-cells completed by 4 empty elementary cells of the same size all around the array in order to remove the absorbing boundary surface far away from the physical array (1.4λ). It gives an in-plane surface of $64 \times 50 = 3200$ cells of dimensions $3.5 \text{ mm} \times 3.5 \text{ mm} \times 9.702 \text{ mm}$. Two layers comprising 3200 empty cells of height $H_1=40.25 \text{ mm}$ and $H_2=48.61 \text{ mm}$ are then introduced so as to place the horn at $F = 100 \text{ mm}$ from the lens. The total number of sub-domains is then $3 \times 3200 + 1 = 9601$ (Fig.2).

The stop criterion in the time domain TLM solver is set to -40 dB. As a matter of fact, the TLM solver injects a transient pulse inside the structure via the port at the horns input, and the results in frequency domain are obtained by a classic Fourier Transform. The precision of the results is directly related to the amount of residual energy that is presented inside the calculation domain. The value of -40 dB is the value of the residual energy that remains in the calculation domain. One should know that the energy is evacuated through three phenomena: radiated energy, reflected energy to the port (mismatch) and dissipated energy (losses).

The convergence of the 2 frequency domain methods (CST/MLFMM and FEM-FETI) is presented in Fig. 6 and Table. VIII where the gain patterns are plotted for a stopping criterion of $\epsilon = 10^{-2}$ and $\epsilon = 10^{-3}$ (residual norm $\frac{\|Ax-b\|}{\|b\|}$ of the global linear system to be solved). It is observed that the level of the main lobe (40°) and reflection lobe (about 140°) is reasonably obtained with the FEM-FETI with a stopping criterion limited to 10^{-2} (orange curve) as the deviation is of 0.5 dBi for the main lobe and 1.2 dBi for the reflection lobe. Furthermore, the MLFMM convergence is not reached with a stopping criterion of 10^{-2} (green curve). Indeed, the level of the main lobe and reflection lobe is 10 dBi lower

than that obtained for a convergence of 10^{-3} (blue curve) and confirmed by the measurements (black curve).

Fig. 6. Gain patterns convergence at 20 GHz for position $a=0 \text{ mm}$

TABLE VIII
MLFMM AND FEM-FETI CONVERGENCE AT 20 GHz FOR POSITION $a=0 \text{ MM}$ AND FOR 10^{-2} AND 10^{-3} CONVERGENCE

F=20 GHz	ϵ	Gain (dBi)		
		MLFMM	FEM-FETI	Meas.
Main lobe (40°)	10^{-2}	10,2	24.7	24.0
	10^{-3}	24.2	24.2	24.0
Reflection lobe (140°)	10^{-2}	5.5	17.4	14.8
	10^{-3}	15.6	16.2	14.8

In terms of memory we observe on Tab. VII and Tab. X that the FETI-2LM method is using 2.5 GBytes/core $\times 9\,601$ cores, that is, approximately 24 Terabytes of memory which is very much higher than the 39 GBytes of the TLM method or even the 79 GBytes of the MLFMM. This particularity comes from the fact that we have chosen to assign a computing core to each sub-domain even to those of smaller size located in the lens. A significant reduction in memory occupation could be obtained by allocating several sub-domains to a computing core as proposed in [21]. Finally, we gain a factor of 10 in simulation time at 20 GHz and a factor of 26 at 30 GHz compared to the MLFMM, but the strategy for using the computational cores (which is not optimal here) for this FETI-2LM simulation requires 303 more memory than MLFMM method. In terms of accuracy, the gain radiation patterns obtained with the Multilevel Fast Multiple Method (MLFMM) [1], the Transmission Line Modeling Method (TLM) [2] and the FETI-2LM, are considered to be globally comparable and acceptable compared to measurements as indicated by the cross comparisons in Fig. 7 and Fig. 8 for the main lobe ($\alpha_0 = 40^\circ$) and the reflection lobe ($\alpha_0 = 140^\circ$) of the TA. However, we can observe in Tab. IX fluctuations of around 1 dBi for the different solutions on the main lobe and reflection lobe for both frequencies 20 GHz and 30 GHz, which attests to

the difficulty of simulation of large-scale dual-band transmit-arrays.

TABLE IX
SUMMARY OF SIMULATED AND MEASURED MAIN LOBE

Freq. (GHz)	a (mm)	Gain (dBi)			
		MLFMM	TLM	FEM-FETI	Meas.
20	-15	22.6	23.6	22.8	22.4
	0	24.2	24.7	24.2	24.0
	+44	25.5	25.6	25.1	24.1
30	-15	26.2	26.8	26.6	27.1
	0	28.2	28.2	28.2	28.8
	+44	28.1	27.6	27.8	28.2

TABLE X
FETI-2LM COMPUTATIONAL STATISTICS

Mesh parameters (unknowns)	
Unit-cell sub-domain (average)	432, 539
Skeleton interface problem	478 millions
Total unknowns of the TA	2.26 billion
Simulation parameters	
Number of cores (Intel Xeon E5-2680v4)	9, 601
Number of iterations (interface problem)	260
Stopping criterion	$\epsilon = 10^{-2}$
Elapse time (1 feed position)	45 minutes
Elapse time (30 feed positions sequential)	22.5 hours
Elapse time (30 feed positions with block Krylov)	2 hours
Memory peak (per core)	2.5 GB

IV. PARALLEL COMPUTATION EFFICIENCY

Tab. X shows the overall computational statistics observed for the FETI-2LM dual-band TA simulation. The multi-scale aspect of this domain decomposition simulation is shown in Tab. III and impacts significantly the FETI-2LM simulation. Indeed, we observe, for example that the height of the different types of subdomains varies between 0.97λ (lens cell) and 7.5λ (horn cell). Likewise, the in-plane unit cell width varies between 0.35λ (lens cell) and 3.6λ (horn cell) and the width loop metallization is very thin ($\frac{\lambda}{200}$). However, we can notice that the effort required to generate the mesh is quite limited. We need only to mesh a few sub-domains with the automatic procedure described previously in section. II. The complete mesh of the TA is indeed never built, which is a considerable advantage of the method.

TABLE XI
SUBDOMAINS UNKNOWNNS RELATIVELY TO FIG. 2

Subdomain	Unknowns
Horn (cyan)	678,704
Empty (green)	227,979
Empty (purple)	189,964
Average unit-cell (red)	432,539

As a consequence to the multi-scale nature of the problem to be solved, the number of unknowns in the sub-domains varies significantly up to 28 % as shown in Tab. XI. It impacts the local matrix factorization time and the load balancing during

FETI-2LM simulations, which is far from perfect in all of the calculation steps (local matrix factorisation, upward/backward local resolutions in the subdomains, and iterative ORTHODIR resolution of the interface problem). Remember that the dimension of the skeleton interface problem is 478 million (Tab. X). The iterative resolution of the interface problem requires, at each iteration, the resolution of the local problems as well as the exchange of the fields at the interfaces of all the subdomains. The imbalance of the local matrices factorizations and local problem solving phases (upward and backward substitutions) resulting from the different local mesh sizes then penalizes the convergence of the ORTHODIR method. This is due to the necessary synchronization of the core processors. Nevertheless, the computation times of 45 minutes for one beam attests the good level of parallelization of the method.

Even if the machines used are different from the ones indicated previously in section III, note that the FETI-2LM simulation time is much lower than that obtained with CST (19 hours and 25 minutes with the MLFMM and 59 hours and 37 minutes with the TLM at 30 GHz). To the best of our knowledge, the effectiveness and scalability of the TLM and MLFMM methods of CST has not been yet demonstrated on parallel machines with more than 9000 cores. Note also that the FETI-2LM method requires 20.5 hours by restarting from scratch for example if 30 beams are requested, but only 2 hours if using the block-Krylov recycling strategy (Tab. VII). It allows us to consider using it in the optimization phase of such type of TA.

Another very important point of the FETI-2LM method is its capability to be scaled for TA simulations which makes it a candidate for TAs of much higher gain. It has already been shown in [12], for example, that significantly increasing the size of periodic arrays has very little impact on the computation time. Indeed, Fig. 3 of reference [12] shows that the elapsed time per frequency observed for 4 metamaterial arrays with progressively increasing size (22x22, 28x28, 56x56 and 85x85) is only varying from 3 minutes (0.3 billion unknowns) to only 10 minutes (2.05 billion unknowns) on the Xeon Haswell cluster for a convergence lower than 10^{-3} . This curve is linear and illustrates the good scalability of the method showing that increasing the number of sub-domains by a factor of 4 is increasing the simulation time by only 3 minutes. Recall that, when using the FEM, the occupation memory depends on the line bandwidth of the matrix (number of neighborhood edges of each edges of the mesh) and can vary considerably according to the regularity of the mesh. Our TA application has many areas with a large scale ratio which considerably affects the quality of the mesh and increases the bandwidth of the matrix. As a consequence it can lead in the sub-domains concerned to an increase of the required memory necessary for the local matrix factorization. Nevertheless, our domain decomposition implementation (number of unknowns in the sub-domain) is not optimal for this TA simulation. As a consequence we can optimize the use of the memory of each of the computational cores following the procedure recently proposed in [21] resulting in important savings in memory and computation time. Indeed, based on the Tab. XI, by increasing the number of unknowns in the sub-domains, it is possible

to reduce the number of "lens" sub-domains by a factor of 2 (1 core allocated to 2 sub-domains) and by a factor of 4 the number of "air" sub-domains (1 core allocated to 4 sub-domains) located between the horn and the lens. The total number of cores required for the TA simulation will then be: $1600 + 2 \times 800 + 1 = 3201$ and the memory required will be reduced to 8 Terabytes. Ka-band Geosynchronous Equatorial Orbit (GEO) satellites requires at least 40 dBi gain antennas [9], corresponding to apertures of 60λ . To achieve a gain of this order we would need to scale 3 times the dimensions of the analysed dual-band TA. In this case, massive parallel computing is the only option to perform the full-wave analysis of these extremely electrical large structures. In fact, we can show as it follows that the presented infrastructure can deal with these large antennas without compromising the accuracy of the results. By extrapolation of our previous simulation results, the lens will be then made up of 22,188 unit-cells of width 3.5 mm. Therefore, by using the optimized fill-in of the cluster with 1 core allocated to 2 "lens" sub-domains and one core allocated to 4 "air" sub-domains, the total number of cores for the full dual-band 40 dBi TA simulation (lens and feed) will be $11,094$ (lens) + $33,182$ (air) + 1 (horn) = $44,377$. Consequently the memory required for this simulation is estimated to 111 Terabyte ($44,377 \times 25$ Gb). The simulation of the 40 dBi dual-band TA is then reasonably accessible with the Occigen cluster already equipped with 50,544 cores.

V. CONCLUSION

This work presents an application of the FETI-2LM Domain Decomposition Method to the calculation of highly complex transmit-array antennas in the Ka-band using FEM. The method is implemented on massively parallel clusters and a block-Krylov recycling algorithm is implemented in order to accelerate the TA simulation with mechanical beam-steering. The orientation of the beam is obtained by in-plane translation of the TA above the primary source, for a large number of beams. The simulation of all required TA positions is efficiently simulated with the FETI-2LM iterative method. An effective strategy based on the implementation of 3D non-periodic grids for fundamentally non-periodic applications with sub-domains of different scales is also proposed. Its main ingredient is the use of an automatic GID batch file procedure and Tcl scripts which allow the limited set of constitutive unit-cell of the TA to be meshed separately. Thus, the implementation of relatively fine meshes due to very fine details ($\frac{\lambda}{200}$ at 30 GHz) in the sub-domains is rendered quite simple making unnecessary the sometimes very slow adaptive mesh refinement techniques. This work confirms the interesting performances in terms of accuracy and calculation time of the FETI-2LM domain decomposition methods in electromagnetism allowing relatively fast computation of transmit-arrays gain radiation patterns for multiple beams. To the best of our knowledge, FETI-2LM methods had not previously been used for the simulation of complete transmit-array antennas with primary sources. We have also shown that the method can be used even when higher gains are required, as in the case of Geosynchronous Equatorial Orbit (GEO) satellite terminals (40 dBi), with apertures of 60λ .

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Fig. 7. Gain patterns at 20 GHz for positions $a=0$ mm, $a=-15$ mm and $a=+44$ mm

Fig. 8. Gain patterns at 30 GHz for positions $a=0$ mm, $a=-15$ mm and $a=+44$ mm