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Chaotic Voltage: An Electronic Entropy Source

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The chaotic terminal voltage dynamics of a semiconductor laser subjected to external optical feedback are utilized to directly generate electronic random number streams with minimal post-processing at rates of 40–120 Gb/s, thus obviating the need for optical-to-electrical conversion and facilitating integration with high-speed computers and devices. Further, a comparison of the terminal voltage to the optical intensity being utilized as entropy sources is performed. It is shown that the voltage dynamics have an inherently larger entropy, a reduction in delay signature, and a more suitable distribution for generating random bit streams.

Random bit sequences are commonly utilized in cryptography, machine learning, and simulations. But, due to the need for fast rates and ease of generation most random bit sequences are generated using deterministic algorithms and not truly random entropy sources. Therefore, fast physical random bit generators that can be easily integrated into computers or electronic systems are of interest for these applications. In this paper, we point the way to such a device by utilizing the chaotic terminal voltage of a laser diode. Specifically, laser diodes undergo a profound modification to their dynamical variables, i.e., carrier density, photon number, and optical phase, when subjected to optical self-feedback. It is well-known that these systems can display chaotic dynamics and have been utilized as an archetypical testbed for nonlinear dynamics [1] and for high-speed information processing applications [2]. In the latter context, the first use of sampled chaotic optical intensity fluctuations as an entropy source utilized for high-speed random number generation was demonstrated by Uchida *et al.* [3]. The generated bit streams were an order of magnitude faster than any other previously utilized physical source. Since then, various realizations and improvements utilizing optical chaos have been demonstrated allowing for faster generation rates up to Tb/s. Various methods based on creating symmetric distributions without bias through post-processing techniques have been proposed, for example, using derivatives [4–6] and finite differences [7]. To further improve the paradigm, other novel chaotic laser sources and schemes have been demonstrated, such as, through bandwidth enhancement [8], in photonic integrated circuits [9], ring lasers [10, 11], heterodyning [12], terahertz optical asymmetric demultiplexers [13], chaotic solitary vertical-cavity surface-emitting lasers [14], polarization rotated feedback [15], as well as, long on-chip optical feedback [16]. Additionally, fast rates have been demonstrated utilizing parallel significant bits [17] and

parallel-laser schemes [18]. The bit streams in most of the previously cited works are post-processed offline. Interestingly in some cases, real-time implementations have been demonstrated, using delayed feedback lasers [19] and photonic integrated circuits [20]. Recently, an interesting concept, *physical white chaos*, has been proposed and the enhanced chaos leads to random number generation at 320 Gb/s [21]. While there have been many improvements and schemes demonstrated in the field of random number generators based on chaotic lasers, in almost all cases advanced post-processing, complex experimental schemes, optical to electronic conversion, and/or exotic components are required for passing the standard randomness tests and for achieving ultrafast bit rates.

In this Letter, we demonstrate that the chaotic laser terminal voltage $V(t)$, as a surrogate for carrier density $n(t)$ in the gain medium [22–25], can be used as a novel, fully electronic, entropy source for random number generation at rates of 40–120 GS/s, where only simple post-processing is required when compared to utilizing the optical intensity $I(t)$. In order to understand the reason behind this improvement, we compare the voltage entropy source with the commonly utilized optical intensity which was measured simultaneously from the same chaotic laser. Specifically, the entropy sources are compared utilizing distributions, delay-correlation, and permutation entropy analysis. It is shown that the simple physical measurement of $V(t)$ is an inherently superior entropy source compared to the optical intensity, $I(t)$. Further, our approach has the benefit of simplicity, while still generating random bit streams at tens to hundreds of Gb/s. Specifically, the system, for the first time, offers a fast electronic entropy source that can be used to generate Gb/s streams without any optical-to-electrical conversion, thus facilitating the use of this entropy source for high-speed computing.

In the experiment, an unpackaged multi-quantum well DFB laser was utilized [25, 26] operating at 1550 nm. The laser was subjected to delayed feedback from a mirror placed at a distance L resulting in a delay time of τ . The sub-nanosecond AC component of $V(t)$ was

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FIG. 1. The relatively simple experimental setup is shown. An unpackaged, MQW laser is subjected to self-feedback from a mirror, after a time delay, τ , and distance L . The AC component of the terminal voltage is measured using a real-time oscilloscope (40 GS/s and 12 GHz).

measured using a high-speed oscilloscope (40 GS/s) connected to a RF probe, bias-T, and amplifier, as shown in Fig. 1. The resulting voltage time-series $V(t)$ were converted to binary sequences by simply retaining the least significant bit (LSB) from the output of the 8-bit analog-to-DC converter (ADC). The generated streams were then subjected to the NIST test for randomness [27]. In Table I, we can see that this simple operation can lead to bit stream at 40 GS/s ($=1 \text{ LSB} \times 40 \text{ GS/s}$), that pass the NIST suite. Furthermore, we studied the maximum number of bits that could be extracted and utilized in real-time. We found that the bit sequence generated by extracting 3 LSBs from the ADC with a bitwise exclusive-or operation with a delayed version of the bit sequence can also pass all the randomness tests in the NIST suite. This means that this still relatively simple scheme allows for a 120 Gb/s ($=3 \text{ LSBs} \times 40 \text{ GS/s}$) real-time bit stream generation with minimal post-processing. The XOR operation is a typical technique used to remove the bias from bit sequences [28]. Of note, the simultaneously measured $I(t)$ does *not* pass the NIST test with a single LSB with or without the XOR. Further, we wanted to verify that the chaotic dynamics are needed in order to pass the NIST test with 1 LSB. Thus, we tested if amplified spontaneous emission noise in $I(t)$ and $V(t)$ would pass by blocking the feedback and they did not.

The bit streams are able to pass the NIST test without any complicated post-processing due to the nature of the voltage entropy source. In the following, we use different tests to confirm the superiority of the voltage as an entropy source. As shown in our previous work [7, 29], the distribution of $I(t)$ is known to be exponential and skewed which leads to bias. This bias can be removed using post-processing techniques and through complex experimental schemes, as demonstrated by the state-of-the-art. But, as we will show, the terminal voltage dynamics have an inherently broader distribution. From Fig. 2, it is clear that the distribution given by $V(t)$ is broader than the exponential distribution of $I(t)$ which leads to significantly less bias in the voltage measurement. It is well known that $n(t)$ follows the optical intensity in the laser cavity as a result of the oscillations between photon and

TABLE I. Results of the NIST test for the terminal voltage measurement using a 1 Gbit stream. For the time series here the cavity length was 30 cm, the injection current was twice threshold, and the feedback strength is chosen such that the laser is operated in coherence collapse [25]. For the tests that produce multiple P-values and proportions the worst case is shown.

STATISTICAL	P-VALUE	PROPORTION	RESULT
Frequency	0.6798	0.992	SUCCESS
Block frequency	0.6392	0.992	SUCCESS
Cumulative sums	0.7095	0.988	SUCCESS
Runs	0.1927	0.993	SUCCESS
Longest run	0.6579	0.986	SUCCESS
Rank	0.4299	0.992	SUCCESS
FFT	0.4578	0.988	SUCCESS
Nonperiodic template	0.0156	0.983	SUCCESS
Overlapping template	0.3994	0.985	SUCCESS
Universal	0.1421	0.990	SUCCESS
Approximate entropy	0.3094	0.988	SUCCESS
Random excursions	0.5070	0.986	SUCCESS
Random variant	0.3917	0.993	SUCCESS
Serial	0.5281	0.994	SUCCESS
Linear complexity	0.4376	0.985	SUCCESS

electron populations. This leads to the expectation that the same distribution would be obtained from both measurements. In fact, neither the magnitude of the optical field nor $n(t)$ are measured directly. Instead, we measure $I(t) = |E(t)|^2$ and $V(t) \propto \ln[n(t)]$, as shown in [23], with $E(t)$ the complex electric field. An explanation for the difference in the distributions can be explained through a change of variable extension of the best fit exponential distribution of $I(t)$ and a natural log relationship of $n(t)$ to the measured $V(t)$. Specifically, if we let $I(t)$ be a continuous random variable with probability density function $f_I(I)$, and let $V(t) = u(I)$ be an invertible function of I with inverse function $I = g(V)$, then the probability distribution of V is $f_V(V) = f_I[g(V)]|g'(V)|$. If $f_I(I)$ is exponential [29]

$$f_I(I) = \frac{1}{b} \exp\left(-\frac{|x - \mu|}{b}\right) \quad (1)$$

then through change of variable we obtain

$$f_V(V) = \frac{1}{b} \exp\left(-\frac{|\exp[kV(t)] - \mu|}{b}\right) \exp[kV(t)] \quad (2)$$

with μ the mean, and k and b distribution fitting parameters.

Figure 2(b), shows the experimental fit to $f_I(I)$, and Fig. 2(a) demonstrates that Eq. 2 is a good fit of the experimental distribution from the voltage, $V(t)$. The goodness of fit was checked using the Kolmogorov-Smirnov test and underlying distributions matched Eqs. 1 and 2 with a 2% and 6% confidence, respectively. This observation confirms that the shape of the observed distribution of $v(t)$ is due to the inherent, physical, log relationship between the carrier density and the voltage. Small

FIG. 2. Comparison of $V(t)$ and $I(t)$ distributions from experimental data. The intensity is fit with the exponential distribution [29] and the voltage is fit with the distribution from Eq. 1. Both distributions were checked using the Kolmogorov-Smirnov test, and were found to match the underlying distributions with a confidence of 6% (a) and 2% (b), respectively. The voltage is broader in nature, thus, it is better suited for generating bit streams. The spreading of the distribution results from the fact the voltage is measured, which is the natural log of the carriers [23].

deviations in the fit of $v(t)$ might arise from experimental error, noise in the amplifier, as well as, propagating error in the original fit of $I(t)$. Nonetheless, the measured $V(t)$ leads to a distribution that has an order of magnitude larger variance and is broader resulting from the natural log relationship between $V(t)$ and $n(t)$. The increased symmetry leads to a decrease in bias of the bit sequence [28], thus explaining the possibility for the LSB of $v(t)$ to pass the NIST test without an additional XOR operation. The increased dispersion or variance means that the distribution is closer to an uniform distribution, leading to a better randomness source. Therefore, the exponential relationship between $n(t)$ and $v(t)$ can be considered as an inherent, physical post-processing that makes the voltage time series more amenable to RNG when compared to $I(t)$.

The second-order statistics of the randomness source also plays a role in its effectiveness. Specifically, reduction of time-delay signatures in the autocorrelation is of importance for generating effective random bit streams. The time-delay signature results from the external-cavity free-spectral range ($1/\tau$) or the boundary condition imposed on the system by the external mirror, as shown in Fig. 1. Figure 3 represents the measured auto-covariance function. The result depicted in this Figure demonstrates that there is less correlation ($\sim 75\%$) in $V(t)$ than in

FIG. 3. Comparison of time-delay signature in $I(t)$ and $V(t)$ chaos. There is a reduction in correlation at the delay for $V(t)$. Dashed line: $I(t)$; solid line: $V(t)$.

FIG. 4. Permutation entropy estimated with an ordinal pattern length $D = 5$ as a function of the embedding delay τ_s/τ . $V(t)$. Dashed line: $I(t)$; solid line: $V(t)$

$I(t)$ at τ , which further illustrates that the voltage entropy source is better suited for generating streams. More broadly, any application that requires the strong suppression or even absence of time-delay signatures [30], for example, chaos-based physical-layer secure communication [31] and chaotic lidar [32], would likely benefit from this chaotic source.

The entropy rate is a crucial parameter that determines the potential and the limits of a chaotic process for RNG [20, 33, 34]. To gain more insight into the advantages of using $V(t)$ over the commonly used $I(t)$, we carry out a permutation-entropy analysis. This analysis is relatively simple to compute and provides convincing information on the complexity even when computed on experimental data [16, 35–38]. It has been widely used for quantifying dynamical complexity in chaotic systems since the seminal work of Bandt and Pompe [38]. In particular, it has been shown that permutation entropy evaluated at specific timescales for external-cavity lasers can recover important features characterized by Kolmogorov-Sinai entropy rate, and is consistent with the notion of weak and strong chaos [35, 36]. Here we compare the results of permutation entropy estimated from the two physical entropy sources, *i.e.*, $I(t)$ and $V(t)$ in chaotic lasers. Of importance, the entropy calculations were per-

formed on the raw data extracted by the ADC without any post-processing, as suggested in [33]. Therefore, the physical entropy rate from the two different sources is compared, and deterministic post-processing is removed from the entropy calculation [33].

For the computation of permutation entropy, we use the ordinal pattern length $D = 5$ and assume the embedding delay τ_s as the control parameter, in order to obtain a measure of the dynamical complexity over multiple timescales. The results are shown in Fig. 4, clearly indicating that $V(t)$ has larger values of permutation entropy than $I(t)$ computed at the embedding delay τ_s matching harmonics and subharmonics of delay time τ . Accordingly, the entropy estimation in conjunction with results in Figs. 2 and 3 further confirm that the proposed terminal voltage $V(t)$ is a promising physical entropy source for fast random number generation, and thus providing a physical explanation of the main results in this work.

Further, the results are robust to experimental conditions if the dynamics are chaotic (coherence collapse). Specifically, L was varied from 30-60 cm, the injection current from 1.5-2.5 times threshold, and the feedback within 10 % and the bit streams generated from the measured chaotic voltage time series passed the NIST test. Contrarily, those extracted from the simultaneously measured chaotic optical intensity and spontaneous emission noise time series did not pass the NIST test with the same post-processing.

In conclusion, we have demonstrated that the simple

terminal voltage measurement of a laser subjected to optical feedback from a mirror can be utilized to generate random bit streams at 40-120 Gb/s thus circumventing the need for optical-to-electrical conversion. Such an electronic entropy source is therefore envisaged to be more easily integrated with high-speed computers. Our results demonstrate that the chaotic terminal voltage is a superior entropy source when compared to optical chaos, benefiting from a broader distribution, a smaller delay signature, and a larger permutation entropy at the delay. More importantly, the scheme demonstrated here is significantly simpler to implement by only requiring a laser, mirror, ADC, and minimal or no post-processing; depending on the speed of the ADC, it is possible to generate random numbers at tens of Gb/s while remaining robust to variations in various experimental parameters. Also, it represents a direct electronic source of chaos that is being utilized for generating these bit streams without optical to electrical conversion. Thus, the system could be implemented as a discrete package into an electronic application that directly needs random number streams at very fast rates. Moreover, any application which requires a reduction in time-delay signature and larger entropy, such as chaos-based secure communications and chaotic lidar, could also benefit from these voltage dynamics.

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