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Synchronized Switch Harvesting on ElectroMagnetic System: a nonlinear technique for hybrid energy harvesting based on active inductance

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Abstract

This paper proposes a hybrid nonlinear interface combining both piezoelectric and electromagnetic effects for energy harvesting purposes. Based on the previously developed Synchronized Switch Harvesting on Inductor (SSHI) scheme, this technique aims at including as much as electroactive parts as possible, thus advantageously replacing the passive inductance in the SSHI approach by an electromagnetic transducer. The proposed scheme, called Synchronized Switch Harvesting on ElectroMagnetic system (SSH-EM) results in a significant increase of the output voltage, thus leading to a consequential increase of the harvested power, especially in the case of low-coupled systems, when compared to the classical SSHI approach and the standard approach (*i.e.*, directly connecting the piezoelectric element to the load through a diode bridge rectifier and a smoothing capacitor). The working principle, along with theoretical developments, and the supporting experimental results, are presented, both considering structures featuring constant displacement magnitude or structures driven at their resonance frequency with a constant excitation magnitude.

Keywords: Vibrational hybrid energy harvesting, piezoelectric, electromagnetic, synchronized switch, nonlinear circuit

1. Introduction

2 The development of wireless sensor networks has received a great deal of interest in research
3 communities over the past two decades [1, 2, 3]. Simultaneously, advancements in low power
4 electronic design and fabrication have drastically reduced the power consumption of small elec-
5 tronic devices opening the possibility for autonomous sensor nodes and, more particularly, in the
6 framework of "Internet of Things" [4]. These sensors are in most cases alimented by conventional
7 batteries which, however, show limitations in terms of ageing phenomenon (self-discharge), whether
8 the battery is used or not, limiting the battery applications, particularly when exposed to relatively
9 harsh environment conditions and/or when placed in confined and hardly accessible locations [5].
10 Hence, harvesting ambient energy to supply such sensor nodes appears to be a key technology in

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11 order to replace the role of such batteries and go towards the development of autonomous self-
12 powered devices [6]. When dealing with energy harvesting from the nearby environment, several
13 sources can be considered; radiations, thermal gradients and vibrations are just few examples [7].
14 Among these, the strong ubiquity and the high power density of mechanical vibrations make these
15 an interesting potential power source for electromechanical systems [8, 9]. Converting the mechan-
16 ical energy from ambient vibrations into electrical energy is performed by a transducer. Among
17 the existing ones, piezoelectric [10, 11, 12, 13, 14, 15] and electromagnetic [16, 17, 18, 19] energy
18 harvesters have received much attention, as they both can reach relatively high power levels with
19 no external voltage source requirement.

20 Piezoelectric energy harvesters, based on the direct piezoelectric effect, have been widely studied
21 and nonlinear electronic treatments (*i.e.* adding a nonlinear element to the circuit, such as switch,
22 as opposite to linear case - which usually consists of a linear load, such as a resistance, connected to
23 the transducer) applied to the piezovoltage have been developed in order to enhance the efficiency
24 of the energy conversion [20, 21, 22]. For instance, Guyomar *et al.* proposed a simple but original
25 treatment, namely the SSHI (Synchronized Switch Harvesting on Inductor), for greatly improving
26 the energy conversion abilities of piezoelectric materials [23], resulting to a typical gain factor of 8
27 in terms of harvested energy in comparison to the classical approach. In order to further increase
28 the conversion abilities of piezoelectric elements, several techniques derived from this method have
29 been successively proposed, although introducing further complexity to the circuit [24, 25, 26, 27].

30 Electromagnetic energy harvesters, based on Faraday's law of induction, usually consists of a
31 coil and a moving magnet, resulting to an alternating magnetic flux. The electricity is generated
32 by the relative movement of the magnet and coil. Generally, the amount of generated electricity
33 depends upon the strength of the magnetic field, the velocity of the relative motion and the number
34 of turns of the coil.

35 Furthermore, it has been demonstrated that nonlinear techniques, particularly those based on
36 the Synchronized Switching techniques, can be derived in several systems and schemes, using dual
37 transformations of the actuator or the controlled electrical quantity [28]. In fact, such energy ex-
38 traction strategies have also been extended to electromagnetic vibration energy harvesters. One
39 good example is the one developed by Arroyo *et al.*, namely the SMFE (Synchronous Magnetic
40 Flux Extraction) technique, which provides a significant optimization of the energy transfer irre-
41 spective of the load impedance [29].

42 Although the majority of energy harvesters that have been so far studied and developed mainly
43 focused on a single energy conversion mechanism, a recent trend consists in combining multiple
44 energy conversion effects, such as piezoelectric and electromagnetic transduction, in a single device,
45 leading to the concept of hybrid energy harvesting [30, 31, 32, 33, 34, 35, 36]. Nevertheless, most of
46 these works mainly focused on the energy conversion mechanism, and very few actually considered
47 the energy harvesting process on the circuit side. Table 1 gives an overview of the voltage and

Table 1: Comparison of voltage and power outputs from hybrid piezoelectric-electromagnetic energy harvesting systems (*Std.* stands for Standard circuit, *Gain PZT* and *Gain EM* stand for the energy gain with respect to the sole use of piezoelectric and electromagnetic system respectively)

Reference	V_p	V_{em}	P_p	P_{em}	P_{hyb}	Gain PZT	Gain EM
	60 V (Std.)		80 mW (Std.)			300 % (Std.)	
[31]	100 V (SSHI)	-	200 mW (SSHI)	-	250 mW	25 % (SSHI)	-
	3.5 V (0.2g)	40 mV (0.2g)	0.085 mW (0.2g)	0.14 mW (0.2g)			
[33]	9.75 V (0.45g)	150 mV (0.45g)	0.5 mW (0.45g)	1.19 mW (0.45g)	-	-	-
[35]	3 V	0.12 V	250 μ W	244 μ W	-	-	-
[32]	8 V	0.4 V	1.65 mW	2.18 mW	2.26 mW	37 %	4 %
[30]	-	-	1.1 mW	0.96 mW	1.2 mW	10 %	30 %
[34]	0.35 V	0.012 V	40 μ W	1.16 μ W	-	-	-

power outputs from such hybrid systems: it is evident the high difference in the voltage outputs between the piezoelectric and the electromagnetic systems, making the latter ones not compatibles with the former ones. Moreover, the peak powers of the two systems are characterized by different values of optimal loads: extracting the energy from the hybrid system would then require an additional stage of load adaptation, hence introducing additional complexity to the circuit and power losses.

To overcome these main drawbacks when considering hybrid piezoelectric-electromagnetic energy harvesters, this paper proposes a hybrid energy harvesting electrical interface based on the coupling of these two energy conversion mechanisms. The model is derived from the previously stated SSHI interface, whose passive inductance is advantageously replaced by an electromagnetic generator, thus making active every constitutive electrical component of the interface. This produces an immediate enhancement of the final energy that can be extracted without adding significant complexity to the systems in terms of electrical components and space. A particular attention is paid to the resulting enhancement of the piezoelectric element output voltage due to the synchronized switching interface. The paper is organized as follows: section II introduces the principles and the operations of the proposed technique; section III exposes in detail the theoretical analysis giving the output energy that can be extracted with such model, either considering constant displacement magnitude or taking into account the damping effect at resonance; a preliminary theoretical discussion is illustrated in section IV; section V aims at exposing the experiments carried out in order to validate such model along with comparative discussion; Finally, section VI briefly concludes the paper.

2. Principles

The aim of this section is to recall the basic principles of energy harvesting using the standard and the SSHI technique and to introduce the main idea and motivation behind the proposed hybrid

72 switching configuration.

73 *2.1. Standard*

74 In the standard interface circuit, a diode rectifier bridge followed by a filtering capacitor C_s and
 75 a load R_L is directly connected to the piezoelectric element, as shown in Fig. 1a. This is basically
 76 the simplest circuit for rectifying an AC voltage, as well as the most used one for energy harvesting
 77 applications. The rectifier bridge is assumed to be perfect and the rectified voltage V_{DC} is assumed
 78 to be constant, which is a good approximation if the time constant $R_L C_s$ is much larger than the
 79 mechanical oscillation half-period. Fig. 1b depicts the typical voltage, displacement and current
 80 waveforms of such configuration:

- 81 • from t_1 to t_2 the rectifier bridge is blocked and the piezovoltage varies with the displacement
 82 with an off-set introduced by the previous energy harvesting process;
- 83 • from t_2 to t_3 the absolute piezovoltage V_p equals the rectified voltage V_{DC} so that the rectifier
 84 bridge is conducting and energy transfer occurs from the piezoelectric element to the load;
- 85 • after t_3 , the bridge rectifier is blocked again and the piezovoltage goes back to open-circuit
 86 condition;
- 87 • from t_3 to t_4 the piezovoltage varies again with the displacement until the condition $|V_p| =$
 88 $-V_{DC}$ is reached so that the rectifier conducts again and a new energy transfer occurs.

89 *2.2. Parallel SSHI*

90 The parallel SSHI interface consists in adding a nonlinear processing circuit in parallel with
 91 the piezoelectric element, as shown in Fig. 2a. This nonlinear processing circuit is composed of
 92 an electronic switch and an inductor L connected in series. The switch is almost always open

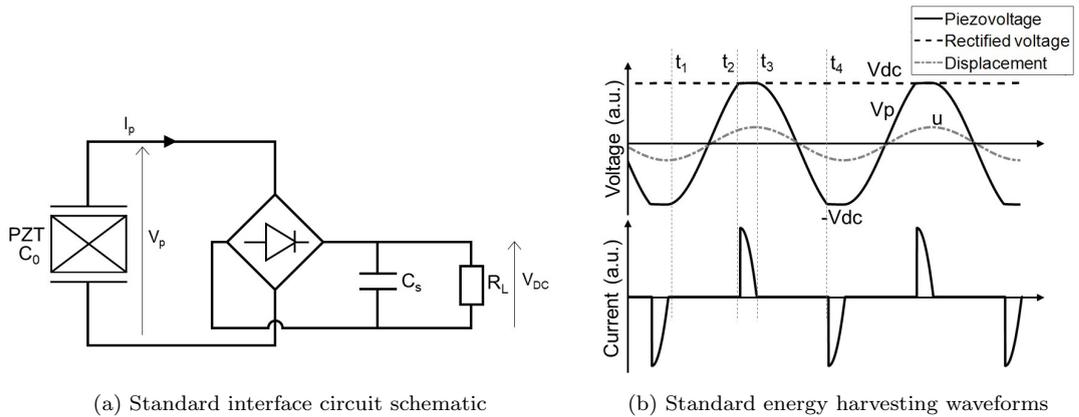
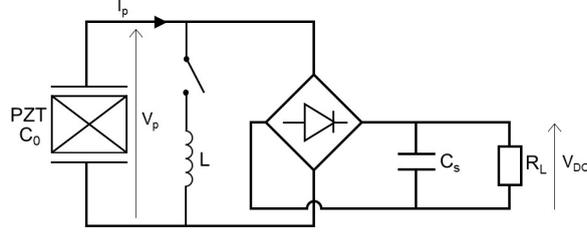
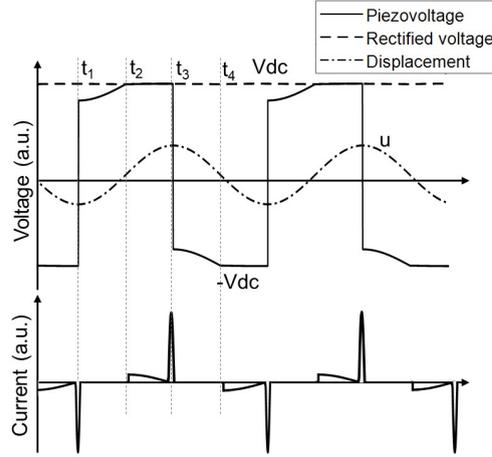


Figure 1: Standard technique



(a) Parallel SSHI circuit schematic



(b) SSHI waveforms

Figure 2: SSHI technique

93 except when an extremum in the displacement occurs. At this instant, the switch is briefly turned
 94 on, establishing an oscillating electrical network made by the inductance L and the piezoelectric
 95 clamped capacitance C_0 . After half an electrical period, defined as equation (1), the switch is again
 96 turned off, resulting in a quasi-instantaneous inversion of the voltage V .

$$t_i = \pi \sqrt{LC_0} \quad (1)$$

97 The voltage inversion is however not perfect (due to the losses in the switching branch) and
 98 characterized by the inversion coefficient defined as:

$$\gamma_0 = e^{-\frac{\pi}{2Q_i}} \quad (2)$$

99 where Q_i is the electrical quality factor of the LC_0 circuit. Fig. 2b depicts the typical voltage,
 100 displacement and current waveforms of the SSHI configuration:

- 101 • from t_1 to t_2 the rectifier bridge is blocked and the piezovoltage varies with the displacement
 102 with an off-set introduced by the previous voltage inversion;

- from t_2 to t_3 the absolute piezovoltage V_p equals the rectified voltage V_{DC} so that the rectifier bridge is conducting and energy transfer occurs from the piezoelectric element to the load;
- at t_3 , which corresponds to a displacement extremum, the switch is turned on (explaining the current spikes) and the piezoelectric voltage is reversed. Its absolute value remains lower than the rectified voltage V_{DC} so the rectifier bridge is blocking again;
- from t_3 to t_4 the piezovoltage varies again with the displacement until the condition $|V_p| = -V_{DC}$ is reached so that the rectifier conducts again and a new energy transfer occurs.

2.3. Hybrid model

Considering the SSHI interface, one can note that the inductor can play an active role if coupled with a permanent magnet that is moving relatively to it. In fact, the idea behind the proposed hybrid model is to convert the inductor from a passive electrical component to an active electromagnetic system leading to the concept of hybrid energy harvesting. Thus, the combination of the two energy conversion effects on the same harvesting system induces an enhancement of the output voltage of the piezoelectric element, as depicted in Fig. 3, resulting in a higher harvested energy without significant loss in terms of space. The general resulting circuit is illustrated in Fig. 4 and its operations will be explained in detail in the following sections. The reason behind the employment of the two transducers in such configuration, rather than extracting the energy from them separately, lies in the duality of the nature of the two transducer systems: the piezoelectric can be electrically modelled as a current source in parallel with a capacitance, whereas an electromagnetic system can be modelled as a voltage source in series with an inductance [37]. The former is characterized by high levels of voltage and low current while the latter is distinguished by high levels of current and low voltage, so interfacing each of the two systems to a single harvesting stage would lead to a more complex electrical interface (for instance, by employing DC/DC converters), seeing the big difference on the voltage/current outputs, and adding complexity to the circuit always translates into energy losses. On the other hand, the proposed SSH-EM (Synchronized Switch Harvesting on ElectroMagnetic system) switching technique is not sensitive to the

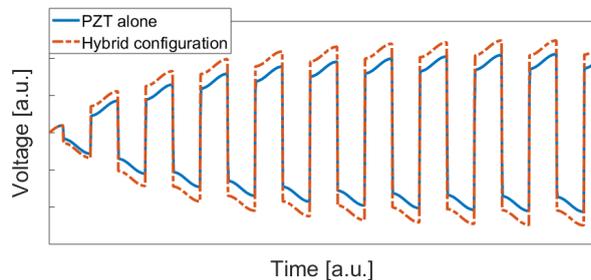


Figure 3: Voltage waveforms of the switching on electromagnetic system strategy and comparison with SSHI technique

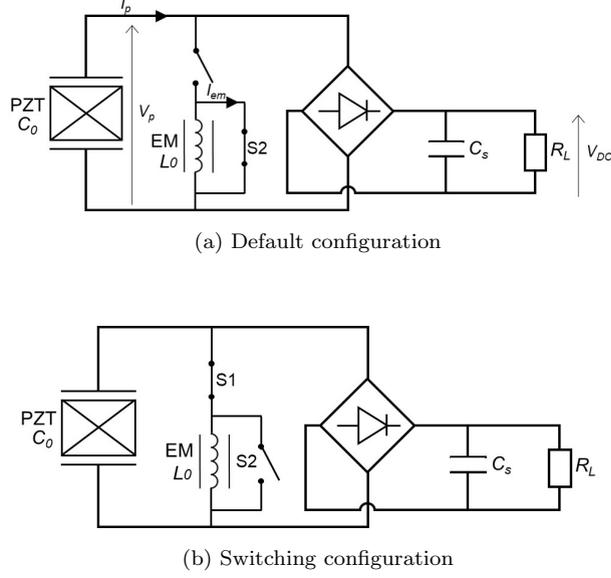


Figure 4: Hybrid model circuit schematic

129 difference in the voltage and current outputs that exists between the two transducers, as they are
 130 made to interact as a LC oscillator, resulting in the magnification of the voltage seen in Fig. 3.

131 3. Theoretical development

132 In the previous section, the basic principles of the proposed model have been introduced. Con-
 133 sequently, it is now possible to set up the theoretical development allowing the prediction of the
 134 extracted energy and output power of such scheme.

135 Assuming linear elasticity, vibrating mechanical structures excited near one resonance frequency
 136 can be modeled by a second-order mass-spring-damper system [38]. The schematic of such inertial
 137 generators can be applied both to piezoelectric and electromagnetic generators [39], as shown in
 138 Fig. 5, where M is the equivalent dynamic mass bonded on a spring K (modeling the stiffness of
 139 the mechanical structure) and a damper C (modeling the mechanical losses).

140 As previously stated, from an electrical point of view, a piezoelectric generator is equivalent

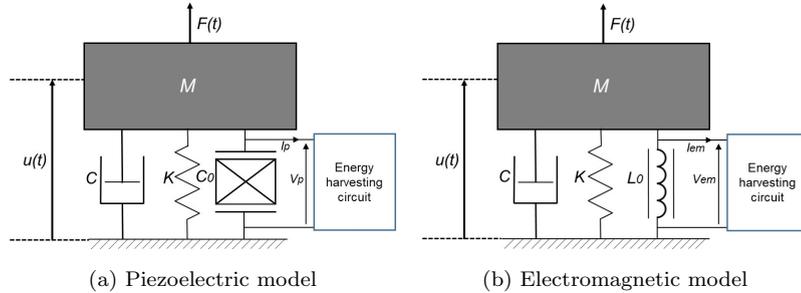


Figure 5: Equivalent mechanical models of piezoelectric and electromagnetic generators

141 to a current source proportional to the relative vibration velocity \dot{u} connected in parallel with a
 142 capacitor while an ideal electromagnetic generator can be modelled as a velocity dependent voltage
 143 source in series with an inductor [28]. Neglecting the transducers' electrical losses (especially in
 144 the electromagnetic case), the constitutive electromechanical equations of such model are therefore
 145 given by:

$$\begin{cases} M\ddot{u} + C\dot{u} + K^{E,H}u = F - \alpha V_p - \beta I_m \\ I_p = \alpha\dot{u} - C_0\dot{V}_p \\ V_{em} = \beta\dot{u} - L_0\dot{I}_{em} \end{cases} \quad (3)$$

146 where u is the displacement of the structure (assumed to be sinusoidal in the followings), V_p and
 147 V_{em} the piezoelectric and electromagnetic voltages respectively and I_p and I_{em} the piezoelectric
 148 and electromagnetic currents respectively; α and β are the force factors of the piezoelectric and
 149 electromagnetic elements respectively; Finally, C_0 represents the piezoelectric clamped capacitance
 150 and L_0 the inductance of the electromagnetic transducer at rest.

151 3.1. Switching process

152 Fig. 6 gives a rough schematic of the proposed hybrid model without the consideration of the
 153 rectifying circuit. This scheme can be divided into two sub-sections, one corresponding to the piezo-
 154 electric (PZT) insert and the other one corresponding to the electromagnetic (EM) element. Most
 155 of the time, the piezoelectric transducer is left in open circuit conditions while the electromagnetic
 156 system is short-circuited, as depicted in Fig. 6a. Under these conditions, the piezoelectric volt-
 157 age and the electromagnetic current vary proportionally to the mechanical displacement, following
 158 equation (4) and (5):

$$I_p = 0 \Rightarrow \dot{V}_p = \frac{\alpha}{C_0}\dot{u} \quad (4)$$

$$V_{em} = 0 \Rightarrow \dot{I}_{em} = \frac{\beta}{L_0}\dot{u} \quad (5)$$

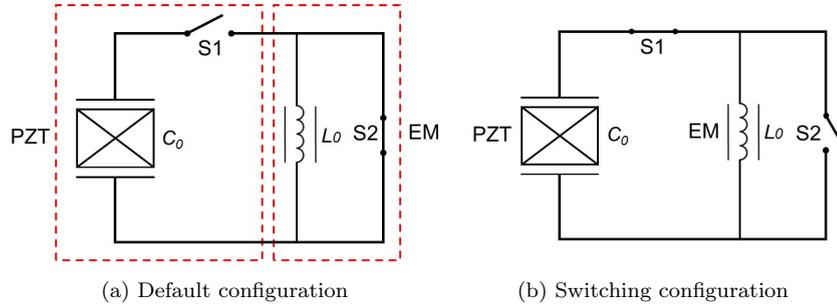


Figure 6: Circuit schematic

159 When the mechanical displacement reaches a maximum or a minimum value (i.e. when the me-
 160 chanical speed cancels), the electronic switches S1 and S2 are briefly turned on and off respectively,
 161 as shown in Fig. 7, so that the piezoelectric and the electromagnetic generators are electrically
 162 connected, establishing an oscillating electrical $L_0 - C_0$ circuit composed by the electromagnetic
 163 inductance L_0 and the piezoelectric clamped capacitance C_0 . As a result, a current flow appears
 164 as:

$$L_0 \ddot{q} + r_s \dot{q} + \frac{q}{C_0} = 0 \quad (6)$$

165 where r_s represents the equivalent linear losses due to switching interface and q is the electric
 166 charge of the piezoelectric element. Differently from standard synchronized switching interfaces,
 167 having null initial current flow, Eq. (6) is solved considering non-zero initial current condition, as
 168 an effect of the addition of the electromagnetic generator, and considering the voltage across the
 169 piezoelectric element:

$$\begin{cases} q_0 = C_0 V_p = \alpha u_M \\ \dot{q}_0 = I_{em} = \frac{\beta}{L_0} u_M \end{cases} \quad (7)$$

170 hence, the voltage and the current oscillation are defined as:

$$V(t) = e^{-\frac{r_s}{2L_0\omega_{sw}}t} \left[V_M \cos(\omega_{sw}t) + \frac{I_M}{C_0\omega_{sw}} \sin(\omega_{sw}t) \right] \quad (8)$$

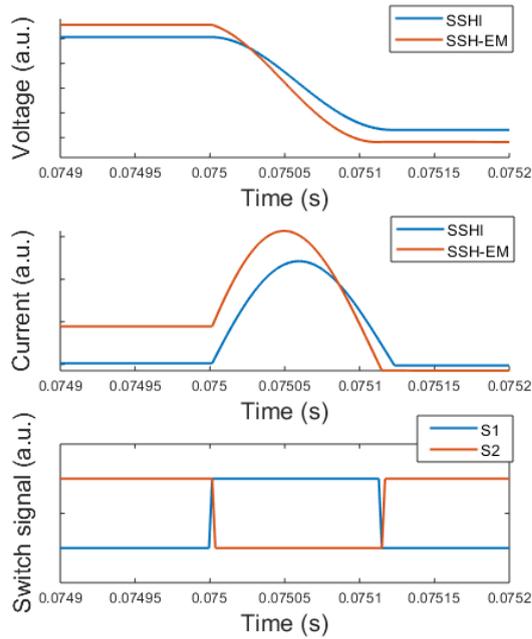


Figure 7: Voltage and current waveforms and switching signals

$$I(t) = e^{-\frac{r_s}{2L_0\omega_{sw}}t} [-C_0V_M\omega_{sw}\sin(\omega_{sw}t) + I_M\cos(\omega_{sw}t)] \quad (9)$$

171 where I_M is the current provided by the electromagnetic element (I_{em}) at the switching instant,
 172 V_M corresponds to the maximum output voltage provided by the piezoelectric component (V_p) at
 173 the same switching instant and ω_{sw} refers to the resonance frequency of the L_0C_0 oscillator (*i.e.*
 174 $1/\sqrt{L_0C_0}$). In order to achieve voltage inversion, the switch S1 must be again turned off (and S2
 175 turned back on) when the current is null. Therefore, as a consequence of the contribution of the
 176 electromagnetic generator (which has lead to non-zero initial current condition), the equivalent
 177 inversion coefficient for the hybrid harvester model is given by:

$$\gamma_P = e^{-\frac{r_s\pi}{2L_0\omega_{sw}}} e^{-\frac{r_s}{2L_0\omega_{sw}}\arctan\left(\frac{I_M}{C_0V_M\omega_{sw}}\right)} \sqrt{1 + \left(\frac{I_M}{C_0V_M\omega_{sw}}\right)^2} \quad (10)$$

178 noting that the parameter $\gamma_0 = e^{-\frac{r_s\pi}{2L_0\omega_{sw}}}$ corresponds to the inversion coefficient of the non-hybrid
 179 synchronized switching case, as defined by equation (2).

180 Consequently, considering the steady-state case, the relation between the output voltage mag-
 181 nitudes before (V_M) and after (V_m) the inversion process are finally given by:

$$\begin{cases} V_M - V_m = 2\frac{\alpha}{C_0}u_M \\ V_m = \gamma_P V_M \end{cases} \quad (11)$$

182 Typical waveforms of the switching event are depicted Fig. 7, giving a comparison with respect
 183 to the classical SSHI technique. It is interesting to point out the improvement in the voltage
 184 inversion and magnification as a consequence of the contribution of the electromagnetic system
 185 that provides initial current.

186 3.2. Derivation of the output power

187 In order to give an estimation of the harvested power, constant displacement magnitude of the
 188 structure is firstly considered. This leads to not considering electromechanical damping effect in
 189 terms of mechanical vibrations, which is a good approximation for low coupled, highly damped
 190 systems, or structures excited out of their resonance frequencies. Secondly, the performance of the
 191 SSH-EM interface will be quantified taking into account this damping effect induced by the energy
 192 harvesting process (which is the case of highly coupled microgenerators excited by a constant
 193 acceleration very close to one of their resonance frequencies).

194 3.2.1. Constant displacement magnitude

195 if the structure is supposed to be driven with a constant displacement magnitude, only the
 196 second and the third constitutive equations of Eq. (3) are actually necessary in order to evaluate
 197 the extracted energy of the proposed scheme. The harvested energy (W_h) for one extraction
 198 event (between the time instants t_1 and t_2 in Fig. 8 can therefore be expressed starting from the

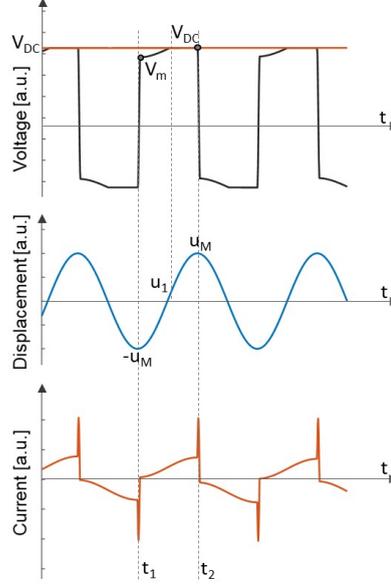


Figure 8: Voltage, displacement and current waveforms in the steady-state case

199 piezoelectric electrical equation of Eq. (3) and leading to the following integral function of the
 200 rectified piezoelectric voltage and current:

$$W_h = \int_{t_1}^{t_2} V_{DC} I_P dt = \alpha V_{DC} \int_{u_1}^{u_M} du = \alpha V_{DC} (u_M - u_1) \quad (12)$$

201 where u_1 and u_M refers to the displacement values when the harvesting process starts and the
 202 displacement magnitude, respectively, as shown in Fig. 8, and V_{DC} the value of the rectified
 203 voltage, assumed to be constant.

204 Considering an energy scavenging process on a positive (resp. negative) voltage, the harvesting
 205 process stops when the current cancels — which occurs on minimum (resp. maximum) displace-
 206 ments. Hence, after energy transfer and inversion process, the system is left open-circuit. Moreover,
 207 in the DC case, $V_M = V_{DC}$ so that $V_m = \gamma_P V_{DC}$. Consequently, as shown by Eq. (13), the equiv-
 208 alent inversion coefficient in that case actually depends on the rectified voltage V_{DC} , so that it will
 209 be noted as $\gamma_P(V_{DC})$ in the followings.

$$\gamma_P(V_{DC}) = \gamma_0 e^{-\frac{r_s}{2L_0\omega_{sw}} \arctan\left(\frac{I_M}{C_0 V_{DC} \omega_{sw}}\right)} \sqrt{1 + \left(\frac{I_M}{C_0 V_{DC} \omega_{sw}}\right)^2} V_{DC} \quad (13)$$

210 Consequently, integrating the second equation of Eq. (3) yields the expression of the
 211 voltage as follows (considering the evolution from $\gamma_P(V_{DC})V_{DC}$ to V_{DC}):

$$\begin{cases} V = \frac{\alpha}{C_0} u + A \\ \gamma_P(V_{DC}) \cdot V_{DC} = -\frac{\alpha}{C_0} u_M + A \end{cases} \quad (14)$$

212 where A is an integration constant.

213 The value of the displacement u_1 when the harvesting process starts can be obtained by con-
 214 sidering the voltage equals to V_{DC} , as provided by the following expressions:

$$V_{DC} = \frac{\alpha}{C_0}(u_1 + u_M) + \gamma_P(V_{DC}) \cdot V_{DC} \quad (15)$$

$$V_{DC}(1 - \gamma_P(V_{DC})) = \frac{\alpha}{C_0}(u_1 + u_M) \quad (16)$$

215 hence, yielding:

$$u_1 = \frac{C_0}{\alpha}V_{DC}(1 - \gamma_P(V_{DC})) - u_M \quad (17)$$

216 Combining Eq. (17) and Eq. (12), gives:

$$W_h = V_{DC}[2\alpha u_M + C_0(V_{DC} - \gamma_P(V_{DC}) \cdot V_{DC})] \quad (18)$$

217 As two energy harvesting processes occur per period, the expression of the output power can
 218 be obtained as a function of the electromechanical parameters as:

$$P = 2f_0 W_h = 2f_0 V_{DC}[2\alpha u_M + C_0 V_{DC}(1 - \gamma_P(V_{DC}))] \quad (19)$$

219 where f_0 denotes the mechanical vibration frequency. Additionally, the harvested power can also
 220 be expressed as a function of the squared value of the voltage V_{DC} and the resistance R_L , giving:

$$P = \frac{V_{DC}^2}{R_L} \quad (20)$$

221 In order to make the analysis independent from the system parameters, the output voltage can
 222 be normalized with respect to the open-circuit voltage amplitude and the resistance with respect
 223 to the optimal resistance of the piezoelectric element in the standard case, as illustrated below:

$$\begin{cases} \tilde{V} = \frac{V_{DC}}{V_{oc}} = \frac{V_{DC}}{\frac{\alpha}{C_0}u_M} \\ \tilde{R} = \frac{R_L}{R_{opt,st}} = \frac{R_L}{1/4f_0 C_0} \end{cases} \quad (21)$$

224 Additionally, substituting the value for I_M following Eq. (5), introducing the ratio r between
 225 the terms $\frac{\beta^2}{L_0}$ and $\frac{\alpha^2}{C_0}$, and substituting Eq. (13) into Eq. (18), we finally obtain the following
 226 non-dimensional 3rd order polynomial expression:

$$2\tilde{V}^3 \left(2 + \tilde{R}(1 - \gamma_0)\right) - 2\tilde{V}^2 \tilde{R} \left(2 + \gamma_0 \frac{1}{2Q_i} \sqrt{r}\right) - \tilde{V} \tilde{R} \gamma_0 r + \tilde{R} \gamma_0 r^{3/2} \frac{1}{2Q_i} = 0 \quad (22)$$

227 of which approximated solution is given by:

$$\tilde{V} = \frac{2\tilde{R} \left(2 + \gamma_0 \frac{1}{2Q_i} \sqrt{r}\right)}{6 \left(2 + \tilde{R}(1 - \gamma_0)\right)} \left(1 + 2 \sqrt[3]{1 + \frac{18 \left(2 + \tilde{R}(1 - \gamma_0)\right) \tilde{R} \gamma_0 r}{8\tilde{R}^2 \left(2 + \gamma_0 \frac{1}{2Q_i} \sqrt{r}\right)}}\right) \quad (23)$$

228 where the expressions used to calculate the voltage expressed in Eq. (23) are detailed in Appendix
 229 A.

230 Consequently, it is possible to express the harvested power combining equations (23), (19) and (20)
 231 and considering the dimensional parameters, yielding:

$$P = \frac{\tilde{V}^2}{\tilde{R}} \frac{4\alpha^2}{C_0} f_0 u_M \quad (24)$$

232 3.2.2. Constant driving force magnitude

233 the operation of converting mechanical energy into electrical energy from a coupled system using
 234 piezoelectric and electromagnetic mechanisms will cause multiple induced damping effects, thus
 235 causing a reduction of the available effective mechanical energy in terms of relative displacement
 236 u and relative velocity \dot{u} , thereby reducing the induced piezoelectric voltage (directly proportional
 237 to the displacement) and the induced electromotive force (directly proportional to the velocity),
 238 respectively; hence, the total harvested power of the coupled system is reduced in return. For sake
 239 of simplicity, from now onwards the analysis will be conducted considering an equivalent value of
 240 the inversion factor γ_P , which takes into account the rectified voltage calculated in the previous
 241 section (written as $\gamma_P(V_{DC})$).

242 In this case, the energy balance of the structure is considered. Multiplying both terms of the first
 243 constitutive equation of Eq. (3) by the velocity and integrating over half a period leads to:

$$M \int_{t_1}^{t_2} \ddot{u} \dot{u} dt. + C \int_{t_1}^{t_2} \dot{u} \dot{u} dt. + K^{E,H} \int_{t_1}^{t_2} u \dot{u} dt. = \int_{t_1}^{t_2} \dot{u} du. - \alpha \int_{t_1}^{t_2} V_p \dot{u} dt. - \beta \int_{t_1}^{t_2} I_{em} \dot{u} dt. \quad (25)$$

244 Particularly, in steady state, it is possible to define the transferred energy of the piezoelectric
 245 element as the sum of electrostatic energy stored on the piezoelectric clamped capacitance and the
 246 energy absorbed by the connected load:

$$\alpha \int_{t_1}^{t_2} V_p \dot{u} dt. = \frac{1}{2} C_0 [V_{DC}^2 - (\gamma_P(V_{DC}) \cdot V_{DC})^2] + \frac{V_{DC}^2 T}{R_L} = \frac{1}{2} C_0 V_{DC}^2 [1 - (\gamma_P(V_{DC}))^2] + \frac{V_{DC}^2 T}{R_L} \quad (26)$$

247 where $\gamma_P(V_{DC})(V_{DC})$ stands for the implicit inversion coefficient in the DC case.

248 As per the mechanical energy converted into electrical energy by the electromagnetic system, the
 249 energy transfer occurs when the electromagnetic system is short-circuited:

$$\beta \int_{t_1}^{t_2} I_{em} \dot{u} dt. = \frac{1}{2} L_0 (2I_{sc})^2 \quad (27)$$

250 When considering steady state conditions, there is no variation of the potential elastic energy
 251 between instant t_1 and t_2 as well as a null kinetic energy at those instants for a system excited at
 252 resonance, thus yielding:

$$\frac{\pi}{2}F_M u_M = \frac{\pi}{2}C u_M^2 \omega + \frac{1}{2}C_0 V_{DC}^2 [1 - (\gamma_P(V_{DC}))^2] + \frac{V_{DC}^2 T}{R_L} \frac{1}{2} + \frac{1}{2}L_0(2I_{sc})^2 \quad (28)$$

253 where F_M stands for the driving force magnitude.

254 Using Eq. (26) and the second constitutive equation of Eq. (3), the rectified voltage can be
 255 expressed according to Eq. (29), while Eq. (30) recalls the electromagnetic current under short-
 256 circuit conditions:

$$V_{DC} = \frac{2R_L \alpha}{RC_0(1 - \gamma_P(V_{DC}))\omega + \pi} \omega u_M \quad (29)$$

$$I_{sc} = \frac{\beta^2}{L_0} u_M \quad (30)$$

257 Hence, the expression of the displacement magnitude at resonance frequency when taking into
 258 account the damping effect leads to:

$$u_m = \frac{F_m}{C\omega + \frac{4R_L \omega \alpha^2 (R_L C_0 (1 - \gamma_P(V_{DC}))\omega + 2\pi)}{\pi} + \frac{4\beta^2}{\pi L_0}} \quad (31)$$

259 and the associated harvested power is thus given by:

$$P = \frac{V_{DC}^2}{R_L} = \frac{4R_L \alpha^2}{(R_L C_0 (1 - \gamma_P(V_{DC}))\omega + \pi)^2} \omega^2 u_M^2 \quad (32)$$

260 Recalling the expression of the displacement and the power from the SSHI case as per Eq.
 261 (33) and (34) respectively [23], the SSH-EM and SSHI systems are in this way easy to correlate
 262 (SSHI being a particular case of SSH-EM with $\beta = 0$ and $\gamma_P = \gamma_0$), thus conveniently allowing a
 263 comparison between the two techniques, as it will be discussed in the next section.

$$u_{MSSH I} = \frac{F_m}{C\omega + \frac{4R_L \omega \alpha^2 (R_L C_0 (1 - \gamma_0)\omega + 2\pi)}{\pi} + \frac{4\beta^2}{\pi L_0}} \quad (33)$$

$$P_{SSH I} = \frac{V_{DC}^2}{R_L} = \frac{4R_L \alpha^2}{(R_L C_0 (1 - \gamma_0)\omega + \pi)^2} \omega^2 u_{MSSH I}^2 \quad (34)$$

264 4. Theoretical discussion

265 Based on the modelling developed in the previous sections, it is proposed here to investigate the
 266 performance of the SSH-EM configuration, and compare it with the original SSHI and standard
 267 interfaces. The performance is first evaluated considering the structure driven with a constant dis-
 268 placement amplitude and then considering a constant force amplitude. It must be noted, however,
 269 that the power losses and the electromagnetic current phase shift due to the coil resistance of the

270 electromagnetic system are not taken into account.

271 As a first theoretical comparison, Fig. 9 evaluates the relation between the ratio r of the param-
 272 eters $\frac{\beta^2}{L_0}$ and $\frac{\alpha^2}{C_0}$ and the improvement of the inversion coefficient γ_P . This figure demonstrates
 273 how incrementing the contribution of the electromagnetic element, in terms of possible transferred
 274 energy, can lead to higher values of inversion coefficient, thus inducing voltage enhancements. As
 275 a result, an increase of the output voltage translates to an increase of the harvested power, as
 276 shown in Fig. 10. The figure plots the evolution of the power generated from the hybrid system,
 277 following to Eq. (25), as a function of the load and the same ratio r coefficient, for different values
 278 of the original SSHI inversion coefficient (*i.e.*, γ_0).

279 In order to conduct the theoretical evaluation as independently as possible from the system

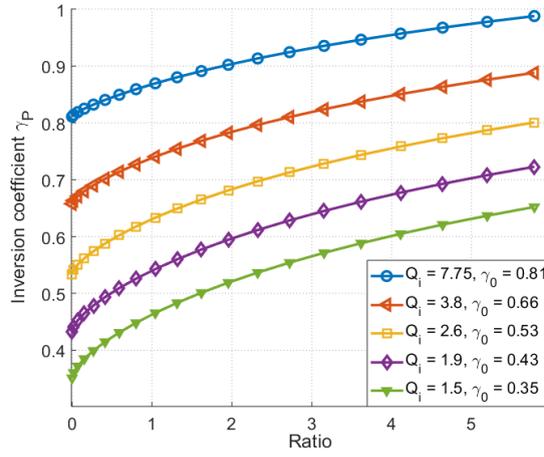


Figure 9: Inversion coefficient γ_P as a function of the ratio for several values of electrical quality factor Q_i

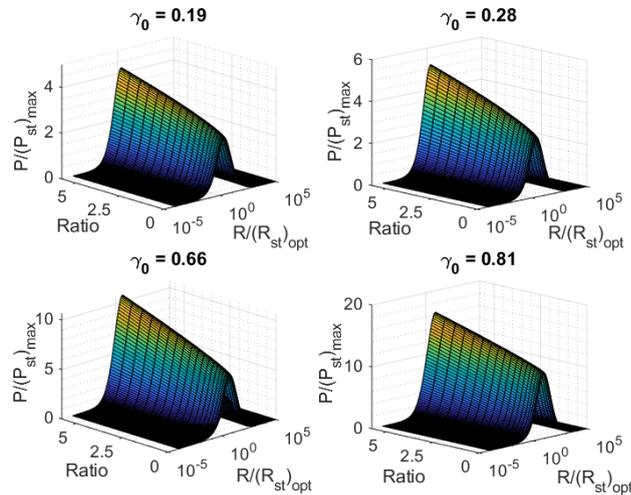


Figure 10: Normalized harvested power (w.r.t. maximal harvested power in the standard approach) as a function of the ratio and the normalized load (w.r.t. the optimal load in the standard approach) for several initial inversion factors

280 parameters, the harvested power has been normalized with respect to the maximum power in the
 281 standard case, and the resistance with respect to the optimal resistance in the standard case, as
 282 defined in Eq. (35),(36) [20]:

$$(P_{st})_{max} = \frac{\alpha^2}{C_0} f_0 u_M^2 \quad (35)$$

$$(R_{st})_{opt} = \frac{1}{4f_0 C_0} \quad (36)$$

283 The chart in Fig. 10 demonstrates that, as the ratio increases (*i.e.*, as the contribution of the
 284 electromagnetic system becomes more relevant), the resulting electrical power increases as well in
 285 a quite similar way (relatively speaking), whatever the initial SSHI inversion coefficient. It can
 286 be noted that for $r = 0$, therefore for $\beta^2/L_0 = 0$, the technique is strictly equivalent to the SSHI
 287 technique.

288 In order to better evaluate the power gain that can be achieved with respect to the SSHI scheme,
 289 Fig. 11 depicts the evolution of this gain as a function of the same ratio term (adopted in the
 290 previous graphs) for different values of the electrical quality factor Q_i . It is interesting to note how
 291 the power gain reaches on optimal value when considering an electrical quality factor Q_i equal to
 292 1.9 (*i.e.* $\gamma_0 = 0.43$).

293 Moreover, the comparison of the SSH-EM approach with the SSHI technique and the sole
 294 use of electromagnetic and piezoelectric elements is shown in Fig. 12. The analysis has been
 295 conducted considering two different electrical quality factors (which corresponds to two different
 296 initial SSHI inversion coefficients) and two values of ratio r (by making varying the electromagnetic
 297 force factor β). In this example, the electromagnetic and piezoelectric powers (curves 4-5-6 of
 298 Fig. 12) have been evaluated considering a simplified linear circuit (*i.e.* without considering the
 299 rectification stage). As a matter of fact, the main issue when using a full bridge rectifier for

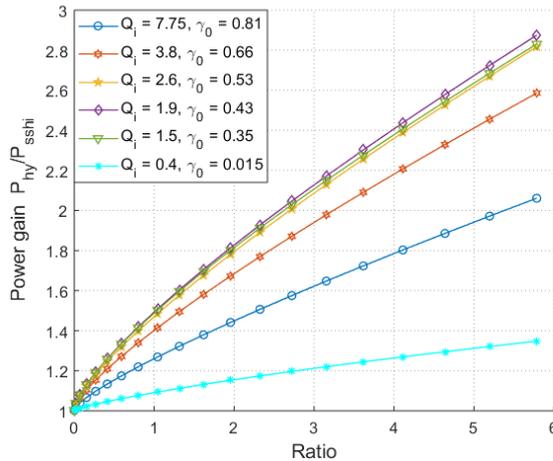


Figure 11: Extracted power gain compared to SSHI interface for several values of electrical quality factor Q_i

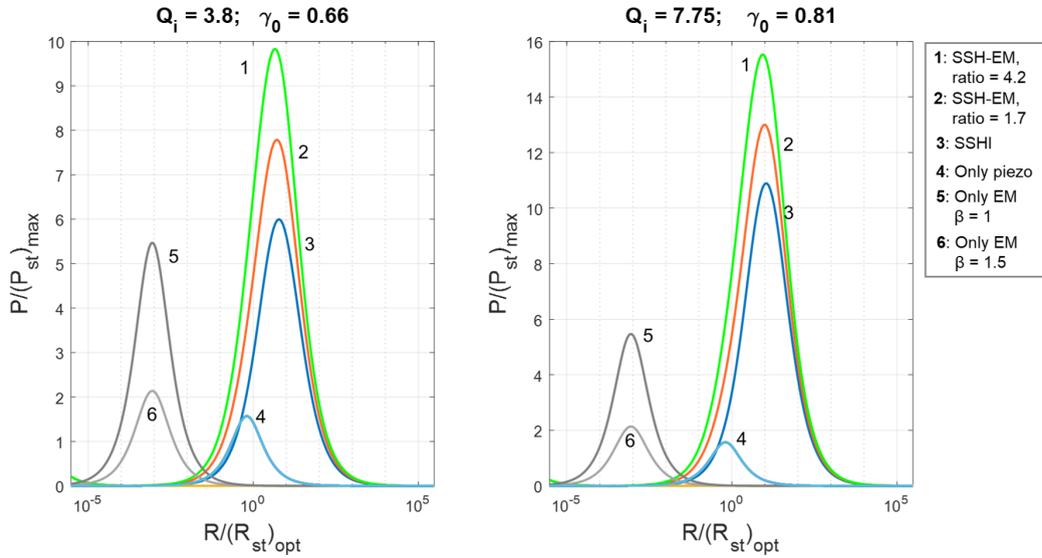


Figure 12: Normalized harvested power (w.r.t. maximal harvested power in the standard approach) as a function of the normalized load (w.r.t. the optimal load in the standard approach) for different techniques, electrical quality factor and ratios

300 electromagnetic energy harvesting systems is that a large amount of energy can be lost in the
 301 diodes if the open-circuit voltage amplitude is lower the diode threshold voltage (which is typical
 302 for many electromagnetic systems - and for the one in this case)¹. Nevertheless, it is evident how the
 303 SSH-EM technique allows achieving power output much greater even when considering the ideal
 304 case of the sum of the single piezoelectric and electromagnetic powers, considering that for the
 305 latter an additional load adaptation stage is required (therefore introducing additional losses and
 306 lowering the hybrid power levels). This confirms the advantage of employing the two transducers
 307 for energy harvesting purposes using the proposed SSH-EM approach.

308 When taking into account the damping effect, two important figures of merit are normally used
 309 to compare different electromechanical systems: the mechanical quality factor Q_m and the squared
 310 coupling coefficient of the structure k_p^2 , the former representing the available amount of energy
 311 while the latter reflecting the available energy that can actually be converted and directly related
 312 to the amount of active material. Fig. 13 thus relates the theoretical maximum powers using the
 313 SSH-EM, SSHI and the linear resistive technique in function of the FoM $k_p^2 Q_m$. Additionally, the
 314 case of the sole use of electromagnetic transducer has been taken into consideration, plotting its
 315 theoretical maximum power in function of the FoM $k_{em}^2 Q_m$. For each technique, the power has

¹It must be noted that a comparison between the output powers that can be achieved with the proposed SSH-EM technique and a combination of nonlinear harvesting for both piezoelectric and electromagnetic transducers (for instance, by employing simultaneously the SSHI and the SMFE technique) has not been considered and it would not be fair, as the latter would require two more additional electrical components (an inductance – for the SSHI – and a capacitor – for the SMFE).

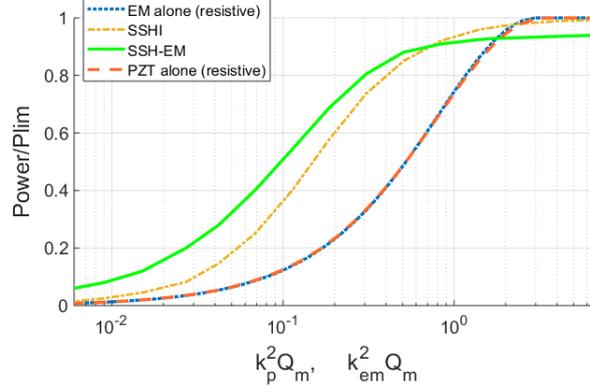


Figure 13: Normalized maximum harvested power (w.r.t. the power limit) for different techniques versus the figure of merit $k_p^2 Q_m$ and $k_{em}^2 Q_m$ when taking into account the damping effect

316 been normalized according to the power limit that can be reached, depending on the mechanical
 317 losses of the structure [23]:

$$P_{lim} = \frac{F_m^2}{8C} \quad (37)$$

318 Based on the trends of Fig. 13, the SSH-EM technique appears to be of particular interest
 319 when considering weakly coupled/highly damped structures, as the damping effect due to the com-
 320 bination of two energy harvesting transducers becomes more relevant in the case of high values of
 321 the figure of merit $k_p^2 Q_m$. This is also confirmed by Fig. 14, plotting the power as a function
 322 of the figure of merit $k_{em}^2 Q_m$, where k_{em}^2 represents the electromagnetic coupling coefficient, for
 323 different values of $k_p^2 Q_m$. The figure clarifies how higher power gains can be achieved for low
 324 coupled/highly damped structures, as well as the existence of an optimum value of the electromag-

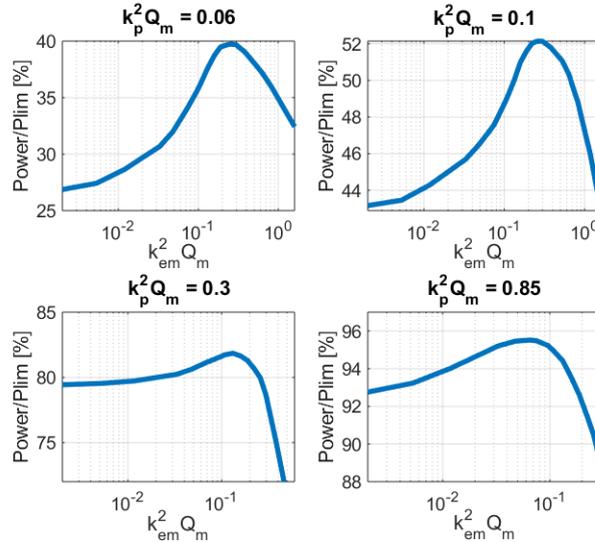


Figure 14: Normalized maximum harvested power (w.r.t. the power limit) as a function of the figure of merit $k_{em}^2 Q_m$ for different values of $k_p^2 Q_m$

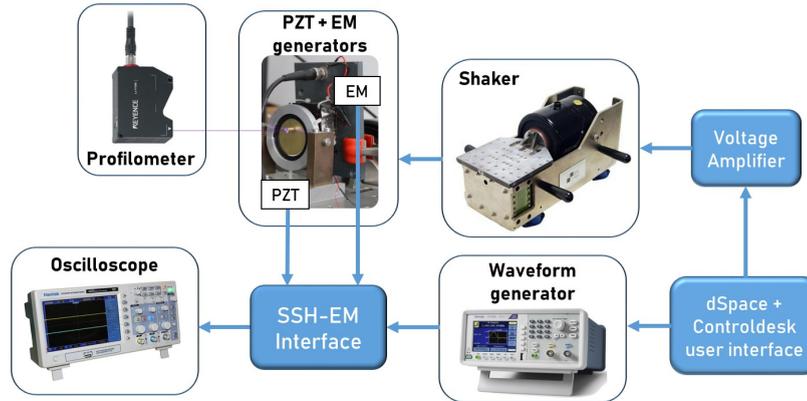
325 netic coupling. Furthermore, as the piezoelectric coupling coefficient increases, this optimal value
 326 of electromagnetic coupling decreases.

327 5. Experimental validations

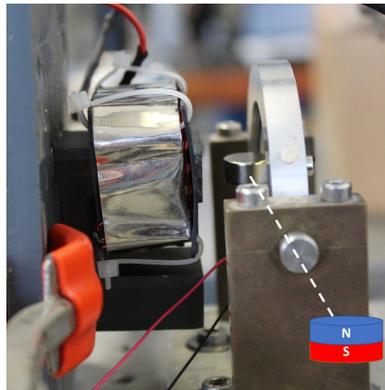
328 The previous discussions theoretically pointed out the advantages of employing the SSH-EM
 329 technique over previously developed nonlinear techniques, particularly when considering low cou-
 330 pled/highly damped structures. Hence, this section proposes to experimentally validate these
 331 theoretical predictions previously discussed.

332 5.1. Experimental set-up

333 Fig. 15a illustrates the schematic of the experimental set-up used in this study. The piezoelectric
 334 generator is composed of a piezoelectric buzzer with an additional permanent magnet (NdFeB)
 335 mounted on it. The permanent magnet allows the coupling of the electromagnetic generator to
 336 the piezoelectric one by placing a coil along the axis of motion of the magnet, as shown in Fig.
 337 15b. The permanent magnet shown in the figure is mounted in this particular way after optimizing



(a) Experimental set-up schematic



(b) EM and PZT transducers

Figure 15: Experimental set-up

338 different orientations of it. Mounting the magnet with the north/south pole completely attached to
 339 the piezoelectric buzzer, would lead to higher clamping of the piezoelectric transducer, thus leading
 340 to less variation of its displacement. Hence, to avoid this effect, the magnet is oriented as shown
 341 in Fig. 15b, which gives the optimal output. Although the magnet has axial magnetization, the
 342 closing through the bottom and the top ferrite legs still allows significant flux variation through
 343 the coil. The ferrite also permits a much higher inductance as compared to the case of generators
 344 without the core, yielding higher reactance than the resistive part of the coil. Moreover, a 0.25mm-
 345 thick nickel-iron soft ferromagnetic alloy with very high permeability, is wrapped around the coil
 346 as a shield against static or low-frequency magnetic fields from the surroundings (the shaker, for
 347 instance). The global system is mounted on a shaker driven by a controlled dSpace card (RTI 1104)
 348 which generates a sinusoidal acceleration with an amplitude of 3.8 ms^{-2} (0.39g) with the help of
 349 a voltage amplifier. The dSpace card is also used to trigger a waveform generator controlling the
 350 switching circuit, at extremum displacements. A laser displacement sensor (Keyence LJ-V7000) is
 351 used to monitor the displacement of the permanent magnet as well as an oscilloscope for monitoring
 352 the voltage and current waveforms. Finally, a digital multimeter is used to measure the *DC* voltage
 353 on the load R_L connected to the SSH-EM interface along with a smoothing capacitor and a rectifier
 354 bridge.

355 5.2. Model parameters identification

356 Preliminary experimental measurements have been carried out in order to determine the model
 357 parameters, listed in Table 2. The parameter identification have been obtained by measuring the
 358 open circuit output voltage V_{OC} , the displacement magnitude u_M and piezoelectric clamped ca-
 359 pacitance C_0 , thus obtaining the value of the piezoelectric force factor α defined as $C_0 \times (V_{OC}/u_M)$.
 360 The same procedure was followed for the identification of the parameters for the electromagnetic
 361 generator, this time measuring the output short-circuit current I_{SC} , the displacement magnitude
 362 u_M and the coil inductance L_0 , thus obtaining the value of the electromagnetic force factor β
 363 defined as $L_0 \times (I_{SC}/u_M)$. C_0 and the coil parameters (inductance and resistance) have been
 364 obtained by the use of a LCR meter. The inversion factor γ_P has been measured from the ratio be-
 365 tween the absolute voltages after and before the inversion process. The electromechanical coupling
 366 of the piezoelectric system k_p^2 is derived from the short-circuit and the open-circuit piezoelectric
 367 resonance frequencies, when the electromagnetic element is open-circuited. On the contrary, the
 368 electromechanical coupling of the electromagnetic element k_{em}^2 is derived from the short-circuit
 369 and the open-circuit electromagnetic resonance frequencies, when the piezoelectric element is short-
 370 circuited. Consequently, it is possible to derive 4 different values of stiffness for the studied system
 371 based on the open-circuit and short-circuit configuration of the two transducers:

$$K_{DH} = \frac{\alpha^2}{C_0} \frac{f_{DH}^2}{f_{DH}^2 - f_{EH}^2} = \frac{\alpha^2}{C_0} \frac{1}{k_p^2} \quad (38)$$

Table 2: Model parameters (PZT and EM respectively refer to piezoelectric and electromagnetic transducers; "oc" and "sc" respectively stand for "open-circuit" and "short-circuit")

PZT clamped capacitance, C_0	67.85 nF
EM coil inductance, L_0	28.17 mH
EM coil resistance, r_L	5.8 Ω
PZT force factor, α	0.0036 NV ⁻¹
EM force factor, β	1.2 NA ⁻¹
Mechanical quality factor, Q_m	30
Resonance frequency, PZT sc, EM oc f_{EH}	152.15 Hz
Resonance frequency, PZT oc, EM oc f_{DH}	155.33 Hz
Resonance frequency, PZT sc EM sc f_{EB}	152.63 Hz
PZT coupling coefficient, k_p^2	0.0402
EM coupling coefficient, k_{em}^2	0.0063
Stiffness (PZT sc, EM oc), K_{EH}	4.52x10 ³ Nm ⁻¹
Stiffness (PZT oc, EM oc), K_{DH}	4.72x10 ³ Nm ⁻¹
Stiffness (PZT sc, EM sc), K_{EB}	8.11x10 ³ Nm ⁻¹
Stiffness (PZT oc, EM sc), K_{DB}	8.06x10 ³ Nm ⁻¹
Dynamic mass, M	5g
Damping coefficient, C	0.335 Nm ⁻¹ s ⁻¹

$$K_{EH} = K_{DH} - \frac{\alpha^2}{C_0} \quad (39)$$

$$K_{EB} = \frac{\beta^2}{L_0} \frac{f_{EH}^2}{f_{EB}^2 - f_{EH}^2} = \frac{\beta^2}{L_0} \frac{1}{k_{em}^2} \quad (40)$$

$$K_{DB} = K_{EB} - \frac{\beta^2}{L_0} \quad (41)$$

372 where the subscripts DH and EH represent the case in which the piezoelectric element is open-
373 circuited and short-circuited respectively, while having the electromagnetic element open-circuited,
374 and the subscripts EB and DB represent the case in which the electromagnetic element is short-
375 circuited while having the piezoelectric element short-circuited and open-circuited, respectively.
376 Finally, the dynamic mass and the damping coefficient of the system are derived as:

$$M = \frac{K_{EH}}{\omega_{EH}^2} \quad (42)$$

$$C = \frac{M\omega_{DH}}{Q_m} \quad (43)$$

377 with Q_m being defined as the mechanical quality factor with respect to the -3dB bandwidth.

378 5.3. Results and discussion

379 In order to conduct the analysis under a constant displacement magnitude, the structure was
 380 excited considering a tip displacement of $110 \mu m$. Moreover, for the sake of investigating the exper-
 381 imental analysis for different values of piezoelectric electromechanical couplings, small capacitors
 382 are added in parallel to the piezoelectric element, artificially decreasing the coupling coefficient.
 383 Considering such conditions, Fig. 16 shows the harvested power for the SSH-EM, SSHI and stan-
 384 dard technique together with the theoretical predictions, when considering an additional capacitor
 385 of $0.22 \mu F$ in parallel to the piezoelectric transducer. For comparative purposes, the theoretical
 386 power when considering the sole use of the electromagnetic transducer has also been plotted. The
 387 experimental results demonstrate good agreement with the theoretical analysis, boosting the power
 388 gain to a value of 10 compared to the maximum harvested power in the standard interface (20%
 389 more compared to the SSHI interface).

390 However, when it comes to realistic applications, where the system is more likely to be subjected
 391 to an external acceleration, one has to keep in mind that extracting energy from mechanical vi-
 392 brations leads to a reduction of the available mechanical energy due to the harvesting process that
 393 yields a mechanical damping effect. This damping effect leads in turn to a power drop, as already

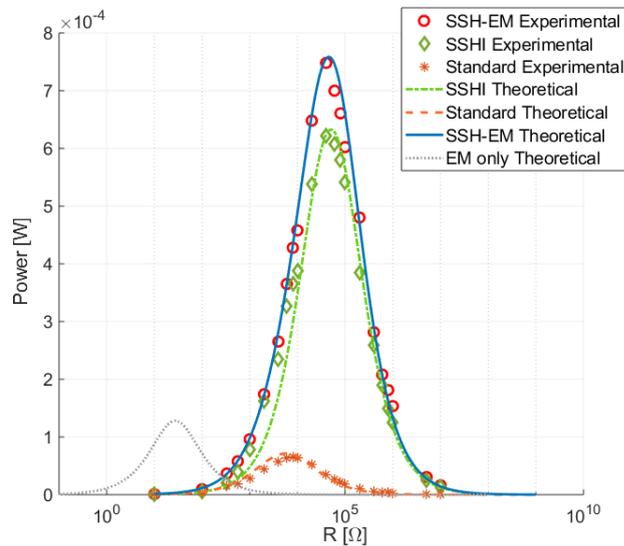


Figure 16: Experimental and theoretical harvested power (considering an additional capacitor of $0.22 \mu F$ in parallel with the piezoelectric element) in the constant displacement magnitude case

394 shown theoretically in Fig. 13. Experimental results in this case, along with the theoretical predic-
 395 tions, are depicted Fig. 17, when the structure is driven at resonance frequency under a constant
 396 sinusoidal acceleration of 3.8 ms^{-2} (0.39g). As expected, the SSH-EM technique leads to higher
 397 harvested power levels for low-coupled systems, while leading to a power drop for higher couplings
 398 because of the mutual damping effect between the piezoelectric and electromagnetic transducers.
 399 Discrepancies between the experimental results and the theoretical predictions attributed to the
 400 diode voltage drop that cannot be longer negligible in low voltage.

401 6. Conclusion

402 This paper proposed a nonlinear electric interface applied to a hybrid piezoelectric-electromagnetic
 403 energy harvesting system employing the synchronized switch approach derived from the SSHI
 404 scheme. Particularly, the proposed technique demonstrated how replacing the passive inductance
 405 with an active electromagnetic generator leads to a power enhancement without a particular loss in
 406 terms of space. Both theoretical predictions and experimental results have demonstrated that the
 407 proposed technique allow boosting the power gain to a factor of 10 when compared to a standard
 408 electrical interface, composed of a rectifier bridge and a smoothing capacitor connected to a load,
 409 with the possibility of increasing this gain by optimizing the energy conversion capabilities of the
 410 electromagnetic transducer.

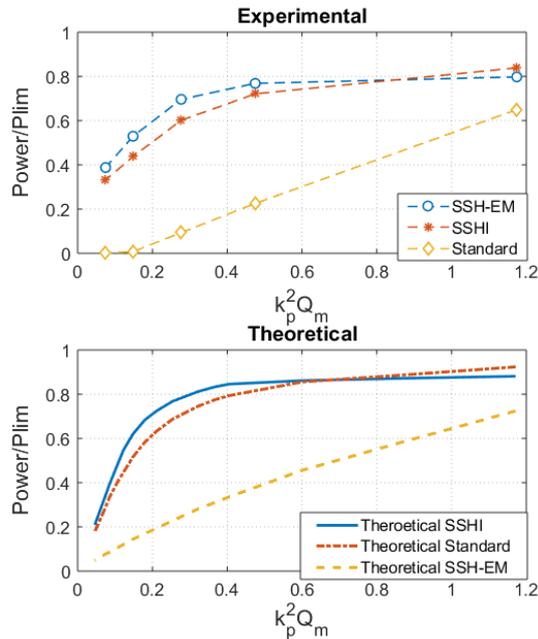


Figure 17: Experimental and theoretical normalized maximum harvested power (w.r.t. the power limit) for each technique versus the squared coupling coefficient and the mechanical quality factor

411 **7. Acknowledgments**

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415 *Appendix .1. Appendix A*

416 The following equation results after the introduction of the normalized terms defined in Eq.
 417 (21) and combining Eq. (19), (13) and (5):

$$-\tilde{V} + \tilde{R} - \tilde{R}\tilde{V}\frac{1}{2}\left\{1 - \gamma_0 \exp\left[-\frac{1}{2Q_i} \arctan\left(\frac{\beta}{\sqrt{L_0}} \frac{1}{\tilde{V}\frac{\alpha}{\sqrt{C_0}}}\right)\right]\sqrt{\left(\tilde{V}\frac{\alpha}{\sqrt{C_0}}\right)^2 + \left(\frac{\beta}{\sqrt{L_0}}\right)^2} \frac{1}{\tilde{V}\frac{\alpha}{\sqrt{C_0}}}\right\} = 0 \quad (1)$$

418 From Eq. (1), the term $\sqrt{\left(\tilde{V}\frac{\alpha}{\sqrt{C_0}}\right)^2 + \left(\frac{\beta}{\sqrt{L_0}}\right)^2}$ can be approximated by Eq. (2) (valid for
 419 $0 \leq (\beta/\sqrt{L_0})/(\tilde{V}\alpha/\sqrt{C_0}) \leq 1/2$), while the arctangent can be approximated as Eq. (3) (valid for
 420 $-1 \leq (\beta/\sqrt{L_0})/(\tilde{V}\alpha/\sqrt{C_0}) \leq 1$), as demonstrated in [40] and [41], respectively, yielding to Eq.
 421 (22).

$$\sqrt{\left(\tilde{V}\frac{\alpha}{\sqrt{C_0}}\right)^2 + \left(\frac{\beta}{\sqrt{L_0}}\right)^2} = \tilde{V}\frac{\alpha}{\sqrt{C_0}} + \frac{\frac{\beta}{\sqrt{L_0}}}{2\tilde{V}\frac{\alpha}{\sqrt{C_0}}} \quad (2)$$

$$\arctan\left(\frac{\beta}{\sqrt{L_0}} \frac{1}{\tilde{V}\frac{\alpha}{\sqrt{C_0}}}\right) = \frac{\beta}{\sqrt{L_0}} \frac{1}{\tilde{V}\frac{\alpha}{\sqrt{C_0}}} \frac{\pi}{4} \quad (3)$$

422 Furthermore, It is known that the explicit solution of the cubic equation $a\tilde{V}^3 + b\tilde{V}^2 + c\tilde{V} + d$
 423 is given by:

$$\tilde{V} = \sqrt[3]{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b}{9a^2}\right)^3}} + \sqrt[3]{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right) - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b}{9a^2}\right)^3}} - \frac{b}{3a} \quad (4)$$

424 However, neglecting the terms $\frac{d}{2a}$ and $\sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b}{9a^2}\right)^3}$ (considered to
 425 be much smaller than the others) it is possible to obtain an approximated solution of the 3rd order
 426 polynomial equation:

$$\tilde{V} = -\frac{b}{3a} \left(1 + 2\sqrt[3]{1 - \frac{9ac}{2b^2}}\right) \quad (5)$$

427 Finally, replacing a, b, c with respect to the actual system parameters, leads to Eq. (23).

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