Entropy Measure of Generating Random Rough Surface for Numerical Simulation of Wave Scattering

Rui Jiang, Kun-Shan Chen, Zhao-Liang Li, Gen-Yuan Du, Wen-Jing Tian

► To cite this version:

Rui Jiang, Kun-Shan Chen, Zhao-Liang Li, Gen-Yuan Du, Wen-Jing Tian. Entropy Measure of Generating Random Rough Surface for Numerical Simulation of Wave Scattering. IEEE Transactions on Geoscience and Remote Sensing, 2021, 59 (5), pp.3623-3641. 10.1109/TGRS.2020.3018961 . hal-03005964

HAL Id: hal-03005964

<https://hal.science/hal-03005964>

Submitted on 1 Dec 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Entropy Measure of Generating Random Rough Surface for Numerical Simulation of Wave Scattering

Rui Jiang, *Member, IEEE*, Kun-Shan Chen[✉], *Fellow, IEEE*, Zhao-Liang Li[✉], *Senior Member, IEEE*, Gen-Yuan Du, and Wen-Jing Tian, *Student Member, IEEE*

Abstract—Numerical simulation of random rough surface finds wide applications in scientific disciplines, e.g., radar remote sensing of terrain and sea. In scattering simulation of rough surface, not only energy conservation must be ensured, but also, perhaps equally important, the surface inherent properties must be preserved. However, the proper choice of surface and grid sizes that are statistically representative poses a problematic issue. This study applied the entropy measure to determine such parameter settings by examining the relative error of sample entropy associated with roughness parameters and by noticing the fact that a rough surface with certain roughness parameters, including power spectrum density function, must have unique sample entropy. It is found that if the two criteria are met, proper choice of surface length and grid size is attainable to warrant minimum uncertainties of rough surfaces and maximum information content for different roughness spectra density functions under different correlation lengths. The feasibility and superiority of the proposed entropy-based method are validated in terms of minimum error of roughness parameters and also the energy conservation in bistatic scattering coefficients of rough surfaces generated using obtained simulation parameters.

Index Terms—Power density functions, radar scattering, random rough surface, sample entropy.

I. INTRODUCTION

NUMERICAL generation of random rough surfaces is desirable for a variety of applications in scientific areas, such as modeling the terrain surfaces in microwave remote

sensing [1]–[3]. It is understood that generating a prespecified random rough surface is crucial to accurate simulation of physical problems, for instance, the energy conservations in numerical simulation of microwave scattering and emission from random rough surfaces [4], instrumental designing for effective and accurate surface parameter measurements [5], [6], and so on. In practice, the random rough surface is statistically described and numerically generated, i.e., using the Monte Carlo spectra method proposed in [7], in which the randomness is qualitatively governed using the random numbers with certain distribution and the roughness is quantitatively specified using moments, and autocorrelation function (ACF) to estimate the mean value, the root-mean-square (rms) height, and the correlation length of surface heights.

In computer generation of random rough surfaces, the surface size and sampling points are finite; hence, the rms heights of the generated surface samples are persistently biased to a certain extent. From a statistical point of view, the bias may be reduced by properly selecting the simulation parameters, namely, the grid size, sampling point, and surface length [8]. The key then lies in how to make the right selection by applying an effective measure, if any. In dealing with the microwave scattering and emission from terrain rough surfaces, for example, 8, 16, or 32 wavelengths or longer surfaces, and 10, 12, 16, or more sampling points per wavelength, as a rule of thumb, were adopted [9]–[12]. Such choices were made by trial and error until the energy conservation is satisfied within a preset accuracy. Reminding the energy conservation should be obeyed for any given surface from which there is microwave scattering or emission. Although these simulation parameters gauged by the rule of energy conservation might be qualified in characterizing the properties of a certain random rough surface accidentally. They still lack sufficient evidence from complete theories, that is, the simulation parameters are better determined by the statistical properties and information contents of the surface itself.

Based on this argument, one alternative is to obtain the simulation parameters by minimizing the relative error of the variance of spatial heights [8]. In this study, we attempt to tackle this issue using the entropy concept, more specifically, using the sample entropy (SampEn) [13], being conceivable from the facts that the following holds:

Manuscript received February 15, 2020; revised May 19, 2020, July 12, 2020, and August 18, 2020; accepted August 19, 2020. This work was supported by the International Joint Laboratory of Polarimetric Sensing and Intelligent Signal Processing, Xuchang University. (*Corresponding author: Kun-Shan Chen.*)

Rui Jiang Gen-Yuan Du are with the School of Information Engineering, Xuchang University, Xuchang 461000, China (e-mail: xcdgy@msn.com).

Kun-Shan Chen is with the School of Information Engineering, Xuchang University, Xuchang 461000, China, also with the College of Geomatic and Geoinformation, Guilin University of Technology, Guilin 541004, China, and also with the State Key Laboratory of Remote Sensing Science, Institute of Remote Sensing and Digital Earth, Chinese Academy of Sciences, Beijing 100012, China (e-mail: chenks@radi.ac.cn).

Zhao-Liang Li is with ICube (UMR 7357) CNRS, Uds, 67412 Illkirch, France (e-mail: lizl@unistra.fr).

Wen-Jing Tian is with the Department of Mathematics and Computer Science, Wuhan Textile University, Wuhan 430200, China (e-mail: wjtian.cs@hotmail.com).

Color versions of one or more of the figures in this article are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TGRS.2020.3018961

- 1) The entropy value is alternate to quantify the information content of a random series.
- 2) In the SampEn algorithm, under a fixed maximum template length (see introductions to SampEn algorithm in Section II), the estimated SampEn of a random series is determined by its standard deviation (SD) value, and in mathematical analysis, the rms height of a 1-D randomly rough surface is equal to the SD value of its spatial heights. Therefore, theoretically speaking, one can come up with the simulation parameters within the desired relative error of SampEn since the uncertainties in roughness parameters are fully considered and can be quantitated by the relative error of SampEn, and such interconnections between the SampEn error and the statistical error are validated in the following.
- 3) Apart from minimizing the uncertainties of roughness parameters in [8] or in 2), the random rough surface with specific power (roughness) spectrum density (PSD) function and roughness parameters should have a unique SampEn (information content).
- 4) Traditional statistical method may be incomplete or even invalid in analyzing complex random series, and the entropy method plays an alternatively effective supplementary to the traditional statistical method [14].

Motivated by the earlier observations, in this article, we propose a novel entropy-based method to select the simulation parameters for microwave scattering from a rough surface. The central idea is that both the minimum errors of SampEn and the unique SampEn are simultaneously satisfied. It should be noticed that due to the truncated Fourier series, a perfectly prespecified rough surface is unattainable. Hence, for the purpose of demonstration, a 10% relative error of SampEn, after trial and error, is selected in this article. Besides, in general, a 2-D random rough surface is statistically or directionally anisotropic, such as seawater surfaces. Then, it is preferable to start with 1-D surface profiles, and the results can be easily applied to a full 2-D surface, especially the directionally independent ones.

The rest of this article is organized as follows. Section II gives some fundamentals of computer generation of random rough surface, including the Monte Carlo spectra method, the basic properties and characterizations of the random rough surface, the basic concept of SampEn algorithm, and the entropy-based method. The relationships between the roughness parameters and the relative error of SampEn are discussed in Section III, followed by the choice of parameter setting based on two criteria: minimum relative error of SampEn and unique SampEn. Section IV discusses the simulation parameters obtained for generating accurate Gaussian and exponential correlated random rough surfaces and examines the performances of the selected simulation parameters in terms of relative errors of variances of surface heights and also the accuracy of the bistatic scattering coefficients (BSCs) of the generated random rough surfaces. Finally, findings from the study are summarized to conclude the presentation.

II. FUNDAMENTALS AND METHODOLOGIES

A. Computer Generation of Random Rough Surfaces

In the Monte Carlo spectra method [7], the spatial height profile of a 1-D surface sample is expressed as

$$z(x_n) = \bar{z} + \frac{1}{L} \sum_{m=-N/2}^{N/2-1} Z(k_m) \exp(ik_m x_n) \quad (1)$$

where $\bar{z}$ is the mean value of surface spatial heights and is usually assumed to be zero for the sake of simplicity, so does in this study; L and N represent the surface length and the sampling points, respectively; $k_m = 2\pi m/L$ represents the spatial frequency components of surface; and $Z(k)$ is a spectral function associated with the random numbers and the PSD function $W(k)$ of surface. Specifically, for a Gaussian distributed random surface, $F(k_m)$ turns to have the expression as

$$Z(k_m) = \sqrt{2\pi L W(k_m)} \cdot \begin{cases} \frac{[\eta_1(0, 1) + i\eta_2(0, 1)]}{\sqrt{2}}, & m \neq 0, \pm N/2 \\ \eta_1(0, 1), & m = 0, \pm N/2 \end{cases} \quad (2)$$

where $\eta_1(0, 1)$, $\eta_2(0, 1)$ are two independent random numbers obeying the standard normal distribution; in realizations, (1) is calculated using the inverse fast Fourier transform (IFFT), to make sure that the generated spatial heights are real numbers, $Z(k_m)$ is furtherly constrained by $Z(k_m) = Z^*(k_{-m})$ and must be a real number at $m = 0, \pm N/2$ simultaneously. In the same manner, the random rough surfaces with other distributions can be obtained through setting the desired distribution for random numbers in (2) but modified with a zero-memory nonlinear filter that transforms the Gaussian sequence into the specific random sequence. Nonetheless, the Monte Carlo spectra method can readily be extended to 2-D or 3-D cases [4], [8]. To quickly understand the sources of numerical error in computer generation of random rough surfaces, the characterizations of random rough surfaces are introduced as follows. Based on the Winner-Khinchin theorem, the PSD function of a stationary process is related to its ACF $\rho(x)$ by

$$\sigma^2 \rho(x) = \int_{-\infty}^{\infty} W(k_x) \exp(ik_x x) dk_x \quad (3)$$

where σ denotes the surface rms height, the vertical roughness scale of a surface. We then have the Parseval theorem readily by noticing that $\rho(0) = 1$

$$\sigma^2 = \int_{-\infty}^{\infty} W(k_x) dk_x. \quad (4)$$

It is evidently seen from (1), (2), and (4) that due to the truncated frequency components, the rms height, or the variance σ^2 of surface heights, is biased in computer generation of rough surfaces. In the natural scene, many rough surfaces or time series were turned to have the power-law PSD functions, say the color noises. For simplicity, but without loss of any generalities, we consider the rough surfaces with Gaussian and

exponential PSD functions in this study. The expressions of ACFs of these two considered rough surfaces are turned to be

$$\rho_g(x) = \exp(-x^2/l) \quad (5)$$

$$\rho_e(x) = \exp(-|x|/l) \quad (6)$$

respectively, with their corresponding PSD functions are

$$W_g(k_x) = \frac{\sigma^2 l}{2\sqrt{\pi}} \exp(-k_x^2 l^2/4) \quad (7)$$

$$W_e(k_x) = \frac{\sigma^2 l}{\pi} \frac{1}{1 + k_x^2 l^2} \quad (8)$$

where l denotes the $1/e$ correlation distance ($\rho(l) = 1/e$) and is named the correlation length. In contrast to the rms height, the correlation length specifies the horizontal roughness scale of a surface. In natural, the correlation length typically depends on the measured surface length L , and a large measured surface length L yields high accuracy in estimating and measuring the correlation length [15]–[18], which means that in computer generation of random rough surfaces, the choice of surface length should guarantee a minimum error in estimating the prespecified correlation length. Fig. 1 shows some generated rough surface samples with different roughness parameters, random seeds, and PSD functions. In brief, their characteristics are summarized as follows, under a PSD function and random seed. The surface roughness increases with the increasing rms height and decreases with the increasing correlation length; for those surfaces having the same PSD function and roughness parameters, the spatial distribution of surface heights varies with a random seed, and finally, under certain roughness parameters and random seed, the surfaces with exponential PSD function have larger roughness than that with the Gaussian PSD function.

B. SampEn Algorithm and the Unique SampEn

On the aspect of evaluating the complexity (information content) of a random series, any definitions of entropy, say, the Shannon entropy, are alternative. Here, without loss of generality, the SampEn, a conditional probability-based algorithm, which is proven to have superior robustness and correctness on analyzing the complexities of a random series [13], is applied. The SampEn of a series $\{x\} = \{x_1, x_2, x_3, \dots\}$ is defined as

$$\text{SampEn}(x, m, r) = -\ln\left(\left(\sum A_k\right)/\left(\sum B_k\right)\right) \quad (9)$$

where k is the ID of the elements in random series, m is a preset template length, which also determines the dimension of the template vector, r is a threshold associated with the SD value of random series, A_k is equal to the number matches of length $m + 1$ with the k th template, and B_k is equal to the numbers of matches of length m with the k th template. In the SampEn algorithm, the k th template vector of length m is defined as

$$X_m(k) = \{x_k, x_{k+1}, x_{k+2}, \dots, x_{k+m-1}\} \quad (10)$$

and the distance between the k th and the j th template vector ($k \neq j$) is defined as $d[X_m(k), X_m(j)]$, where $k \neq j$ means that the self-matches are excluded. To give a comprehensive

Fig. 1. Generated rough surface samples with roughness parameters, PSD functions, and random seeds.

Fig. 2. Illustration of the k th template vectors $X_m(k)$ and $X_{m+1}(k)$ for the case of $m = 2$ ($X_m(k)$: light blue block and $X_{m+1}(k)$: light orange block).

guide to the above-defined variables, Fig. 2 shows a case of the k th template vector $X_m(k)$ and $X_{m+1}(k)$ at $m = 2$.

Following the original SampEn algorithm [13], in this study, the Euclidian distance between two template vectors is applied. Then, B_k is the number of template pairs satisfying $d[X_m(k), X_m(j)] < r$ and A_k is equal to the number of template pairs that satisfies $d[X_{m+1}(k), X_{m+1}(j)] < r$. Consequently, for the k th template vector, $A_k \leq B_k$ is always established, and so does $\sum A_k / \sum B_k \leq 1$ for all template vectors. It is noted that the choices of m and r dominate the estimated SampEn value of a random series evidently [19], [20]. However, qualitatively, under a certain m and r , the weakly or nonfluctuated random series, such as the constant terms (dc components or flat surfaces), always have smaller estimated SampEns than that of the strongly fluctuated random series since the latter has larger complexities. Without loss of generality, the suggested and defaulted parameters,

Fig. 3. Estimated SampEns of the white noise series with increasing sampling points (300 realizations).

$m = 2$ and $r = 0.15 \cdot sd\{x\}$, are applied to the estimated SampEn of random series concerned in this study. Under these parameters, Fig. 3 shows the estimated SampEns of the white noise series with increasing sampling point. To guarantee the accurateness, 300 realizations are conducted and ensemble-averaged.

As shown in Fig. 3, the estimated SampEn of white noise series is infinitely large at small sampling points, then decreases with the increase in sampling point, and saturates at sampling points at around 500 or above. It instantly can be explained by the information theories that less sampling points afford less information content to characterize the white noise and, therefore, yield large uncertainties and estimate large SampEns, whereas more sampling points afford more information, decrease the uncertainties, and estimate smaller SampEns. Therefore, the saturated SampEn can be regarded as the unique SampEn of white noise, which means that, in temporal, no matter how long the white noise series is, the sampled series with sampling points at 500 or above contains the whole information content of the white noise. Once the sampling rate is given, the temporal length of the sample series can be obtained immediately.

We reason the use of the SampEn algorithm as follows. It is noted that in numerical simulations of wave scattering from rough surfaces, the unsatisfactory accuracies are mainly due to the lack of surface details. As an example, Fig. 4 shows the real and the simulate surfaces at two different sampling rates, respectively. It is seen that compared with the poor one, finer sampling characterizes the real surface vividly, notably for regions with high local curvatures, where the differentials varied rapidly. Thus, within the difference approximation in numerical simulations, finer samplings find higher accuracies in quantifying these finer scales.

These differentials are crucial components in forming the impedance matrix element in the method of moment (MOM) and, therefore, dictate the energy conservations [4]. The SampEn algorithm is effective in capturing these high local curvatures since the following holds. First, the Euclid distance larger than the threshold value means that a drastic change may occur between two adjacent points in compared templates. These template pairs are recorded as unqualified matches, which yield a smaller probability in (9) and therefore estimate a higher SampEn value. Second, the stepped sliding window in

Fig. 4. Real and simulate rough surfaces at two different sampling rates.

the SampEn algorithm ensures the changes before and after a point to be evaluated. Note that with certain template lengths, the residual points at the end of the series are evaluated by expanding the series periodically.

C. Criteria of Determining Simulation Parameters

Based on the proceeding discussions, we propose the following criteria must be met to determine the minimum sampling point at a certain grid size Δx .

Step 1: Evaluating the relative error of SampEn $\varepsilon(N, \sigma, l, \Delta x)$ with prespecified roughness parameters σ and l , the minimum sampling point N_1 can be obtained within an acceptable relative error of SampEn, namely

$$|\varepsilon(N, \sigma, l, \Delta x)| \leq A \quad (N_1 \geq N) \quad (11)$$

with

$$|\varepsilon(N, \sigma, l, \Delta x)| = \left| \frac{SE(N, \sigma, l, \Delta x) - \langle SE_s(N, \sigma', l, \Delta x) \rangle}{SE(N, \sigma, l, \Delta x)} \right| \quad (12)$$

where SE is the SampEn estimated using the prespecified rms height and SE_s are estimated using the actual rms height σ' of generated surface samples, A denotes the acceptable threshold of relative error of SampEn, and $A = 0.1$ is selected in this study as mentioned in Section I. The bracket $\langle \rangle$ denotes the ensemble averages over samples generated using different random seeds.

Step 2: Using the unique SampEn SE_{unique} to determine the minimum sampling point N_2 via

$$SE(N, \sigma, l, \Delta x) = SE_{\text{unique}} \quad (N \geq N_2). \quad (13)$$

Step 3: Comparing the two obtained minimum sampling point N_1 and N_2 , then the large one is set to meet both criteria in Steps 1 and 2, meaning that the candidate minimum sampling point is

$$N_{\min} = \max\{N_1, N_2\}. \quad (14)$$

A flowchart illustrating the main procedures of the proposed entropy-based method is shown in Fig. 5, and the realization of each step will be discussed in the following.

Then, at a certain grid size Δx , once its minimum sampling point $N_{\min}$ is obtained, the corresponding minimum surface length is then measured by $L_{\min} = N_{\min} \Delta x$. It is noticed that as early discussed in Section I, criteria 1 is in some sense that

Fig. 5. Flowchart illustrating the main procedures of the proposed entropy-based method.

the same as a statistical method since the estimated SampEn value is determined by the SD value of a random series; therefore, the differences between the actual rms height and the prespecified rms height are fully contained in the relative error of SampEn, as it will be proved in the following. The relative error of SampEn is feasible to evaluate the uncertainties of roughness parameters.

III. SELECTING SIMULATION PARAMETERS USING THE PROPOSED ENTROPY-BASED METHOD

A. Relative Error of SampEn With Rms Height

To explore variations of relative error of SampEn with the rms height, the correlation length must be fixed. Fig. 6 shows the relative error of SampEn as a function of the ratio of the correlation length to rms height for the surfaces with Gaussian and exponential PSD functions. In Fig. 6, the correlation length is fixed at 1 unit length, and the grid size is set at $\Delta x = 0.01$ unit length, which corresponds to 100 sampling points per unit length. To guarantee the accurateness, 100 realizations are conducted and ensemble-averaged. It is found that in Fig. 6, the relative errors of SampEn are negative. The negative relative errors of SampEn mean that the rms heights are underestimated, that is, the rms heights of generated samples are smaller than the prespecified one and, therefore, yield less match numbers A_k and estimate larger SampEn than the prespecified rms height. Meanwhile, increasing sampling points tends to reduce the relative errors of SampEn; however,

Fig. 6. Relative error of SampEn with varied rms heights. (a) Gaussian PSD. (b) Exponential PSD, $\Delta x = 0.01$, correlation length: 1 unit length, and 100 realizations).

at all selected sampling points, the relative errors of SampEn in Fig. 6 are unchanged regardless of rms height.

At this point, we make the following remarks on the invariant relative errors of SampEn with rms height. First, as stated in Section II, the rms height can be regarded as a constant in both PSD function and IFFT process. Therefore, under certain correlation lengths, sampling points, PSD functions, and seeds, the rough surface samples generated by different rms heights are geometrically similar. The geometrical similarity of two rough surface samples generated by different rms heights only is shown in Fig. 1(a). From information perspectives, the information content of a series remains unchanged when it is multiplied by a constant since these two series are geometrically similar, and the unchanged SampEn of geometrically similar series was proved and even was extended to unchanged multiscale SampEn case for the self-linked fractal structures in [21]. More vividly, by using the natural properties, namely, the linearity and the invariant information content, of an ideally linear amplifier as prior knowledge, one can come up with the meaning of similarity in Fig. 1(a) by regarding these two surface samples as the input and output signals of a linear amplifier.

Second, under certain roughness parameters and PSD functions, the surface samples generated with different random seeds are turned to have the same SampEn, statistically (the proof is given in the Appendix). It is noted that in Monte Carlo simulations, the random selection process cannot avoid producing the same seed over large amounts of realizations, although our subjective will be, more or less, to use different random seeds and generate surface samples with different geometry shapes. Thus, for a given rms height, two surface

Fig. 7. Relative errors of SampEn with varied correlation lengths at selected grid sizes (Gaussian PSD, 100 realizations, and rms height: 0.1-unit length). (a) $\Delta x = 0.5$. (b) $\Delta x = 0.2$. (c) $\Delta x = 0.125$. (d) $\Delta x = 0.1$. (e) $\Delta x = 0.02$. (f) $\Delta x = 0.01$.

samples with the same geometry shape can always be found over large amounts of realizations; also, due to the random selection process, two geometrically similar surface samples can always be found among two sets of surface samples that generated by different rms heights. Joint with the earlier discussions, it is reasonable to conclude that under fixed correlation lengths and PSD functions, the estimated SampEn values of surface samples keep invariant regardless of the rms height.

Hence, the invariant relative error of SampEn also means that the SampEn values $SE(N, \sigma, l, \Delta x)$ of generated surface samples are invariant when they are evaluated using different prespecified rms heights. The independence on prespecified rms heights can be explained mathematically, as the variance of the estimated rms height $\hat{\sigma}$ may be expressed as [17], [18]

$$\text{var}(\hat{\sigma}) = \frac{\sigma^2}{L} \int_0^\infty \rho^2(\xi) d\xi = \frac{\sigma^2}{N \Delta x} \int_0^\infty \rho^2(\xi) d\xi. \quad (15)$$

Such a representation of uncertainty is associated with the prespecified rms height. When the relative error of variance of surfaces is introduced to quantify the bias, then the bias is no longer dependent on the prespecified rms height. For instance, for surfaces with an exponential PSD function, the relative error of variance of surface heights is written as [8]

$$\varepsilon_{\text{rel}} = \frac{\sigma^2 - \langle \sigma_s^2 \rangle}{\sigma^2} = \left(\frac{2}{\pi^2} \right) \left(\frac{\Delta x}{l} \right) + 2 \left(\frac{l}{L} \right) - \frac{1}{N}. \quad (16)$$

Therefore, the invariant relative errors of SampEn in Fig. 5 shows in accordance with (16) when the bias is evaluated in terms of the information content. Due to the entropy property, the unchanged relative errors of SampEn with rms height offer an advantage, that is, we only need to concern the relative error of SampEn with correlation length.

B. Relative Error of SampEn With Correlation Length

Following the fact that the rms height has no effects on the relative error of SampEn, and without loss of generality, the rms height is selected at 0.1 unit length hereafter. The ratio of correlation length to the fixed rms height ranges from 1 to 100 is considered in this section, which corresponds to a range of correlation length varying from 0.1 to 10 unit length. For each selected grid size, Figs. 7 and 8 show the relative errors of SampEn as a function of correlation length for the surfaces with Gaussian and exponential PSD function, respectively.

As shown in Figs. 7 and 8, for surfaces with both PSD functions and at all grid sizes, the relative errors of SampEn are larger at small sampling points than at large sampling points. At large sampling points, say, 8192, the relative error of SampEn tends to be invariant regardless of correlation length, especially at small grid sizes. Once the 10% relative errors of SampEn is selected, it is clearly seen from Figs. 7 and 8 that the minimum sampling point strongly depends on the range of correlation length, in particular, which is dominated by the maximum correlation length. For instance, in Fig. 7(b), the sampling point 128 meets the 10% relative error of SampEn in the range of correlation length varying from 0.1 to 5.0 unit length, but it is out of consideration at correlation lengths larger than 5.0 unit length. For each selected grid size, the minimum sampling points N_1 gauged by criteria 1 with their corresponding surface lengths L_{m1} are listed in Tables I and II for surfaces with Gaussian and exponential PSD functions, respectively.

C. Simulation Parameters Gauged by Criteria 2

In what follows, we turn to obtain the minimum sampling point N_2 gauged by the unique SampEn of a surface. Unlike criteria 1, the roughness parameters must be fixed for

TABLE I

MINIMUM SAMPLING POINTS AND THEIR CORRESPONDING SURFACE LENGTHS FOR SURFACES WITH GAUSSIAN PSD FUNCTION (WITHIN 10% RELATIVE ERROR OF SAMPEN FOR CRITERIA 1)

	$l \in [0.1, 1.0]$		$l \in [0.1, 2.5]$		$l \in [0.1, 5.0]$		$l \in [0.1, 7.5]$		$l \in [0.1, 10.0]$	
Δx	N_1	L_{m1}	N_1	L_{m1}	N_1	L_{m1}	N_1	L_{m1}	N_1	L_{m1}
0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.2	128	25.6	128	25.6	128	25.6	256	51.2	256	51.2
0.125	128	16	128	16	256	32	512	64	512	64
0.1	128	12.8	256	25.6	512	51.2	512	51.2	512	51.2
0.02	256	5.12	512	10.24	1024	20.48	1024	20.48	2048	40.96
0.01	512	5.12	1024	10.24	2048	20.48	2048	20.48	4096	40.96

TABLE II

MINIMUM SAMPLING POINTS AND THEIR CORRESPONDING SURFACE LENGTHS FOR SURFACES WITH EXPONENTIAL PSD FUNCTION (WITHIN 10% RELATIVE ERROR OF SAMPEN FOR CRITERIA 1)

	$l \in [0.1, 1.0]$		$l \in [0.1, 2.5]$		$l \in [0.1, 5.0]$		$l \in [0.1, 7.5]$		$l \in [0.1, 10.0]$	
Δx	N_1	L_{m1}	N_1	L_{m1}	N_1	L_{m1}	N_1	L_{m1}	N_1	L_{m1}
0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.2	128	25.6	128	25.6	256	51.2	256	51.2	512	102.4
0.125	128	16	128	16	256	32	512	64	1024	128
0.1	128	12.8	256	25.6	512	51.2	512	51.2	1024	102.4
0.02	512	10.24	1024	20.48	2048	40.96	4096	81.92	4096	81.92
0.01	1024	10.24	2048	20.48	4096	40.96	8192	81.92	8192	81.92

searching the unique SampEn of a certain surface; meanwhile, for comparing the obtained parameters by criteria 1 and 2, the roughness parameters in this section must be selected with the same manner in criteria 1 either. Following this, the rms height is set to be 0.1 unit length, while for correlation length, five ratios of the correlation length to the fixed rms height, namely, 1.0, 25, 50, 75, and 100, are considered. It is not difficult to realize that the correlation lengths at these five selected ratios correspond to the maximums of the five selected ranges of correlation length in criteria 1, respectively. The expedience of such an option will be presented later. Under the selected roughness parameters, and for each selected grid size, the SampEn of generated surface samples is estimated, ensemble-averaged over 100 realizations, and depicted with increasing sampling points for surfaces with Gaussian and exponential PSD function in Figs. 9 and 10, respectively.

As shown in Figs. 9 and 10, at each grid size, the estimated SampEn at each selected ratio decreases and then saturates with increasing sampling points as it was expected in former discussions. Then, the saturated SampEn is, therefore, the unique SampEn of the surface with certain roughness parameters and PSD function. Following this, the minimum sampling point N_2 and their corresponding surface length L_{m2} are obtained, as listed in Tables III and IV at each selected grid size and for surfaces with Gaussian and exponential PSD function, respectively. It is clearly seen in Figs. 9 and 10 that the estimated SampEn values at large ratios saturate more slowly than that at small ratios, especially for surfaces with exponential PSD function, and the minimum sampling points gauged by the unique SampEn monotonically increase with the increase in correlation length. Consequently, it is deduced that in a selected range of correlation length, the minimum sampling points, which are determined by the maximum of the selected range, are large enough to acquire the unique

SampEn of a surface under the range of correlation lengths. Therefore, the choice of correlation length meets the rules of setting roughness parameters.

Besides, through a comparison in Figs. 9(a)–(f) and 10(a)–(f), it is found that at fixed ratios, the estimated SampEn decreases with the increase in sampling density, and for large ratios, it even approaches to zero. However, the decreased SampEn cannot be regarded as a signature of saturation of SampEn since the following holds. First, for certain sampling points, the surface length decreases with the increase in sampling density, and the unchanged sampling points, therefore, affords more information content in characterizing the shorter surfaces and estimates lower SampEn values. Second, at large ratios (correlation lengths), the shorter surfaces tend to be specular, its spatial heights gradually approach to a constant series, and a constant series contains no information.

D. Parameter Settings Obtained by Criteria 1 and 2

Up to now, the minimum sampling points $N_{\min}$ that meet both criteria 1 and 2 can be obtained through comparing the minimum sampling points N_1 and N_2 listed in Tables I–IV and selecting the larger one to be qualified. At each selected grid size, within the 10% relative error of SampEn, Tables V and VI present the obtained minimum sampling points $N_{\min}$ and their corresponding surface lengths L_m at the selected ranges of correlation length and for surfaces with Gaussian and exponential surfaces, respectively. It is noted that for grid size $\Delta x = 0.5$, although the unique SampEns of surfaces with certain roughness parameter are attainable, however, as shown in Figs. 7 and 8 and Tables I and II, in the ranges of correlation length, the acceptable relative error of SampEn cannot be met by any minimum

Fig. 8. Relative errors of SampEn with varied correlation lengths at selected grid sizes (exponential PSD, 100 realizations, and rms height: 0.1 unit length). (a) $\Delta x = 0.5$. (b) $\Delta x = 0.2$. (c) $\Delta x = 0.125$. (d) $\Delta x = 0.1$. (e) $\Delta x = 0.02$. (f) $\Delta x = 0.01$.

Fig. 9. SampEn with increasing sampling points at different grid sizes (Gaussian PSD, 100 realizations, and rms height: 0.1 unit length). (a) $\Delta x = 0.5$. (b) $\Delta x = 0.2$. (c) $\Delta x = 0.125$. (d) $\Delta x = 0.1$. (e) $\Delta x = 0.02$. (f) $\Delta x = 0.01$.

sampling points. Therefore, the minimum sampling points at this grid size are unavailable. Meanwhile, it is further found in Tables V and VI that with the increase in grid size (or sampling density equivalently), the minimum surface length $L_{\min}$ decreases and then saturates at large grid sizes. The saturated minimum surface is the smallest one among the minimum surface lengths under different grid sizes and is regarded as the least surface length L_m in characterizing the rough surface with certain roughness parameters and PSD

functions. From Tables V and VI, the least surface lengths are associated with the correlation length. At selected ranges of correlation length, the least surface lengths are obtained and listed in Table VII in terms of the maximum correlation length $l_{\max}$ of the selected ranges of correlation lengths. For example, for the surface with maximum correlation length $l_{\max} = 2.5$ unit length, the least surface lengths are $L_{\min} = 4.096l_{\max}$ and $L_{\min} = 16.384l_{\max}$ for Gaussian and exponential PSD function, respectively.

TABLE III

MINIMUM SAMPLING POINTS AND CORRESPONDING SURFACE LENGTHS GAUGED BY UNIQUE SAMPEN (GAUSSIAN PSD FUNCTION FOR CRITERIA 2)

	$l \in [0.1, 1.0]$		$l \in [0.1, 2.5]$		$l \in [0.1, 5.0]$		$l \in [0.1, 7.5]$		$l \in [0.1, 10.0]$	
Δx	N_2	L_{m2}	N_2	L_{m2}	N_2	L_{m2}	N_2	L_{m2}	N_2	L_{m2}
0.5	256	128	256	128	512	256	512	256	512	256
0.2	256	51.2	256	51.2	512	102.4	512	102.4	512	102.4
0.125	256	32	256	32	512	64	512	64	512	64
0.1	256	25.6	256	25.6	512	50.12	512	51.2	512	51.2
0.02	512	10.24	512	10.24	512	10.24	512	10.24	512	10.24
0.01	1024	10.24	1024	10.24	1024	10.24	1024	10.24	1024	10.24

TABLE IV

MINIMUM SAMPLING POINTS AND THEIR CORRESPONDING SURFACE LENGTHS GAUGED BY UNIQUE SAMPEN (EXPONENTIAL PSD FUNCTION FOR CRITERIA 2)

	$l \in [0.1, 1.0]$		$l \in [0.1, 2.5]$		$l \in [0.1, 5.0]$		$l \in [0.1, 7.5]$		$l \in [0.1, 10.0]$	
Δx	N_2	L_{m2}	N_2	L_{m2}	N_2	L_{m2}	N_2	L_{m2}	N_2	L_{m2}
0.5	512	256	512	256	512	256	1024	512	1024	512
0.2	512	102.4	1024	204.8	1024	204.8	2048	409.6	2048	409.6
0.125	512	64	1024	128	2048	256	4096	512	4096	512
0.1	512	51.2	1024	102.4	2048	204.8	4096	409.6	4096	409.1
0.02	512	10.24	2048	40.96	4096	81.92	4096	81.92	4096	81.92
0.01	512	5.12	4096	40.96	8192	81.91	8192	81.92	8192	81.92

TABLE V

MINIMUM SAMPLING POINTS AND THEIR CORRESPONDING SURFACE LENGTHS FOR SURFACES WITH GAUSSIAN PSD FUNCTION (WITHIN 10% RELATIVE ERROR OF SAMPEN)

	$l \in [0.1, 1.0]$		$l \in [0.1, 2.5]$		$l \in [0.1, 5.0]$		$l \in [0.1, 7.5]$		$l \in [0.1, 10.0]$	
Δx	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$
0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.2	256	51.2	256	51.2	512	102.4	512	102.4	512	102.4
0.125	256	32	256	32	512	64	512	64	512	64
0.1	256	25.6	256	25.6	512	50.12	512	51.2	512	51.2
0.02	512	10.24	512	10.24	1024	20.48	1024	20.48	2048	40.96
0.01	1024	10.24	1024	10.24	2048	20.48	2048	20.48	4096	40.96

TABLE VI

MINIMUM SAMPLING POINTS AND THEIR CORRESPONDING SURFACE LENGTHS FOR SURFACES WITH EXPONENTIAL PSD FUNCTION (WITHIN 10% RELATIVE ERROR OF SAMPEN)

	$l \in [0.1, 1.0]$		$l \in [0.1, 2.5]$		$l \in [0.1, 5.0]$		$l \in [0.1, 7.5]$		$l \in [0.1, 10.0]$	
Δx	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$	$N_{\min}$	$L_{\min}$
0.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.2	512	102.4	1024	204.8	1024	204.8	2048	409.6	2048	409.6
0.125	512	64	1024	128	2048	256	4096	512	4096	512
0.1	512	51.2	1024	102.4	2048	204.8	4096	409.6	4096	409.1
0.02	512	10.24	2048	40.96	4096	81.92	4096	81.92	4096	81.92
0.01	1024	10.24	4096	40.96	8192	81.92	8192	81.92	8192	81.92

IV. VALIDATIONS AND DISCUSSION

In this section, the feasibility and performance of the simulation parameters obtained by the proposed entropy-based method are analyzed by revealing the statistical error of rms height and the results of scattering from rough surfaces generated using the obtained simulation parameters.

A. Statistical Error of Obtained Simulation Parameters

Unlike estimating the rms height and correlation function of a surface from *in situ* measured surface heights and

lengths [15]–[18], in computer generation of random rough surfaces, only the uncertainty of rms height or variance is primarily concerned. Therefore, in a statistical sense, the way to validate the obtained simulation parameters with a 10% relative error of SampEn is to check their relative errors of the variance of surface heights by substituting the obtained simulation parameters into (16). Due to the lack of an analytical expression of relative error of variance of surfaces with Gaussian PSD function at this point, only the simulation parameters for surfaces with exponential PSD function are validated here. Fig. 11 shows the relative errors of the variance

Fig. 10. SampEn with increasing sampling points at different grid sizes (exponential PSD, 100 realizations, and rms height: 0.1 unit length). (a) $\Delta x = 0.5$. (b) $\Delta x = 0.2$. (c) $\Delta x = 0.125$. (d) $\Delta x = 0.1$. (e) $\Delta x = 0.02$. (f) $\Delta x = 0.01$.

of the simulation parameters obtained within 10% relative error of SampEn; as an example, the simulation parameters that achieve the least surface length at each correlation length range are selected. It is seen from Fig. 11 that the relative errors of variance are associated with the correlation length, notably, dominated by the maximum of the correlation length ranges.

Only presenting the relative errors of variance is not enough to show the feasibility of the proposed entropy-based method. As shown in Figs. 7 and 8, the minimum number of sampling points strongly depends on the choice of relative error of SampEn, that is, a smaller relative error of SampEn yields a larger minimum number of sampling points and a larger minimum surface length under a fixed grid size. Meanwhile, as noted in (16), large surface lengths and the number of sampling points reduce the relative error of variance at a certain grid size. Thus, theoretically speaking, a smaller relative error of variance is attained by setting a smaller of SampEn in criteria 1. To illustrate this, the relative errors of variance under the simulation parameters obtained within 5% relative error of SampEn are presented in Fig. 12. Apparently, at each selected correlation length range, the relative errors of variance in Fig. 12 are smaller than that in Fig. 11, especially at large correlation lengths. In particular, within 5% relative error of SampEn, the maximum relative errors of variance are all at around or below 10%. Thus, Figs. 11 and 12 show that smaller relative errors of SampEn correspond to smaller statistical errors of variance and fully prove that the relative error of SampEn can be regarded as an effective signature in determining the simulation parameters.

Fig. 11. Relative errors of variance of the obtained simulation parameters within 10% relative error of SampEn.

B. Performances on Scattering From Rough Surfaces

In what follows, we check the performances of the obtained simulation parameters within 10% relative error of SampEn in dealing with the issue of scattering from rough surface. This study suggests a procedure of setting simulation parameters in numerical simulation of scattering from the rough surface as follows.

- 1) For correlation length range, both the selected surface length and the grid size must be sufficiently containing good information content in characterizing inherent surface properties.
- 2) The energy conservation must be obeyed under such selected simulation parameters in 1).

Notably, the choice of surface length has the highest priority, that is, in a considered correlation length range, it must be

TABLE VII

LEAST SURFACE LENGTHS WITH MAXIMUM CORRELATION LENGTHS
(WITHIN 10% RELATIVE ERROR OF SAMPEN)

Maximum Correlation Length $l_{\max}$	Least Surface Length (in $l_{\max}$)	
	Gaussian	Exp
1.0	10.24	10.24
2.5	4.096	16.384
5.0	2.048	16.384
7.5	2.731	10.932
10.0	4.096	8.192

chosen such that it is larger than the least surface lengths listed in Table VII and then followed by the corresponding sampling rate (grid size). For instance, it is seen from Tables V–VII, for correlation lengths varying from 0.1 to 1.0 wavelength, the least surface length should be 10.24 wavelengths for surfaces with both PSD functions; then, the sampling rate should be 50 sampling points per wavelength or even larger. In numerical simulations [10], [12], 8.0 wavelengths' long surface was investigated with a correlation length varying from 0.1 to 1.0 wavelength, which might be reasonable when the selected correlation length is much smaller than 1.0 wavelength, but for larger correlation lengths approaching to 1.0 wavelength, it is no longer suitable regardless of the sampling rate. The common choice of short surface length and small sampling rates is mainly due to the limitation of computational resources at that time. Commonly, it has been argued that a sampling rate of 10 points per wavelength (PPW) was widely adopted in setting simulation parameters [9]; however, as discussed earlier, such a sampling rate is only applicable to a 25.6 wavelength or longer Gaussian correlated surface or a 51.2 wavelength or longer exponential correlated surface. For short surface lengths, say, the widely selected 16 or 32 wavelengths long, the traditional sampling rate of sampling 10 PPW lacks sufficient information content to preserve inherent surface properties.

Nonetheless, as briefly mentioned in Section I, the issues of surface inherent properties and energy conservation should be treated independently, that is, for long surfaces, though the parameter setting strategy of sampling 10 PPW offers sufficient information content, however, it is not clear that whether such a sampling rate could guarantee a high accuracy of energy conservation. Under the premise of ensuring sufficient surface information content, for the issue of scattering from long surfaces, the sampling rate of 10 sampling points per wavelength probably should be enhanced to pursue higher accuracies of energy conservation, as have been proved in previous studies [11], [12], [22], where the sampling rates were larger than 10 sampling points per wavelength. However, the surface lengths applied in these studies were almost smaller than 51.2 wavelengths; meanwhile, the results in Tables V–VII suggest a larger sampling rate for surfaces shorter than 51.2 wavelengths. This means that, in some sense, the obtained simulation parameters in Tables V–VII have the potentials in explaining why the sampling rate must be increased in these studies.

Hence, it is preferable, for the purpose of illustration, to select short surfaces and compare the performances of the

obtained simulation parameters in this study and the traditional parameter strategy. Following this, the BSCs of PEC surfaces with Gaussian and exponential PSD functions are calculated. Without loss of generality, it is sufficient to consider the H-pol. scattering (TE case). Following the derivations in [4], the surface integral equation (SIE) and the point match method are applied in this study, namely, for a PEC surface $z = z(x)$ [see (1)] and the TE case, the electric-field integral equation (EFIE) turns to be

$$\psi_{\text{inc}}(\bar{r}) = \int_S g(\bar{r}, \bar{r}') \hat{n} \cdot \nabla \psi(\bar{r}') ds \quad (17)$$

where the Green's function is

$$g(\bar{r}, \bar{r}') = \frac{i}{4} H_0^{(1)}(k|\bar{r} - \bar{r}'|) \quad (18)$$

k is the wavenumber in free space, i is the imaginary unit, $\hat{n}$ is the outward (upper hemisphere) unit norm vector, $\psi(\bar{r})$ denotes the surface field, $\bar{r} = (x, z(x))$ and $\bar{r}' = (x', z(x'))$ are the observation point and source point, respectively. Then, using the point match method, (17) is deduced into a matrix equation, and the small argument approximation is applied to eliminate the singularity of green function when $\bar{r} \rightarrow \bar{r}'$ and the expression of the elements in impedance matrix Z is

$$Z_{mn} = \begin{cases} \Delta x g(x_m, z(x_m); x_n, z(x_n)), & \text{for } m \neq n \\ \frac{i \Delta x}{4} \left[1 + i \frac{2}{\pi} \ln \left(\frac{\gamma k}{4e} \Delta l_m \right) \right] & \text{for } m = n \end{cases} \quad (19)$$

where $\Delta l_m = \Delta x(1 + (z'(x_m))^2)^{1/2}$, $\gamma = 1.78107$, and $e = 2.71828$.

The left-hand side of (17) denotes the incident field and is usually assumed to be a plane wave in general cases. However, to avoid the potential edge effects induced by the limited surface lengths, the incident plane wave is tapered via [4]

$$\psi_{\text{inc}}(\bar{r}) = \exp(ik(x \sin \theta_i - z \cos \theta_i)) \cdot (1 + w(\bar{r})) \exp\left(-\frac{(x + z \tan \theta_i)^2}{g^2}\right) \quad (20)$$

and

$$w(\bar{r}) = \left[2 \frac{(x + z \tan \theta_i)^2}{g^2} - 1 \right] / (kg \cos \theta_i)^2. \quad (21)$$

In this study, the commonly used tapering factor $g = L/4$ is selected, which means that the ratio of the incident beamwidth to the surface length L is always fixed. It is not clear at this point that such a choice of tapering factor is applicable to extremely short surface cases; however, the obtained minimum surface lengths in this article show all larger than the commonly used surface lengths (say, eight wavelengths) compared to those by previous studies [10], [12], in which the same choice of tapering factor was applied. Therefore, such a choice is adaptable to produce convincing simulation results. From Table VII, the surface length is fixed at 10.24 wavelengths and the correlation length is selected at 1.0 wavelength (the maximum in selected correlation length range). Then, from Tables V and VI, the number of sampling points is $N = 512$ for surfaces with both PSD functions.

Fig. 12. Relative errors of variance of the obtained simulation parameters within 5.0% relative error of SampEn.

Once (17) is solved numerically, the scattered fields $\psi_s^{(N)}(\theta_s)$ can be evaluated by

$$\begin{aligned} \psi_s^{(N)}(\theta_s) &= -\exp[-ik(\sin\theta_s x + z(x)\cos\theta_s)] \\ &\cdot \int_{-\infty}^{\infty} dx \left\{ -U(x) + \psi(x)ik \left[\frac{dz}{dx} \sin\theta_s - \cos\theta_s \right] \right\}. \end{aligned} \quad (22)$$

and the BSCs $\sigma^\circ(\theta_s)$ are calculated by using

$$\sigma^\circ(\theta_s) = \frac{|\psi_s^{(N)}(\theta_s)|}{8\pi k g \sqrt{\frac{\pi}{2}} \cos\theta_i \left[1 - \frac{1+2\tan^2\theta_i}{2k^2 g^2 \cos^2\theta_i} \right]}. \quad (23)$$

A total of 100 realizations are carried out to obtain the averaged BSC at 30° of incident angle, which is plotted in Figs. 13 and 14 for surfaces with Gaussian and exponential PSD function, respectively. For the PEC surface case, the energy conservation means that the reflectivity $r(\theta_s)$ satisfies

$$r(\theta_s) = \int_{-\pi/2}^{\pi/2} \sigma^\circ(\theta_s) d\theta = 1 \quad (24)$$

As shown in Fig. 13, at small rms heights, say, 0.1 or 0.35 wavelengths, the energy conservation is satisfied at a sampling rate of 10 PPW ($N = 128$) and the sampling points $N = 512$ obtained within 10% relative error of SampEn. Besides, it is found that at small rms heights, even the sampling points $N = 64$ still guarantees high accuracy of energy conservation. As discussed earlier, sampling points smaller than 512 are likely a lack of sufficient information content in characterizing inherent surface properties, though satisfying the energy conservation. Thus, it suggests from Fig. 13(a) and (b) that the issues of inherent surface properties and the energy conservation should be treated separately.

Note that, the rms height turns to have no effects on estimating the SampEn in the proposed entropy-based method. Therefore, theoretically, the simulation parameters chosen from within a 10% relative error of SampEn should be valid for any given rms heights. Following this, an extremely steep rough surface, namely, both the rms height and correlation length are set at 1.0 wavelength, is considered. As shown in Fig. 13(c), the accuracy of energy conservation clearly improves with the increase in sampling points, and the sampling points ($N = 512$) obtained from within 10% relative

errors of SampEn have the highest accuracy, implying that for surfaces with large roughness, the sampling rate of 10 PPW is both invalid in terms of energy conservation and information content.

Similarly, the results in Fig. 14 show the same tendencies with those in Fig. 13 qualitatively. For extremely steep rough surfaces, the sampling points so selected in this study fail to guarantee satisfactorily energy conservation in both cases of Figs. 13(c) and 14(c). The error is probably given rise from the selected 10% relative error of SampEn. On the contrary, comparing the results in Figs. 13 and 14 shows that using the proposed entropy-based selection of simulation parameters, both preserving information content and guaranteeing energy conservation over a wide range of surface roughness are simultaneously attainable and ensured.

Two dielectric cases, a low lossy medium with permittivity of $3.22 + i0.33$ and a high lossy medium with permittivity of $20.8 + i3.75$, are examined; the simulation parameters are set to be the same with those in Figs. 13 and 14, and H polarization is considered. For dielectric surfaces and the TE polarization, based on the extinction theorem, the dual EFIEs are turned to be [4], for upper hemisphere (medium 0, free space in this study)

$$\begin{aligned} \psi_{\text{inc}}(\bar{r}) &= \frac{1}{2} \psi(\bar{r}) - \int_{PV} \psi(\bar{r}') \hat{n} \cdot \nabla g_0(\bar{r}, \bar{r}') ds \\ &+ \int_S g_0(\bar{r}, \bar{r}') \hat{n} \cdot \nabla \psi(\bar{r}') ds \end{aligned} \quad (25)$$

and for lower hemisphere (medium 1)

$$\begin{aligned} \frac{1}{2} \psi(\bar{r}) &+ \int_{PV} \psi(\bar{r}') \hat{n} \cdot \nabla g_1(\bar{r}, \bar{r}') ds \\ &- \int_S g_1(\bar{r}, \bar{r}') \hat{n} \cdot \nabla \psi(\bar{r}') ds = 0. \end{aligned} \quad (26)$$

In the above, $g_0(\bar{r}, \bar{r}')$, and $g_1(\bar{r}, \bar{r}')$ stand for the green function in the medium 0 and the lossy medium 1, respectively; the corner subscript denotes the principal value of the integration since the high-order singularity induced by the gradient of Green's function is excluded when $\bar{r} \rightarrow \bar{r}'$.

Similarly, using the point match method, two impedance matrix equations are acquired for medium 0 and medium 1, respectively. For (25)

$$\sum_{n=1}^N A_{mn}^{(0)} u_n + \sum_{n=1}^N B_{mn}^{(0)} \psi_n = b_m \quad (27)$$

and for (26)

$$\sum_{n=1}^N A_{mn}^{(1)} u_n + \sum_{n=1}^N B_{mn}^{(1)} \psi_n = 0 \quad (28)$$

where $u(x)$ is defined as

$$u(x) = \sqrt{1 + \left(\frac{dz}{dx} \right)^2} (\hat{n} \cdot \nabla \psi(\bar{r}))_{z=z(x)} \quad (29)$$

and b_m stands for the discretized incident field; furthermore, the expressions of elements in the impedance matrix A_{mn} and

Fig. 13. Bistatic scattering coefficient of PEC surfaces with Gaussian PSD function (100 realizations, zenith angle: 30°).

B_{mn} are derived as

$$A_{mn}^{(j)} = \begin{cases} \Delta x K_j(x_m, x_n), & \text{for } m \neq n \\ \frac{i \Delta x}{4} \left[1 + \frac{i2}{\pi} \ln \left(\frac{\gamma k_j}{4e} \Delta l_m \right) \right], & \text{for } m = n \end{cases} \quad (30)$$

and

$$B_{mn}^{(j)} = \begin{cases} -\Delta x K_{jN}(x_m, x_n; 0), & \text{for } m \neq n \\ -\frac{1}{2} - \frac{z''(x_m)}{4\pi} \frac{\Delta x}{1 + (f'(x_m))^2}, & \text{for } m = n \end{cases} \quad (31)$$

where $j = 0, 1$ denotes the upper hemisphere medium 0 and the lower hemisphere medium 1, respectively, and

$$K_j(x, x') = g_j(x, z(x); x', z(x')) \quad (32)$$

$$K_{jN}(x, x') = \frac{ik_j}{4} \frac{H_j^{(1)}(k_j|r - \bar{r}'|)}{|\bar{r} - \bar{r}'|} \cdot \{z'(x)(x - x') - (z(x) - z(x'))\}. \quad (33)$$

Once the impedance matrix equations (28) and (29) are solved, jointly, the surface fields are obtained, and the BSCs can be calculated using the expression given in (24). For the lossy medium, energy conservation is different from that of the PEC surfaces. The absorption in the lower half-space medium should be considered, that is, for medium cases, the energy conservation is governed by

$$a(\theta_s) + r(\theta_s) = 1 \quad (34)$$

where $a(\theta_s)$ denotes the absorptivity of the lower hemisphere medium.

The results of energy conservation are presented in Tables VIII and IX. For surfaces with Gaussian PSD function, satisfactory energy conservations are obtained for all selected sampling points, even for the steep rough surface. Again, the satisfactory energy conservations under selected simulation parameters show the validity of the proposed entropy-based method. It is again seen that a smaller sampling point per wavelength may satisfy the energy conservation but lack sufficient information content of inherent surface properties.

However, for dielectric surfaces with exponential PSD function, the results of steep rough surface are dramatically against the energy conservation law at all selected sampling points. For slightly rough surfaces, it seems that the selected simulation parameters perform well, but not much, in high

lossy medium than that in low lossy medium. In other words, the selected simulation parameters within a 10% relative error of SampEn are somehow invalid. By comparing the performance in Gaussian correlated surfaces and in exponentially correlated surfaces, it is found that the exponential surface contains much finer roughness scales that are responsible for a little poorer performance. Hence, a higher criterion, e.g., 5.0% relative error of SampEn, might be preferred if computation resources allow. Following this, we carried out the numerical simulation with simulation parameters given in Fig. 11, namely, sampling points at $N = 1024$ and least surface length at 20.48 wavelengths. The results of energy conservation are listed in Table X. It is found that the simulation parameters within 5.0% relative error of SampEn improve the accuracy of energy conservation, though not so significant. Despite this, such a tendency further confirms the validity of the proposed entropy-based method in selecting the surface parameters for numerical simulation of wave scattering of the rough surface.

Now, both the simulation results of PEC and dielectric surfaces suggest that the inherent surface properties should be preserved when setting simulation parameters. In some sense, highly accurate energy conservation at small sampling points is meaningless because, under small sampling points, the surfaces with fixed length are artificially smoothed due to the underestimated rms heights, especially at large correlation lengths [8], and the distribution of surface current density also affects the choice of sampling points. This can be observed from the significantly improved the accuracy of energy conservation in Figs. 13(c) and 14(c), that is, for PEC surfaces, the free charges evoked by external fields are mainly concentrated at the tips of a surface and, therefore, dominates the scattered fields. Thus, if the details of these minor tips on the surface are not characterized, accurate energy conservation cannot be acquired, whereas, for dielectric surfaces, the polarized charges are quite different and are generally fixed at a certain position on the surface, that is why highly accurate energy conservation is easily attainable for Gaussian correlated surfaces at small sampling points as shown in Tables VIII and IX, even with steep roughness. Hence, at small sampling points, the high accuracy of energy conservation of exponentially correlated surfaces in Tables VIII and IX can also be explained by combining two conditions. Nevertheless, it is not difficult to explain why the so selected simulation parameters produce

TABLE VIII
ENERGY CONSERVATION OF A DIELECTRIC SURFACE WITH MEDIUM PERMITTIVITY $3.22 + i0.33$
(THE SAME SIMULATION PARAMETERS AS IN FIGS. 12 AND 13)

RMS height	Gaussian			Exponential		
	0.1	0.35	1.0	0.1	0.35	1.0
N=64	0.9993	0.9993	1.0100	0.9905	0.9871	1.30
N=128	0.9998	0.9996	0.9984	0.9709	0.9607	1.38
N=256	0.9999	0.9992	0.9965	0.9458	0.9257	1.13
N=512	0.9999	0.9992	0.9959	0.9182	0.8891	1.99

TABLE IX
ENERGY CONSERVATION OF A DIELECTRIC SURFACE WITH MEDIUM PERMITTIVITY $20.8 + i3.75$
(THE SAME SIMULATION PARAMETERS AS IN FIGS. 12 AND 13)

RMS height	Gaussian			Exponential		
	0.1	0.35	1.0	0.1	0.35	1.0
N=64	1.005	1.004	1.018	1.012	1.073	2.34
N=128	0.997	0.999	0.996	1.006	1.058	1.56
N=256	0.9998	0.998	0.988	0.9912	1.016	1.43
N=512	0.9998	0.998	0.993	0.9751	1.004	1.84

better performance in a high lossy medium than in a low lossy medium. It also suggests that for high lossy medium, such as seawater, increasing sampling points is an effective means to enhance the accuracy of energy conservation. Consequently, preserving sufficient information content is the result of a finer sampling, and the number of sampling points is dictated by the surface inherent properties. Then, using the pulse basis function in MOM, more sampling points perform better than the empirical rules of parameter settings, especially for very rough surfaces with exponential PSD function.

Following the above account, we used the pulse basis function (point matching) in computing the scattered field to demonstrate the use of entropy-based SampEn as a measure to determine the sampling density that best preserves the surface information content. Within a selected number of sampling points, the roof-top basis function or other choices of numerical techniques may be applied.

We also note that even combining the high sampling rate and numerical techniques does not warrant a highly accurate energy conservation for a very rough surface. Given this situation, the finer scales, which are much shorter compared to the incident wavelength and out of resonance with incident waves, were ignored and replaced by the interpolated curves [23]. With this in mind, pursuing finer representations of the rough surface seems to lose its standing point. However, if one looks deeper into this method, say, in [23], the key still lies in exploring the optimal discretization strategies of a rough surface. In other words, one still needs to find the least sampling rate first and then connect these sampling points using interpolated curves.

To further take the abovementioned two issues into account, the Nystrom method [22] is selected as an accurate computation to demonstrate the sampling effects in numerical simulations of wave scattering from a rough surface. Details of the Nystrom method are omitted here for the sake of simplicity and can be referred to [22]. Following the same manner as in [22], fifth-order Legendre polynomials are applied in the Nystrom method. The two selected media in the former

TABLE X
ENERGY CONSERVATION OF DIELECTRIC SURFACES WITH EXPONENTIAL PSD FUNCTION (WITHIN 5.0% RELATIVE ERROR OF SAMPEN)

RMS Height (wavelength)	Permittivity	
	$3.22+i0.33i$	$20.8+i3.75$
0.1	0.9223	0.9753
0.35	0.8974	1.0002

discussions are first conducted, with the energy conservations for both polarizations and at different sampling rates (PPW) are presented in Tables XI and XII, in which the rms height, correlation length, and surface length were fixed at 0.35, 1.0, and 12 wavelengths, respectively. Within these simulation parameters, the least sampling rate in this study was 50 PPW.

It is readily seen from Tables XI and XII that the energy conserves well at low sampling rates for both polarizations, that is, under the sampling rates of 16 PPW for a low lossy medium and 20 PPW for a highly lossy medium. However, these are somewhat meaningless owed to the relatively lower sampling rates that artificially smoothed the rough surfaces, and the results induced by these low sampling rates should be excluded in later analysis. Thus, as discussed at the beginning of this study, using energy conservation as a signature only is somehow unreasonable, not at low sampling rates, whereas, as shown in Fig. 4, a relatively higher sampling rate preserves much better surface's finer scales. However, at a sampling rate of 16 PPW for low lossy medium and 20 PPW for highly lossy medium, the energy conservations are still unsatisfied and even worse than that at low sampling rates, which reveals that such sampling rates, insufficient to characterize surface details, must be increased. Then, for the low lossy medium, the sampling rate of 20 PPW performs better than 16 PPW and even much better so at a sampling rate of 50 PPW as suggested in this study. For the highly lossy medium, only the sampling rate at 50 PPW can satisfy energy conservation. In summary, we may perceive that the following holds.

TABLE XI

ENERGY CONSERVATION OF A DIELECTRIC SURFACE WITH MEDIUM PERMITTIVITY $3.22 + i0.33$ (RMS HEIGHT 0.35, CORRELATION LENGTH: 1.0, SURFACE LENGTH 12.0, AND 100 REALIZATIONS)

PPW	3	5	10	16	20	50
V (TM)	0.9993	0.9994	0.9996	0.9939	1.0002	0.9998
H (TE)	0.9991	0.9993	0.9993	0.9923	0.9993	0.9995

TABLE XII

ENERGY CONSERVATION OF A DIELECTRIC SURFACE WITH MEDIUM PERMITTIVITY $20.8 + i3.75$ (RMS HEIGHT 0.35, CORRELATION LENGTH: 1.0, SURFACE LENGTH 12.0, AND 100 REALIZATIONS)

PPW	3	5	10	16	20	50
V (TM)	0.9934	0.9981	1.0049	0.9990	0.9986	0.9993
H (TE)	0.9994	0.9989	0.9989	0.9988	0.9987	0.9989

- 1) The surface with larger roughness reveals larger SampEn value and therefore demands a higher sampling rate, that is why the sampling rates below 50 PPW in Cases A and B well satisfy the energy conservation.
- 2) The suggested least sampling rate applies in a wide range of surface roughness. From Cases A and B, even for the same roughness parameters, the sampling rates, which satisfy the energy conservation, are different. Then, besides the surface roughness, the obtained simulation parameters in this study are also applicable to different dielectric media.
- 3) The concept of reserving sufficient surface inherent properties (information content) physically explains the reason for pursuing satisfying energy conservation by increasing the sampling rate and can be regarded as a unique criterion in selecting the simulation parameters when numerically generating rough surfaces for scattering study. By doing so, we can avoid desperate searching for optimal stimulation parameters for different roughness parameters and mediums individually.

V. CONCLUSION

Based on the entropy measure of the minimum error of SampEn and the unique SampEn, we investigate the surface parameter settings in computer generation of random rough surfaces for applications, e.g., the numerical simulation of wave scattering. Using the minimum error of SampEn and a unique SampEn, the minimum number of sampling points and their corresponding surface length are consistently determined for surfaces with Gaussian and exponential PSD functions. The proposed choice of a minimum surface length ensures a statistically representative characterization of a randomly rough surface with specified PSD function under different correlation lengths. The validated interconnections between the statistical error of variance and the relative error of SampEn of the chosen simulation parameters show that the relative error of SampEn is effective in determining the simulation parameters for generating a statistically accurate rough surface. Together with preserving the inherent surface properties, a smaller relative error of SampEn guarantees a more statistically accurate surface. Numerical simulations of wave scattering from rough surfaces suggest that the inherent surface properties

and the energy conservation can be simultaneously attainable using the proposed simulation parameters within a preset 10% relative error of SampEn. These simulation parameters are more effective and yet efficient under a wide range of surface roughness compared with the commonly used rule of thumb with a sampling rate of 10 PPW. In practice, a preset relative error of SampEn is suggested to ensure both preserving the inherent surface properties and high accuracy of energy conservation; a 10% relative error of SampEn is demonstrated in this article. A smaller value is also viable at the user's preference. In summary, this study proposes a consistent way of surface parameter setting based on the entropy measure for generating statistically accurate random rough surface and, as an example, demonstrates the parameter settings in numerical simulation of wave scattering of rough surfaces.

APPENDIX

The appendix aims at showing the correctness of the proposition, which under certain roughness parameters and PSD functions, the surface samples generated by randomly selected seeds have the same SampEn value.

It is noted that in [19] and [21], the estimated SampEn value of a random series has a bias, and the bias is evaluated and represented using a 95% confidence interval. Thus, theoretically speaking, under certain roughness parameters and PSD functions, the estimated SampEns of surface samples generated with different random seeds fluctuate and with their 95% confidence intervals overlapped. Using ten different random seeds and under certain roughness parameters, Fig. 15 shows the estimated SampEn values with a 95% confidence interval for surface samples with Gaussian and exponential PSD functions.

In Sections III and IV of this study, we follow the ensemble averaging process in the Monte Carlo simulation, and the estimated SampEn value denotes the mean value of the estimated SampEn values over large amounts of realizations. When it comes to the case of exploring the dependencies of estimated SampEn value on random seed, under certain roughness parameters and PSD functions, as shown in Fig. 15, the SampEn value of surface samples must be estimated separately for each random seed and then compared with each other.

Fig. 14. Bistatic scattering coefficient of PEC surfaces with exponential PSD function (100 realizations, zenith angle: 30°).

Fig. 15. Estimated SampEn values (with a 95% confidence interval) of ten surface samples generated with different random seeds. (a) Gaussian. (b) Exponential.

Due to the fluctuations and the overlapped 95% confidence intervals, the key then lies in how to statistically prove that the estimated SampEn values can be regarded as the same under different random seeds. Here, to intuitively and rigorously show the correctness of this proposition, we propose a statistically based method, which fully takes the advantages of the mse concepts and the independences induced by the random seeds. The details and interpretations are presented as follows.

The mse, a SampEn-based algorithm, was first introduced in analyzing complex physiological time series [21]. Two main procedures in the mse are the coarse-graining process and the SampEn estimation for each coarse-grained subseries. For an original series $\{x\}$ with length N , the coarse-graining process can be mathematically expressed as

$$y_j^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau}^{j\tau} x_i \quad 1 \leq j \leq N/\tau \quad (\text{A-1})$$

Fig. 16. Diagram illustrating the coarse-graining processes with scale factor up to three.

where τ is the introduced scale factor denoting the times of coarse graining and therefore dominates the length of each coarse-grained subseries, and i and j denote the ID of elements in original series and in the coarse-grained subseries, respectively. Fig. 16 shows the coarse-graining process with scale factor up to three, and it is noted that the original series corresponds to the case of $\tau = 1$.

Once a scale factor τ is prespecified, the SampEn value of each subseries is then estimated to follow the SampEn algorithm introduced in Section II-B. As an example, Fig. 17 shows the estimated SampEn values (with a 95% confidence interval) of both the white noise series and the $1/f$ noise ($p = 1.5$) series with length $N = 3000$ and a scale factor at $\tau = 20$. It is seen from Fig. 17 that with the increase in scale factor, the estimated SampEn values of white noise gradually decreased, whereas the estimated SampEn values of the $1/f$ noise keep invariant regardless of the scale factor. Fig. 17 shows the superiority of mse algorithm, that is, the mse deeply mines the complexity of a random series, and the $1/f$ noise is regarded equipping higher complexity than that of the white noise, although the latter one has larger estimated SampEn values at small scale factors up to three [21].

Hence, to rigorously show the correctness of the proposed proposition, we decide to prove that under certain roughness parameters and PSD functions, the mse of surface samples keeps invariant with varied random seed. This is equivalent to the proposed proposition, but more theoretically complete and technically sound. Meanwhile, it is convenient to test the

Fig. 17. Estimated mses with 95% confidence interval for the white noise and the $1/f$ noise (series length: 3000, scale factor: 20, and $m = 2$).

Fig. 18. Estimated mses with 95% confidence interval for two surface samples generated with different random seeds. (a) Gaussian. (b) Exponential.

significance of difference between two series (mses shown in Fig. 18), rather than the separate numbers (estimated SampEn values in Fig. 14). As an example, following the mse algorithm, the mse with a 95% confidence interval of two surface samples generated with different random seeds is shown in Fig. 18 with a scale factor at $\tau = 10$. If the estimated mses of two surface samples shown in Fig. 18 were not significantly different, and then, the random seed can be regarded as having no effects on estimating the MSE and nor does the SampEn. Due to the independence induced by the randomly selected seeds, the estimated mse series in Fig. 18 are therefore independent of each other, and thus, one alternative method to test the significance of the difference is using the Mann–Whitney U test (also named the rank-sum test after his name).

From the principles discussed earlier, in what follows, we validate the correctness of the proposed proposition for general cases, that is, test the significance of difference of MSE series over large amounts of realization. In the rank-sum

Fig. 19. Calculated p values in testing the significance of difference of mses of surface samples generated with different random seeds. (a) Gaussian. (b) Exponential.

test, without loss of generality, $p < 0.05$ is considered to be significant. For both Gaussian and exponential functions, 30 realizations, which yields 435 comparing pairs (number of p values), are conducted. The rms height and correlation length are both set at 1.0 to correspond to the largest roughness case, the sampling points and grid size at 8192 and 0.01 obtained from the former sections are followed to make sure that the surface samples are accurately generated, respectively. The calculated p values of these 435 comparing pairs are presented in Fig. 19.

As shown in Fig. 19, under the selected roughness parameters, for the Gaussian PSD function, the calculated p values are all larger than the threshold 0.05. Thus, there are no significant differences between the mses of surface samples generated with randomly selected seeds and can be statistically regarded as the same, while for exponential PSD function, almost all p values are larger, but only four of them are smaller than the threshold 0.05. The appearance of these four smaller p values may probably owes to the largest roughness case or the simulation parameters obtained within a 10% relative error of SampEn. However, this will not prevent us from concluding that there are no significant differences between the mses of surface samples with exponential PSD function since, from possibility perspectives, the possibility of the appearance of these four smaller p values ($P = 4/435 = 0.0092$) is much smaller than the possibility of the impossible event ($P = 0.05$), and these four smaller p values can be ignored.

Thus, it is conceivable to conclude that under certain roughness parameters and PSD functions, the estimated mses

of surface samples generated with different random seeds can be regarded as the same statistically. Then, the randomly selected seed is turned to have no effects on estimating the SampEn of surface samples, which also means that in Monte Carlo simulations, under certain roughness parameters and PSD functions, the surface samples generated with randomly selected seeds have the same information content. Together with remarks on the geometry similarity and the Monte Carlo method in Section III-A, the invariant relative error of SampEn with rms height is illustrated.

ACKNOWLEDGMENT

The authors would like to thank the MIT laboratory of Computational Physiology Group for sharing the fundamental and advance studies of sample entropy and multiscale entropy to commons. They would like to thank Prof. Leung Tsang and Dr. Jiyue Zhu for the fruitful grateful technical discussions. They would also like to thank the anonymous reviewers for their comments and suggestions that helped improve this quality of this article.

REFERENCES

- [1] F. T. Ulaby, *Handbook of Radar Scattering Statistics for Terrain*, 2nd ed., Boston, MA, USA: Artech House, 2019.
- [2] L. Tsang, J. A. Kong, and K. H. Ding, *Scattering of Electromagnetic Waves: Theories and Applications*, 1st ed. Hoboken, NJ, USA: Wiley, 2000.
- [3] A. K. Fung and K. S. Chen, *Microwave Scattering and Emission Models for Users*, 1st ed. Boston, MA, USA: Artech House, 2010.
- [4] L. Tsang, J. A. Kong, K. H. Ding, and C. O. Ao, *Scattering of Electromagnetic Waves: Numerical Simulations*, 1st ed. Hoboken, NJ, USA: Wiley, 2001.
- [5] V. Y. Mendeleyev, "Dependence of measuring errors of RMS roughness on stylus tip size for mechanical profilers," *Appl. Opt.*, vol. 36, no. 34, pp. 9005–9009, 1997.
- [6] J.-J. Wu, "Spectral analysis for the effects of stylus tip curvature on measuring isotropic rough surfaces," *Meas. Sci. Technol.*, vol. 13, no. 5, pp. 720–730, May 2002.
- [7] E. I. Thorsos, "The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum," *J. Acoust. Soc. Amer.*, vol. 83, no. 1, pp. 78–92, Jan. 1988.
- [8] C. A. Mack, "Generating random rough edges, surfaces, and volumes," *Appl. Opt.*, vol. 52, no. 7, pp. 1472–1480, 2013.
- [9] J. V. Toporkov, R. T. Marchand, and G. S. Brown, "On the discretization of the integral equation describing scattering by rough conducting surfaces," *IEEE Trans. Antennas Propag.*, vol. 46, no. 1, pp. 150–161, Jan. 1998.
- [10] L. Zhou, L. Tsang, V. Jandhyala, Q. Li, and C. H. Chan, "Emissivity simulations in passive microwave remote sensing with 3-D numerical solutions of maxwell equations," *IEEE Trans. Geosci. Remote Sens.*, vol. 42, no. 8, pp. 1739–1748, Aug. 2004.
- [11] S. Huang and L. Tsang, "Electromagnetic scattering of randomly rough soil surfaces based on numerical solutions of Maxwell equations in three-dimensional simulations using a hybrid UV/PBTG/SMCG method," *IEEE Trans. Geosci. Remote Sens.*, vol. 50, no. 10, pp. 4025–4035, Oct. 2012.
- [12] J. S. Yang, Y. Du, and J. C. Shi, "Polarimetric simulations of bistatic scattering from perfectly conducting ocean surfaces with 3 m/s wind speed at L-band," *IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens.*, vol. 9, no. 3, pp. 1176–1186, Mar. 2016.
- [13] J. S. Richman and J. R. Moorman, "Physiological time-series analysis using approximate entropy and sample entropy," *Amer. J. Physiol.-Heart Circulatory Physiol.*, vol. 278, no. 6, pp. 2039–2049, 2000.
- [14] M. Costa, A. L. Goldberger, and C.-K. Peng, "Multiscale entropy analysis of complex physiologic time series," *Phys. Rev. Lett.*, vol. 89, no. 6, Jul. 2002, Art. no. 068102.
- [15] Y. Oh and Y. Chul Kay, "Condition for precise measurement of soil surface roughness," *IEEE Trans. Geosci. Remote Sens.*, vol. 36, no. 2, pp. 691–695, Mar. 1998.
- [16] W. J. Malcolm Davidson, T. L. Toan, F. Mattia, G. Satalino, T. Manninen, and M. Borgeaud, "On the characterization of agricultural soil roughness for radar remote sensing studies," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 2, pp. 630–640, Mar. 2000.
- [17] F. T. Ulaby and D. Long, *Microwave Radar and Radiometric Remote Sensing*. Ann Arbor, MI, USA: University of Michigan Press, 2014.
- [18] Y. Yang, K. S. Chen, and G. F. Shang, "Surface parameters retrieval from fully bistatic radar scattering data," *Remote Sens.*, vol. 11, no. 5, p. 596, 2019.
- [19] J. S. Richman, "Sample entropy," in *Computer Numerical Method: Part E*, vol. 383. Oxford, U.K.: Elsevier, 2004.
- [20] V. V. Nikulin and T. Brismar, "Comment on 'Multiscale entropy analysis of complex physiologic time series,'" *Phys. Rev. Lett.*, vol. 92, no. 8, 2004, Art. no. 089803.
- [21] M. Costa, A. L. Goldberger, and C.-K. Peng, "Multiscale entropy analysis of biological signals," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 71, no. 2, Feb. 2005, Art. no. 021906.
- [22] T. Qiao, Y. Du, and L. Tsang, "Electromagnetic scattering and emission by ocean surfaces based on neighborhood impedance boundary condition (NIBC) with dense grid: Accurate emissivity and sensitivity to salinity," *Prog. Electromagn. Res. B*, vol. 81, pp. 141–162, 2018.
- [23] P. Xu and L. Tsang, "Bistatic scattering and emissivities of lossy dielectric surfaces with exponential correlation functions," *IEEE Trans. Geosci. Remote Sens.*, vol. 45, no. 1, pp. 62–72, Jan. 2007.

Rui Jiang (Member, IEEE) received the Ph.D. degree in radiophysics from Wuhan University, Wuhan, China, in 2018.

He was a Research Assistant with Electronic Information School, Wuhan University, from 2013 to 2014, a Visiting Student with the Institute of Digital Earth and Remote Science, Chinese Academy of Sciences, Beijing, China, from 2015 to 2018, and a Visiting Scholar with Radiation Laboratory, Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA, from 2019 to 2020. He is with Xuchang University, Xuchang, China. His research interests include the wave scattering from rough surface, physical modeling in microwave remote sensing, and microwave imaging techniques.

Kun-Shan Chen (Fellow, IEEE) received the Ph.D. degree in electrical engineering from The University of Texas at Arlington, Arlington, TX, USA, in 1990.

From 1992 to 2014, he was a Professor with the National Central University, Taoyuan, Taiwan. Since 2014, he has been with the Institute of Remote Sensing and Digital Earth, Chinese Academy of Sciences, Beijing, China. He has authored or coauthored over 170 refereed journal articles, contributed ten book chapters, coauthored (with A. K. Fung) the book titled *Microwave Scattering and Emission Models for Users* (Artech House, 2010), authored the book titled *Principles of Synthetic Aperture Radar: A System Simulation Approach* (CRC Press, 2015), co-edited (with X. Li, H. Guo, and X. Yang) *Advances in SAR Remote Sensing of Ocean* (CRC Press, 2018), and authored a recent book on *Radar Scattering and Imaging of Rough Surface: Modeling and Applications with MATLAB®* (CRC Press, 2020). His research interests include microwave remote sensing theory, modeling, system, and measurement, and intelligent signal processing and data analytics for radar.

Dr. Chen served as an Editorial Board Member of the PROCEEDINGS OF THE IEEE from 2014 to 2019. He has been an Editorial Board Member of the IEEE ACCESS since 2020. His academic activities include as a Guest Editor of the Special Issue on Remote Sensing for Major Disaster Prevention, Monitoring, and Assessment (2007) of the IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING, a Guest Editor of the Special Issue on Remote Sensing for Natural Disasters (2012) of PROCEEDINGS OF THE IEEE, a member of the IEEE GRSS Adcom from 2010 to 2014, the Founding Chair of the GRSS Taipei Chapter, and the Founding Deputy Editor-in-Chief of the IEEE JOURNAL OF SELECTED TOPICS IN APPLIED EARTH OBSERVATIONS and *Remote Sensing* from 2008 to 2010. He has been an Associate Editor of the IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING since 2000. He has served as a Guest Editor for *Remote Sensing* of Special Issue on Data Restoration and Denoising of Remote Sensing Data and the Special Issue on Radar Imaging Theory, Techniques, and Applications. He was the Co-Chair of the Technical Committee for IGARSS 2016 and IGARSS 2017.

Zhao-Liang Li (Senior Member, IEEE) received the B.A. degree in photogrammetry from the Wuhan Technical University of Surveying and Mapping, Wuhan, China, in 1985, and the M.S. degree in imaging processing and computer graphics and the Ph.D. degree in terrestrial environmental physics from the University of Louis Pasteur, Strasbourg, France, in 1987 and 1990, respectively.

Since 1992, he has been a Research Scientist with CNRS, Illkirch, France. He joined the Institute of Agricultural Resources and Regional Planning, Chinese Academy of Agricultural Sciences, Beijing, China, in 2013. He has participated in many national and international projects, such as the National Aeronautics and Space Administration (NASA)-funded Moderate Resolution Imaging Spectroradiometer, the European Community (EC)-funded program Exploitation of angular effects in land surface observations from satellites (EAGLE), and the ESA-funded programs SPECTRA. He has authored more than 200 articles in international refereed journals. His main research interests include thermal infrared radiometry, parameterization of land surface processes at large scale, and assimilation of satellite data to land surface models.

Wen-Jing Tian (Student Member, IEEE) is pursuing the bachelor's degree in computer science with the School of Mathematics and Computer Science, Wuhan Textile University, Wuhan, China.

Her studies mainly include digital image processing, entropy methods in texture analysis, and machine learning.

Gen-Yuan Du received the B.S. degree in computer science and technology from Henan Normal University, Xinxiang, China, in 1997, the M.E. degree in signal and information processing from the Chengdu University of Technology, Chengdu, China, in 2005, and the Ph.D. degree from the College of Information Engineering, Chengdu University of Technology, in 2011.

He is a Professor with Xuchang University, Xuchang, China. His research interests include parallel computing and remote sensing image processing.