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Vortex dynamics behind a self-oscillating inverted flag placed in a channel flow: Time-resolved particle image velocimetry measurements

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The unsteady flow behind an inverted flag placed in a water channel and then excited into a self-oscillating state is measured using time-resolved particle image velocimetry. The dynamically deformed profiles of the inverted flag are determined by a novel algorithm that combines morphological image processing and principle component analysis. Three modes are discovered with the successive decrease in the dimensionless bending stiffness: the biased mode, the flapping mode, and the deflected mode. The distinctly different flow behavior is discussed in terms of instantaneous velocity field, phase-averaged vorticity field, time-mean flow field, and turbulent kinetic energy. The results demonstrated that the biased mode generated abundant vortices at the oscillating side of the inverted flag. In the deflected mode, the inverted flag is highly deflected to one side of the channel and remains almost stationary, inducing two stable recirculation zones and a considerably inversed flow between them. In the flapping mode, the strongly oscillating flag periodically provides a strengthened influence on the fluid near the two sidewalls. The reverse von Kármán vortex street is well formed and energetic in the wake, and a series of high-speed impingement jets between the neighboring vortices are directed toward the sidewalls in a staggered fashion.

NOMENCLATURE

B	Flexural rigidity (N m)
C_p	Pressure coefficient
d	Mean diameter of glass beads used for PIV
E	Young's modulus of the inverted flag (Gpa)
f	Flapping frequency of the inverted flag (Hz)
f^*	Dimensionless frequency
h	Thickness of the inverted flag (m)
H	Height (or span) of the inverted flag (m)
K_B	Dimensionless bending stiffness
l	Length of the clamp plate (m)
L	Length (or chord) of the inverted flag (m)
m_s	Structure-to-fluid mass ratio
P	Fluid pressure (Pa)
P_0	Free stream pressure (Pa)
ΔP	Pressure difference between the two sides of the inverted flag (Pa)
ΔP^*	Dimensionless pressure difference
Re	Reynolds number (based on the length of the inverted flag)
t	Time (s)
t^*	Non-dimensional time
u	Streamwise velocity (m s^{-1})
u'	Fluctuating part of the streamwise velocity (m s^{-1})
u'_0	Fluctuating part of the streamwise velocity in the absence of the inverted flag (m s^{-1})
U	Flow velocity (m s^{-1})

U_0	Averaged velocity of the free stream (m s^{-1})
U_s	Advection velocity of the vortices (m s^{-1})
v	Wall-normal velocity (m s^{-1})
v'	Fluctuating part of the wall-normal velocity (m s^{-1})
v'_0	Fluctuating part of the wall-normal velocity in the absence of the inverted flag (m s^{-1})
W	Width of the channel (m)
X^*	Normalized wall-normal coordinate
Y^*	Normalized streamwise coordinate

Greek symbols

α	Angle of attack (m s^{-1})
λ	Wavelength of the undulating wake
ρ_f	Fluid density (kg m^{-3})
ρ_s	Density of the inverted flag (kg m^{-3})
σ	Poisson's ratio of the inverted flag
ω	Vorticity (s^{-1})

Abbreviations

FFT	Fast Fourier transform
LEV	Leading edge vortex
PCA	Principle component analysis
PET	Polyethylene terephthalate
TEV	Trailing edge vortex
TKE	Turbulence kinetic energy, $\text{TKE} = 1/2 (\overline{u'^2} + \overline{v'^2})$
TKE^*	Dimensionless turbulence kinetic energy, $\text{TKE}^* = (\overline{u'^2} + \overline{v'^2}) / (\overline{u_0'^2} + \overline{v_0'^2})$
TR-PIV	Time-resolved particle image velocimetry

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I. INTRODUCTION

A self-oscillating flag placed in the channel has been recently proposed to substantially enhance the wall heat transfer.^{1–3} At appropriate flow condition, the flag generates periodic oscillations with large flapping amplitudes; the resulting well-organized vortices sweep out the thermal boundary layer and enhance the thermal mixing between the fluid near the heated wall and the channel core flow.^{1,4} The insightful understanding of unsteady flow behaviors excited by the self-oscillating flag would fundamentally benefit such applications.

The flapping dynamics of an inverted flag placed in the uniform flow were first studied by Kim *et al.*⁵ An inverted flag was clamped at the trailing edge and free to oscillate at the leading edge. The flapping dynamics of the inverted flag were characterized by low critical speed, along with high oscillating amplitude in water and air flow, which could generate large-scale coherent vortical structures. Three modes were reported as the bending stiffness decreases: the straight mode, the flapping mode, and the deflected mode with a highly curved shape. Bistable states were observed in the transition regime between the modes, along with a small hysteresis loop. Using the immersed boundary-lattice Boltzmann method for the fluid flow at $Re = 100$ – 500 and a finite element method for the plate deformation, Tang *et al.*⁶ reproduced the phenomenon observed in experiments. Moreover, the oscillating amplitude and the frequency were considerably increased as the aspect ratio (height/length) increased from $H/L = 0.5$ to 1.5 , and they tended to saturate at $H/L = 2$. Sader *et al.*⁷ extended the measurements to study the effect of aspect ratio; the critical flow speed in the flapping mode decreased as H/L increased, and a convergence of the critical speed was reached beyond $H/L = 2$. In combination with theoretical analysis, they concluded that the oscillation exhibited the characteristics of vortex induced vibration.

Vortex dynamics are essential to elucidate vortex induced vibration behaviors of flexible flags. Ryu *et al.*⁸ used the immersed boundary method to determine flow dynamics of an inverted flag at $Re \leq 250$ in uniform flow. Different modes of the evolving vortical structures were established in terms of the dimensionless flapping frequency defined by $f^* = fL/U$ (here, the f and U correspond to the flapping frequency and the free stream velocity, respectively): (1) At low frequency $f^* = 0.12$, the vortical structures had sufficient time to grow, forming two pairs of vortices in half cycle, with a leading edge vortex (LEV) followed by a trailing edge vortex (TEV) in each pair; (2) at higher frequency $f^* = 0.19$, a TEV sandwiched by two LEVs shed during each stroke; (3) as f^* was greater than 0.26 , the vortices were more coherent and only one pair of LEV and TEV detached in each stroke, which corresponded to the flow visualized by Kim *et al.*⁵ The direct numerical simulation (DNS) of the uniform flow at $Re \leq 1000$ by Gurugubelli and Jaiman⁹ suggested more dispersed vortical structures at lower f^* . In half cycle of the flapping mode, the inverted flag generated three pairs plus one single vortices at $f^* = 0.05$ and two pairs plus one single vortices at $f^* = 0.10$. Dynamics of the vortical structures in the Poiseuille channel flow at $Re \leq 800$ were simulated by Park *et al.*,¹ in which the

width of the channel was two times the length of the inverted flag. A group of LEVs sandwiched by two TEVs were generated in each stroke due to the interaction between the flag and the surrounding fluid; this behavior in the flow channel actually corresponded to the abovementioned second mode in the uniform flow by Ryu *et al.*⁸ In the uniform flow without wall constraint, the fluid at the leading edge of the inverted flag moved faster due to the flag's acceleration; the LEV took the position of the front vortex. However, in the channel flow configuration, the fluid velocity was lower at the leading edge than the trailing edge as the inverted flag reached the stroke extreme; correspondingly, the TEV transported faster than the LEV, and detachment of the LEV induced another TEV. As this group of three vortices moved downstream, the front TEV drew another pair of vortices near the channel wall.

Notwithstanding the huge potential application for channel heat transfer enhancement,² the vortex dynamics of a self-oscillating inverted flag in turbulent channel flow are relatively unexplored. In the present study, the highly unsteady flow field interacting with a self-oscillating inverted flag placed in a turbulent channel flow is experimentally determined using time-resolved particle image velocimetry (TR-PIV). An inverted flag configuration is used due to its low critical speed of instability, beyond which coherent vortical structures form in the wake.^{5,8} To accurately determine and quantify the dynamically deformed flag, an algorithm combining morphological image processing and principle component analysis (PCA) was first developed. Three distinctly different modes of the oscillating flag were observed as the dimensionless bending stiffness of the inverted flag decreased: the biased, flapping, and deflected modes, respectively.

II. EXPERIMENTAL METHOD

A. Experimental apparatus

The experiments were conducted in a water channel, as schematically shown in Fig. 1. A smooth contraction fairing with a contraction ratio of 9.375:1 was designed to straighten the flow. The test section was 40 mm in width, 80 mm in height, and 900 mm in length. The length (or chord), span, and

FIG. 1. Schematic diagram of the experimental setup.

thickness of the inverted flag were $L = 20$ mm, $H = 60$ mm, and $h = 100$ μ m, respectively. The high aspect ratio of the inverted flag $H/L = 3$ allowed two-dimensional flow near the mid-span of the inverted flag. The thickness of the turbulent boundary layer was $\delta_{99} = 0.3L$, where L was the length of the inverted flag. The free stream turbulence level was below 8%. The averaged free stream velocity was varied between $U_0 = 0.23$ m/s and 0.38 m/s, which resulted in a Reynolds number (based on the length of the inverted flag) $Re = U_0 L / \nu = 4660\text{--}7600$. The inverted flag was made of transparent polyethylene terephthalate (PET), which facilitated access to the whole flow field. The inverted flag was free at the leading edge and clamped by a transparent plate with a length of $l = 10$ mm and thickness of 2 mm. The clamped plate spanned the full height of the channel vertically.

In terms of Newton's second law, vibration of a two dimensional elastic flag is given by¹⁰

$$\rho_s h \frac{\partial^2 X}{\partial t^2} + B \frac{\partial^4 X}{\partial Y^4} = \Delta P, \quad (1)$$

where ρ_s is the density of the inverted flag, the wall-normal deflection of the flag (X) is a function of the streamwise direction Y and the time t , $B = Eh^3/12(1 - \sigma^2)$ is the flexural rigidity of the flag, E and σ are, respectively, Young's modulus and Poisson's ratio, and ΔP is the pressure difference between two sides of the flag. The forces corresponding to the viscoelastic damping of the material and the tension due to the viscous boundary layers around the flag are negligible as observed by Connell and Yue.¹¹ Scaling space on the length of the inverted flag L , time on L/U_0 , pressure difference on $\rho_f U_0^2$, and the dimensionless quantities are given by

$$X^* = \frac{X}{L}, \quad Y^* = \frac{Y}{L}, \quad t^* = \frac{t U_0}{L}, \quad \Delta P^* = \frac{\Delta P}{\rho_f U_0^2}, \quad (2)$$

where ρ_f is the fluid density. Equation (1) is then non-dimensionalized as

$$m_s \frac{\partial^2 X^*}{\partial t^{*2}} + K_B \frac{\partial^4 X^*}{\partial Y^{*4}} = \Delta P^*, \quad (3)$$

where the dimensionless bending stiffness K_B represents the ratio of the flag's bending force to the fluid's inertial force, i.e.,

$$K_B = \frac{B}{\rho_f U_0^2 L^3}. \quad (4)$$

This was claimed to be the primary parameter determining the oscillating modes of the flag.^{5,9} The flexural rigidity of the flag was $B = 2.16 \times 10^{-4}$ N m. As Eq. (4) shows, the increase in the Reynolds number indicates the fast reduction in K_B , which ranges from 0.19 to 0.43. As for the flexible flag,¹⁰ the structure-to-fluid mass ratio m_s was another important parameter characterizing the interaction between the fluid flow and an elastic sheet,

$$m_s = \frac{\rho_s h}{\rho_f L}. \quad (5)$$

The density of the inverted flag in experiment was $\rho_s = 1.38 \times 10^3$ kg/m³, giving a mass ratio $m_s = 0.007$. According to Kim *et al.*,⁵ the effect of mass ratio on the range of dimensionless bending stiffness corresponding to the

flapping mode was negligible when the mass ratio varied between $O(10^{-3})$ and $O(1)$.

In experiments, the mid-span of the inverted flag was illuminated by a 1-mm-thick laser sheet (532 nm). The laser sheet was produced by a 5W continuous-wave semiconductor laser combined with cylindrical lenses. A 12-bit high-speed CMOS camera (dimax HS4, pco., USA) was installed perpendicular to the laser sheet as schematically shown in Fig. 1, to capture the varying profiles of the inverted flag and the seeded flow fields. In the experiment, the camera was operated at 2000×1200 pixels with a sampling rate of 2000 fps. Glass beads ($\rho_p = 1.05$ g/cc, $d \approx 10$ μ m) were used as tracer particles. The standard cross-correlation PIV algorithm, in combination with window offset, multi-pass scheme (two iteration steps) in which the displacements were refined from their previous estimates, sub-pixel recognition by Gaussian fitting, and sub-region distortion, was used to determine the velocity fields.¹² The interrogation window size was 16×16 pixels with a 50% overlap, and the spatial resolution of the velocity vectors was 0.57×0.57 mm². Physically irrelevant flow vectors that traversed the flag were not considered in the present study. A total of 20 000 instantaneous velocity fields were obtained from 20 001 sequential images. The ratios of the rejected vectors in measurements were typically about 2.8% of the total number of vectors per vector map. These rejected spurious vectors were then substituted through 3-point by 3-point neighborhood interpolations to arrive at the final velocity vector fields. The error in measuring the particle displacement between two images was less than 0.1 pixels, and the uncertainty in the measurement of the velocity field was less than 2%.¹³

B. Structure boundary detection algorithm

To determine the dynamically deformed profiles of the inverted flag, which interacted with the fluid, a novel structure boundary detection algorithm was first developed. The algorithm provided self-adapted identification for the changing image intensity and morphology of the inverted flag at various instants. Figure 2 illustrates the procedure of the structure boundary detection algorithm. Logarithmic transform of image intensity was implemented to increase the contrast between the foreground and the dark background.¹⁴ Figure 2(a) gives a close-up view of the curved flag at an instant of the flapping mode. The illuminated flag was immersed in the flow seeded with particles for PIV measurement. The flow was from top to bottom. The first step was to separate the structure boundary from the bright dot particles. A binary image was generated from the grayscale image using a pre-defined threshold. Here, the method proposed by Otsu¹⁵ was used to adaptively determine the threshold value, which minimized the intra-class variance of the thresholded black and white pixels. The regions of high image intensity, including the flag and some of the bright particles, were set to one, while the dark background and particles with image intensities lower than the threshold were set to zero. The connected-component labeling¹⁶ of 1 pixel was performed to the binary image. Accordingly, the connected regions were identified and their areas were measured. The flag was characterized by a large connected region, so the remaining particles were filtered based on the area of the regions. And the mask was generated

FIG. 2. Illustration of the structure boundary detection algorithm: (a) deflected flag immersed in the seeded particle image; (b) mask generated by binarizing the image through thresholding and filtering particles based on the connection area protocol; (c) fitted flag shape along the principle axis determined from principle component analysis; (d) projection of the fitted line back to the original coordinate system. (The yellow cross corresponds to the monitored point on the inverted flag.)

as shown in Fig. 2(b). The mask comprised the flag but was actually thicker due to the reflection of light. More weights should be given to the high-intensity centerline of the flag. To this end, a Gaussian function fit was conducted to the intensity histogram of the pixels within the region of the mask, and the pixels whose intensities were higher than the mean of the Gaussian distribution were then chosen to fit the profile of the inverted flag. However, a direct fitting in the present coordinate system, as defined in Fig. 1, gave a poor estimation, as the profile of the inverted flag was not monotonous in many of the snapshots. To select the principle axes that maximized standard deviation automatically as the profiles changed, PCA¹⁷ was implemented. The points were projected to the principle axes, and a fourth-order polynomial curve fitting was used, as represented by the red line in Fig. 2(c). Finally, the fitted curve was projected back to the original coordinate system, and curve integration was performed to detect the location of particular monitor points. The red line in Fig. 2(d) shows the detected profile of the inverted flag, while the yellow cross depicts the three-quarter-length monitored location on the inverted flag, which is discussed later.

III. RESULTS AND DISCUSSIONS

The inverted flag's flapping dynamics were dependent on the dimensionless bending stiffness and Reynolds number. The tip-to-tip flapping amplitude of the inverted flag is plotted in Fig. 3, in which the measurements corresponding to the Re -increasing loop ($Re = 4660$ – 7600) and the Re -decreasing loop ($Re = 7600$ – 4660) are, respectively, indicated by black diamond and red circle. At large bending stiffness $K_B > 0.44$ ($Re < 4940$), the straight inverted flag was buckled with a slight inclination but almost remained static ($A/L < 0.3$). The flapping amplitude increased with the decrease in the bending stiffness, and the moderate flapping amplitude ($0.3 \leq A/L \leq 1.2$) was reached at $0.42 \geq K_B \geq 0.3$ ($5100 \leq Re \leq 6000$) with the increase in Re , where the inverted flag entered the biased mode. In this mode, the inverted flag oscillated at one side of the channel as shown by the superimposed view of the detected profiles; the top and bottom positions of the detected profile correspond to the free leading edge and the clamped trailing edge of the flag, respectively. The further decrease in the bending stiffness $0.30 > K_B \geq 0.21$ ($6000 < Re \leq 7122$) resulted in the flapping mode of the inverted flag with a large

oscillation amplitude around $A/L = 1.8$. The inverted flag suddenly deflected to one side of the channel when the bending stiffness was below $K_B = 0.20$ ($Re = 7400$). In the Re -decreasing loop, the flapping mode was reached until the bending stiffness increased to $K_B = 0.22$, and the large flapping amplitude was maintained till $K_B = 0.31$. Such hysteresis phenomenon was also observed by Kim *et al.*⁵ The unsteady flow fields corresponding to the biased mode, flapping mode, and deflected mode are examined in Secs. III A–III C, respectively.

A. Biased mode

To analyze the inverted flag's flapping dynamics and determine the phase-dependent flow behavior, a location at three quarters of the length of the inverted flag was selected as the monitor point [Fig. 2(d)]. Figure 4(a) shows the temporal variation of the wall-normal coordinate X^* and streamwise coordinate Y^* of the monitor point at $K_B = 0.32$ ($Re = 5800$). The movement of the inverted flag was periodic, as shown in Fig. 4(a). The coordinate X^* was always positive, indicating that the flapping of the inverted flag was biased to one side. Close observation revealed that the evolutions of X^* and Y^* were not sinusoidal. The maxima peaks of X^* were flatter than the minima peaks. X^* was characterized by a moderate increase and a relatively steep decrease, while Y^* decreased slowly

FIG. 3. The dependency of the flapping amplitude and flapping dynamics of the inverted flag on the Reynolds number Re and the dimensionless bending stiffness K_B . The black diamond and red circle correspond to the Re -increasing loop and the Re -decreasing loop, respectively.

FIG. 4. Coordinates of the monitor point in the biased mode at $K_B = 0.32$ ($Re = 5800$): (a) temporal variation, (b) spectra of the wall-normal coordinate X^* and the fluid velocity at the location ($X^* = 0$, $Y^* = -1$), (c) phase plot of X^* and Y^* for numerous periods. (The phases highlighted by red dots and blue triangles were selected to analyze the unsteady flow field interacted with the flapping inverted flag in the following figure.)

compared with its increase. The inverted flag thus appeared to bend slowly but rebounded quickly. We revisit this point in our discussion of the unsteady flow field (Fig. 5) (Multimedia view). The dimensionless frequency was defined by

$$f^* = \frac{fL}{U_0}, \quad (6)$$

where f was the flapping frequency of the inverted flag. The spectra which were identified from fast Fourier transform (FFT) analysis of the wall-normal coordinate X^* and the fluid velocity at the location ($X^* = 0$, $Y^* = -1$) are shown in Fig. 4(b). The dominant frequency at $f_0^* = 0.12$ was determined to be the flapping frequency of the inverted flag. As for the velocity spectrum, similar peak frequencies corresponding to the dominant frequency and the second harmonic frequency of the inverted flag were discerned. However, higher frequencies were characterized by numerous peaks apart from the integral higher order harmonics, which was due to the shedding of a series of vortices in every cycle (this will be illustrated in Fig. 7) (Multimedia view). The phase plot of X^* and Y^* for

many periods exhibits an arc shape in Fig. 4(c). The trajectories of the right and left strokes overlapped. Eight key phases, highlighted by red dots (a–e) for the right stroke and blue triangles (f–h) for the left stroke, were selected to analyze how the flow field interacted with the flapping dynamics of the inverted flag.

A preliminary view of the eight representative instantaneous streamline patterns and velocity contours for the phases identified in Fig. 4(c) is provided in Figs. 5(a)–5(f) (Multimedia view). Here, the streamline pattern was visualized by the line integral convolution (LIC) technique,¹⁸ and the contour denoted the velocity magnitude U/U_0 . At the left extreme of the inverted flag in Fig. 5(a) (Multimedia view), i.e., the minimum angle of attack of the inverted flag $\alpha = 5.34$, two low velocity regions $U/U_0 < 0.5$ were observed: one at the right side of the inverted flag in the near wake and the other close to the right wall between $-1L$ and $-2.5L$. Many small-scale eddies were formed in the turbulent wake, and a leading edge vortex (LEV) C0 elevated the flow velocity at its right side to $U/U_0 > 1.5$, as the circulation of the clockwise vortex aligned with the approaching flow. A counterclockwise vortex accelerated the fluid and resulted in another high velocity region, $1.5U_0$ – $1.7U_0$, presented at around $Y^* = -5$ at the left side of the channel. However, the vortex dissipated farther downstream, and only the corrugated pattern was recognized. As the inverted flag moved toward the right side, the two low-velocity regions transported downstream and the areas reduced, as shown in Figs. 5(b) and 5(c) (Multimedia view). The convection of the vortices in the near wake led to a travelling wavy pattern between $-1L$ and $-3.5L$ in the middle of the channel. The flow around the inverted flag reattached at the middle stroke of the flag, whose angle of attack was large, $\alpha = 21.84$, as shown in Fig. 5(c) (Multimedia view). As the flag bent further, another LEV C1₁ was initiated, as shown in Fig. 5(d) (Multimedia view). A small low-velocity region S1' close to the wall was also observed. The areas of the low-velocity regions expanded at the right stroke extreme of the inverted flag, as in Fig. 5(e) (Multimedia view). During the left stroke of the flag, as shown in Figs. 5(f) and 5(g) (Multimedia view), the large-scale vortices fed small-scale eddies in the turbulent wake, and large low-velocity regions were formed. As the flow separated massively around the flag in the left stroke, the inverted flag suffered a considerable reduction in lift. The lift in aerodynamics referred to the force perpendicular to the free stream velocity, which in this case pointed to the right during the left stroke. The reduction of lift around the inverted flag accounted for the steep bounce back compared with the moderate bending process, shown in Fig. 4(a). The bending force in the biased mode was relatively small due to the limited deformation of the inverted flag, and instead the reduction of lift resulting from the flow separation contributed to the self-oscillating mechanism of the inverted flag. The flow in Fig. 5(h) (Multimedia view) was actually at the same phase as that in Fig. 5(a) (Multimedia view), showing a similar global pattern, while the small-scale structures differed due to the flow turbulence. Figure 5 (Multimedia view) includes a video of the phase-averaged velocity field with the oscillating motion of the inverted flag. The video consists of 60 phases in one period.

FIG. 5. Instantaneous streamline pattern and velocity contour in a period of the inverted flag in the biased mode at $K_B = 0.32$ ($Re = 5800$): $t^* =$ (a) 0, (b) 1.28, (c) 2.39, (d) 3.54, (e) 4.55, (f) 5.66, (g) 6.62, and (h) 8.08. Multimedia view: <https://doi.org/10.1063/1.5001967.1>

Although the channel flow was entirely changed by the dynamically behaving flag, the distributions of the time-mean flow quantities could assist in the general understanding of its influence on the wall heat removal performance. Figure 6 displays the time-mean velocity U/U_0 and the dimensionless turbulence kinetic energy (TKE) in the biased mode at $K_B = 0.32$ ($Re = 5800$). Here, $TKE = 1/2 (\overline{u'^2} + \overline{v'^2})$ is normalized by the counterpart of the flow in the absence of the flag; consequently, the dimensionless $TKE^* = (\overline{u'^2} + \overline{v'^2}) / (\overline{u_0'^2} + \overline{v_0'^2})$ implies the turbulence enhancement induced by the flag instabilities, where u' and v' are the fluctuating parts of the streamwise velocity and the wall-normal velocity, respectively, and u_0'

and v_0' are the fluctuating parts of the streamwise velocity and the wall-normal velocity in the absence of the inverted flag, respectively. The mean flow velocity at the left side of the channel and the tip of the inverted flag were accelerated to around $1.2U_0$, while most of the fluid at the right side was characterized by a low mean flow speed of around $0.6U_0$ – $0.8U_0$ due to the oscillation of the inverted flag, as shown in Fig. 6(a). The mean flow velocity in the near wake of the inverted flag was particularly low, at below $0.2U_0$, due to the shedding of the vortices. In contrast, the formation and shedding of unsteady vortices enlarged $TKE^* = 12$ – 20 behind the leading edge of the inverted flag, as shown in Fig. 6(b). The trailing vortices entrained into the wake induced a belt-shaped high TKE region

FIG. 6. Contour plots of time-mean flow quantities in the biased mode at $K_B = 0.32$ ($Re = 5800$): (a) dimensionless velocity U/U_0 and (b) dimensionless turbulence kinetic energy TKE^* .

at the left side of the clamp plate. As the vortices broke up and dissipated in the farther wake $Y^* < -2.5$, TKE^* was almost reduced to $TKE^* < 4$.

To resolve the phase-dependent variation of unsteady flow behavior, we plot phase-averaged vorticity contours in Fig. 7 (Multimedia view). The boundaries of the large-scale unsteady turbulent swirling flows were identified from the topology of the velocity field, according to the Γ_2 criterion.¹⁹ Providing that $\Gamma_2 > 2/\pi$, the flow was dominated by the swirling motion. The dimensionless vorticity was found within the identified vortex boundary, which provided a better visualization for the development and decay of vortex than Γ_2 . The positive value denoted a counterclockwise vortex, and the negative value denoted a clockwise vortex. Figure 7 (Multimedia view) shows the phase-averaged vorticity in the biased mode of the inverted flag. At $t^* = 0$, the clockwise LEV $C0$ and counterclockwise $S0'$ were shedding, which corresponded to the low-velocity wake near the inverted flag in Fig. 5(a) (Multimedia view). An individual counterclockwise vortex $S0$ whose vorticity was

around 2 was discovered farther downstream at around $Y^* = -5$. The two vortices $C0$ and $S0'$ tore each other up into the two pairs $S0'_1$ and $C0_1$ and $S0'_2$ and $C0_2$, respectively, at $t^* = 1.28$. As they shed downstream, another counterclockwise vortex $S0'_3$ was entrained into the wake at $t^* = 2.39$. This train of vortices resulted in the wavy pattern in the middle of the channel shown in Figs. 5(b)–5(d) (Multimedia view). The vorticity of the newly developed clockwise LEV ($C1_1$) reached -4 at $t^* = 3.54$. $C1_1$ rolled up a counterclockwise vortex $S1'$ from the right wall, which corresponded to the small low-velocity region near the wall as shown in Fig. 5(d) (Multimedia view). The vortex near the channel wall, which was induced by another opposite sign vortex, was also captured by the numerical work.¹ The structure $S1'$ moved away from the channel wall when it grew at $t^* = 4.55$, and the individual counterclockwise vortex $S1$ separated into the wake from the trailing edge of the inverted flag. $S1$ was distorted by $S1'$ when $S1'$ rushed toward $S1$ at $t^* = 5.10$. $S1$ was transported downstream fast at $t^* = 5.56$ and 6.62 , which was consistent with the existence

FIG. 7. Phase-averaged vorticity contours in a period of the inverted flag in the biased mode at $K_B = 0.32$ ($Re = 5800$). The vortex boundary was determined by Γ_2 criterion $\Gamma_2 > 2/\pi$. Multimedia view: <https://doi.org/10.1063/1.5001967.2>

of the large-velocity region $1.5U_0$ – $1.7U_0$ in the wake, as shown in Figs. 5(g) and 5(h) (Multimedia view). By comparison, the transportation speed of the LEV $C1_1$ was so low that another LEV, $C1_2$, which was formed at $t^* = 6.62$, caught up with $C1_1$ and merged into the single vortex $C0$ at the beginning of a new period. Again, $C1$ and $S1'$ broke down into a train of vortices in the new period as $t^* = 1.28, 2.39$, and 3.54 . Two clockwise LEV ($C1_1$ and $C1_2$) and four counterclockwise vortices ($S0'_1, S0'_2, S0'_3$, and $S1$) were formed in one period. Figure 7 (Multimedia view) includes a video of the phase-averaged vorticity with the oscillating motion of the inverted flag. The video consists of 60 phases in one period.

B. Flapping mode

Figure 8 displays the coordinate of the monitor point at $K_B = 0.28$ ($Re = 6200$). The period of X^* was two times that of Y^* as shown in the temporal variation in Fig. 8(a), i.e., the flapping of the inverted flag was two-sided. The minimum Y^* approached 0.05, which means the inverted flag endured nearly 90° bending. The extremes of X^* were flat, indicating the nearly translational motion and low speed of the leading edge of the inverted flag during stroke reversal. The speed with which the inverted flag crossed over the equilibrium line was high, leading to a large slope of X^* . A close observation shows that the variation of Y^* was not strictly sinusoidal. The increase of Y^* was relatively moderate compared with the steep drop, which indicates that the bounce back of the inverted flag was slower than its deviation from the equilibrium position. This will be revisited in the estimated pressure field (Fig. 13). The spectrum in Fig. 8(b) shows that the fundamental frequency of the inverted flag increased to $f_0^* = 0.18$ along with the elevated amplitude. Recall that the dimensionless frequency is the ratio of the time scale of flow (L/U_0) to the oscillation time scale of the inverted flag ($1/f$). A larger dimensionless frequency denotes a longer time scale of flow or vortices compared to the oscillation time scale of the inverted flag; this indicates the generation of fewer vortices for every oscillating period, but the vortices are more coherent. This is consistent with the

FIG. 8. Coordinates of the monitor point in the biased mode at $K_B = 0.28$ ($Re = 6200$): (a) temporal variation, (b) spectra of the wall-normal coordinate X^* and the fluid velocity at the location ($X^* = 0, Y^* = -1$), (c) phase plot of X^* and Y^* for numerous periods. (The phases highlighted by red dots and blue triangles were selected to analyze the unsteady flow field interacted with the flapping inverted flag in Fig. 9.)

claim that the lower flapping frequency provided vortices with sufficient time to form and grow.⁸ Compared to the counterpart in the biased mode in Fig. 4(b), the even harmonics remained approximately constant, while the amplitudes of the odd harmonics ($3f_0^*, 5f_0^*$) of the inverted flag were amplified due to the flat extremes of the wall-normal coordinate X^* in Fig. 8(a). In the velocity spectrum, the amplitude at $2f_0^*$ was larger than that at f_0^* because the shedding frequency of the vortices were two times the flapping frequency of the inverted flag, which will be shown in Figs. 9 and 11 (Multimedia view). The amplitudes of higher order harmonics $3f_0^* - 5f_0^*$ in the velocity spectrum were also increased compared to that in the biased mode. The phase plot of X^* and Y^* presents a half-ellipse shape in Fig. 8(c). Ten representative phases highlighted by red dots that are marked by the alphabet (a–e) for the right stroke and blue triangles marked by the alphabet (f–j) for the left were selected to analyze the flow field interacting with the oscillating motion of the inverted flag.

The instantaneous streamline pattern and velocity contour in the flapping mode are shown in Fig. 9 (Multimedia view).

FIG. 9. Instantaneous streamline pattern and velocity contour for the inverted flag in a period in the flapping mode at $K_B = 0.28$ ($Re = 6200$): $t^* =$ (a) 0, (b) 0.71, (c) 1.53, (d) 2.12, (e) 2.6, (f) 3.09, (g) 3.52, (h) 4.41, (i) 4.93, and (j) 5.63. Multimedia view: <https://doi.org/10.1063/1.5001967.3>

The counterclockwise LEV S1 formed just before the flag reached the right stroke extreme, as shown in Fig. 9(a) (Multimedia view). The LEV grew and remained attached to the inverted flag when the flag rebounded to mid-stroke and then bent to the right side, as shown in Figs. 9(b)–9(d) (Multimedia view). The clockwise vortex C1 also grew and combined with the counterclockwise vortex S0, which was shed during the last period to form a reverse von Kármán vortex pair. The fluid flow between S0 and C1 accelerated to around $1.5U_0$ – $2U_0$, generating an impinging jet toward the left wall. The direction of flow turned to the lower right and impinged toward the right wall further downstream due to the reverse von Kármán vortex pair S0 and C0. Accordingly, an undulating wave-like flow pattern was formed. The vortex S1 transferred to bounded circulation around the flag, as shown in Fig. 9(e) (Multimedia view), while another clockwise LEV C2 formed at the right stroke extreme of the flag. A large velocity gradient was formed at the trailing edge of the inverted flag, and the bounded circulation then developed into a trailing edge vortex (TEV) at the right side of the channel. The velocity gradient served to feed on the counterclockwise S1, which in turn transferred momentum and energy to the clockwise LEV C2 in Figs. 9(g)–(h)

(Multimedia view) until the inverted flag crossed over the equilibrium position during the left stroke. The reverse von Kármán vortex street transported downstream and S1 joined its wake, as shown in Figs. 9(i)–9(j) (Multimedia view). Figure 9 (Multimedia view) includes a video of the phase-averaged velocity field with the oscillating motion of the inverted flag. The video consists of 60 phases in one period.

The dynamics of LEV in the confined channel were fundamentally different from those in the configuration of unbounded flow. As the inverted flag rebounded, the LEV did not shed directly from the leading edge as the case in the unbounded flow configuration.⁵ Instead, the counterclockwise LEV generated at the left side of the channel evolved into bounded circulation and was eventually shed at the right side as counterclockwise TEV. Similarly, the clockwise vortex formed at the right side shed at the left side. Consequently, the reverse von Kármán vortex street was then formed as shown in the instantaneous flow field rather than the classic von Kármán vortex street behind the bluff body. Both the von Kármán street and the reverse von Kármán vortex street consisted of two rows of counter-rotating vortices with a jet meandering between them. However the jet formed between the von Kármán streets

pointed upstream, whereas the jet between the reverse von Kármán vortex streets pointed downstream, as shown in Fig. 9 (Multimedia view). The reverse von Kármán vortex street wake resembled the thrust wake of swimming fish, as shown by Müller²⁰ and Shelley and Zhang.²¹ Furthermore, the high-speed impingement jets between the neighboring vortices were directed toward the sidewalls in a staggered fashion, as shown in Figs. 9(a)–9(j) (Multimedia view). These jets dynamically swept the channel walls from the near to far wake areas as the vortices convected downstream, which could have served as a favorable wall heat removal mechanism (jet impingement).

Figure 10 shows the time-mean velocity and turbulence kinetic energy in the flapping mode at $K_B = 0.28$ ($Re = 6200$). The mean flow velocity around the tips of the inverted flag was $1.0U_0$ – $1.2U_0$, as shown in Fig. 10(a). As the large bounded circulation transferred to the TEV at the trailing edge of the inverted flag, the mean velocity of the flow was also large in this region. However, for the LEV region attached near the body of the inverted flag, the flow velocity decreased to around $0.6U_0$ – $0.8U_0$. The shedding of the vortex also resulted in the

low velocity region between $-3 < Y^* < -1.5$. The mean flow near the channel walls in the downstream was also low because the vortex cores were close to the wall when the vortices convected downstream, as shown in Fig. 9 (Multimedia view). The flow velocity in the center of the channel increased gradually to around U_0 at $Y^* < -3$, and the high-speed region expanded toward the channel wall farther downstream. The path of the attached LEV, transitioning to bounded circulation and then to LEV, led to a ring with a high $TKE^* > 70$, as shown in Fig. 10(b). The envelope encircled by the tips of the inverted flag showed a slightly lower TKE^* of around 40. The distribution of TKE contrasted with that of the mean flow field where the mean velocity of the envelope was higher than that near the body of the inverted flag, as shown in Fig. 10(a). As the LEVs were relatively weak when they were born at the leading edge of the inverted flag, the TKE was significantly enhanced by the LEVs as they further developed and convected downstream toward the trailing edge of the inverted flag. An elongated tail was formed in the center of the channel from $Y^* = -1.7$ to $Y^* = -4.0$, where TKE^* was attenuated from 40 to 20 as the

FIG. 10. Contour plots of time-mean flow quantities in the flapping mode at $K_B = 0.28$ ($Re = 6200$): (a) velocity U/U_0 and (b) dimensionless turbulence kinetic energy TKE^* .

vortices dissipated. TKE^* close to the channel walls was also high at around 10 farther downstream.

The transformation of the LEV to bounded circulation and then to TEV was again envisioned from the phase-averaged vorticity contours in Fig. 11 (Multimedia view). During the right stroke of the inverted flag at $t^* = 1.82$ and 2.18, the counterclockwise LEV S1 at the left side of the channel was transported to the trailing part of the inverted flag and then rotated around the flag at $t^* = 2.53$. The clockwise LEV C2 was also formed. Although its boundary was small, the vorticity was strong: $\omega L/U_0 < -4$. The bounded circulation of S1 transferred to TEV gradually at $t^* = 3.08$, and the TEV developed at the beginning of the left stroke of the inverted flag. The TEV was fed by the velocity gradient, and the vorticity remained beyond $\omega L/U_0 > 4$ and the boundary broadened at $t^* = 3.69$ –4.29. It was eventually shed at the right side of the channel. The LEV C2 continued to grow and remained attached to the inverted flag when the flag rebounded. Due to the transformation of the LEV at one side of the channel to the TEV at the other side, a reverse von Kármán vortex street was formed in the wake. The boundary of the vortices C1, S0, and C0 enlarged, and the vorticity decayed as they convected downstream at $t^* = 1.82$ –4.29. The vorticity fell to -2 to 2 at $3L$ in the downstream. A video of the phase-averaged vorticity for the flapping mode of the inverted flag are provided in Fig. 11 (Multimedia view).

The trajectory of vortex core was then quantified. The vortex core was determined according to the scalar functions Γ_1 derived from the velocity fields.^{19,22} The threshold of Γ_1 was set to 0.6, and the geometry center weighted by Γ_1 was defined as the center of the vortex core. As shown in Fig. 12, (a) counterclockwise vortex was initiated at $(X^*, Y^*) = (-0.60, -0.11)$. An examination of the vortex's trajectory showed that X^* decreased to -0.77 while Y^* held constant, as the LEV followed the leading edge of the inverted flag and bounced back. The vortex remained attached to the leading edge of

FIG. 11. Phase-averaged vorticity contours of the inverted flag in the flapping mode at $K_B = 0.28$ ($Re = 6200$). The vortex boundary was determined by the Γ_2 criterion $\Gamma_2 > 2/\pi$. Multimedia view: <https://doi.org/10.1063/1.5001967.4>

FIG. 12. Trajectory of the counterclockwise vortex core in the flapping mode at $K_B = 0.28$ ($Re = 6200$).

the inverted flag to move upstream and draw an arc trajectory. X^* then increased from -0.5 to 0 at $Y^* = 0.34$. The process was characterized by the attached LEV. When the LEV transferred to bounded circulation, the Y^* position of the vortex core moved downstream quickly. The bounded circulation then transferred to TEV, which developed and was shed at $t^* = 4.05$ ($t/T = 0.7$). The shedding trajectory of the vortex core rolled down to a larger X^* due to the shear effect of the counterclockwise vortex when convected downstream. The trajectory formed a sigmoid shape. Although the boundary of the vortex was largely extended beyond the centerline of the channel, as shown in Fig. 11 (Multimedia view), the vortex core centralized along $X^* = 0.9$ after $Y^* < -2$. The mean convection velocity of the vortex in the shedding stage was determined to be $U_s = 0.54U_0$. Given the flapping frequency of the inverted flag f , the mean wavelength of the undulating wake was determined as $\lambda = U_s/f = 3.4L$.

A fuller understanding of the periodic flow in the flapping mode was gained from the pressure field. This was estimated from the velocity field by following the algorithm adopted by Dabiri *et al.*²³ The pressure field was determined based

on median polling of several integration paths through the pressure gradient field, to reduce the errors that accumulated along individual integration paths. Integration paths in the flow were nullified for those that crossed the fluid-solid interface identified using the aforementioned structure boundary detection algorithm. Hence, the pressure value for those paths also became undefined and therefore did not contribute to the median calculation. The phase-averaged flow field was used to reduce the error occurring when taking the derivative of velocity in the algorithm. For grid spacing of less than $L/16$, the error fell below 5%.²³ Figure 13 shows the contour of the phase-averaged pressure coefficient, and the arrows depict the down-sampled phase-averaged velocity vectors. The pressure coefficient is defined as

$$C_p = \frac{P - P_0}{1/2 \rho_f U_0^2}, \quad (7)$$

where P is the fluid pressure and P_0 is the free stream pressure. A global view of Figs. 13(a)–13(j) reveals a low-pressure region with $C_p = -2.5$ for the vortex, and high pressure up to $C_p = 0.5$ for the upwards of the leading edge of the inverted flag, due to blockage and movement of the flag opposite the approaching flow. The attached LEV S1 in Figs. 13(a)–13(c) and C2 in Figs. 13(e)–13(h) induced a low-pressure region with $C_p = -2$ to -2.5 close to the inverted flag during the rebound process. Thus, the lift force was elevated by the attached LEV, which slowed down the bounce-back process of the inverted flag, while a rapid deviation of the flag was observed as shown in Fig. 8. The bounce-back process was mainly driven by the bending stiffness of the dramatically deformed inverted flag, meaning that the elastic energy of the flag was released to overcome the kinetic energy of flow and the drag exerted on the flag due to the pressure differences between its two sides. In contrast, the pressure differences on the two sides

FIG. 13. Phase-averaged pressure contour and velocity vector in the flapping mode at $K_B = 0.28$ ($Re = 6200$): $t^* =$ (a) 0, (b) 0.71, (c) 1.53, (d) 2.12, (e) 2.6, (f) 3.09, (g) 3.52, (h) 4.41, (i) 4.93, and (j) 5.63.

contributed to the increase in the elastic energy of the flag during the bending process [Figs. 13(c)–13(e) and 13(h)–(j)]. The periodic large-amplitude self-oscillation was manifested by the energy transfer mechanism between the flow and flag. The transition of LEV to bounded circulation also introduced a low-pressure region around the flag due to the increase in the local fluid's flow speed. After the TEV formed, the low pressure rose quickly in the shedding stage. For example, the pressure of vortex C1 increased from $C_p = -2$ to -2.5 as shown in Fig. 13(d) to $C_p = -1.5$ when TEV was transported to $Y^* = -1.5$ as shown in Fig. 13(e). The low-pressure area of the vortex C1 shrank along with the increase of the pressure coefficient $C_p = -1$ to -0.5 as C1 convected downstream as shown in Figs. 13(f)–13(j). The pressure recovered back to around 0 beyond the station $Y^* = -4.5$.

C. Deflected mode

When the dimensionless bending stiffness decreased to $K_B = 0.187$ ($Re = 7600$), the coordinate of the monitor point became highly stabilized at approximately $X^* = 0.66$, $Y^* = 0.34$, with small fluctuations due to the turbulence of flow, as shown in Figs. 14(a) and 14(b). The inverted flag was deflected to one side because the force of the fluid was overwhelmingly strong so that the inverted flag could not rebound.

The instantaneous streamline pattern and velocity contour in Fig. 15 (Multimedia view) show that the flow velocity at the left side of the channel increased by over two times the mean free stream velocity between $-4.5 < Y^* < 0$. By comparison, the flow velocity at the deflected side was low due to the blockage of the deflected inverted flag. The strong shear established by the high-speed flow at the left side of the channel and the low-speed flow at the right side led to numerous vortical structures. Apart from many turbulent small scale eddies, a counterclockwise recirculation was identified, marked by R1

FIG. 14. Displacement of the monitor point in the deflected mode at $K_B = 0.187$ ($Re = 7600$): (a) temporal variation and (b) phase plot.

in Fig. 15 (Multimedia view). A secondary clockwise recirculation zone marked by R2 anchored behind the leading edge of the inverted flag in all instants. A clear view of the unsteady behavior can be seen in the Multimedia view, which compiles 60 consecutive frames of the instantaneous velocity fields.

Figure 16(a) displays the time-mean velocity in the deflected mode. Two recirculation zones were observed, between which the inversed flow with a large mean speed of around $0.6U_0$ was formed. The turbulence kinetic energy in Fig. 16(b) shows that a long belt-shaped high $TKE^* = 10$ – 20

FIG. 15. Instantaneous streamline pattern and velocity contour in the deflected mode at $K_B = 0.187$ ($Re = 7600$): $t^* =$ (a) 0, (b) 1, (c) 2, and (d) 3. Multimedia view: <https://doi.org/10.1063/1.5001967.5>

FIG. 16. Contour plots of time-mean flow quantities in the deflected mode at $K_B = 0.187$ ($Re = 7600$): (a) velocity U/U_0 and (b) dimensionless turbulence kinetic energy TKE^* .

region near $X^* = -0.5$ stretched from $Y^* = 0$ to $Y^* = -5$. This again resulted from the vortices superimposed in the strong shear region. On the deflected side, the area behind the flag was featured by the low-speed fluid flow, stable recirculation zones, and low TKE.

IV. CONCLUDING REMARKS

A highly unsteady flow field interacting with a self-oscillating inverted flag placed in a channel flow was experimentally measured using the TR-PIV technique. The dynamically deformed profiles of the inverted flag were determined by a novel algorithm that combined morphological image processing and PCA. Three modes were discovered with the successive decrease in the dimensionless bending stiffness: the biased, flapping, and deflected modes, respectively. In the range of $0.42 \geq K_B \geq 0.3$ ($5100 \leq Re \leq 6000$) in the Re -increasing loop, the inverted flag oscillated in the biased mode, generating two clockwise and four counterclockwise vortices. In the range of $0.30 > K_B \geq 0.21$ ($6000 < Re \leq 7122$), the

large oscillating amplitude of the inverted flag was reached in the flapping mode, periodically providing a strengthened influence on the fluid near the two sidewalls. The reverse von Kármán vortex street was well formed and energetic in the wake; a series of high-speed impingement jets between the neighboring vortices was directed to impinge onto the sidewalls in a staggered fashion. For the pressure field estimated from the velocity field, the LEV-induced low pressure region ($C_p = -2$ to -2.5) elevated the lift force; therefore, the bounce back process of the inverted flag was slower than the bending process. This was distinctly different from the biased mode in which the rebound process was faster due to the flow separation. At $K_B = 0.20$ ($Re = 7400$), the inverted flag deflected to one side of the channel and remained almost stationary. The flow was blocked at the deflected side and accelerated at the other side, inducing two stable recirculation zones behind the flag. A considerably inversed flow was generated between these two recirculation areas.

As the scales of the vortical structures are proportional to the flapping amplitude, the present study focused on the

inverted flag with the largest confinement $2L/W = 1$. This configuration is shown to induce very large vortical structures strongly interacting with the boundary layer in the channel, which would benefit the channel heat transfer enhancement. When the confinement decreases by reducing the flag's length, the interaction of the vortical structures with the boundary layer becomes weak. At the limit of very low confinement, the flow would resemble that in the unbounded flow, where the LEV sheds directly from the leading edge of the inverted flag in the flapping mode. The effort to quantify the transition of flow pattern from the limit of the very low confinement (unbound flow) to the present limit ($2L/W = 1$) would be an interesting avenue for future research. In addition, further experimental studies will be undertaken on wall heat transfer management using the self-oscillating flag in the channel flow.

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